



BY
TMDS FOR THE SIMPLE-MINDED

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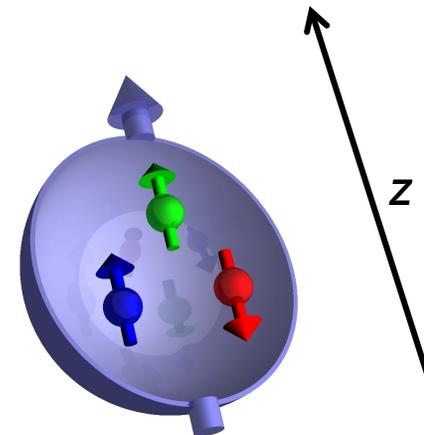
OUTLINE

- Introduction
- 1-D parton structure of the Nucleon
- From 1-D to 3-D
- What can we measure (in principle)?
- Observables
- What can we measure (in practice)?
- Conclusion
- PAID ADVERTISEMENT

INTRODUCTION

The familiar (?) 1D world of Nucleon longitudinal structure:

- Take a nucleon
- Move it real fast along z
⇒ light cone momentum
 $P_+ = P_0 + P_z (>>M)$
- Hit a “parton” (q, g,...) inside
- Measure **its** l.c. momentum
 $p_+ = p_0 + p_z (m \approx 0)$
- ⇒ Momentum Fraction $\xi = p_+ / P_+^*$
- In DIS: $\xi = (q_z - \nu)/M \approx x_{Bj} = Q^2/2M\nu$
- Probability: $F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x)$



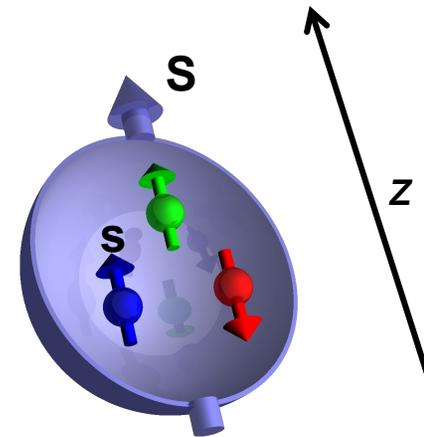
In the following, will often write “ $f_1(x)$ ” for $q(x)$

*) Advantage: Boost-independent

INTRODUCTION

Introduce two more quantities of interest:

- Proton spin \mathbf{S}
- Parton spin \mathbf{s}
- Now we have 3 vectors: $\hat{z}, \vec{S}, \vec{s}$
- **But:** Every observable must be a scalar
- **And:** Spins are axial vectors!
- **Finally:** Must treat longitudinal and transverse directions differently (boost)
- Can form two pseudoscalars: $H = \vec{S} \cdot \hat{z}, h = \vec{s} \cdot \hat{z}$
and one scalar: Transversity T (see later) $T = \vec{S}_\perp \cdot \vec{s}_\perp$ *)
- 2nd Structure function $g_1(x) = \frac{1}{2} \sum_i e_i^2 \langle hH \rangle q_i(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x)$
 $\Delta q_i = q_{\uparrow\uparrow}(x) - q_{\uparrow\downarrow}(x)$

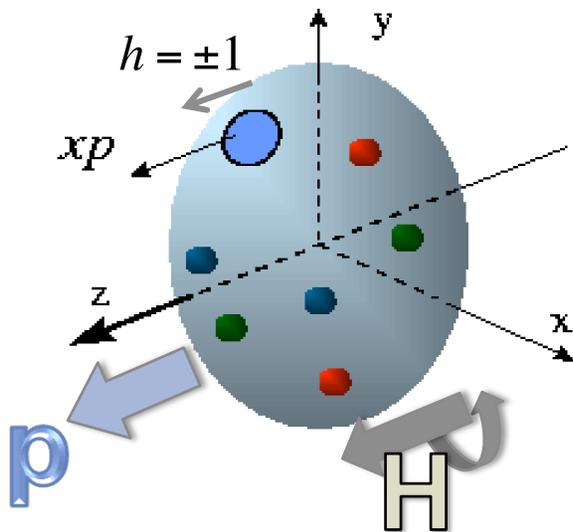


In the following, will often write “ $g_1(x)$ ” for $\Delta q(x)$

*) Not measurable in DIS

INTRODUCTION

So there we are:



$$q(x; Q^2), \langle h \cdot H \rangle q(x; Q^2)$$

Traditional “1-D” Parton Distributions (PDFs) (inclusive, integrated over many variables)

Parton model: DIS can access

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x) \text{ and } F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \text{ and } g_2(x) = 0$$

Complications: pQCD evolution and radiation

$$g_1(x, Q^2)_{pQCD} = \frac{1}{2} \sum_q^{N_f} e_q^2 [(\Delta q + \Delta \bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_q) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G \otimes \frac{\delta C_G}{N_f}]$$

⇒ access to gluons. $\delta C_q, \delta C_G$ – Wilson coefficient functions

SIDIS: allows flavor tagging ⇒ separated

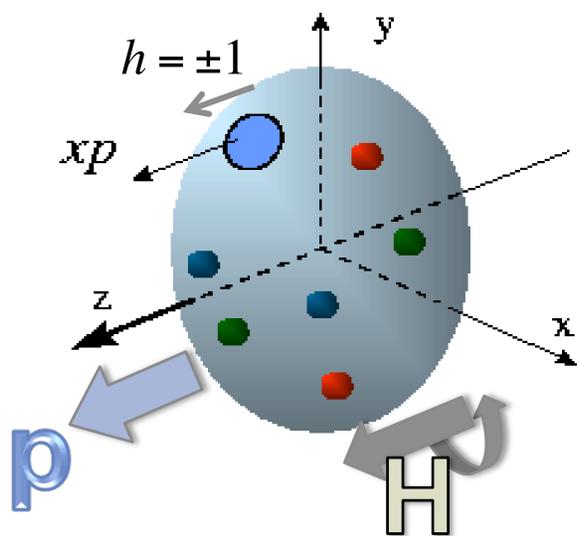
$$q_i, \Delta q_i$$

Complications: Higher Twist and resonances:

- Non-zero $R = F_L/2xF_1, g_2$
- Further Q^2 -dependence (power series in $\frac{1}{Q^n}$)

INTRODUCTION

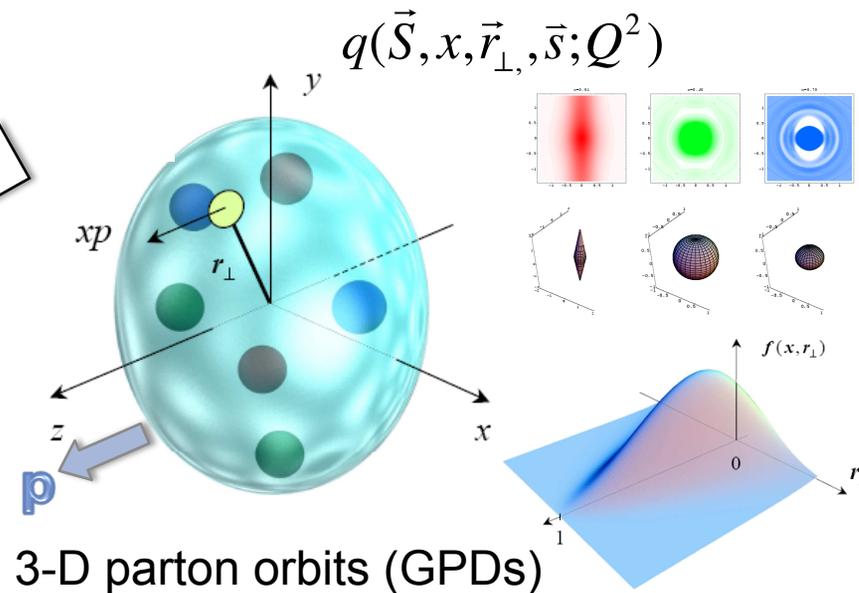
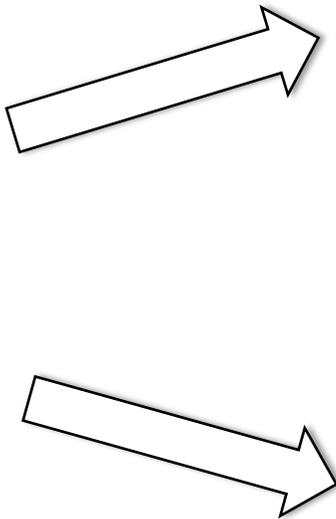
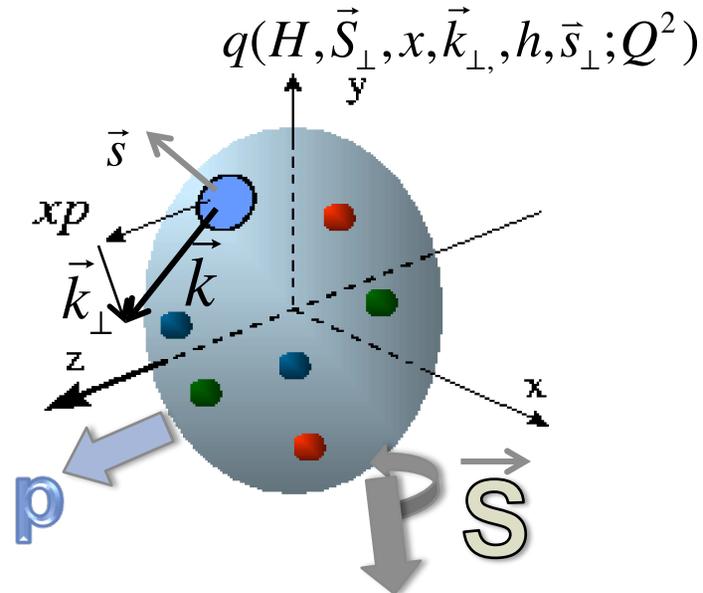
Now let's put our 3-D glasses on!



$$q(x; Q^2), \langle h \cdot H \rangle q(x; Q^2)$$

Traditional "1-D" Parton Distributions (PDFs)
(inclusive, integrated over many variables)

3-D Picture of parton flavor, spin and momentum (TMDs)

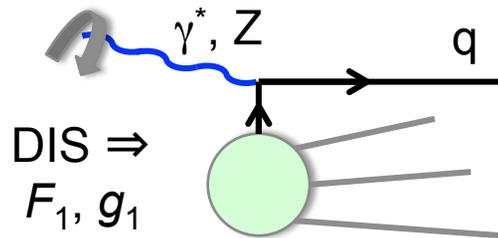


3-D parton orbits (GPDs)

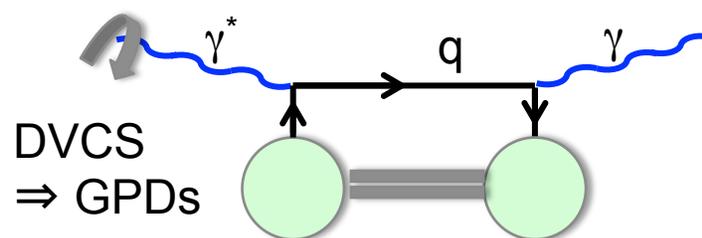
TOWARDS A COMPLETE PICTURE:

HOW can we access the full 3-D spin-flavor structure?

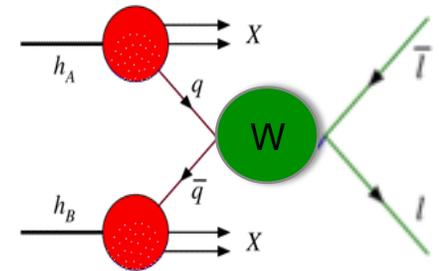
Electroweak Probe to “see” partons...



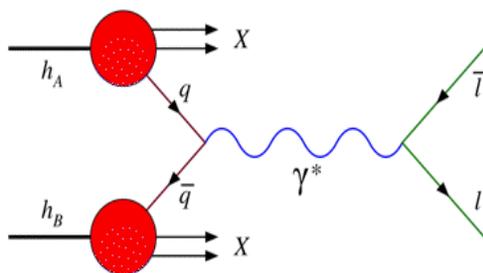
(1D only, no flavor tagging)



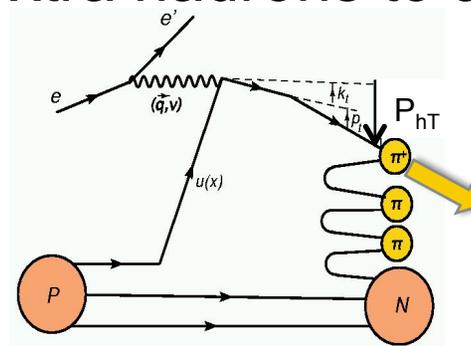
W production



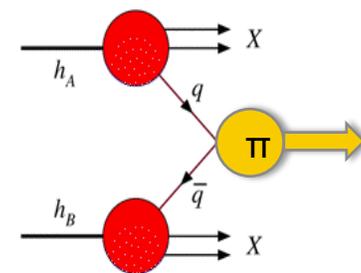
...+ extra hadrons to access all observables:



Drell-Yan



SIDIS



Hadron Production

THE THREE-DIMENSIONAL PICTURE

What CAN we observe?

TMDs

- Have 2 pseudoscalars (h, H), 2 vectors (\hat{z}, \vec{k}_\perp) and 2 axial vectors ($\vec{S}_\perp, \vec{s}_\perp$)
- Observables depend on x (and Q^2) and must be scalars!
- 3 possible moments without \vec{k}_\perp :
 - 1 (ordinary PDFs \Rightarrow unpolarized structure function f_1)
 - $H \cdot h$ (helicity PDFs \Rightarrow spin structure function g_1)
 - $\vec{S}_\perp \cdot \vec{s}_\perp$ (transversity PDFs \Rightarrow structure function h_1)
- 4 possible moments linear in \vec{k}_\perp :
 - $H(\vec{k}_\perp \cdot \vec{s}_\perp), h(\vec{k}_\perp \cdot \vec{S}_\perp)$ (“Worm gear” PDFs g_{1T}, h_{1L}^\perp)
 - $\vec{S}_\perp \cdot (\vec{k}_\perp \times \hat{z}), \vec{s}_\perp \cdot (\vec{k}_\perp \times \hat{z})$ (Sivers, Boer-Mulders PDFs $f_{1T}^\perp, h_{1T}^\perp$)
- Further possibilities quadratic in \vec{k}_\perp :
 - $\langle k_\perp^2 \rangle$ moments of first three...
 - “Pretzelosity” $h_{1T}^\perp = (\vec{k}_\perp \cdot \vec{S}_\perp) \cdot (\vec{k}_\perp \cdot \vec{s}_\perp) - \frac{1}{2}(\vec{S}_\perp \cdot \vec{s}_\perp) \vec{k}_\perp^2$

$(\vec{S}_\perp, \vec{s}_\perp)$

	1	h	s_T
1			
H			
S_T			

A MORE DETAILED LOOK

		quark		
		U	L	T
n u c l e o n	U	f_1  *)		h_1^\perp  - 
	L		g_1  -  *)	h_{1L}^\perp  - 
	T	f_{1T}^\perp  - 	g_{1T}^\perp  - 	h_1  -  *) h_{1T}^\perp  - 

*) ...and $\langle k_\perp^2 \rangle$ moments

A MORE DETAILED LOOK

		quark		
		U	L	T
n u c l e o n	U	f_1 		h_1^+  - 
	L		g_1  - 	h_{1L}^+  - 
	T	f_{1T}^+  - 	g_{1T}^+  - 	h_1  -  h_{1T}^+  - 

“Transversity” h_1 $\vec{S}_\perp \cdot \vec{s}_\perp$

*) ...and $\langle k_\perp^2 \rangle$ moments

- 0th moment of transverse momentum, akin to f_1 and g_1
- Different from g_1 because Lorentz boost does not commute with rotation
- Interesting in its own right: no mixing with gluons, sum rules and bounds exist
- Inaccessible in DIS, but as fundamental as the other two
- Requires SIDIS, Drell-Yan or hadronic reactions to access
- First results from HERMES, COMPASS, Jefferson Lab, ...

