Gluon quasi PDFs: Renormalization and one-loop matching

Shuai Zhao

Based on arXiv:1904.00978 by Wei Wang, Jian-Hui Zhang, SZ and Ruilin Zhu

Theory center cake seminar, JLab, 06/12/2019

Contents

1. LaMET and gluon quasi-PDFs
2. Renormalization of gluon quasi-PDFs
3. One-loop matching
4. Summary

Parton distribution function

❖ Parton distribution functions:

$$f_{q/H}(x,\mu^2) \ = \int \frac{d\xi^-}{4\pi} e^{-i\xi^- xP^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \mathcal{P}\exp\left(-ig\int_0^{\xi^-} d\eta A^+(\eta)\right)\psi(0)|P\rangle,$$

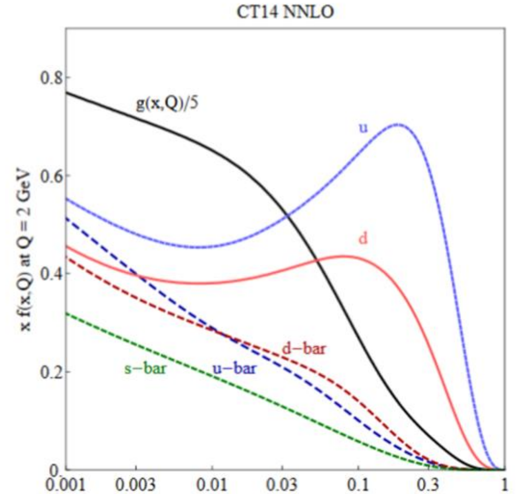
light-cone correlations
nonperturbative

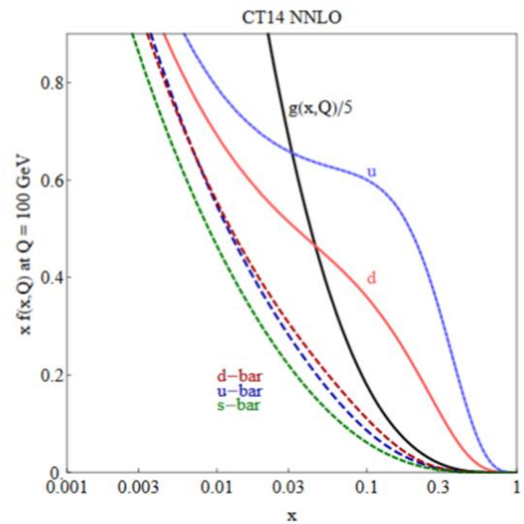
❖ Factorization theorems:
Foundation of applying perturbation theory in QCD.
e.g., for DIS

$$d\sigma \sim C_q(x,Q,\mu) \bigotimes f_{q/P}(x,\mu) + \cdots$$

❖ Can be extracted from experimental data

❖ Can not be calculated by LQCD directly





Large Momentum Effective Theory

- Large momentum effective theory (LaMET): A novel approach of accessing parton physics on the lattice (Ji,2013, 2014)

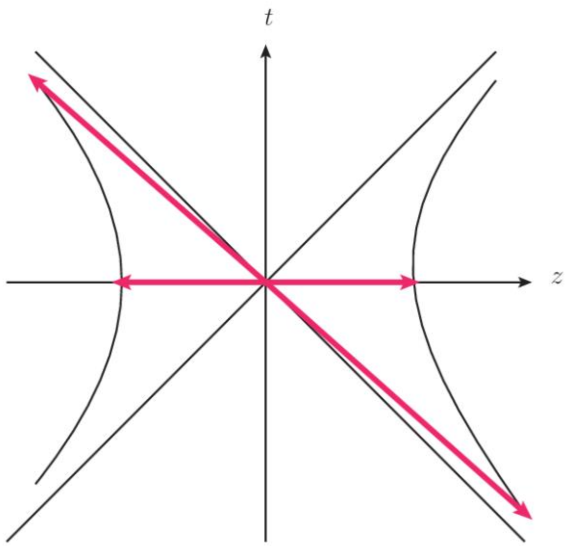
- The quasi-PDFs: light-cone correlation matrix elements->equal-time correlation matrix element

$$\tilde{q}(x, \Lambda, P_z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixzP_z} \left\langle P \middle| \bar{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A^z(z')\right) \psi(0) \middle| P \right\rangle$$

- Quasi-PDF is defined by equal time operator.

- Can be evaluated in LQCD

- Boosting to infinite momentum frame (large P^z): normal PDF



LaMET

- QCD is asymptotic freedom theory. Pz provides a hard scale, similar to m_b in HQET.
- One can expect a factorization formula, which connect light-cone PDF and quasi-PDF:

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

$$Z(x,\mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P^z) + \dots$$

Z: matching coefficient, can be calculated perturbatively.

- Quasi distributions: quantities in "full theory"

  Light cone distributions: quantities in "effective theory"

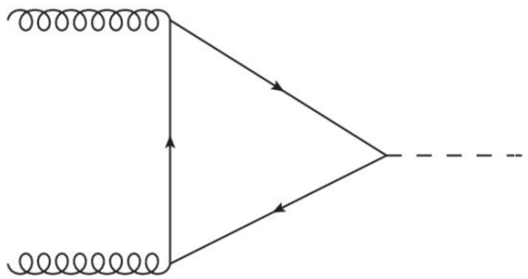
- The factorization in terms of Feynman diagrams can be proved to all orders Ma and Qiu, 2014

- Reduced Ioffe time distribution approach Radyushkin,2017; Orginos, Karpie, Radyushkin, Karpie, Zafeiropoulos, 2017
- Lattice cross sections Ma and Qiu, 2014, 2017 Sufian, Karpie, Egerer, Orginos, Qiu, Richards,2019

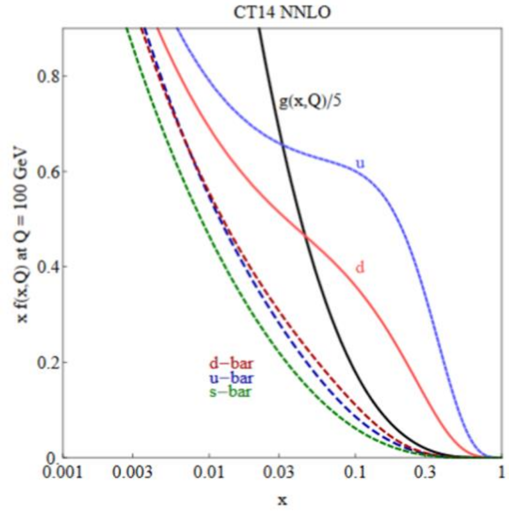
Gluon quasi-PDF

- Gluon PDFs are key input parameters for physics at hadron colliders

- Important for various physical processes, e.g., higgs and quarkonium production at hadron colliders.



- At ultrahigh energy, gluon PDF dominates.



- Crucial in the study of hadron structure and spin physics.

- Gluon quasi PDF is also important to extract quark PDFs.

Gluon quasi-PDF

- Gluon PDF

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ixP^+\xi^-} \langle P|F_a^{+i}(\xi^-)\mathcal{W}(\xi^-,0)F_a^{+i}(0)|P\rangle$$

- Gluon quasi-PDF

$$\tilde{f}_{g/H}(x,P^z) = \int \frac{dz}{2\pi x P^z} e^{izxP^z} \langle P \big| F_i^z(z) W(z,0;L_{n^z}) F^{iz}(0) \big| P\rangle$$