A MORE DETAILED LOOK

		quark		
		U	L	T
nucleon	U	$f1$		$h1^+$ -
	L		$g1$ -	$h1^+$ -
	T	$f1^+$ -	$g1^+$ -	$h1^+$ -

May explain large single spin asymmetries in hadronic reactions; may be related to orbital motion of quarks

Odd under time reversal T:

- all 3 vectors change sign
- But not a problem: T reverses initial state \leftrightarrow final state
- Predict sign change between SIDIS and DY

“Sivers” $\vec{S}_\perp \cdot (\vec{k}_\perp \times \hat{z})$

Slice at fixed x

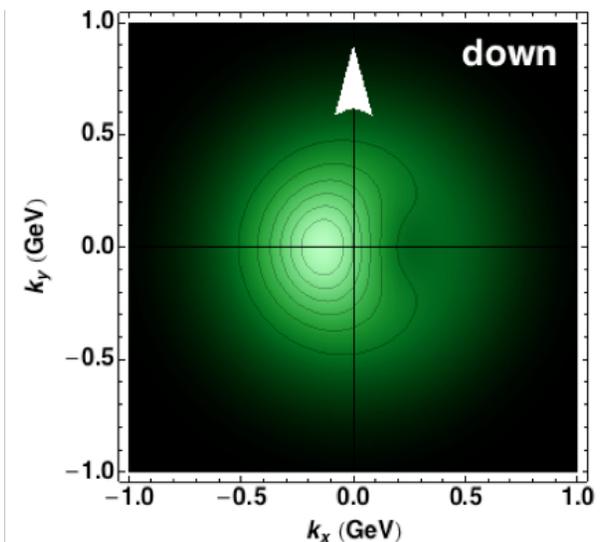
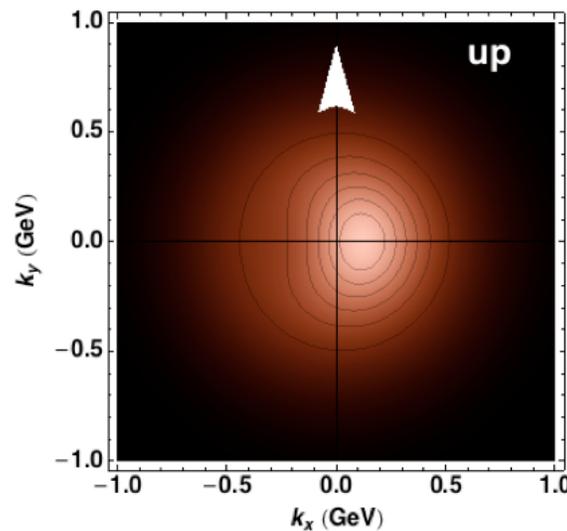


Fig. Credit: A. Bacchetta, M. Contalbrigo, "The proton in 3D," to appear in "Il Nuovo Saggiatore," bulletin of Italian Physical Society

A MORE DETAILED LOOK

		quark		
		U	L	T
n u c l e o n	U	f_1		h_1^+ -
	L		g_1 -	h_{1L}^+ -
	T	f_{1T}^+ -	g_{1T}^+ -	h_1 - h_{1T}^+ -

“Boer-Mulders” $\vec{s}_\perp \cdot (\vec{k}_\perp \times \hat{z})$

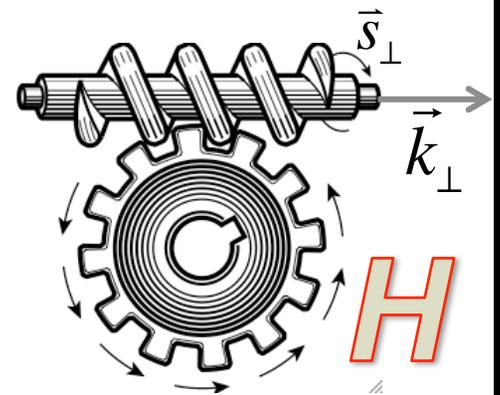
- Correlation between quark spin and transverse momentum even in unpolarized nucleon
- Has subtle effect on unpolarized SIDIS (cos2φ moment)

A MORE DETAILED LOOK

		quark		
		U	L	T
n u c l e o n	U	f_l		h_r^+ -
	L		g_l -	h_{ll}^+ -
	T	f_{lr}^+ -	g_{lr}^+ -	h_l - h_{lr}^+ -

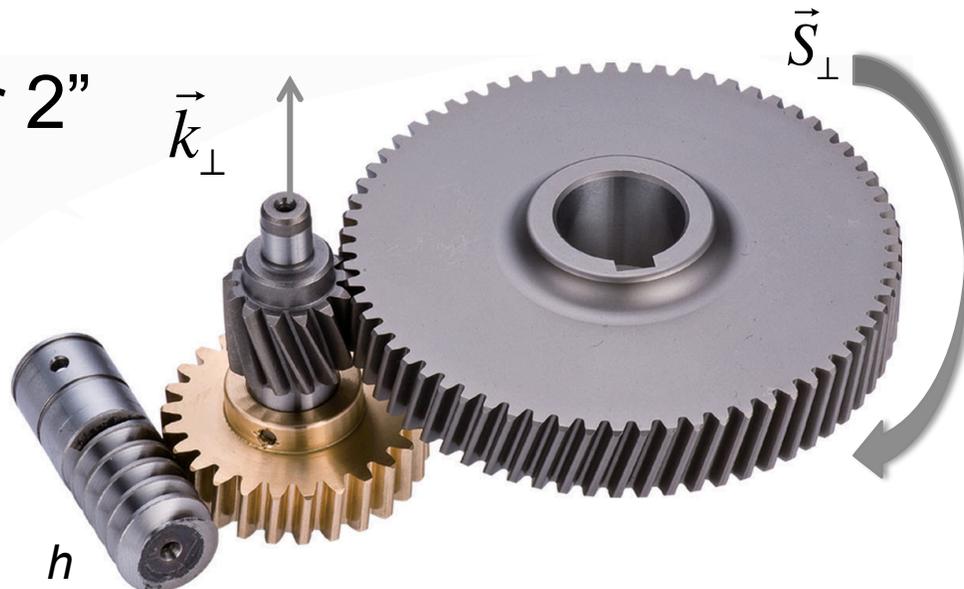
“Worm Gear 1”
long. pol. target

$$H(\vec{k}_\perp \cdot \vec{s}_\perp)$$



“Worm Gear 2”

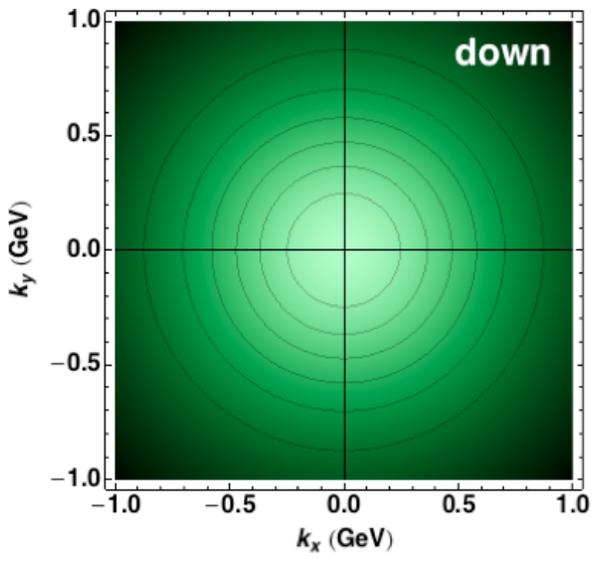
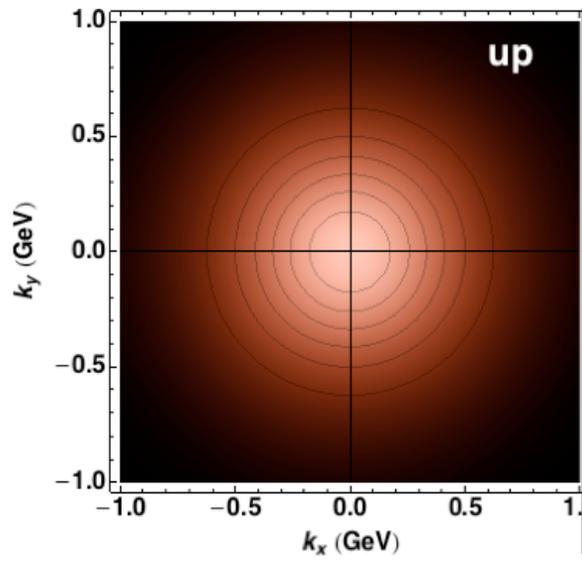
$$h(\vec{k}_\perp \cdot \vec{S}_\perp)$$



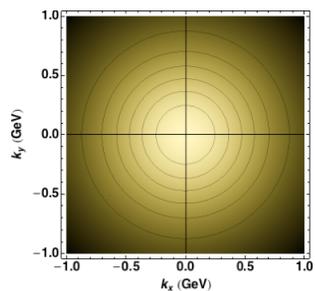
A MORE DETAILED LOOK

$\langle k_{\perp}^2 \rangle$ moments

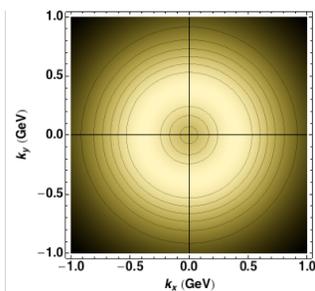
		quark		
		U	L	T
nucleon	U	f1		h_1^{\perp}
	L		g1	h_{1T}^{\perp}
	T	f_{1T}^{\perp}	g_{1T}^{\perp}	h_1



Beyond $\langle k_{\perp}^2 \rangle$...

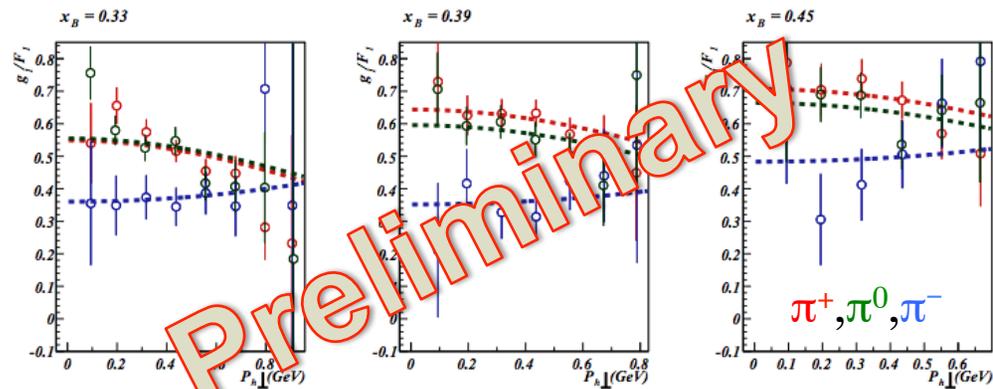


Gaussian bell...



...or donut?

Flavor dependence? Spin ($h \cdot H$) dependence?

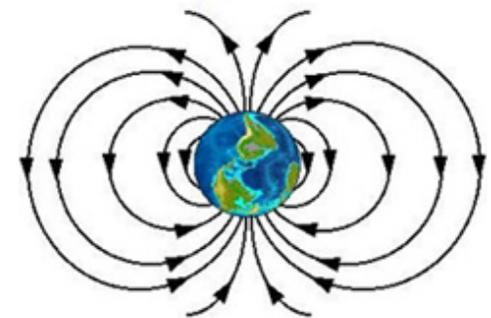


A MORE DETAILED LOOK

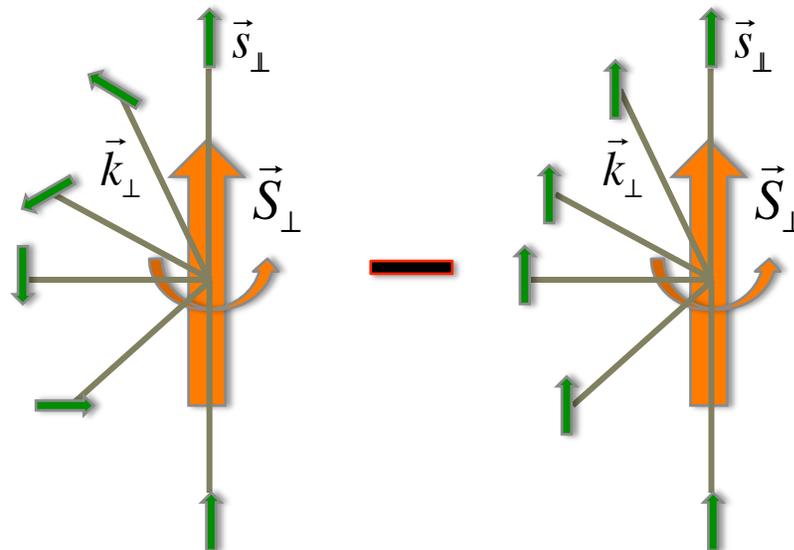
		quark		
		U	L	T
n u c l e o n	U	f1		h_T^+ -
	L		g1 -	h_{TL}^+ -
	T	f_{TT}^+ -	g_{TT}^+ -	h_{TT}^+ -

“Pretzelosity”

$$(\vec{k}_\perp \cdot \vec{S}_\perp) \cdot (\vec{k}_\perp \cdot \vec{s}_\perp) - \frac{1}{2} (\vec{S}_\perp \cdot \vec{s}_\perp) k_\perp^2$$



Connected to other TMDs



OBSERVABLES

Electron Scattering - what can we measure?

What is the likelihood to find the electron scattered into the detector?

$$P \sim n_T \cdot L = \frac{N_T}{AL} \cdot L = \frac{N_T}{A}$$

\Rightarrow call $\Delta\sigma = P / \left(\frac{N_T}{A}\right)$ (cross section)

$\Delta\sigma$ DEPENDS on the kinematics (E, E', θ_e) and is \approx proportional to SIZE of kinematic bin spanned by the detector

* Note: $\frac{N_T}{A} = \rho \left[\frac{g}{cm^3}\right] \cdot L [cm] \cdot \frac{\text{Avogadro}}{\text{Atomic Weight [u]}}$

Count rate ($L =$ luminosity):

$$\dot{N} = P \cdot \dot{n}_{el} = \Delta\sigma \cdot \frac{N_T}{A} \dot{n}_{el} = \Delta\sigma \cdot \frac{N_T}{A} \frac{I}{e} = \Delta\sigma \cdot L$$

In general:

$$\Delta\sigma = \Delta\sigma(E' \dots E' + \Delta E', \theta_e \dots \theta_e + \Delta\theta_e, \varphi \dots \varphi + \Delta\varphi)$$

Limit of infinitesimal acceptance:

$$\Delta\sigma = \frac{d^3\sigma}{dE' d\theta_e d\varphi}(E', \theta_e, \varphi) \Delta E' \Delta\theta_e \Delta\varphi = \frac{d\sigma}{dE' d\Omega} \Delta E' \Delta\Omega$$

(use Jacobian to transform variables)

In case of more particles/observables and finite phase space:

$$\Delta\sigma = \iiint_{\text{Phase Space}} \frac{d^n\sigma}{dk_1 dk_2 \dots dk_n}(k_1 \dots k_n) \text{Acc}(k_1 \dots k_n) dk_1 dk_2 \dots dk_n$$

where Acceptance is defined by detector boundaries and bin sizes as well as number of observed particles

(inclusive = only 1, semi-inclusive, exclusive)

KINEMATIC VARIABLES

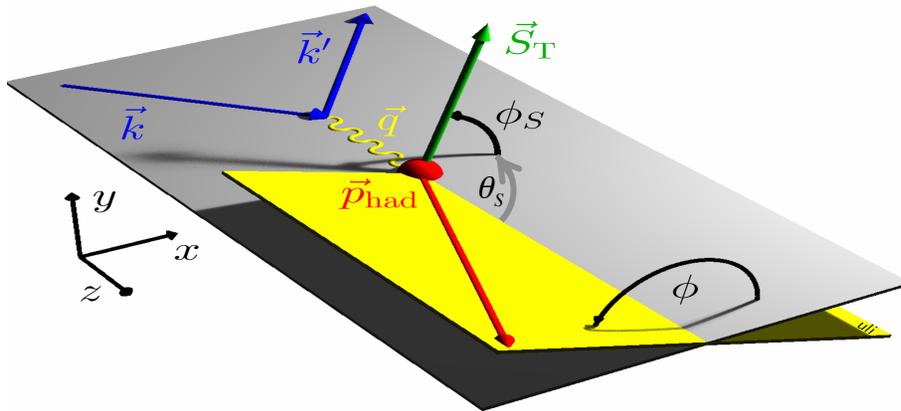
$$v = E - E' = |\vec{k}| - |\vec{k}'|$$

$$\vec{q} = \vec{k} - \vec{k}'; \quad q^\mu = (v, \vec{q}) = k^\mu - k'^\mu$$

$$Q^2 = -(k - k')^2 = \vec{q}^2 - v^2 \approx 4EE' \sin^2 \frac{\theta_e}{2}$$

$$y = \frac{q^\mu P_\mu}{k^\mu P_\mu} = \frac{v}{E}; \quad x = \frac{Q^2}{2q^\mu P_\mu} = \frac{Q^2}{2Mv}$$

$$W = \sqrt{(P_\mu + q^\mu)^2} = \sqrt{M^2 + (1/x - 1)Q^2}$$



Inclusive

Semi-Inclusive

Lepton variables

Hadron variables

$$P_{had}^\mu = \left(z\nu, \sqrt{z^2\nu^2 - m_h^2 - P_{hT}^2}, \vec{P}_{hT} \right)$$

$$z = E_h / \nu$$

$$\vec{P}_{hT} = |\vec{P}_{hT}| (\sin \theta_h \cos \phi_h, \sin \theta_h \sin \phi_h)$$

$$x_F = \frac{\vec{P}_h^{c.m.} \cdot \hat{q}}{|\vec{P}_h^{c.m.}|(\max)}$$

$$\text{Rapidity} \quad \text{artanh} \frac{|\mathbf{p}|c}{E} = \frac{1}{2} \ln \frac{E + |\mathbf{p}|c}{E - |\mathbf{p}|c}$$

$$\text{Pseudo-Rapidity} \quad \eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right]$$

A NOTE ABOUT SPIN OBSERVABLES

1. “Little known” fact: A single spin-1/2 particle is always “100% polarized” along **some** direction \rightarrow can be represented by \hat{P}
2. An ensemble of spin-1/2 particles (target, beam) can be fully described by the average of all \hat{P} ’s $\Rightarrow \bar{P}$ ($0 \leq \text{length} \leq 1$)
(related to density matrix ρ ; any ensemble with same \bar{P} is indistinguishable)
3. Any observable can only depend on $\bar{A} \cdot \bar{P}$ (\bar{A} = analyzing power)
4. If ensemble has a quantization axis (e.g. B-field in target), \bar{P} will be along that axis (e.g. z-direction), and we can write $P_z = (N^\uparrow - N^\downarrow) / (N^\uparrow + N^\downarrow)$
5. Spin-1 ensembles (like polarized deuteron targets) have both a vector polarization (P_z) and a tensor polarization (P_{zz}) along quantization axis. In general, one has a vector and a symmetric tensor to describe all components; they rotate following usual rules.

Example: $P_{zz} = (N^\uparrow + N^\downarrow - 2N^0) / (N^\uparrow + N^0 + N^\downarrow)$. General form of cross section:

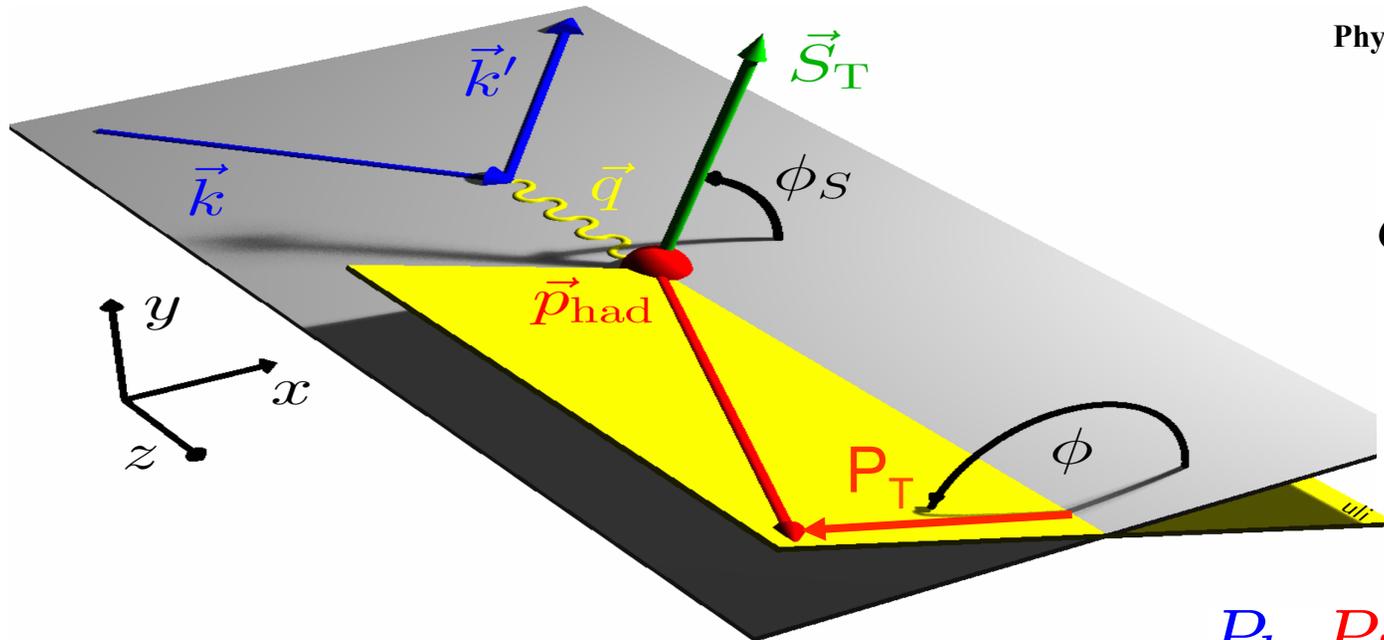
$$\sigma = \sigma_0 \left[1 + \frac{3}{2} P_y A_y + \frac{1}{2} P_{zz} A_{zz} + \frac{2}{3} P_{xz} A_{xz} + \frac{1}{6} (P_{xx} - P_{yy}) (A_{xx} - A_{yy}) \right] \quad (\text{x in the scattering plane})$$

6. This can be generalized to double-spin asymmetries, e.g.

$$\sigma = \sigma_0 \left[1 + \frac{3}{2} P_z (A_d^z + h A_{ed}^z) + \frac{1}{2} P_{zz} (A_d^{zz} + h A_{ed}^{zz}) \right]$$

7. Rotational and parity symmetries dictate spatial dependence of A’s: A_{UL} and A_{LU} must be odd functions of ϕ , A_{LL} must be even in ϕ etc.

ON TO SIDIS



Trento Conventions
 Phys.Rev. D70, 117504 (2004).

$$\nu = E - E'$$

$$Q^2 = (k - k')^2$$

$$y = \nu/E$$

$$x = Q^2/2M\nu$$

$$z = E_h/\nu$$

P_b, P_t

- U** unpolarized
- L** long.polarized
- T** trans.polarized

Target polarization

Beam polarization

$$\sigma = \sigma_{UU} + P_t \sigma_{UT} \sin(\phi - \phi_S) + P_b P_t \sigma_{LT} \cos(\phi - \phi_S) \dots$$

$$A_{UT}^{\sin(\phi - \phi_S)} = \frac{\sigma_{UT}}{\sigma_{UU}} \longrightarrow$$

$\sin(\phi - \phi_S)$ moment of the cross section for unpolarized beam and transverse target

SIDIS CROSS SECTION

Bacchetta, et al., JHEP 2(2007)093

$$\begin{aligned}
 \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \\
 & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
 & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\
 & + S_{\parallel} \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
 & \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
 & + |S_{\perp}| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\},
 \end{aligned}$$



Leading Twist



Sub-Leading Twist
(extra factor of 1/Q)



0 (i.e. $R = \sigma_L / \sigma_T = 0$)

$A_{UL} = \{\text{UL terms}\} / \{\text{UU terms}\}$

$A_{LL} = \{\text{LL terms}\} / \{\text{UU terms}\}$

etc.

Each F now depends also on transverse hadron momentum $P_{h\perp}$

STRUCTURE FUNCTIONS vs TMDs

$$F_{UU,T} = C[f_1 D_1] \quad F_{UU,L} = 0 \quad F_{UU}^{\cos 2\phi_h} = C \left[-\frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_1^\perp H_1^\perp \right]$$

$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} C \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x h_1^\perp H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x f_1^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right]$$

$$F_{LU}^{\sin \phi_h} = \frac{2M}{Q} C \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x e_1^\perp H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x g_1^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

$$F_{UL}^{\sin 2\phi_h} = C \left[-\frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_{1L}^\perp H_1^\perp \right]$$

$$F_{UL}^{\sin \phi_h} = \frac{2M}{Q} C \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x h_L^\perp H_1^\perp + \frac{M_h}{M} g_{1L} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x f_L^\perp D_1 - \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{H}}{z} \right) \right]$$

$$F_{LL} = C[g_{1L} D_1]$$

$$F_{LL}^{\cos \phi_h} = \frac{2M}{Q} C \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x e_L^\perp H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right]$$

+ ...

WHICH OBSERVABLES ARE POSSIBLE?

- Use similar symmetry consideration as for the TMDs
- Have now 3 vectors:

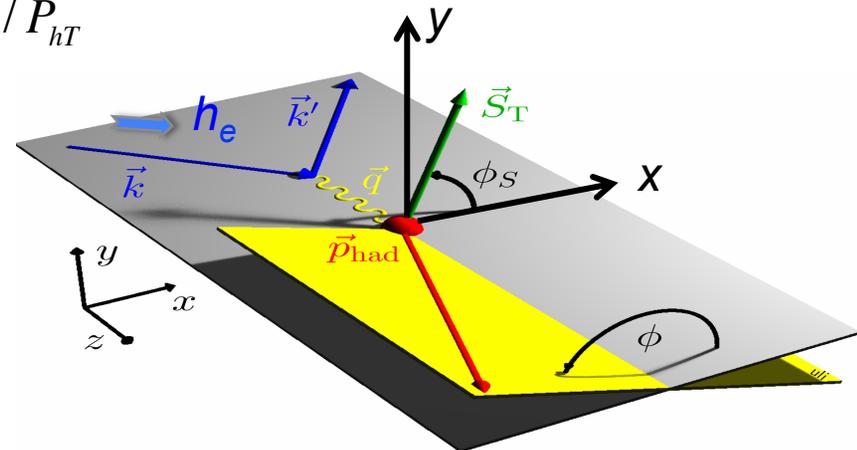
$$\hat{z} = \vec{q} / q; \hat{x} \propto (\vec{k} + \vec{k}') - \hat{z} \cdot (\vec{k} + \vec{k}') \hat{z}; \vec{P}_{hT} / P_{hT}$$

2 axial vectors:

$$\hat{y} = \vec{k} \times \vec{k}'; \vec{S}_{\perp}$$

2 pseudoscalars:

$$h_e \rightarrow h_{\gamma}; H = \vec{S} \cdot \hat{z}$$



- Once again, observables must be scalars...

MORE POSSIBLE COMBINATIONS

- Can also combine pseudoscalars with h, H :

- $h_\gamma H$;
 $h_\gamma H \cdot \cos\phi$
 $\rightarrow A_{LL}$
- $h_\gamma \cdot \sin\phi$
 $\rightarrow A_{LU}$
- $H \cdot \sin\phi$;
 $H \cdot \sin 2\phi$
 $\rightarrow A_{UL}$

...and more cosines and sines

$$\frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right.$$

Bacchetta, et al., JHEP 2(2007)093

$$+ \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+ S_{\parallel} \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right]$$

$$+ S_{\parallel} \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right]$$

$$+ |S_{\perp}| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right.$$

$$+ \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)}$$

$$\left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right]$$

$$+ |S_{\perp}| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right.$$

$$\left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right\},$$

Legend:

- Leading Twist
- Sub-Leading Twist (extra factor of 1/Q)
- 0 (i.e. R=σ_L/σ_T=0)

$A_{UL} = \{UL \text{ terms}\} / \{UU \text{ terms}\}$
 $A_{LL} = \{LL \text{ terms}\} / \{UU \text{ terms}\}$
 etc.