F: gluon field strength tensor

W: Wilson line, links 0 and ξ^- (or z), adjoint representation

- But the choice is not unique：

$$O_g^{\mu\nu}(z,0) = F^{\mu\alpha}(z)\mathcal{W}(z,0)F_\alpha^\nu(0)$$

t or z

transverse directions x, y or all directions t, x, y, z

Gluon quasi-PDF

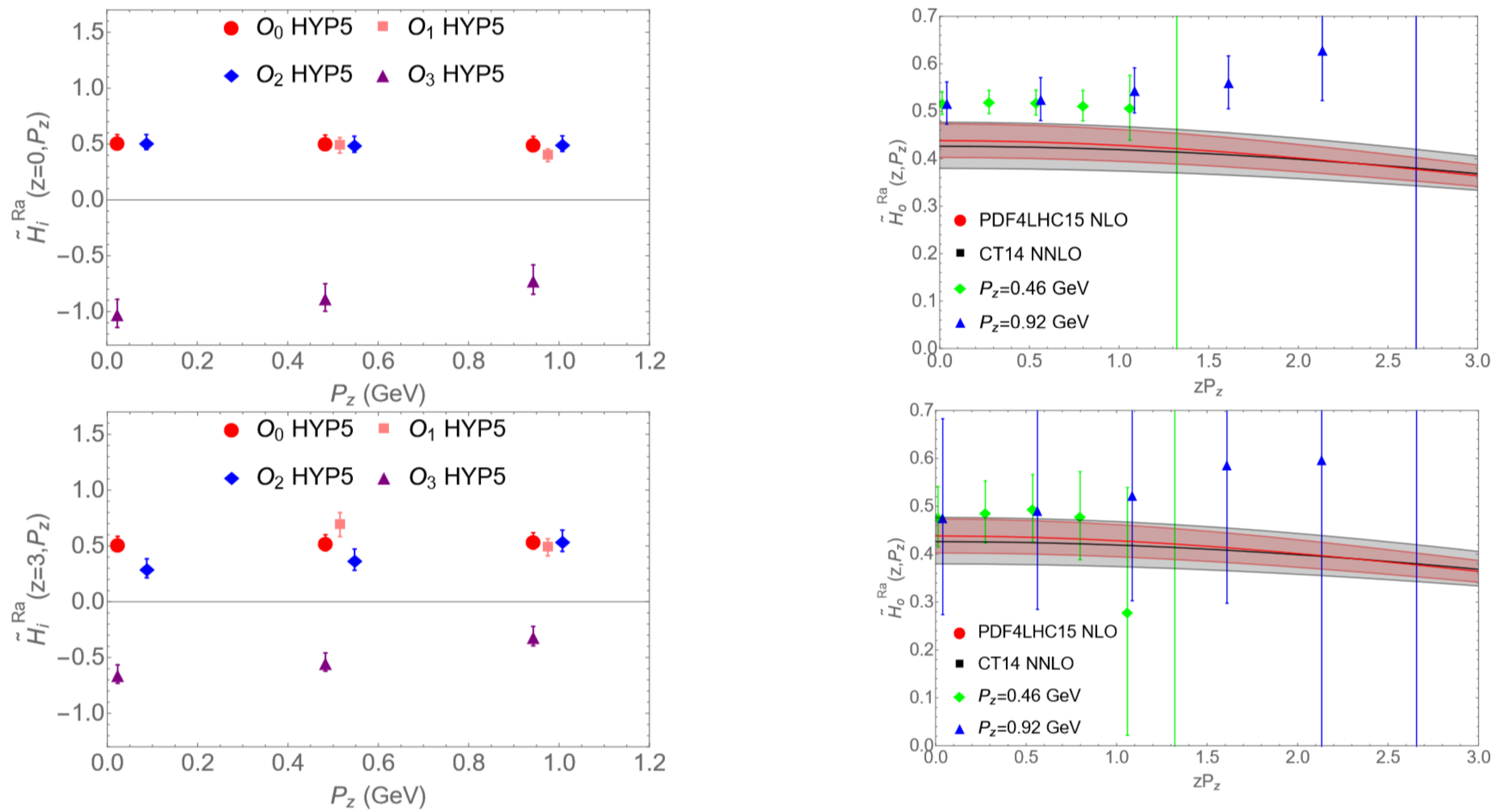
- Allowed operators:

$$O_g^{(1)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ t}(0), \quad O_g^{(2)}(z,0) \equiv F^{zi}(z)\mathcal{W}(z,0)F_i^{\ z}(0),$$

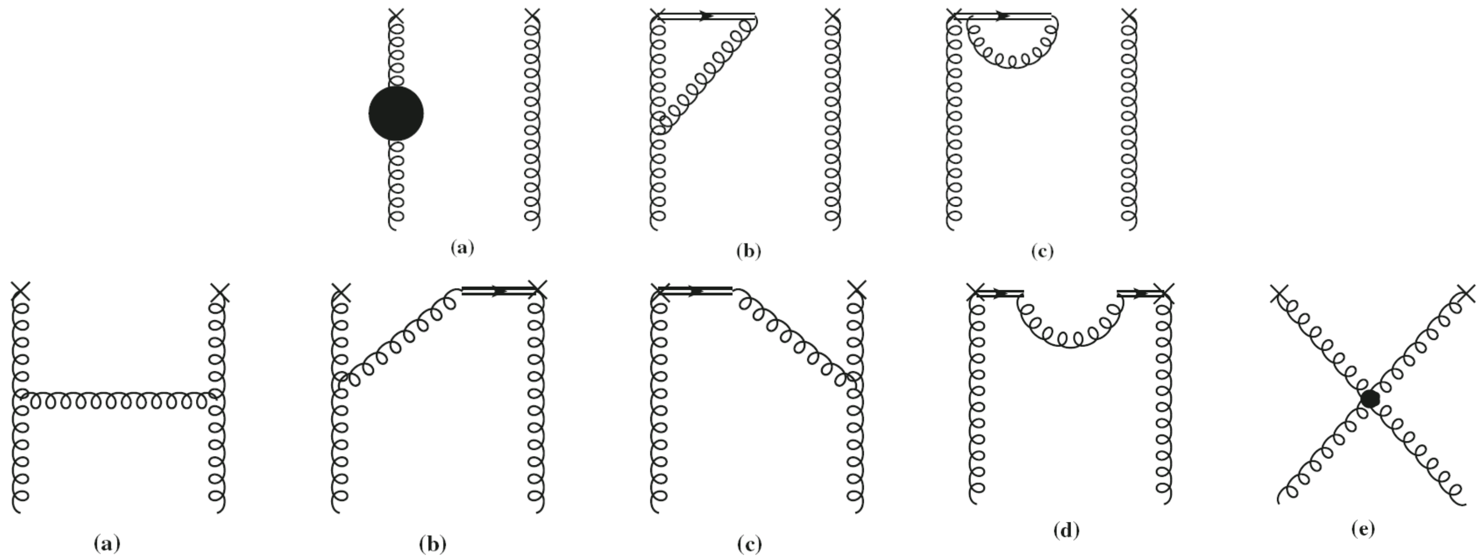
$$O_g^{(3)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ z}(0), \quad O_g^{(4)}(z,0) \equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu^{\ z}(0),$$

$$O_g^{(5)}(z,0) \equiv F^{t\mu}(z)\mathcal{W}(z,0)F_\mu^{t}(0)$$

- Lattice simulation: Z.Y. Fan, Y. B. Yang, A. Anthony, H.W. Lin, K.F. Liu, PRL121 (2018) 242001



Perturbative matching



❖ One loop matching with dimensional regularization (DR) and UV cutoff

W.Wang, SZ, R.L.Zhu, Eur. Phys. J. C (2018)78: 147
W.Wang, SZ, JHEP 1805 (2018) 142

$$\tilde{f}_{i/H}(x, P^z) = \int_0^1 \frac{dy}{y} Z_{ij}\left(\frac{x}{y}, \frac{\mu}{P^z}\right) f_{j/H}(y, \mu) \equiv Z_{ij}\left(\xi, \frac{\mu}{P^z}\right) \otimes f_{j/H}(y),$$

One-loop matching coefficients

❖ Matching coefficients

$$Z_{qq}^{(1)}\left(\xi,\frac{\mu}{P^z}\right)=C_F\begin{cases}\left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}+1\right]_+, & \xi>1\\ \left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{4(P^z)^2\xi(1-\xi)}+\frac{2-5\xi+\xi^2}{1-\xi}\right]_+, & 0<\xi<1\\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}-1\right]_+, & \xi<0\end{cases}$$

$$Z_{qg}^{(1)}\left(\xi,\frac{\mu}{P^z}\right)=T_F\begin{cases}(\xi^2+(1-\xi)^2)\ln\frac{\xi}{\xi-1}-2\xi+1, & \xi>1\\ -(\xi^2+(1-\xi)^2)\ln\frac{\mu^2}{4\xi(1-\xi)(P^z)^2} & \\ +1+2\xi-4\xi^2, & 0<\xi<1\\ -(\xi^2+(1-\xi)^2)\ln\frac{\xi}{\xi-1}+2\xi-1, & \xi<0\end{cases}$$

$$Z_{gq}^{(1)}\left(\xi,\frac{\mu}{P^z}\right)=C_F\begin{cases}\frac{1+(1-\xi)^2}{\xi}\ln\frac{\xi}{\xi-1}-1+\frac{5}{2\xi}, & \xi>1\\ -\frac{1+(1-\xi)^2}{\xi}\ln\frac{\mu^2}{4\xi(1-\xi)(P^z)^2} & \\ +3-\frac{1}{2\xi}, & 0<\xi<1\\ -\frac{1+(1-\xi)^2}{\xi}\ln\frac{\xi}{\xi-1}+1-\frac{5}{2\xi}, & \xi<0\end{cases}$$