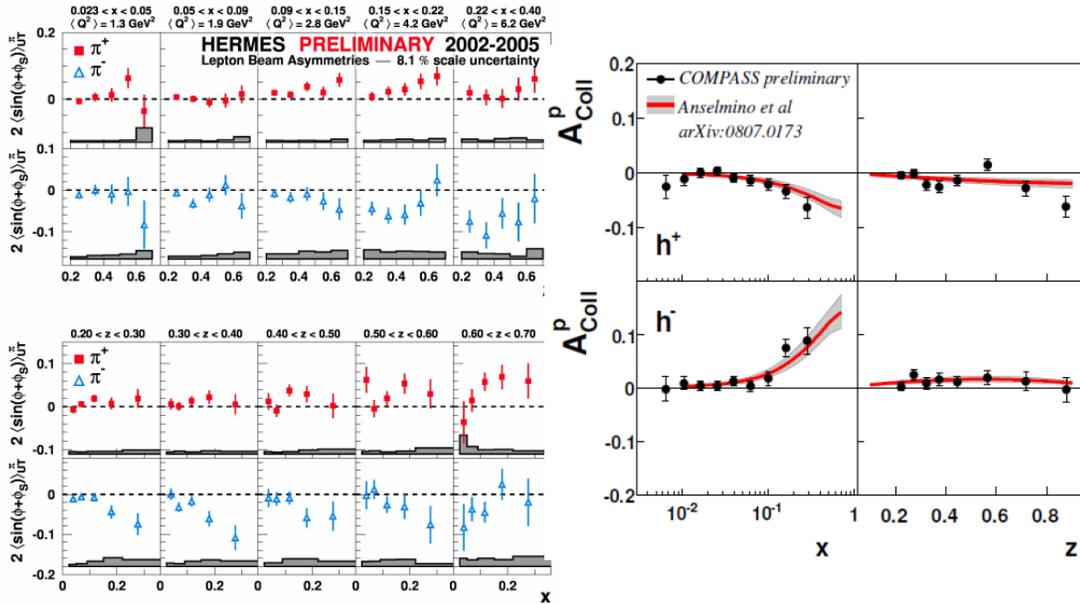
Each F now depends also on transverse hadron momentum $P_{h\perp}$

BUT WHICH OBSERVABLE MEASURES WHAT TMD?

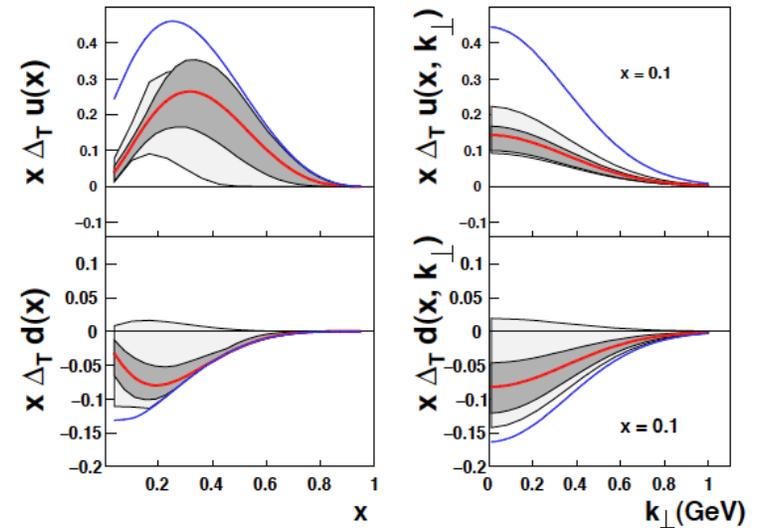
This is where I have to leave it to the experts... (time's up ;-)

- **The measured final state hadron momentum is a complicated convolution of initial k_T , hard scattering, final state interaction between struck quark and nucleon remnant, and hadronization – cannot study independently (except in e^+e^-)**
- **Need models not only for TMDs but also for FFs**
- **Things get even more messy: pQCD evolution (actively worked on), higher twist (lots more TMD PDFs!), current vs. target fragmentation, “gauge link”...**
- **Experiments are also challenging!**
- **... so it ain't as easy as I (hopefully) made it seem**

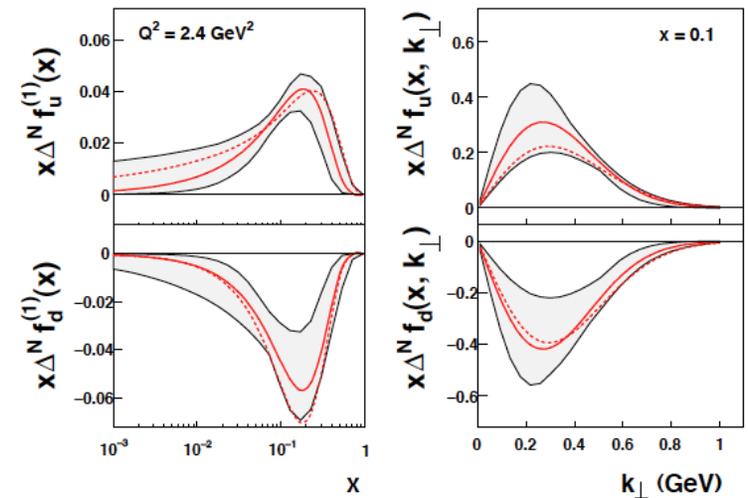
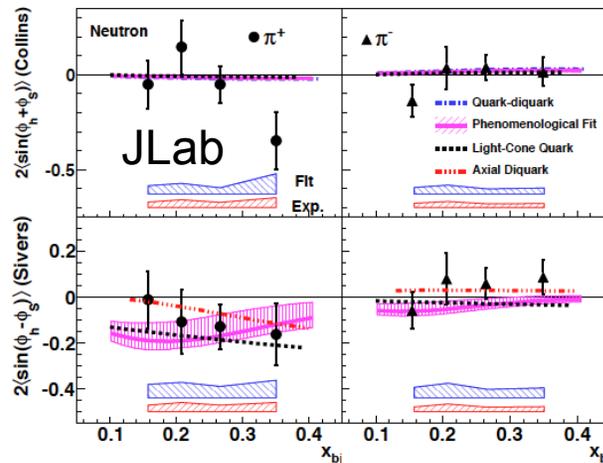
THE PRESENT SITUATION - TMDs



Fit by Anselmino et al.



A full program to measure 8 TMDs has just begun...



Sivers Functions

WHAT'S MISSING?

- **Tensor charge of the nucleon (analog to vector and axial charges)**
- **Full mapping of all 11 TMD PDFs as a function of quark flavor in the valence region**
- **Test of universality**
- **Test of prediction that time-odd TMDs (e.g., Sivers asymmetry) change sign in Drell-Yan processes**
- **TMDs of sea quarks and gluons**
- **Towards a “wave function” of the nucleon including angular orbital momentum (Wigner distribution? Lattice comparison?)**

WHAT DO WE NEED?

(Polarized) Beam

Electrons or muons $\rightarrow \gamma^*$; pions, protons, antiprotons?...

(Polarized) Targets (or counterrotating beams)

Protons, deuterons, ^3He [for n], antiprotons?...

Detectors

For scattered/produced electrons/muons and hadronic debris

- Need to distinguish $e^-/\mu^-/\pi^-/K^-...$ and $e^+/\mu^+/\pi^+/K^+/p...$, measure γ/n , reconstruct $\pi^0, \eta, ...$
- Cover relatively large acceptance for both scattered particle and produced hadron/photon; usually require moderate kinematic resolution

Facilities

SLAC



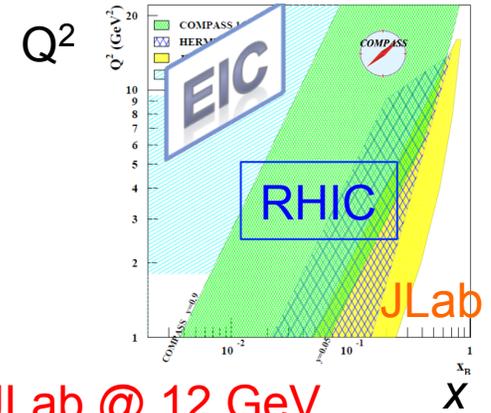
RHIC

Jefferson Lab



EIC

SUMMARY: COMPLETING THE PICTURE



EIC

- $\Delta u/u$ and $\Delta d/d$ at high x still poorly constrained JLab @ 12 GeV
- What is happening with the strange sea polarization? $>0?$ $=0?$ $<0?$ Zero crossing? (Tension DIS – SIDIS) COMPASS, JLab
- Is the sea polarization isospin-symmetric? JLab high x ; RHIC W, COMPASS
- Gluon helicity distribution at large x and a small x ? What is the integral ΔG ? JLab + COMPASS (NLO), RHIC, COMPASS
- What happens at really small $x \ll 0.01$?
- Tensor charge of the nucleon ALL
- Full mapping of all TMD PDFs as a function of quark flavor in the valence region JLab @ 12 GeV
- Test of universality RHIC
- Test of prediction that time-odd TMDs (e.g., Sivers asymmetry) change sign in Drell-Yan processes RHIC, COMPASS, FAIR, JPARC...
- TMDs of sea quarks and gluons

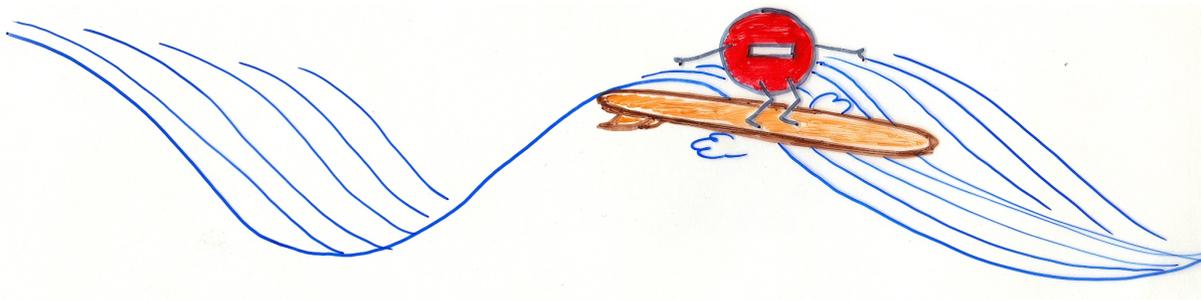
EIC

ADVERTISEMENT



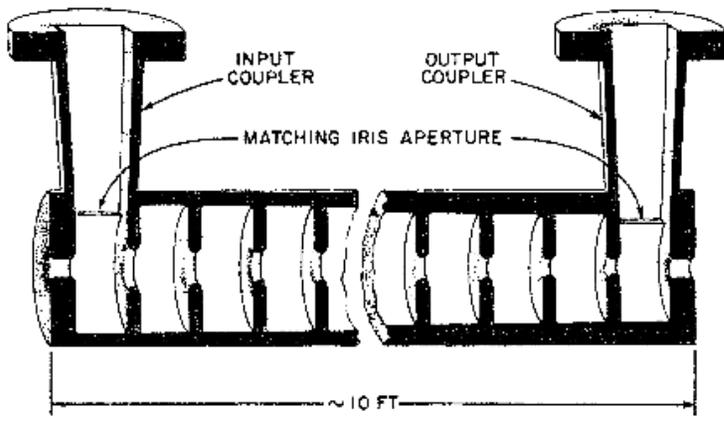


ACCELERATORS

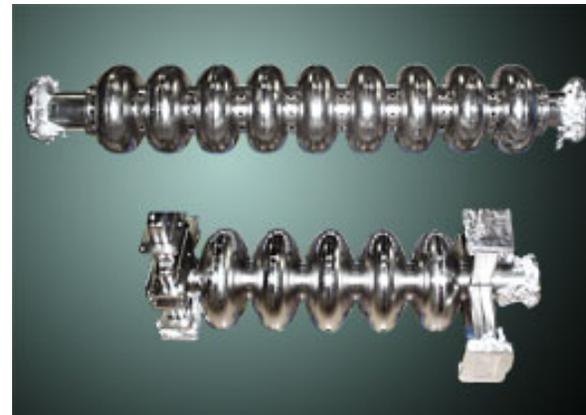


Surf the
microwaves!

Accelerating cavity: disk loaded cylindrical wave guide
use TM_{01} mode to get a longitudinal electric field
match phase and velocity



DESIGN



new

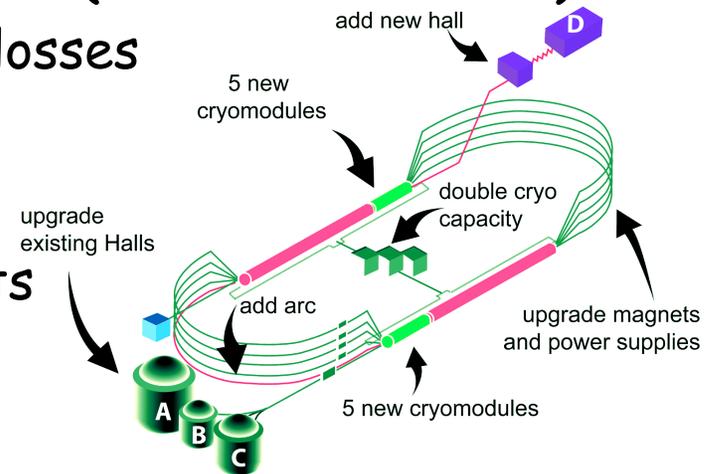
old

Jefferson Lab

ACCELERATORS – 2 EXAMPLES

- Superconducting Linear Accelerators (CEBAF at JLab)

- 2K niobium cavities, very low resistive losses
- Recirculate few times, 100's of μA
- High gradient (5-50 MeV/m \Rightarrow 4-12 GeV)
- CW extracted beam on external targets
- Thick targets \Rightarrow high luminosity

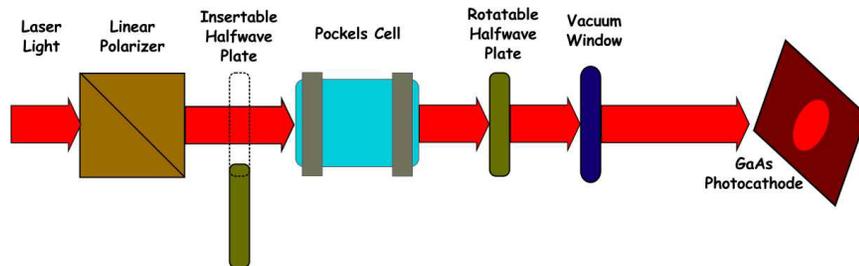


- Storage rings (HERA at DESY, RHIC at BNL)

- Large circulating currents (mA)
- Recirculate millions of times
- Require only modest (re)acceleration
- CW internal beam on thin gas targets or counterrotating beams (typically lower Luminosity)



POLARIZED ELECTRON BEAMS



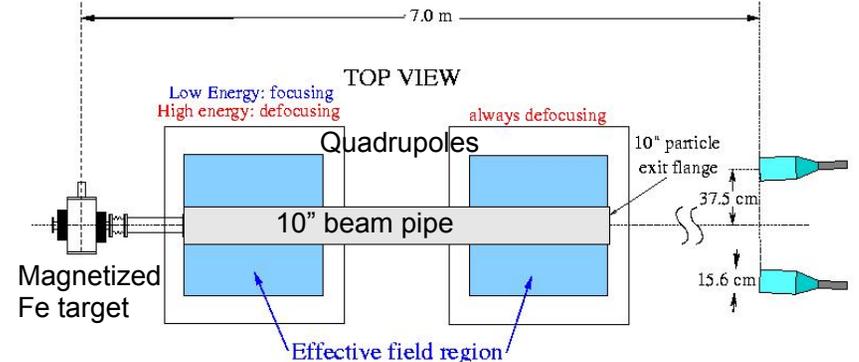
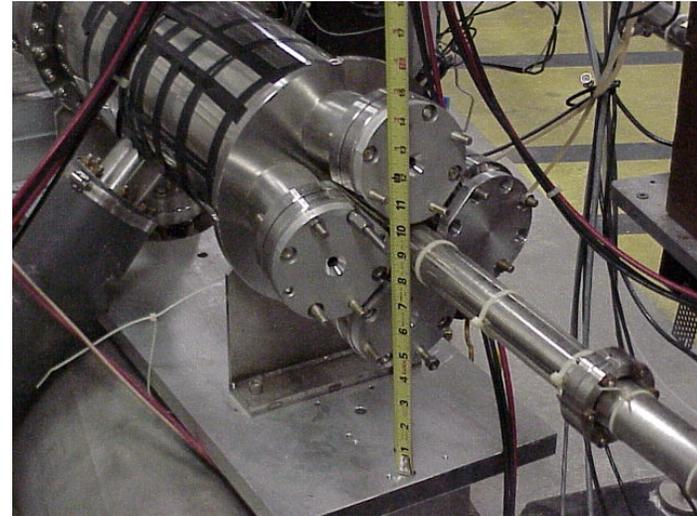
JLab polarized electron source

List of Ingredients:

- Circularly polarized laser light
 - Ti-Sapphire or Diode lasers
 - Polarizers, Pockel cell, $\lambda/2$ plate
- Photocathode (Semiconductor covered by Alkalide, e.g. Cs)
 - Historically: Bulk GaAs (<50% Pol.)
 - Strained GaAs (breaks degeneracy of 2 energy levels)
 - State of the art: “Superlattice” - Thin GaAs on top of GaAsP (> 85% Pol.)
- Beam transport
 - Wien filter - crossed E and B fields to rotate spin direction arbitrarily
 - Injection into main accelerator
- Acceleration (account for precession)
- Beam diagnostics (emittance, phase lock, polarization...)

POLARIMETRY

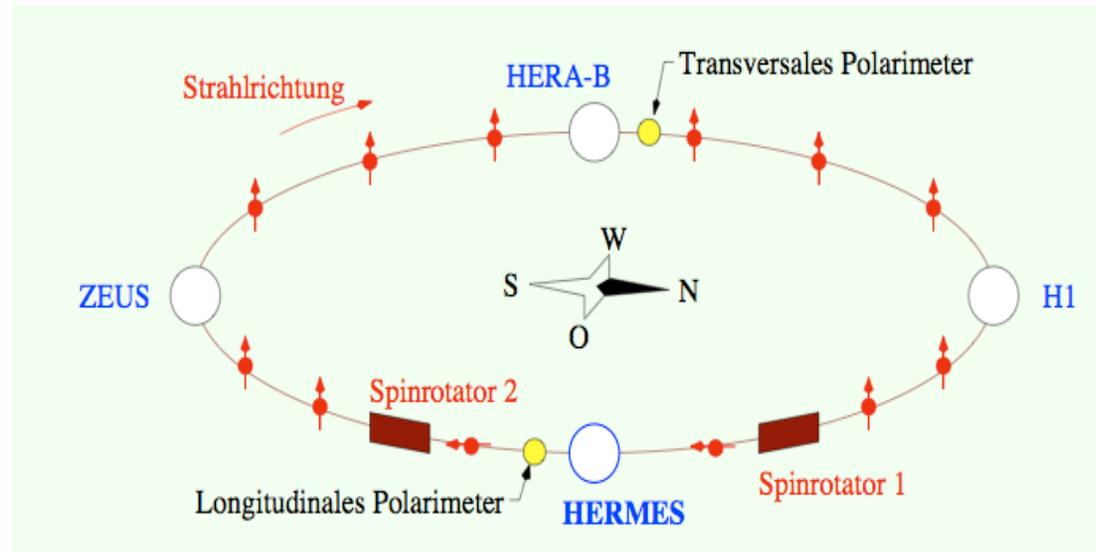
- Mott polarimeter (e-Nucleus scattering) at low energies (injector)
 - Spin-orbit coupling in electromagnetic interaction (electron magnetic moment “sees” current due to moving nuclear charge in its rest frame)
- Møller polarimeter (e-e scattering) in experimental Hall; use magnetized iron foils (spin-polarized e’s)
 - Spin-spin coupling in electromagnetic interaction
 - Asymmetry can be calculated as function of electron energy and scattering angle from QED
- Compton polarimeter - polarized laser light interacting with electrons
 - Similar to spin structure functions – electron can only absorb photon with opposite spin direction



Don Jones, 2nd prize 2011 Poster Competition JLab UG meeting

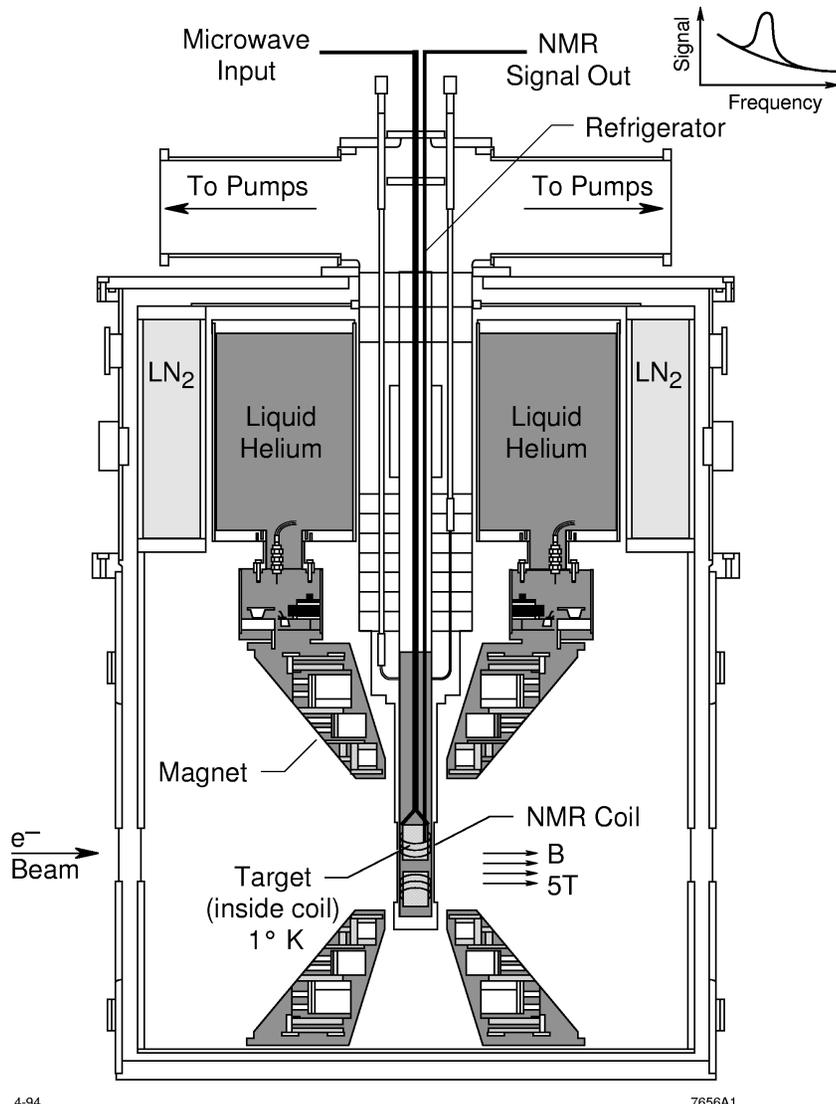
OTHER POLARIZED BEAMS

- Polarized muons: Create pion beam (proton-nucleus interaction), pions decay via weak interaction => muons are “naturally” polarized via parity violation in weak IA (select polarization kinematically). Example: CERN
- Electrons and positrons in storage rings: Spin-dependent part of synchrotron radiation emission leads over time to build-up of transverse polarization (Sokolov-Ternov effect). Spin rotators at interaction points -> longitudinal polarization. Example: HERA/HERMES



- Polarized protons: Atomic Beam sources (HFS of H, D...); avoid depolarizing resonances while accelerating!
- More exotic: Polarized anti-protons (spin filters),...

POLARIZED PROTON AND DEUTERON SOLID TARGETS



List of Ingredients:

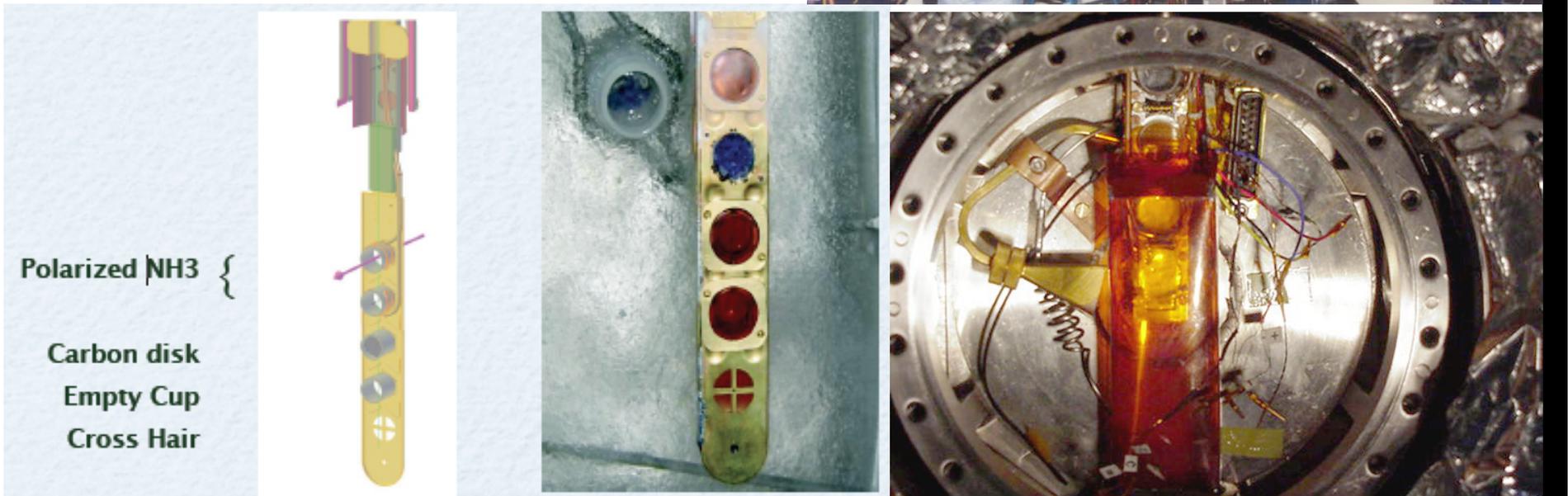
- Polarizable Material (high H content) with paramagnetic centers = unpaired e⁻ (irradiated or chemically doped)
 - Alcohols (e.g. butanol)
 - Ammonia - ¹⁵NH₃ and ¹⁵ND₃
 - HD ice
- (Very) Low Temperature
 - About 1 K for (continuous) dynamic polarization - pumped-on liquid ⁴He bath at low pressure
 - 100 mK or less for frozen spin mode - ³He/⁴He dilution refrigerator
- Dynamic Polarization in high B-field
 - About 2.5 - 5 T -> unpaired e⁻ 100% polarized
 - Polarization transferred to nuclei via HF transitions (simultaneous electron and nuclear spin flip); requires 70-140 GHz microwaves (few 100 mW)
- Holding Field (for frozen spin mode; can be lower)
- NMR system to monitor polarization
- Insulation vacuum, beam and scattered particles ports

Considerations:

- Possibly significant “dilution” by “inactive” nuclei
- Beam must be rastered to avoid local depolarization
- Target must be annealed repeatedly to alleviate radiation damage (electron beam)
- Total dose and current limitations (e.g., 100 nA max)
- Transverse DNP targets -> large deflection of electrons

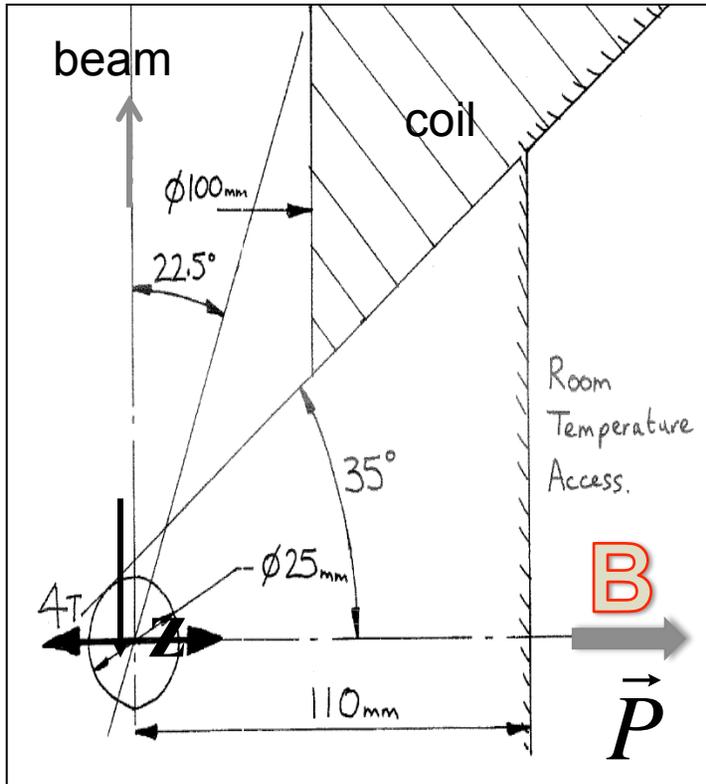
POLARIZED PROTON AND DEUTERON SOLID TARGETS

Example:
CLAS polarized NH_3/ND_3 target.
Typical polarization 0.7-0.9 (p) and
0.3-0.4 (d)



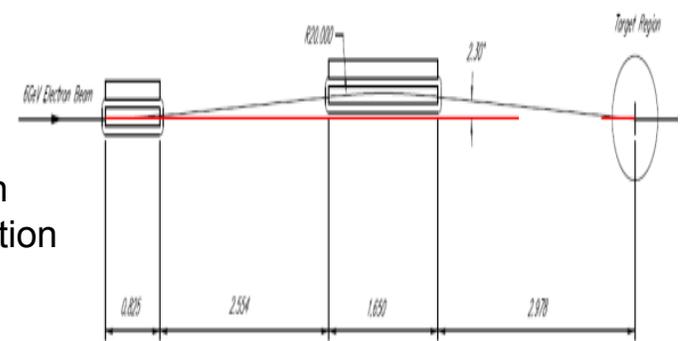
TRANSVERSE POLARIZATION?