$$Z_{gg}^{(1)}\left(\xi,\frac{\mu}{P^z}\right)=C_A\begin{cases}\frac{2\xi^3-3\xi^2+2\xi-2}{\xi}\ln\frac{\xi-1}{\xi} & \\ +\xi\left[\frac{1+\xi}{\xi-1}\ln\frac{\xi-1}{\xi}+1\right]_++\xi-1+\frac{8}{3\xi}, & \xi>1\\ \frac{2\xi^3-3\xi^2+2\xi-2}{\xi}\ln\frac{\mu^2}{4\xi(1-\xi)(P^z)^2} & \\ +\xi\left[\frac{1+\xi}{\xi-1}\ln\frac{\mu^2}{4\xi(1-\xi)(P^z)^2}\right]_+ & \\ +\delta(1-\xi)\left(\frac{5}{3}C_A-\frac{4}{3}T_F n_f\right)\ln\frac{(P^z)^2}{\mu^2} & \\ -\left[\frac{2\xi^2-\xi+1}{1-\xi}\right]_++\frac{10\xi^2}{3}-4\xi+4-\frac{2}{3\xi}, & 0<\xi<1\\ -\frac{2\xi^3-3\xi^2+2\xi-2}{\xi}\ln\frac{\xi-1}{\xi} & \\ -\xi\left[\frac{1+\xi}{\xi-1}\ln\frac{\xi-1}{\xi}+1\right]_+-\xi+1-\frac{8}{3\xi}, & \xi<0\end{cases}$$

$$\xi\equiv x/y,$$

x is the momentum fraction in quasi PDF, y is the momentum fraction in PDF.

W.Wang,SZ, R.L.Zhu, Eur. Phys. J. C (2018)78: 147

One-loop matching: DR vs cutoff

DR

- Preserve gauge invariance
- Lattice evaluation needs cutoff. Unpractical
- Power divergences only exist in Wilson line's self energy

Naïve cutoff

- Lattice provides cutoff. More "practical" than DR
- Power divergence. Mixes with other operators
- Gauge symmetry is broken

Renormalization of quasi-PDFs

- Well defined continuum limit call for renormalization

- Renormalization of quasi PDF on the Lattice

- Lattice perturbation:

  Complicated Feynman rules, hard to evaluate

- Nonperturbative methods

  gradient flow K. Orginos and C. Monohan, 2016

  Regularization independent momentum subtraction (RI/MOM)

  I.Stewart and Y.Zhao, Phys.Rev. D97 (2018) no.5, 054512

  LP3, Phys.Rev. D97 (2018) no.1, 014

  M. Constantinou and H. Panagopoulos, Phys.Rev. D96 (2017) no.5, 054506

  ETMC, Nucl.Phys. B923 (2017) 394-415

RI/MOM

- Renormalization condition

$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

- Renormalized quark quasi-PDF

$$\tilde{h}^R(z, P_z, p_z^R, \mu_R) = \tilde{Z}^{-1}(z, p_z^R, a^{-1}, \mu_R)\tilde{h}(z, P_z, a^{-1})\Big|_{a\to 0},$$

$$\tilde{h}(z, P_z, a^{-1}) = \frac{1}{2P^0}\langle P|O_{\gamma_t}(z)|P\rangle$$

$$\tilde{q}(x, P_z, p_z^R, \mu_R) = P_z \int \frac{dz}{2\pi}\, e^{ixP_z z}\tilde{h}^R(z, P_z, p_z^R, \mu_R)\,.$$

- The renormalized matrix element does not depend on regularization scheme
- Matching in RI/MOM: convert lattice regularized quasi-distribution to MSbar light-cone distribution
- Has been used to: quark PDFs, meson DAs, GPDs, …

Renormalization of quasi-PDFs

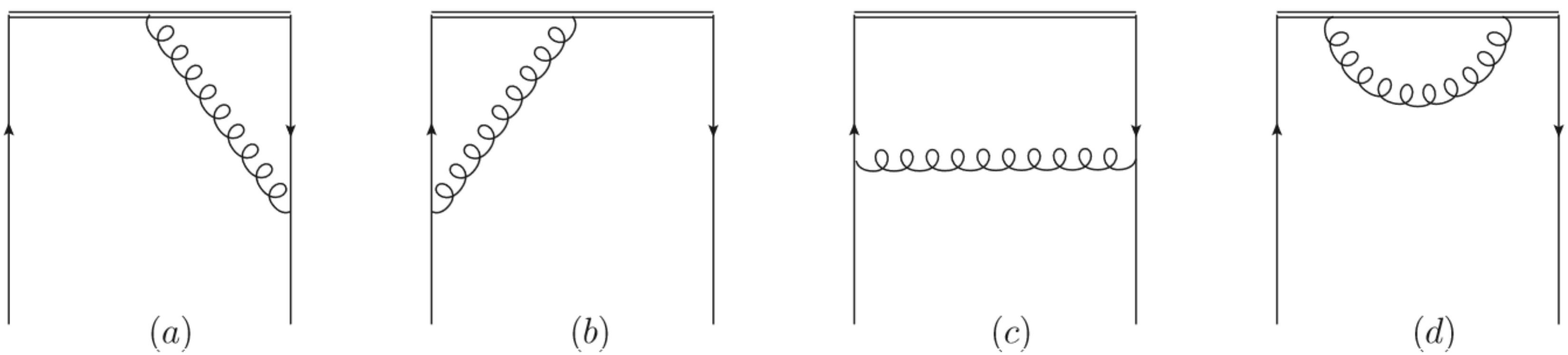
❖ Quark quasi-PDF: multiplicatively renormalizable

X. Ji, J.-H. Zhang, and Y. Zhao, Phys. Rev. Lett. 120, 112001 (2018)

Ishikawa, Ma, Qiu, Phys.Rev. D96 (2017) no.9, 094019

J. Green, K. Jansen and F. Steffens, Phys. Rev. Lett. 121, 022004 (2018)

$$\overline{\psi}_B(z)\gamma^z W^B(z,0)\psi_B(0) = Z_{\psi,z} e^{\delta m|z|}\overline{\psi}_R(z)\gamma^z W^R(z,0)\psi_R(0)$$



Auxiliary field formalism

- Auxiliary field formalism can be used to study the renormalization of gauge invariant bilocal operators

- One-dimensional auxiliary field

J. L. Gervais and A. Neveu, Nucl. Phys. B **163**, 189 (1980).

Dorn, Fortsch. Phys. **34**, 11 (1986).

- Auxiliary heavy quark field

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{Q}(x) in \cdot DQ(x),$$

$$\int \mathcal{D}\overline{Q}\mathcal{D}Q\, Q(x)\overline{Q}(y) e^{i\int d^4x \mathcal{L}} = S_Q(x,y) e^{i\int d^4x \mathcal{L}_{\rm QCD}}.$$

$$S_Q(x,y) = \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp) W(x^z, y^z),$$

- Renormalizability of quasi-PDF operators →→ renormalizability of HQET

Bagan and Gosdzinsky, 1994

Renormalization of quark quasi PDF operator

X.Ji, J.H.Zhang, Y.Zhao, PRL 120, 112001 (2018)

$$O_{q_i}(z_2,z_1) = \bar{q}_i(z_2)\Gamma W(z_2,z_1)q_i(z_1)$$

$$\Downarrow$$

$$\mathcal{O}_{q_i}(z_2,z_1) = \bar{q}_i(z_2)\Gamma Q(z_2)\overline{Q}(z_1)q_i(z_1) \equiv \bar{j}(z_2)j(z_1),$$

$$\bar{j}(z_2) = \bar{q}_i(z_2)\Gamma Q(z_2), \quad j(z_1) = \overline{Q}(z_1)q_i(z_1).$$

- Renormalization factor of the original operator is given by the renormalization factor of the composite operators
- In DR,

$$O(z_2,z_1) = Z_{\bar{j}}Z_j O_R(z_2,z_1) \qquad Z_j = Z_q^{1/2}Z_Q^{1/2}Z_{V_j}$$

- In cutoff schemes (e.g., lattice regularization)

$$\delta\mathcal{L}_m = -\delta m\overline{Q}Q \qquad O_R = Z_{\bar{j}}^{-1}Z_j^{-1}e^{\delta\bar{m}|z_2-z_1|}\overline{\psi}(z_2)\Gamma L(z_2,z_1)\psi(z_1).$$