Top View



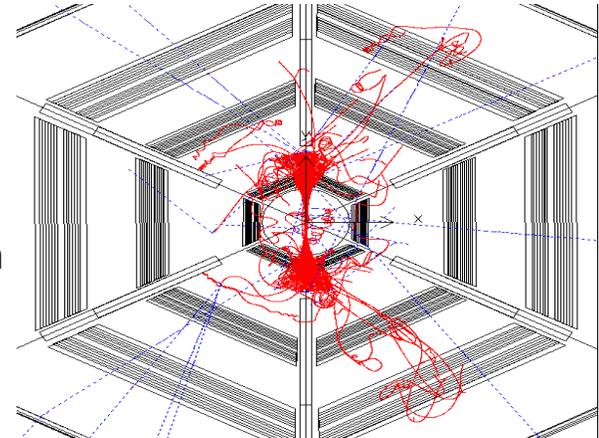
Acceptance:
 $\Theta_{\text{Trans}} < 22.5$ degree (forward cone)
 $55 < \Theta_{\text{Trans}} < 125$ (transverse)
 $\Theta_{\text{BendPlane}} \sim$ full coverage (except for support structure)

Chicane system

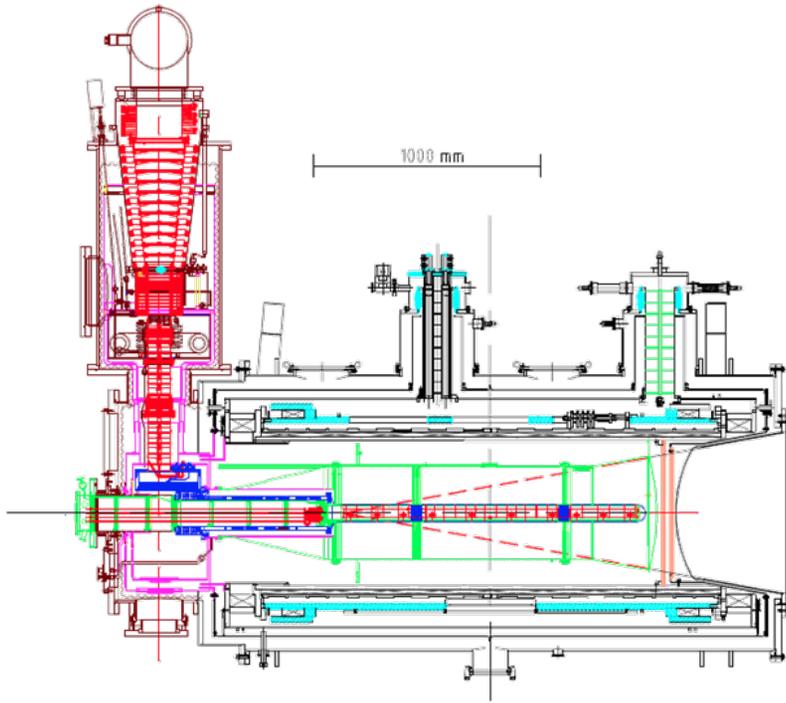


Goal:
 Let beam exit in horizontal direction

Issue:
 Strong electromagnetic background from synchrotron radiation



OTHER EXAMPLES FOR SOLID TARGETS



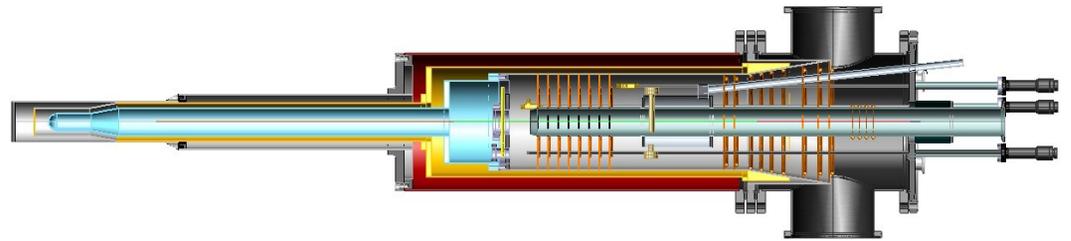
SMC/COMPASS target –
Largest in the world

(3 cells with opposite polarization)

- $^3\text{He} - ^4\text{He}$ dilution refrigerator ($T \sim 50\text{mK}$)
- ^6LiD or NH_3 50/90% polarization
- 2.5 T solenoid

Future longitudinally polarized target for CLAS12 (11 GeV program at Jefferson Lab)

- Horizontal ^4He evaporation cryostat
- 5 T B-field provided by central detector



OTHER EXAMPLES FOR SOLID TARGETS – HD ICE

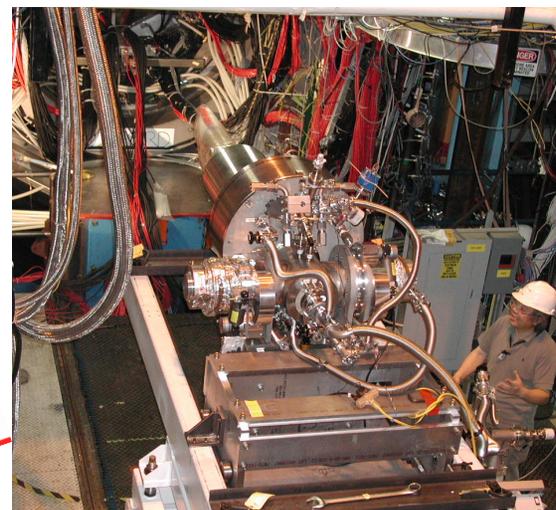
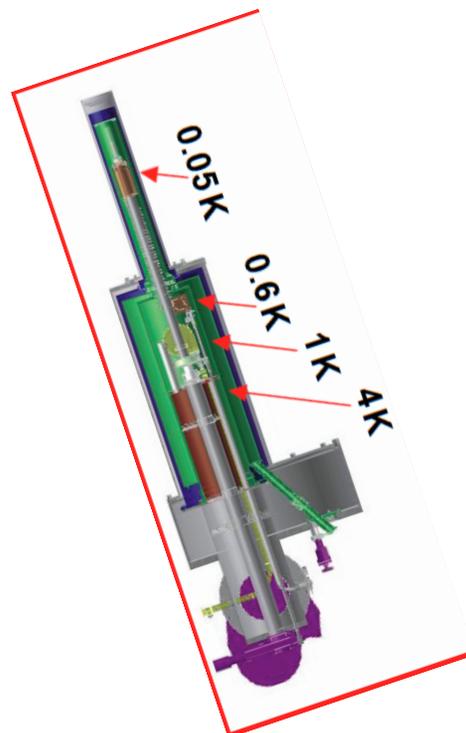
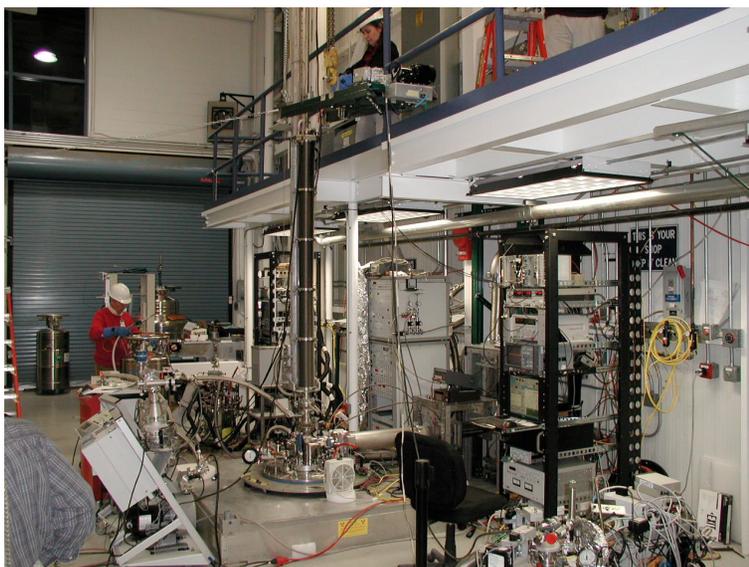
Frozen solid of HD molecules

Polarize (brute force) at 0.010 K and 15 T

Store + transfer at 2K, run at 0.05K

Advantage: little background material, can measure H and D simultaneously

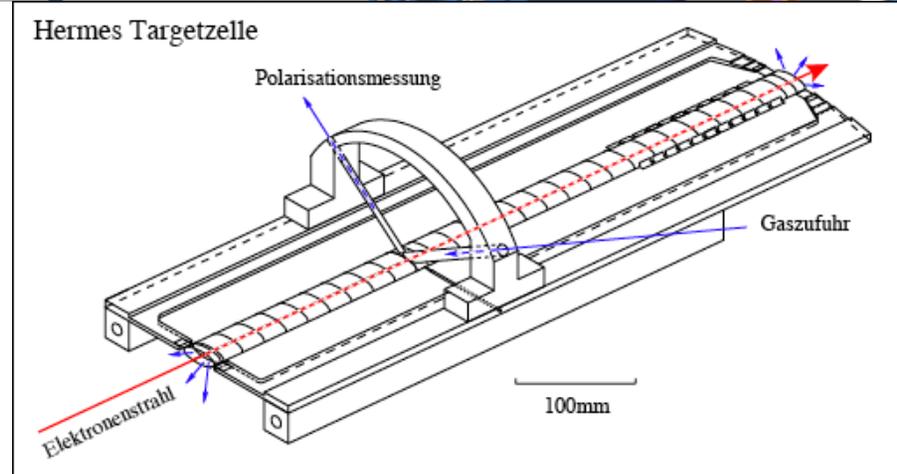
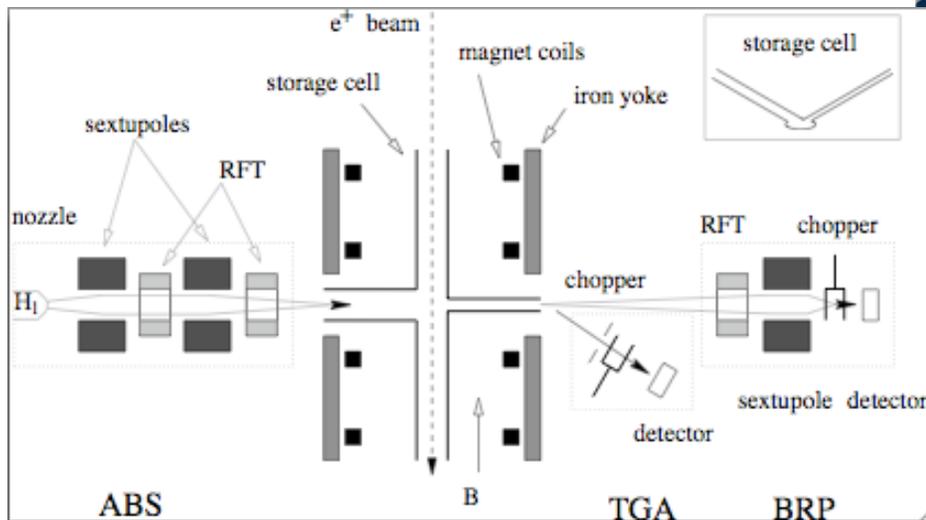
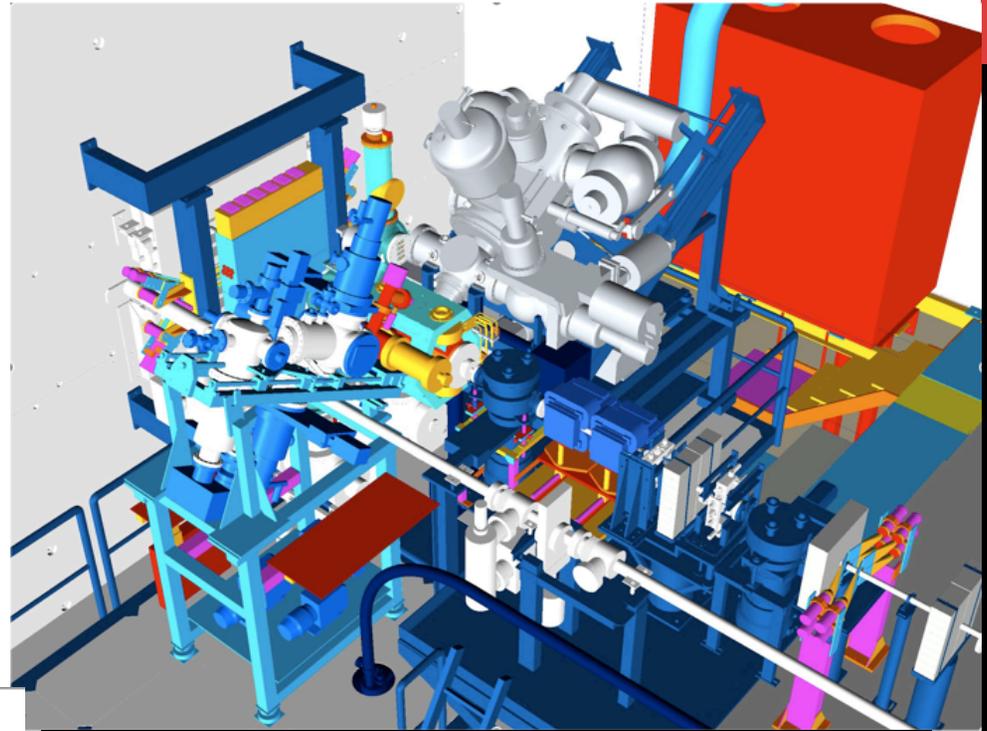
Disadvantage: Don't know yet whether it will work with electron beam...