- Linear divergence is absorbed by δm

Polyakov, 1980

Renormalization of gluon quasi PDF operator

- In auxiliary field formalism,

$$F_a^{\mu\nu}(z_2)L_{ab}(z_2,z_1)F_b^{\rho\sigma}(z_1) \Longrightarrow J_1^{\mu\nu}(z_2)\bar{J}_1^{\rho\sigma}(z_1)$$

$$J_1^{\mu\nu}(z) = F_a^{\mu\nu}(z)\mathcal{Q}_a(z)$$

- Building blocks of quasi PDF operator: $J_1^{\mu z},\ J_1^{\mu i},\ J_1^{t\mu}, J_1^{ti}$
- Gauge-invariant local operators mixing Joglekar, Lee, Annals Phys. 76
  - -Gauge-invariant operators
  - -BRST exact operators
  - -Operators that vanish by equation of motion
- Different from quark case, $J_1^{\mu\nu}$ get mixed with other operators

$$\begin{pmatrix} J_{1,R}^{\mu\nu} \\ J_{2,R}^{\mu\nu} \\ J_{3,R}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{\mu\nu} \\ J_2^{\mu\nu} \\ J_3^{\mu\nu} \end{pmatrix}$$

$$J_2^{\mu\nu} = n_\rho(F_a^{\mu\rho}n^\nu - F_a^{\nu\rho}n^\mu)\mathcal{Q}_a/n^2,$$

$$J_3^{\mu\nu} = (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in\cdot D - m)\mathcal{Q})_a/n^2$$

Dorn, Fortsch. Phys. **34**, 11 (1986)

Renormalization of gluon quasi PDF operator

- When $\nu = z$, $J_1^{\mu z} = J_2^{\mu z}$

$$Z_{11} + Z_{12} = Z_{22}, \qquad Z_{13} = Z_{23}.$$

- We also have

$$J_2^{ti} = 0,\ J_3^{ti} = 0 \qquad J_2^{ij} = 0,\ J_3^{ij} = 0$$

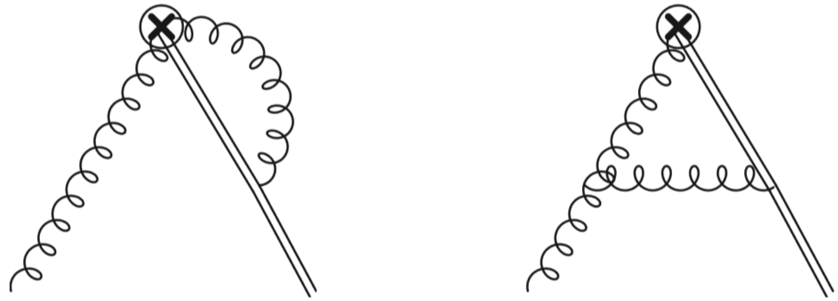
- The renormalization of “building blocks”

$$\begin{pmatrix} J_{1,R}^{z\mu} \\ J_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{z\mu} \\ J_3^{z\mu} \end{pmatrix}, \qquad J_{1,R}^{ti} = Z_{11} J_1^{ti}, \quad J_{1,R}^{ij} = Z_{11} J_1^{ij}.$$

Different component has different renormalization, due to Lorentz symmetry breaking in the presence of a four-vector along the z direction

Renormalization of gluon quasi PDF operator

❖ One loop correction to J_1



$$I_1^{\rho\nu} = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{d-4}(A_a^\nu n^\rho - A_a^\rho n^\nu) n\cdot\partial\mathcal{Q}_a/n^2 + \frac{\pi\mu}{d-3}\big(n^\rho A_a^\nu - n^\nu A_a^\rho\big)\mathcal{Q}_a + reg.\right\},$$

$$I_2^{\rho\nu} = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{d-4}\left[\frac{1}{4}F_a^{\rho\nu}\mathcal{Q}_a + \frac{1}{2}\big(F_a^{\rho\sigma}n_\nu n_\sigma - F_a^{\nu\sigma}n_\rho n_\sigma\big)\mathcal{Q}_a/n^2 + \frac{1}{2}(A_a^\rho n^\nu - A_a^\nu n^\rho)n\cdot\partial\mathcal{Q}_a/n^2\right]\right.$$
$$\left. - \frac{\pi\mu}{d-3}\big(n^\rho A_a^\nu - n^\nu A_a^\rho\big)\mathcal{Q}_a + reg.\right\},$$

-Linear divergence (pole at d=3) cancels

-Show up in naïve cutoff

-The only linear divergence comes from the Wilson line self energy

Renormalization of gluon quasi PDF operator

- J_3 Operator is irrelevant to the renormalization of nonlocal operator, since it only yields contact terms when auxiliary fields are integrated out.
- Building blocks of multiplicatively renormalizable operators:

$$J^{zi}_{1,R},\ J^{ti}_{1,R},\ J^{z\mu}_{1,R}.$$

- Multiplicatively renormalizable composite operators:

$$\mathcal{O}^1_R(z_2,z_1) \equiv J^{ti}_{1,R}(z_2)\overline{J}^{ti}_{1,R}(z_1).$$

$$\mathcal{O}^2_R(z_2,z_1) \equiv J^{zi}_{1,R}\overline{J}^{zi}_{1,R},$$

$$\mathcal{O}^3_R(z_2,z_1) \equiv J^{ti}_{1,R}(z_2)\overline{J}^{zi}_{1,R}.$$

$$\mathcal{O}^4_R(z_2,z_1) \equiv J^{z\mu}_{1,R}(z_2)\overline{J}^{z}_{1,R,\mu},$$

Renormalization of gluon quasi PDF operator

- Integrating out the auxiliary field:

  Four multiplicatively renormalizable quasi-PDF operators

$$O_R^1(z_2, z_1) = Z_{11}^2 e^{\overline{\delta m}\Delta z} F^{ti}(z_2) L(z_2, z_1) F^{ti}(z_1),$$
$$O_R^2(z_2, z_1) = Z_{22}^2 e^{\overline{\delta m}\Delta z} F^{zi}(z_2) L(z_2, z_1) F^{zi}(z_1),$$
$$O_R^3(z_2, z_1) = Z_{11} Z_{22} e^{\overline{\delta m}\Delta z} F^{ti}(z_2) L(z_2, z_1) F^{zi}(z_1)$$
$$O_R^4(z_2, z_1) = Z_{22}^2 e^{\overline{\delta m}\Delta z} F^{z\mu}(z_2) L(z_2, z_1) F^{z}{}_{\mu}(z_1),$$

$$O_{g,R}^{(5)}(z_2, z_1) \equiv (F^{t\mu}(z_2) \mathcal{W}(z_2, z_1) F^{t}{}_{\mu}(z_1))_R = -O_{g,R}^{(1)}(z_2, z_1) - O_{g,R}^{(2)}(z_2, z_1) - O_{g,R}^{(4)}(z_2, z_1)$$

- Proof of multiplicative renormalizability with diagrammatic method

  Li, YQM, Qiu, 1809.01836, PRL 122 (2019) no.6, 062002

  All the 36 components of $\mathcal{O}_{bg}^{\mu\nu\rho\sigma}(\xi) = F^{\mu\nu}(\xi)\,\Phi^{(a)}(\{\xi, 0\})\,F^{\rho\sigma}(0)$ are multiplicatively renormalizable

Renormalization in RI/MOM

- Renormalization equation

$$\begin{pmatrix} O_g^{(n)}(z,0) \\ O_q^s(z,0) \end{pmatrix} = \begin{pmatrix} Z_{11}(z) & Z_{12}(z)/z \\ zZ_{21}(z) & Z_{22}(z) \end{pmatrix} \begin{pmatrix} O_{g,R}^{(n)}(z,0) \\ O_{q,R}^s(z,0) \end{pmatrix} \qquad \bar{\mathcal{Z}} = \begin{pmatrix} \bar{Z}_{11}(z) & \bar{Z}_{12}(z)/z \\ z\bar{Z}_{21}(z) & \bar{Z}_{22}(z) \end{pmatrix} = \begin{pmatrix} Z_{11}(z) & Z_{12}(z)/z \\ zZ_{21}(z) & Z_{22}(z) \end{pmatrix}^{-1}$$

- Renormalization condition:

$$\left.\frac{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_R}{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2 \\ p_z=p_z^R}} = 1, \qquad \left.\frac{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_R}{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2 \\ p_z=p_z^R}} = 1,$$

$$\left.\mathrm{Tr}[\Lambda_{12}(p,z)\mathcal{P}]_R\right|_{\substack{p^2=-\mu_R^2 \\ p_z=p_z^R}} = 0, \qquad \left.[P_{ij}^{ab}\Lambda_{21}^{ab,ij}(p,z)]_R\right|_{\substack{p^2=-\mu_R^2 \\ p_z=p_z^R}} = 0,$$

P: projectors
Λ: offshell amputated Green’s functions

- Renormalized matrix element

$$h_{g,R}^{(n)}(z,P^z,\mu_R,p_z^R) = \bar{Z}_{11}(z,\mu_R,p_z^R,1/a)h_g^{(n)}(z,P^z,1/a) + \bar{Z}_{12}(z,\mu_R,p_z^R,1/a)/z\ h_q^s(z,P^z,1/a),$$
$$h_{q,R}^{s}(z,P^z,\mu_R,p_z^R) = \bar{Z}_{22}(z,\mu_R,p_z^R,1/a)h_q^s(z,P^z,1/a) + z\bar{Z}_{21}(z,\mu_R,p_z^R,1/a)\ h_g^{(n)}(z,P^z,1/a).$$

Matching equation

- Matching equation:

$$\tilde{f}^{(n)}_{g/H}(x,P^z,p^R_z,\mu_R)=\int_{-1}^{1}\frac{dy}{|y|}\Big[C_{gg}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{g/H}(y,\mu)+C_{gq}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{q_j/H}(y,\mu)\Big]$$
$$+\mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),$$
$$\tilde{f}_{q_i/H}(x,P^z,p^R_z,\mu_R)=\int_{-1}^{1}\frac{dy}{|y|}\Big[C_{q_iq_j}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{q_j/H}(y,\mu)+C_{qg}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{g/H}(y,\mu)\Big]$$
$$+\mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),$$

- Polarized PDFs also have same matching equation but with different matching coefficients

- Can be derived by OPE

Unpolarized PDF: quark in quark

- The amputated Green's function has structure

$$\Lambda_{\gamma^t}(p,z)=\widetilde{f}_t(p,z)\gamma^t+\widetilde{f}_z(p,z)\frac{p^t\gamma^z}{p^z}+\widetilde{f}_p(p,z)\frac{p^t\not{p}}{p^2}$$

- Minimal Projection:

  Project out $\widetilde{f}_t(p,z)$ LP3, 1807.06566

  Project out $\widetilde{f}_t(p,z)$ and $\widetilde{f}_z(p,z)$ This work

- Bare quark quasi PDF

$$\left[\tilde{f}^{(1)}_{q/q}(x,\rho\to 0)\right]_+=\left[\tilde{f}^{(1)}_{q/q,t}(x,\rho\to 0)\right]_+ + \left[\tilde{f}^{(1)}_{q/q,z}(x,\rho\to 0)\right]_+ \qquad \rho=-p^2/p_z^2$$

- Matching coefficient

$$C^{(1)}_{qq}\left(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}\right)=\left[\tilde{f}^{(1)}_{q/q}(x,\rho\to 0)-f^{(1)}_{q/q}\left(x,\frac{\mu^2}{-p^2}\right)-(\tilde{f}^{(1)}_{q/q})_{C.T.}\right]_+$$

$$(\tilde{f}^{(1)}_{q/q})_{C.T.}=\left|\frac{p_z}{p_z^R}\right|\tilde{f}^{(1)}_{q/q,t}\left(\frac{p_z}{p_z^R}(x-1)+1,r\right)+\left|\frac{p_z}{p_z^R}\right|\tilde{f}^{(1)}_{q/q,z}\left(\frac{p_z}{p_z^R}(x-1)+1,r\right)$$

$$r=\mu_R^2/(p_z^R)^2.$$

Unpolarized PDF : gluon in gluon

- Projector: $P_{ij}^{ab} = \delta^{ab} g_{\perp,ij}/(2-D)$
- At one-loop level, the partonic quasi-PDF can be written as

$$x\tilde{f}_{g/g}^{(n)}(x,\rho) = \left[x\tilde{f}_{g/g}^{(n)}(x,\rho)\right]_+ + \tilde{c}^{(n)}\delta(x-1), \qquad \tilde{c}^{(n)} = \frac{1}{p_z^2} N^{(n)} \langle g(p)|O_g^{(n)}(0,0)|g(p)\rangle$$

- Offshell gluon matrix element can mix with gauge variant operators

$$\begin{aligned}
\tilde{c}^{(1,g)} &= \frac{\alpha_s C_A}{12\pi\epsilon}\frac{p^2}{p^2+p_z^2} + \mathcal{O}(\epsilon^0), \\
\tilde{c}^{(2,g)} &= -\frac{\alpha_s C_A}{12\pi\epsilon}\frac{p^2}{p_z^2} + \mathcal{O}(\epsilon^0), \\
\tilde{c}^{(3,g)} &= \mathcal{O}(\epsilon^0), \\
\tilde{c}^{(4,g)} &= \frac{\alpha_s C_A}{3\pi\epsilon}\frac{p^2}{p_z^2} + \mathcal{O}(\epsilon^0),
\end{aligned}$$

- O_3 is a “good operator” because it is the “t z” component of the gluon energy-momentum tensor
- For O_1, O_2, O_4, the large x behavior depends on the offshellness of external gluon

Unpolarized PDF : gluon in gluon

- Asymptotic region of quark quasi-PDF

$$\lim_{\xi\to\infty} C_B\left(\xi, \frac{p^z}{\mu}\right) = -\frac{3}{2|\xi|} \qquad \lim_{\xi\to\infty} \left|\frac{p^z}{p_z^R}\right| h\left(1+\frac{p^z}{p_z^R}(\xi-1), r\right) = -\frac{3}{2|\xi|}$$

Yong Zhao, talk at BNL, 2019

- For gluon quasi-PDF defined by O_1

$$\lim_{\xi\to\infty} C_B\left(\xi, \frac{p^z}{\mu}\right) \propto -\frac{1}{|\xi|} \qquad \lim_{\xi\to\infty} \left|\frac{p^z}{p_z^R}\right| h\left(1+\frac{p^z}{p_z^R}(\xi-1), r\right) \propto -\frac{1}{(1-r)|\xi|}$$

The UV divergence is subtracted but the asymptotic behavior is not

- Mixing with gauge variant operators, even for local operator

J. C. Collins and R. J. Scaliset, PRD50,1994

Unpolarized PDF : gluon in gluon

❖ Bare gluon quasi-PDF

$$\left[x\tilde{f}^{(3,1)}_{g/g}(x,\rho\to 0)\right]_+ = \frac{\alpha_s C_A}{2\pi}\begin{cases}\left[\frac{2(1-x+x^2)^2}{x-1}\ln\frac{x-1}{x}+\frac{4x^3-6x^2+8x-5}{2(x-1)}\right]_+, & x>1\\ \left[\frac{2(1-x+x^2)^2}{x-1}\ln\frac{\rho}{4}+\frac{12x^4-24x^3+30x^2-17x+5}{2(x-1)}\right]_+, & 0<x<1\\ \left[-\frac{2(1-x+x^2)^2}{x-1}\ln\frac{x-1}{x}-\frac{4x^3-6x^2+8x-5}{2(x-1)}\right]_+, & x<0.\end{cases}$$

❖ Light-cone PDF

$$\left[xf^{(1)}_{g/g}\left(x,\frac{\mu^2}{-p^2}\right)\right]_+ = \theta(x)\theta(1-x)\left\{\frac{\alpha_s C_A}{2\pi}\left[\frac{2(1-x+x^2)^2}{x-1}\ln\frac{-p^2x(1-x)}{\mu^2}+2x^3-2x^2+3x-2\right]_+ - \frac{\alpha_s C_A}{4\pi}\left[\frac{x}{1-x}\right]_+\right\},$$

❖ Matching coefficient

$$xC^{(3,1)}_{gg}\left(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}\right) = \left[x\tilde{f}^{(3,1)}_{g/g}(x,\rho\to 0) - xf^{(1)}_{g/g}\left(x,\frac{\mu^2}{-p^2}\right) - (x\tilde{f}^{(3,1)}_{g/g})_{C.T.}\right]_+ + \left(\tilde{c}^{(3,g)}_{\mathrm{RI/MOM}} - c^{3,g}_{\overline{\mathrm{MS}}}\right)\delta(x-1),$$

$$(x\tilde{f}^{(3,1)}_{g/g})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| x\tilde{f}^{(3,1)}_{g/g}\left(\frac{p_z}{p_z^R}(x-1)+1,r\right)$$