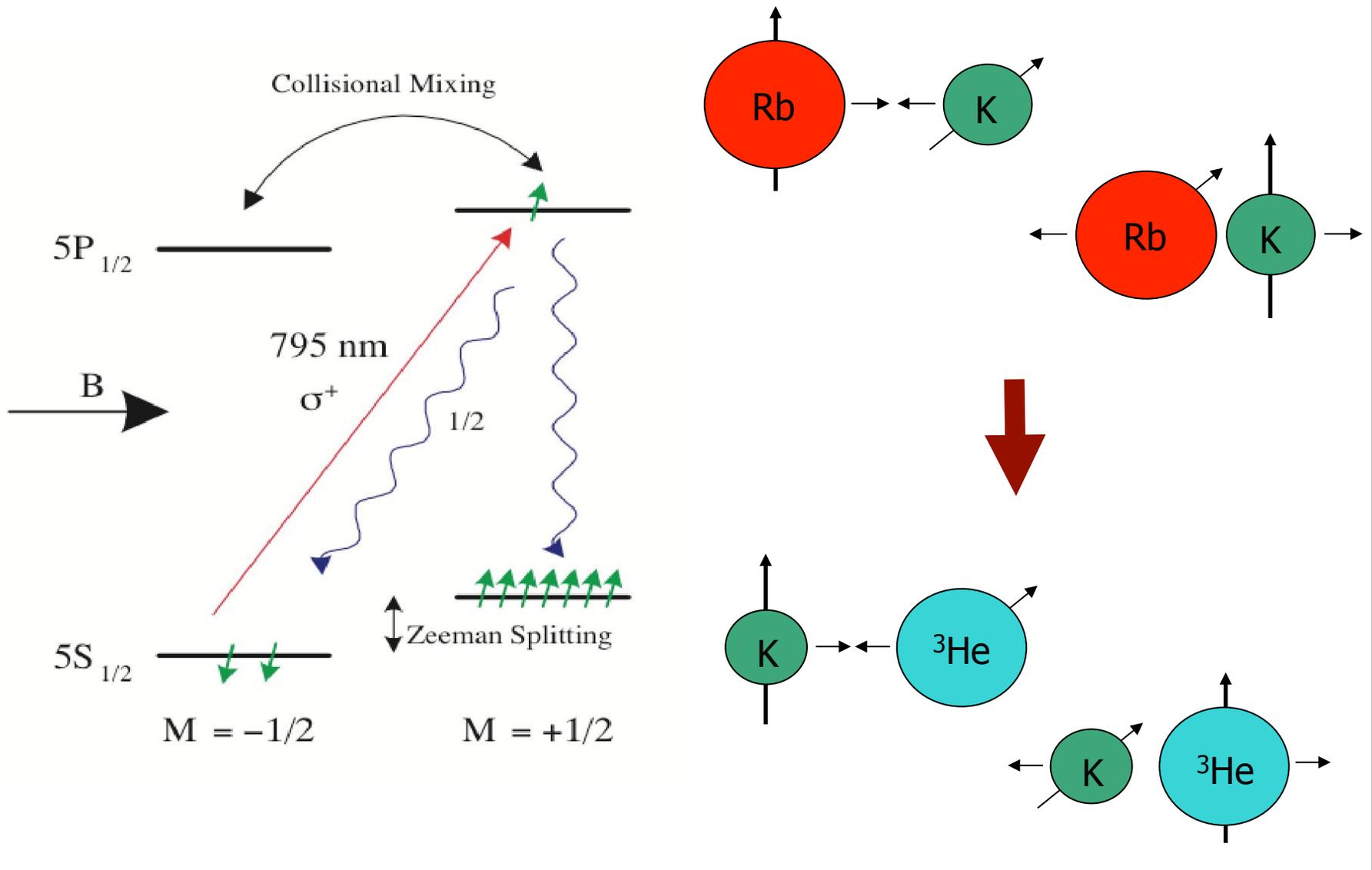
POLARIZED PROTON AND DEUTERON GAS JET TARGETS

Example: HERMES

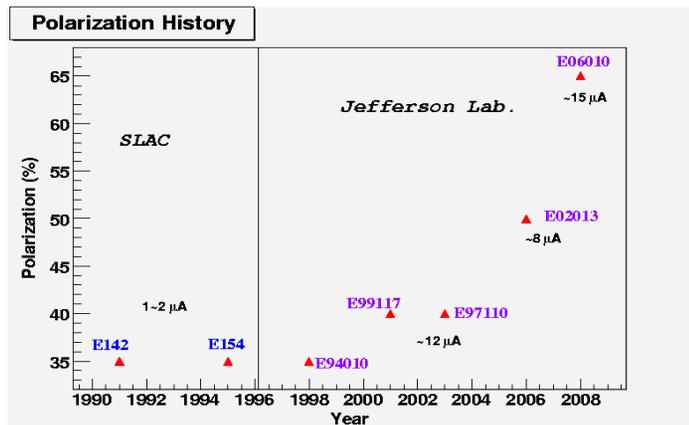
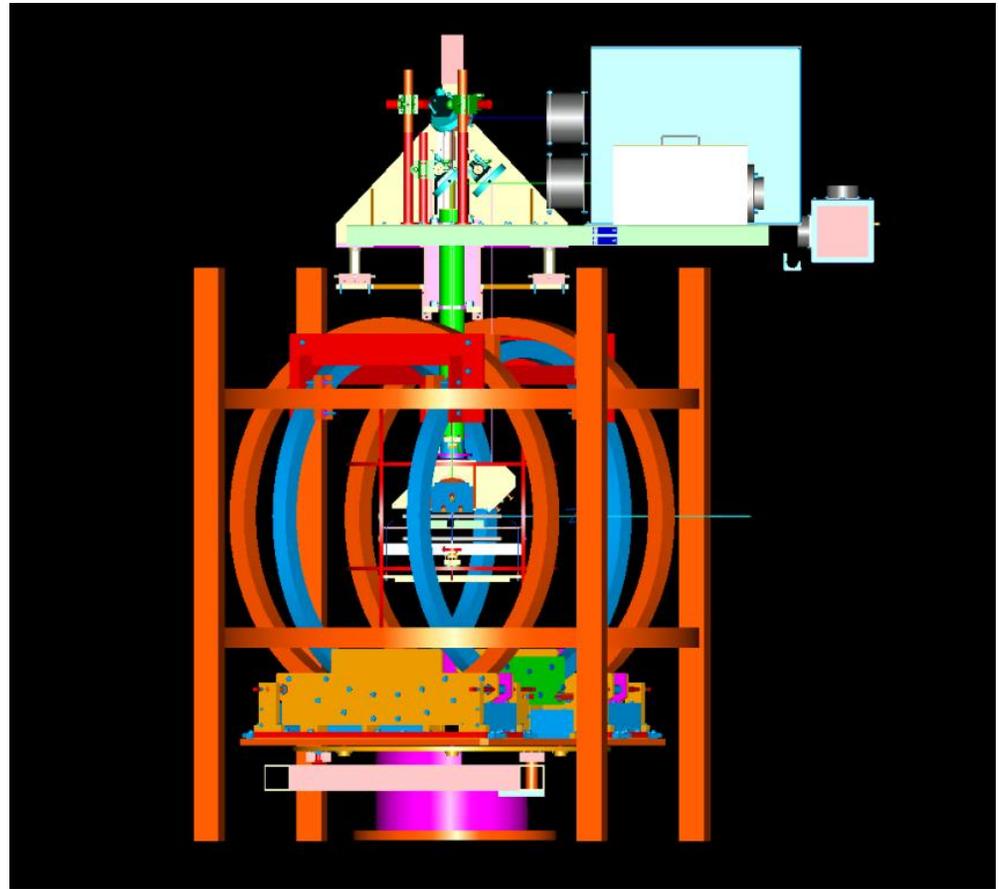
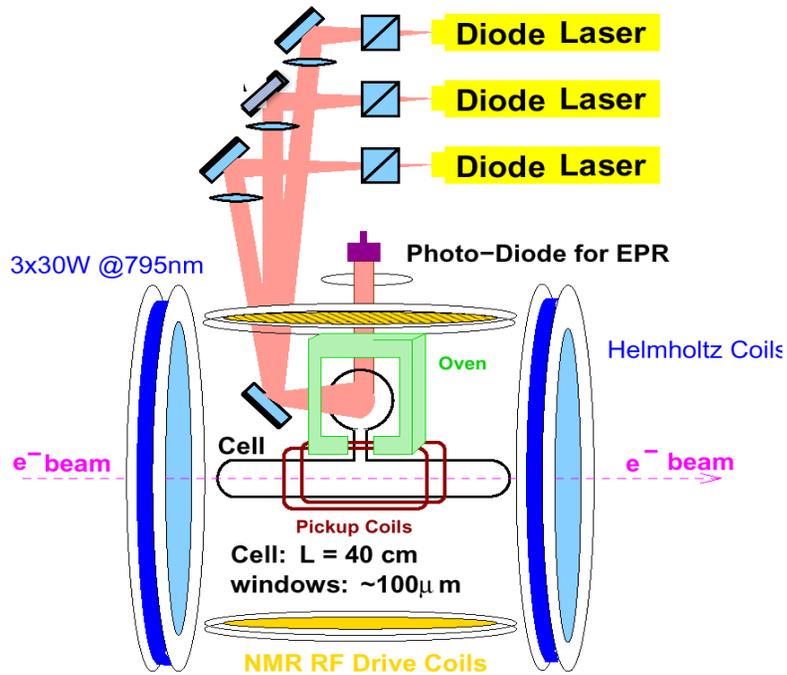
- Atomic Beam Source (Stern-Gerlach separator, HF transitions using RF circuits)
- Open storage cell to get densities of up to a few times 10^{14} nucleons/cm²
- Differential pumping system
- Circulating beam



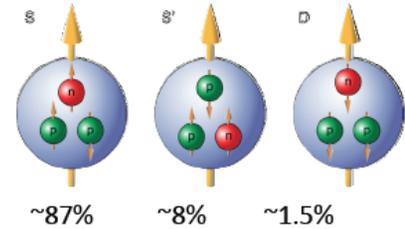
POLARIZED ^3He TARGETS (APPROX. POL. NEUTRON)



POLARIZED ^3He TARGETS (APPROX. POL. NEUTRON)



Polarized ^3He Gas Target



To extract information on neutron,
one would assume :

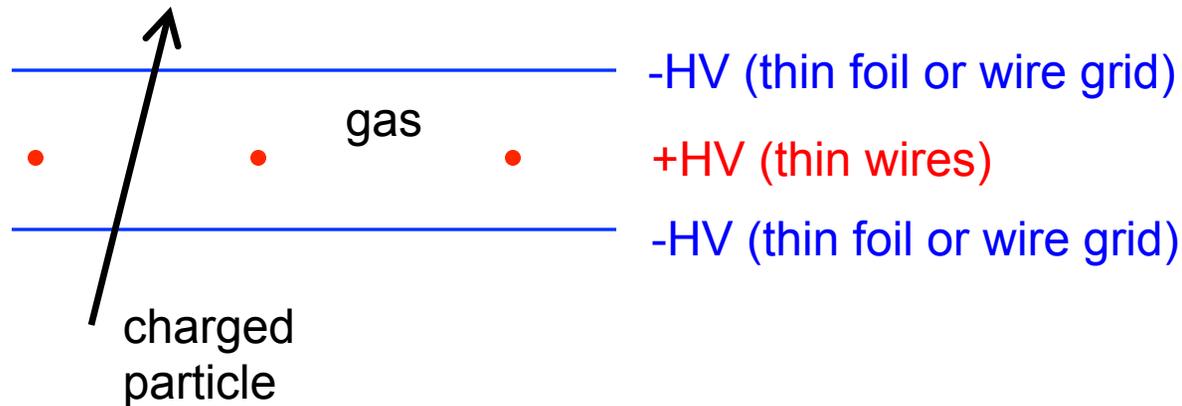
$$^3\text{He}^\uparrow = 0.865 \cdot n^\uparrow - 2 \times 0.028 \cdot p^\uparrow$$



- 10 atm ^3He , Rb/K mix. 15 μA electron beam: $L(n) = 10^{36}$ cm 2 /s
- 3D holding field, fast spin flip (20 min). Polarization $\sim 60\%$.
- 4 target spin configurations: up, down, left, right.

TYPICAL DETECTOR ELEMENTS

Wire chambers measure position (and angle)



1. Charged particle passes through wire chamber and knocks out electrons from the gas.
2. Electrons drift in the E field to the cathode wire, colliding with gas molecules
3. Close to the wire, the mean free path times the electric field is large enough to ionize the gas molecules. **Avalanche!**
4. Read the signal on the cathode wire (time gives distance)

Similar: Gas Electron Multipliers, μ MEGAs, ...

Applications: VDC, Multi-layer drift chamber (track $\rightarrow \vec{p}$), Time Projection Chamber

TYPICAL DETECTOR ELEMENTS

Scintillators: time ($\Rightarrow \beta \Rightarrow$ particle type) and energy measurement

(typical resolution: down to 50-100 ps for plastic)

- Typically a doped plastic or crystal (eg: Ge, NaI, BaF₂)
- Charged particle passes through scintillator (or neutral particle interacts) and excites atomic electrons. These de-excite and emit light.
- Minimum energy loss (when $\beta\gamma \approx 1$) is $dE/dx = 2 \text{ MeV}/(\text{g}/\text{cm}^2)$

Cherenkov counter: threshold velocity measurement.