Unpolarized PDF : gluon in gluon

- The counter term

$$
\begin{aligned}
&\left[x\tilde{f}^{(3,1)}_{g/g}(x,\rho)\right]_+ \\
&= \frac{\alpha_s C_A}{2\pi}\begin{cases}
\Bigg[\frac{-(\rho-4)^2(\rho-1)+8(\rho+2)x^4-16(\rho+2)x^3-2\left(\rho^2+8\rho-24\right)x^2+\left(6\rho^2+20\rho-32\right)x}{8(\rho-1)^2(x-1)}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} & \\
+\frac{4x^3}{(2x-1)(\rho+4x^2-4x)}+\frac{8x^4-16x^3-22x^2+34x-9}{4(\rho-1)(x-1)(2x-1)}-\frac{8x^3(x-1)}{(\rho+4x^2-4x)^2}+\frac{3(2x-1)x}{2(\rho-1)^2}-\frac{4x+1}{4(x-1)}\Bigg]_+, & x>1 \\
\Bigg[\frac{-(\rho-4)^2(\rho-1)+8(\rho+2)x^4-16(\rho+2)x^3-2\left(\rho^2+8\rho-24\right)x^2+\left(6\rho^2+20\rho-32\right)x}{8(\rho-1)^2(x-1)}\frac{1}{\sqrt{1-\rho}}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}} & \\
+\frac{-30x^2+34x-9}{4(\rho-1)(x-1)}+\frac{3\left(4x^3-4x^2+x\right)}{2(\rho-1)^2}+\frac{6x+1}{4(x-1)}\Bigg]_+, & 0<x<1 \\
\Bigg[-\frac{-(\rho-4)^2(\rho-1)+8(\rho+2)x^4-16(\rho+2)x^3-2\left(\rho^2+8\rho-24\right)x^2+\left(6\rho^2+20\rho-32\right)x}{8(\rho-1)^2(x-1)}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} & \\
-\frac{4x^3}{(2x-1)(\rho+4x^2-4x)}+\frac{-8x^4+16x^3+22x^2-34x+9}{4(\rho-1)(x-1)(2x-1)}+\frac{8x^3(x-1)}{(\rho+4x^2-4x)^2}-\frac{3(2x-1)x}{2(\rho-1)^2}+\frac{4x+1}{4(x-1)}\Bigg]_+, & x<0.
\end{cases}
\end{aligned}
$$

Unpolarized PDF :Gluon in quark

- Light-cone splitting function

$$x f^{(1)}_{g/q}\left(x, \frac{\mu^2}{-p^2}\right) = \frac{\alpha_s C_F}{2\pi}\left[(1+(1-x)^2)\ln\frac{\mu^2}{-p^2 x(1-x)} + x(1-x) - 2\right].$$

- Bare splitting function

$$\mathrm{Tr}\left[\left(x f^{(3,1)}_{g/q,t}(x,\rho)\gamma^t + x f^{(3,1)}_{g/q,z}(x,\rho)\frac{p^t}{p^z}\gamma^z + x f^{(3,1)}_{g/q,p}(x,\rho)\frac{p^t \not{p}}{p^2}\right)\mathcal{P}\right],$$

$$x\tilde{f}^{(3,1)}_{g/q}(x,\mu,P^z) = \frac{\alpha_s C_F}{2\pi}\begin{cases} -\left(1+(1-x)^2\right)\ln\frac{x-1}{x} - x + 2, & x > 1 \\ -\left(1+(1-x)^2\right)\ln\frac{\rho}{4} - 4x^2 + 6x - 2, & 0 < x < 1 \\ \left(1+(1-x)^2\right)\ln\frac{x-1}{x} + x - 2, & x < 0. \end{cases},$$

- Matching coefficient

$$x C^{(3,1)}_{g/q}\left(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}\right) = \left[x\tilde{f}^{(3,1)}_{g/q}(x,\rho\to 0) - x f^{(1)}_{g/q}\left(x, \frac{\mu^2}{-p^2}\right) - (x\tilde{f}^{(3,1)}_{g/q})_{C.T.}\right],$$

$$(x\tilde{f}^{(3,1)}_{g/q})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| x\tilde{f}^{(3,1)}_{g/q}\left(\frac{p_z}{p_z^R}(x-1)+1, r\right).$$

Unpolarized PDF :Quark in gluon

- Light-cone splitting function

$$f^{(1)}_{q/g}\left(x,\frac{\mu^2}{-p^2}\right)=\frac{\alpha_s T_f}{2\pi}\left[(x^2+(1-x)^2)\ln\frac{\mu^2}{-p^2x(1-x)}-1\right],$$

- Bare splitting function

$$\tilde{f}^{(1)}_{q/g}(x,\rho\to 0)=\frac{\alpha_s T_f}{2\pi}\begin{cases}-(x^2+(1-x)^2)\ln\frac{x-1}{x}-2x+1, & x>1\\ -(x^2+(1-x)^2)\ln\frac{\rho}{4}-6x^2+6x-2, & 0<x<1\\ (x^2+(1-x)^2)\ln\frac{x-1}{x}+2x-1, & x<0.\end{cases}$$

- Matching coefficient

$$C^{(1)}_{qg}\left(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}\right)=\left[\tilde{f}^{(1)}_{q/g}(x,\rho\to 0)-f^{(1)}_{q/g}\left(x,\frac{\mu^2}{-p^2}\right)-(\tilde{f}^{(1)}_{q/g})_{C.T.}\right],$$

$$(\tilde{f}^{(1)}_{q/g})_{C.T.}=\left|\frac{p_z}{p_z^R}\right|\tilde{f}^{(1)}_{q/g}\left(\frac{p_z}{p_z^R}(x-1)+1,r\right).$$

Polarized quasi-PDFs

❖ Polarized gluon PDF

$$\Delta f_{g/H}(x,\mu) = i\epsilon_{\perp ij}\int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- x P^+} \langle P|F^{+i}(\xi^- n_+)\mathcal{W}(\xi^- n_+, 0; L_{n_+})F^{j+}(0)|P\rangle,$$

❖ Multiplicatively renormalizable polarized gluon quasi-PDFs:

$$\begin{aligned}
\Delta O_g^1(z,0) &= i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{tj}(z_1),\\
\Delta O_g^2(z,0) &= i\epsilon_{\perp,ij}F^{zi}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),\\
\Delta O_g^3(z,0) &= i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),
\end{aligned}$$

$$\mathcal{P}_{\perp,ij} = \frac{i}{D-2}\epsilon_{\mu\nu ij}n_t^\mu n^\nu$$

Zhang, Ji, Schäfer, Wang, SZ, Phys.Rev.Lett. 122 (2019) no.14, 142001

Polarized PDF: gluon in gluon

- Projector: $\mathcal{P}_{\perp,ij} = \frac{i}{D-2}\epsilon_{\mu\nu ij} n_t^\mu n^\nu .$

- At one-loop level, the partonic quasi-PDF can be written as

$$x\Delta\tilde{f}^{(n)}_{g/g}(x) = [x\Delta\tilde{f}]_+ + \Delta\tilde{c}^{(n)}\delta(x-1)$$

$$\Delta\tilde{c}^{(n)} = \frac{1}{(p^z)^2}\Delta N^{(n)}\langle g(p)|\Delta O^{(n)}_{g,R}(0,0)|g(p)\rangle,$$

$$\Delta N^{(1)} = \frac{(p^z)^2}{(p^t)^2},\ \ \Delta N^{(2)} = 1,\ \ \Delta N^{(3)} = \frac{p^z}{p^t}.$$

with

$$\Delta\tilde{c}^{(1)} = -\frac{\alpha_s C_A (p^2 + 6(p^z)^2)}{24\pi\epsilon(p^2 + (p^z)^2)},$$

$$\Delta\tilde{c}^{(2)} = -\frac{\alpha_s C_A (5p^2 + 6(p^z)^2)}{24\pi\epsilon(p^z)^2},$$

$$\Delta\tilde{c}^{(3)} = -\frac{\alpha_s C_A}{4\pi\epsilon},$$

Offshell gluon matrix element may mix with gauge variant operators

- We use ΔO_g^3 to define quasi-PDF

Polarized PDF: gluon in gluon

- Light-cone PDF

$$x\Delta f^{(1)}_{g/g}(x,\mu) = \frac{\alpha_s C_A}{2\pi}\left\{\frac{x}{x-1}\left[\left(4x^2-6x+4\right)\ln\frac{-p^2(1-x)x}{\mu^2}+8x^2-11x+7+\frac{(1-\xi)}{2}\right]\right\}_+$$

- Bare quasi-PDF

$$x\Delta\tilde{f}^{(3,1)}_{g/g}(x,\rho) = \frac{\alpha_s C_A}{2\pi}\begin{cases}\frac{8x^2+4\left(2x^2-3x+2\right)x\ln\frac{x-1}{x}-8x+1}{2(x-1)}, & x>1\\ \frac{4\left(2x^2-3x+2\right)x\ln\frac{\rho}{4}+20x^3-28x^2+15x-1}{2(x-1)}, & 0<x<1\\ -\frac{8x^2+4\left(2x^2-3x+2\right)x\ln\frac{x-1}{x}-8x+1}{2(x-1)}, & x<0.\end{cases}$$

The virtual contribution is the same as the unpolarized case

- Matching coefficient

$$x\Delta C^{(3,1)}_{gg}\left(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}\right) = xC^{(3,1)}_{gg}\left(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}\right)+\left[\left(x\Delta\tilde{f}^{(3,1)}_{g/g}(x,\rho\to 0)-x\tilde{f}^{(3,1)}_{g/g}(x,\rho\to 0)\right)\right.$$
$$\left.-\left(x\Delta f^{(3,1)}_{g/g}\left(x,\frac{\mu^2}{-p^2}\right)-xf^{(3,1)}_{g/g}\left(x,\frac{\mu^2}{-p^2}\right)\right)-(x\Delta\tilde{f}^{(3,1)}_{g/g})_{C.T.}\right]$$

$$(x\Delta\tilde{f}^{(3,1)}_{g/g})_{C.T.} = \left|\frac{p_z}{p_z^R}\right|\left[x\Delta\tilde{f}^{(3,1)}_{g/g}\left(\frac{p_z}{p_z^R}(x-1)+1,r\right)-x\tilde{f}^{(3,1)}_{g/g}\left(\frac{p_z}{p_z^R}(x-1)+1,r\right)\right]$$

polarized PDF: Quark in quark

- Light-cone PDF

$$\Delta f^{(1)}_{q/q}\left(x, \frac{\mu^2}{-p^2}\right) = f^{(1)}_{q/q}\left(x, \frac{\mu^2}{-p^2}\right)$$

- Quasi PDF

$$\Delta \tilde{f}^{(1)}_{q/q,z}(x,\rho)|_{\rho\to 0} = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{(x^2+1)\ln\frac{x-1}{x}+x-1}{x-1}, & x>1 \\ \frac{2(x^2+1)\ln\frac{\rho}{4}+4x^2+1}{2(x-1)}, & 0<x<1 \\ -\frac{(x^2+1)\ln\frac{x-1}{x}+x-1}{x-1}, & x<0. \end{cases}$$

- Matching coefficient

$$\Delta C^{(1)}_{qq}(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}) = \left[\Delta \tilde{f}^{(1)}_{q/q,z}(x, \rho\to 0) - \Delta f^{(1)}_{q/q}\left(x, \frac{\mu^2}{-p^2}\right) - (\Delta \tilde{f}^{(1)}_{q/q})_{C.T.}\right]_+$$

$$(\Delta \tilde{f}^{(1)}_{q/q})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| \Delta \tilde{f}^{(1)}_{q/q,z}\left(\frac{p_z}{p_z^R}(x-1)+1, r\right)$$

LP3, 1807.06566, 1807.07431

polarized PDF :Gluon in quark

- Light-cone PDF

$$x\Delta f^{(1)}_{g/q}(x,\mu) = \frac{\alpha_s C_F}{2\pi}\left(x(x-2)\ln\frac{-p^2(1-x)x}{\mu^2} + x^2 - 5x\right)$$

- Quasi PDF

$$x\Delta \tilde{f}^{(3,1)}_{g/q}(x,\rho\to 0) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{2}\left(2x + 2(x-2)x\ln\frac{x-1}{x} - 1\right), & x>1 \\ \frac{1}{2}\left(2(x-2)x\ln\frac{\rho}{4} + 6x^2 - 8x + 1\right), & 0<x<1 \\ \frac{1}{2}\left(-2x - 2(x-2)x\ln\frac{x-1}{x} + 1\right), & x<0. \end{cases}$$

- Matching coefficient

$$x\Delta C^{(3,1)}_{g/q}(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}) = \left[x\Delta\tilde{f}^{(3,1)}_{g/q}(x,\rho\to 0) - x\Delta f^{(1)}_{g/q}\left(x,\frac{\mu^2}{-p^2}\right) - (x\Delta\tilde{f}^{(3,1)}_{g/q})_{C.T.}\right]$$

$$(x\Delta\tilde{f}^{(3,1)}_{g/q})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| x\Delta\tilde{f}^{(3,1)}_{g/q}\left(\frac{p_z}{p_z^R}(x-1)+1, r\right)$$

polarized PDF : Quark in gluon

- Light-cone PDF

$$\Delta f^{(1)}_{q/g}(x,\mu) = \frac{\alpha_s T_f}{2\pi}\left((1-2x)\ln\frac{-p^2(1-x)x}{\mu^2} - 4x + 1\right)$$

- Quasi PDF

$$\Delta \tilde{f}^{(1)}_{q/g}(x,\rho\to 0) = \frac{\alpha_s T_f}{2\pi}\begin{cases} (1-2x)\ln\frac{x-1}{x} - 2, & x>1 \\ (1-2x)\ln\frac{\rho}{4} - 4x + 1, & 0<x<1 \\ (2x-1)\ln\frac{x-1}{x} + 1, & x<0. \end{cases}$$

- Matching coefficient

$$\Delta C^{(1)}_{qg}(x,r,\frac{p_z}{\mu},\frac{p_z}{p_z^R}) = \left[\Delta \tilde{f}^{(1)}_{q/g}(x,\rho\to 0) - \Delta f^{(1)}_{q/g}\left(x,\frac{\mu^2}{-p^2}\right) - (\Delta \tilde{f}^{(1)}_{q/g})_{C.T.}\right]$$

$$(\Delta \tilde{f}^{(1)}_{q/g})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| \Delta \tilde{f}^{(1)}_{q/g}\left(\frac{p_z}{p_z^R}(x-1)+1, r\right)$$

Summary

- The renormalization of gluon quasi PDFs can be studied in auxiliary field formalism. Four unpolarized and three polarized quasi PDF operators are multiplicatively renormalizable.

- The RI/MOM matching for gluon and singlet quark quasi PDF is studied. Operators that can avoid mixing with gauge variant operators are identified. The matching coefficient in RI/MOM scheme is determined at one-loop accuracy

- Similar calculation can be performed to the GPDs

- Can be used to extract quark and gluon PDFs from lattice simulation

Thank you!

Backup: counter terms

$$\tilde{f}^{(1)}_{q/q,t}(x,\rho) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{2x^2}{(2x-1)(\rho+4x^2-4x)} + \frac{4x-3}{2(\rho-1)(2x-1)} - \frac{3}{2(x-1)} - \frac{\left(3\rho+4x^2+(\rho-8)x\right)\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}}{4(1-\rho)^{3/2}(x-1)}, & x>1 \\ \frac{4x-3}{2(\rho-1)} + \frac{3}{2(x-1)} - \frac{\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1+\rho}}\left(3\rho+4x^2+(\rho-8)x\right)}{4(1-\rho)^{3/2}(x-1)}, & 0<x<1 \\ -\frac{2x^2}{(2x-1)(\rho+4x^2-4x)} + \frac{3-4x}{2(\rho-1)(2x-1)} + \frac{3}{2(x-1)} + \frac{\left(3\rho+4x^2+(\rho-8)x\right)\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}}{4(1-\rho)^{3/2}(x-1)}, & x<0, \end{cases}$$

$$\tilde{f}^{(1)}_{q/q,z}(x,\rho) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{\left(2\rho^2+3\rho+4(\rho+2)x^2-(13\rho+8)x+4\right)\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}}{4(1-\rho)^{5/2}(x-1)} + \frac{2\left(3x^2-2x\right)}{(2x-1)^3(\rho+4x^2-4x)} \\ -\frac{8\left(x^3-x^2\right)}{(2x-1)(\rho+4x^2-4x)^2} + \frac{8x^4-34x^3+40x^2-17x+2}{(\rho-1)(x-1)(2x-1)^3} + \frac{3(4x-3)}{2(\rho-1)^2(2x-1)}, & x>1 \\ \frac{\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1+\rho}}\left(2\rho^2+3\rho+4(\rho+2)x^2-(13\rho+8)x+4\right)}{4(1-\rho)^{5/2}(x-1)} + \frac{2-3x}{(\rho-1)(x-1)} + \frac{3(4x-3)}{2(\rho-1)^2}, & 0<x<1 \\ -\frac{\left(2\rho^2+3\rho+4(\rho+2)x^2-(13\rho+8)x+4\right)\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}}{4(1-\rho)^{5/2}(x-1)} - \frac{2\left(3x^2-2x\right)}{(2x-1)^3(\rho+4x^2-4x)} \\ +\frac{8\left(x^3-x^2\right)}{(2x-1)(\rho+4x^2-4x)^2} + \frac{-8x^4+34x^3-40x^2+17x-2}{(\rho-1)(x-1)(2x-1)^3} - \frac{3(4x-3)}{2(\rho-1)^2(2x-1)}, & x<0. \end{cases}$$

$$x\tilde{f}^{(3,1)}_{g/q}(x,\mu,P^z) = \frac{\alpha_s C_F}{2\pi}\begin{cases} -\frac{5\rho^2-10\rho+(8\rho+4)x^2-4(\rho+2)x+8}{4(\rho-1)^2}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\ -\frac{(\rho-4)\rho+8(2\rho+1)x^3-4\left(\rho^2+2\rho+6\right)x^2+2\left(3\rho^2-2\rho+8\right)x}{2(1-\rho)^2(\rho+4x^2-4x)}, & x>1 \\ -\frac{5\rho^2-10\rho+(8\rho+4)x^2-4(\rho+2)x+8}{4(\rho-1)^2}\frac{1}{\sqrt{1-\rho}}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}} \\ -\frac{(2x-1)(\rho+2(\rho+2)x-4)}{2(1-\rho)^2}, & 0<x<1 \\ \frac{5\rho^2-10\rho+(8\rho+4)x^2-4(\rho+2)x+8}{4(\rho-1)^2}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\ +\frac{(\rho-4)\rho+8(2\rho+1)x^3-4\left(\rho^2+2\rho+6\right)x^2+2\left(3\rho^2-2\rho+8\right)x}{2(1-\rho)^2(\rho+4x^2-4x)}, & x<0. \end{cases}$$

Backup: counter terms

$$
\tilde{f}^{(1)}_{q/g}(x,\rho)
= \frac{\alpha_s T_f}{2\pi}
\begin{cases}
-\frac{\rho^2-2\rho+4(\rho+2)x^2-4(\rho+2)x+4}{4(1-\rho)^{3/2}} \frac{1}{\sqrt{1-\rho}} \ln \frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} - \frac{(2x-1)\left(-(\rho-4)\rho+4(\rho+2)x^2-4(\rho+2)x\right)}{2(1-\rho)^{3/2}(\rho+4x^2-4x)}, & x>1 \\
-\frac{\rho^2-2\rho+4(\rho+2)x^2-4(\rho+2)x+4}{4(1-\rho)^{3/2}} \frac{1}{\sqrt{1-\rho}} \ln \frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}} - \frac{-\rho+12x^2-12x+4}{2(1-\rho)^{3/2}}, & 0<x<1 \\
\frac{\rho^2-2\rho+4(\rho+2)x^2-4(\rho+2)x+4}{4(1-\rho)^{3/2}} \frac{1}{\sqrt{1-\rho}} \ln \frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} - \frac{(2x-1)\left((\rho-4)\rho-4(\rho+2)x^2+4(\rho+2)x\right)}{2(1-\rho)^{3/2}(\rho+4x^2-4x)}, & x<0.
\end{cases}
$$

$$
x\Delta\tilde{f}^{(3,1)}_{g/g}(x,\rho) = \frac{\alpha_s C_A}{2\pi}
\begin{cases}
-\frac{\rho\left(\rho^2-3\rho+8\right)+8(\rho-4)x^3+8\left(\rho^2-\rho+6\right)x^2-2\left(9\rho^2-10\rho+16\right)x}{8(1-\rho)^{5/2}(x-1)} \ln \frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\
+\frac{4x^3}{(2x-1)(\rho+4x^2-4x)} + \frac{-8x^3-8x^2+14x-3}{4(\rho-1)(x-1)(2x-1)} \\
-\frac{8\left(x^4-x^3\right)}{(\rho+4x^2-4x)^2} + \frac{3(2x-1)}{2(\rho-1)^2} - \frac{4x+1}{4(x-1)}, & x>1 \\
-\frac{\rho\left(\rho^2-3\rho+8\right)+8(\rho-4)x^3+8\left(\rho^2-\rho+6\right)x^2-2\left(9\rho^2-10\rho+16\right)x}{8(1-\rho)^{5/2}(x-1)} \ln \frac{1-\sqrt{1-\rho}}{1+\sqrt{1+\rho}} \\
+\frac{3\left(4x^2-4x+1\right)}{2(\rho-1)^2} + \frac{-16x^3+8x^2+6x-3}{4(\rho-1)(x-1)} + \frac{6x+1}{4(x-1)}, & 0<x<1 \\
\frac{\rho\left(\rho^2-3\rho+8\right)+8(\rho-4)x^3+8\left(\rho^2-\rho+6\right)x^2-2\left(9\rho^2-10\rho+16\right)x}{8(1-\rho)^{5/2}(x-1)} \ln \frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\
-\frac{4x^3}{(2x-1)(\rho+4x^2-4x)} + \frac{8x^3+8x^2-14x+3}{4(\rho-1)(x-1)(2x-1)} \\
+\frac{8\left(x^4-x^3\right)}{(\rho+4x^2-4x)^2} - \frac{3(2x-1)}{2(\rho-1)^2} + \frac{4x+1}{4(x-1)}, & x<0.
\end{cases}
$$

Backup: counter terms

$$\Delta\tilde{f}^{(1)}_{q/q,z}(x,\rho)=\frac{\alpha_s C_F}{2\pi}\begin{cases} -\frac{3\rho-2x^2-2}{2(1-\rho)^{3/2}(x-1)}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\ +\frac{4x^2}{(2x-1)(\rho+4x^2-4x)}+\frac{1-2x^2}{(\rho-1)(x-1)(2x-1)}-\frac{8\left(x^3-x^2\right)}{(\rho+4x^2-4x)^2}-\frac{3}{2(x-1)}, & x>1 \\ -\frac{3\rho-2x^2-2}{2(1-\rho)^{3/2}(x-1)}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1+\rho}}+\frac{1-2x^2}{(\rho-1)(x-1)}+\frac{3}{2(x-1)}, & 0<x<1 \\ -\frac{-3\rho+2x^2+2}{2(1-\rho)^{3/2}(x-1)}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\ -\frac{4x^2}{(2x-1)(\rho+4x^2-4x)}+\frac{2x^2-1}{(\rho-1)(x-1)(2x-1)}+\frac{8\left(x^3-x^2\right)}{(\rho+4x^2-4x)^2}+\frac{3}{2(x-1)}, & x<0. \end{cases}$$

$$x\Delta\tilde{f}^{(3,1)}_{g/q}(x,\rho)=\frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{(2x-1)\left(\rho(\rho+2)+4(\rho+2)x^2+4\left(\rho^2-2\rho-2\right)x\right)}{4(\rho-1)^2(\rho+4x^2-4x)} \\ +\frac{\left(-(\rho-4)\rho+4(\rho+2)x^2-4\left(\rho^2-2\rho+4\right)x\right)\ln\left(\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}\right)}{8(1-\rho)^{5/2}}, & x>1 \\ \frac{\rho+12x^2+4(\rho-4)x+2}{4(\rho-1)^2}+\frac{\ln\left(\frac{1-\sqrt{1-\rho}}{\sqrt{\rho+1}+1}\right)\left(-(\rho-4)\rho+4(\rho+2)x^2-4\left(\rho^2-2\rho+4\right)x\right)}{8(1-\rho)^{5/2}}, & 0<x<1 \\ -\frac{(2x-1)\left(\rho(\rho+2)+4(\rho+2)x^2+4\left(\rho^2-2\rho-2\right)x\right)}{4(\rho-1)^2(\rho+4x^2-4x)} \\ +\frac{\left((\rho-4)\rho-4(\rho+2)x^2+4\left(\rho^2-2\rho+4\right)x\right)\ln\left(\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}\right)}{8(1-\rho)^{5/2}}, & x<0. \end{cases}$$

$$\Delta\tilde{f}^{(1)}_{q/g}=\frac{\alpha_s T_f}{2\pi}\begin{cases} -\frac{\rho+8x^2+2(\rho-4)x}{1-\rho(\rho+4x^2-4x)}-\frac{\rho+4x-2}{2(1-\rho)^{3/2}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}, & x>1 \\ \frac{1-4x}{1-\rho}-\frac{\rho+4x-2}{2(1-\rho)^{3/2}}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1+\rho}}, & 0<x<1 \\ \frac{\rho+8x^2+2(\rho-4)x}{1-\rho(\rho+4x^2-4x)}+\frac{\rho+4x-2}{2(1-\rho)^{3/2}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}, & x<0. \end{cases}$$