- Typically an empty box with smoke (ie: a gas) and mirrors
- Local light speed is $v = c/n < c$
- Particles travelling faster than v will emit Cherenkov light (an electromagnetic 'sonic boom') \Rightarrow threshold CC (yes/no)
- The opening angle of the Cherenkov cone is related to the particle's velocity $\Rightarrow R_{\text{imaging}} C_{\text{Cherenkov}}$ (measure $\beta \Rightarrow$ particle type)

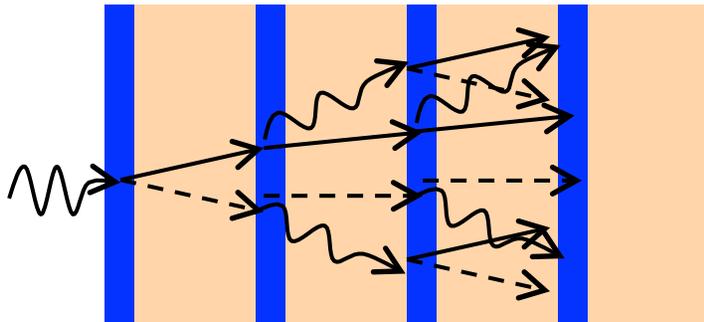
Also: $T_{\text{ransition}} R_{\text{adiation}} D_{\text{etector}}, \dots$

TYPICAL DETECTOR ELEMENTS

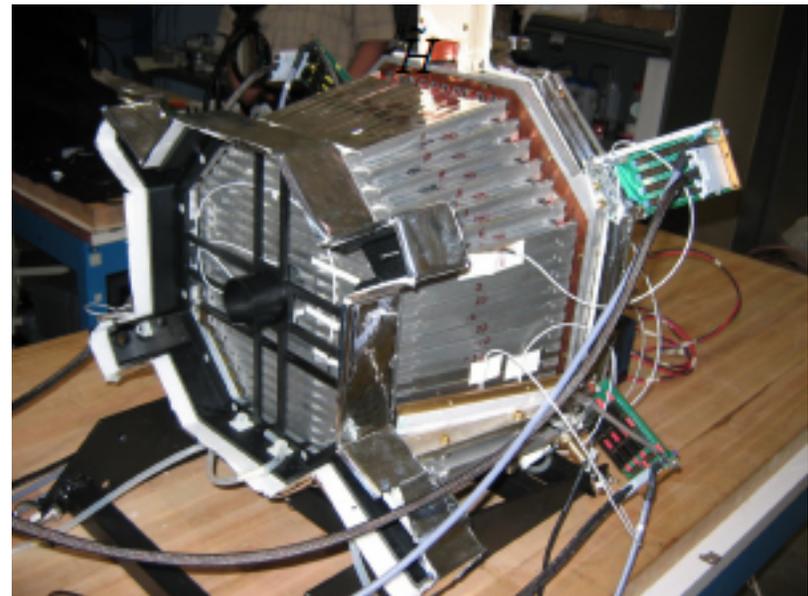
Electromagnetic shower counters:

measure energy (also time), discriminate electrons and detect neutral particles

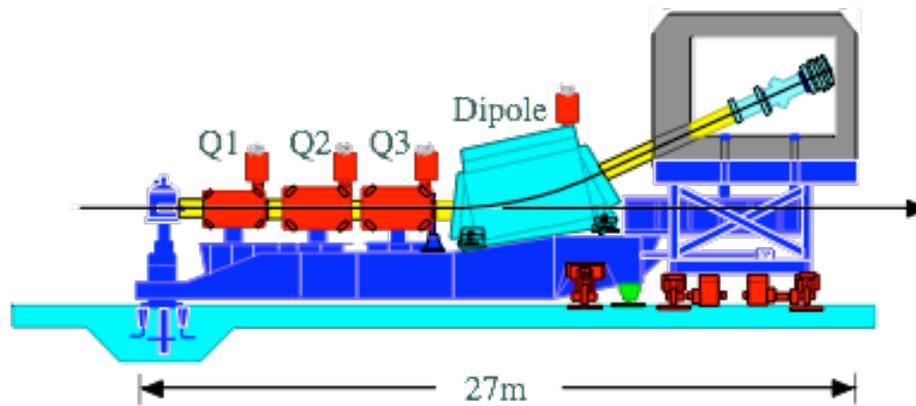
- Electrons and photons passing through material shower
- After one radiation length of material on average:
 - Electrons emit a bremsstrahlung photon
 - Photons convert to an electron/positron pair or Compton-scatter
- After ≈ 10 radiation lengths, one e^- or γ is now ~ 1000 particles
- Simple design: alternating layers of lead ($R_L = 6$ mm) and scintillant
- Higher resolution: Heavy metal glass (Pb glass, PbWO_4) combine both
 - Particles shower in the lead
 - Charged particles deposit energy in the scintillant



Also: Hadronic Calorimeter, μ counter...

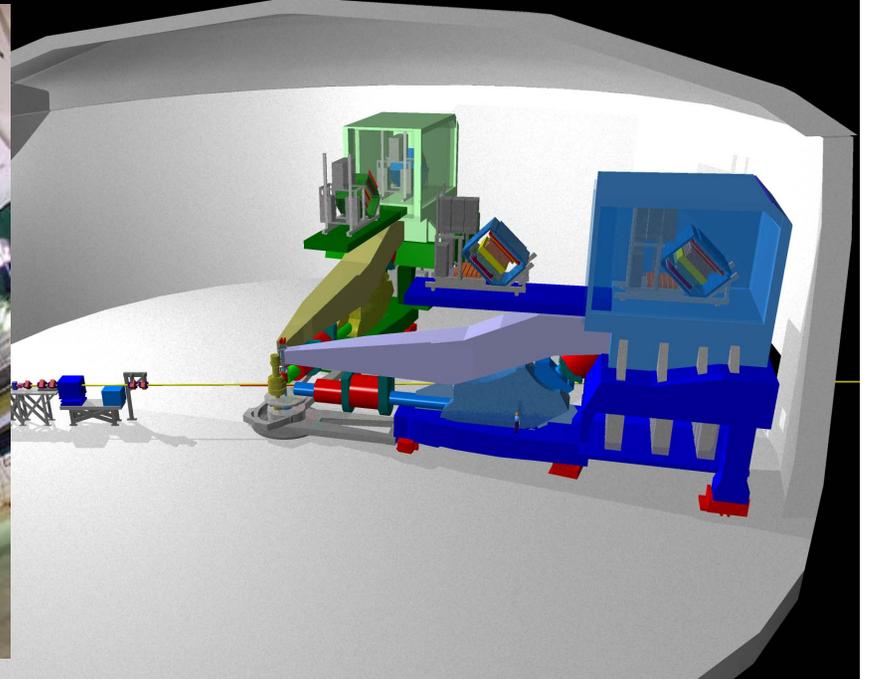
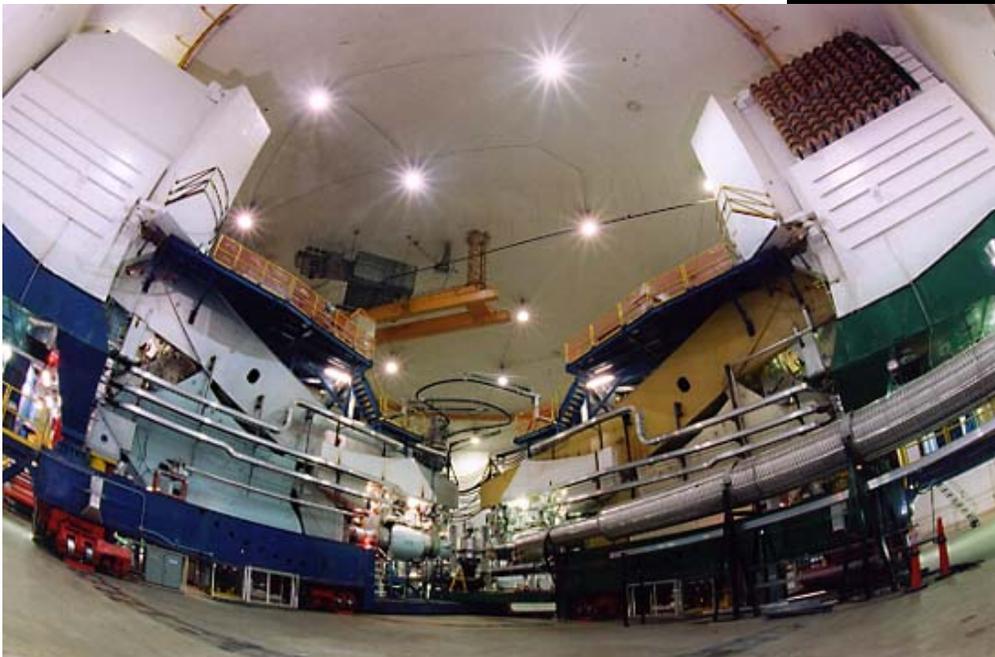


HIGH RESOLUTION SPECTROMETERS

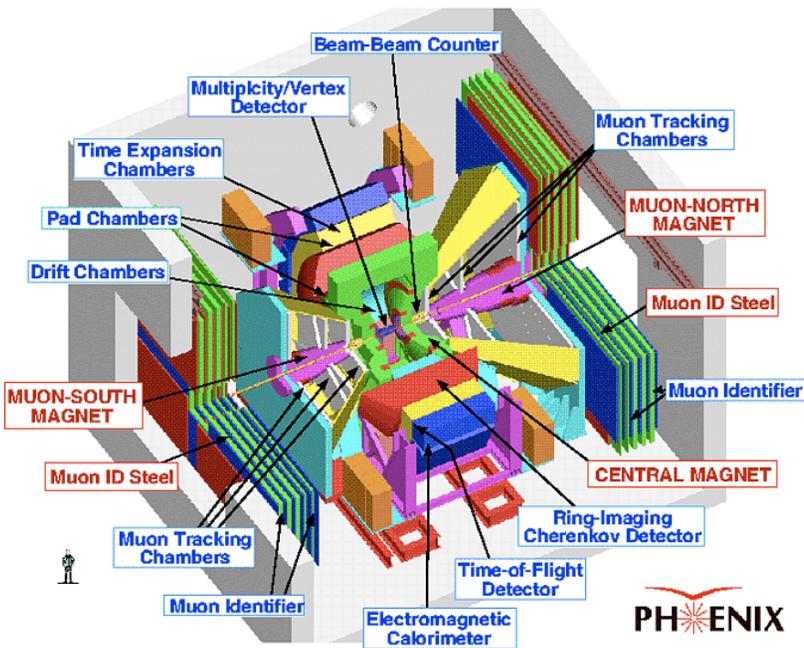
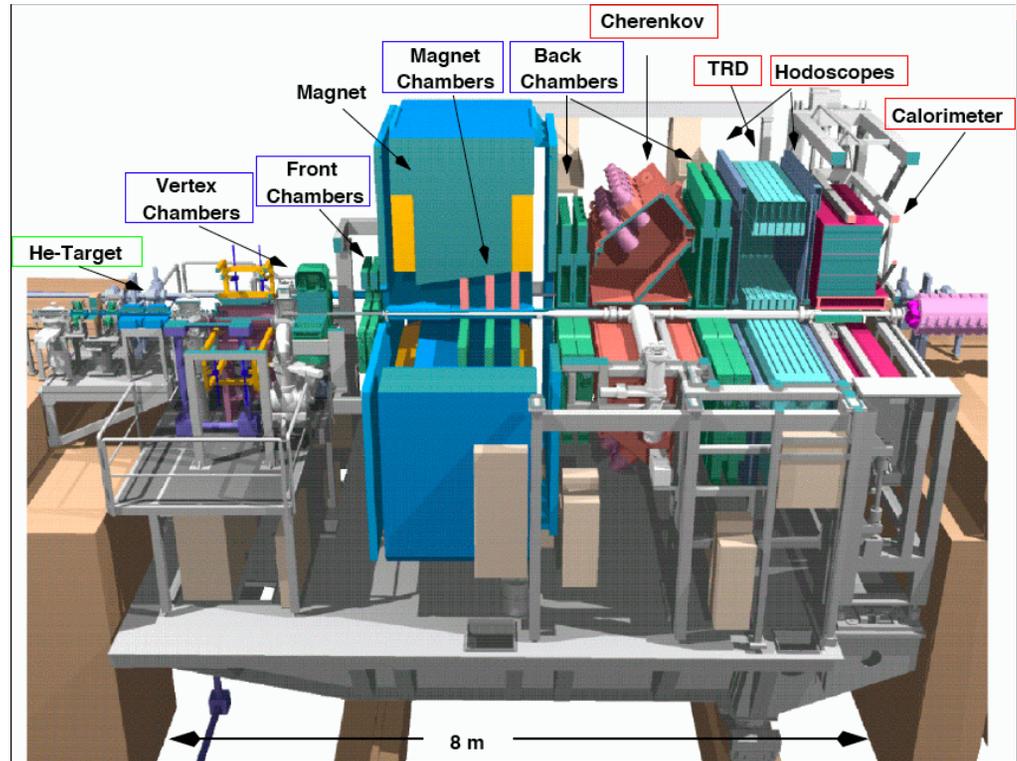
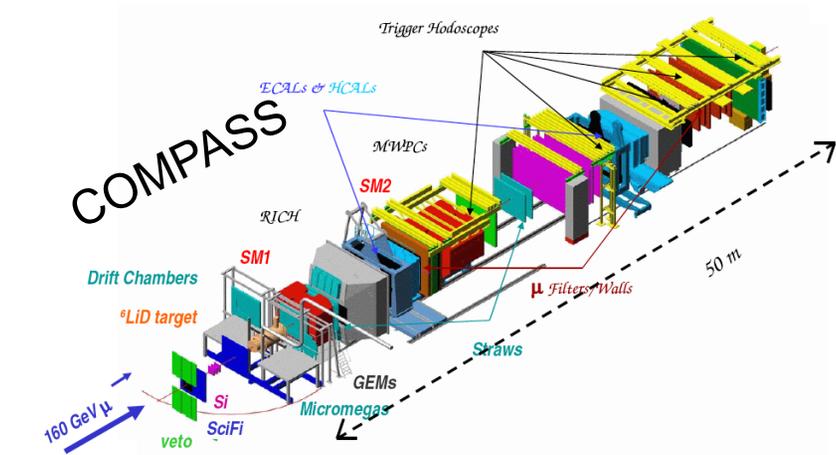


Jefferson Lab Hall A

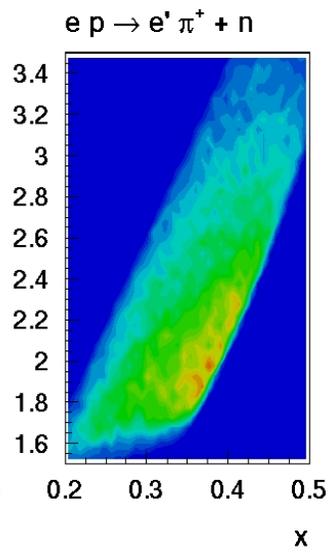
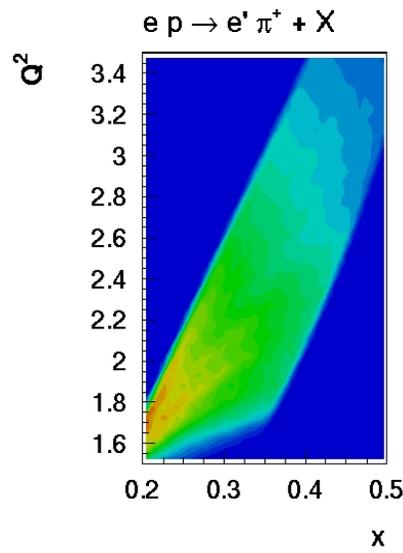
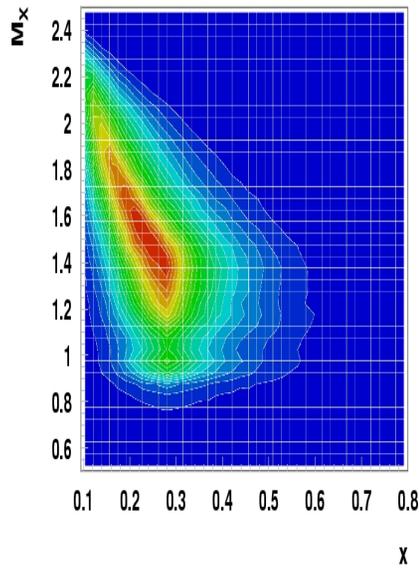
Typically small acceptance but high resolution, very good shielding
(→ high luminosity)



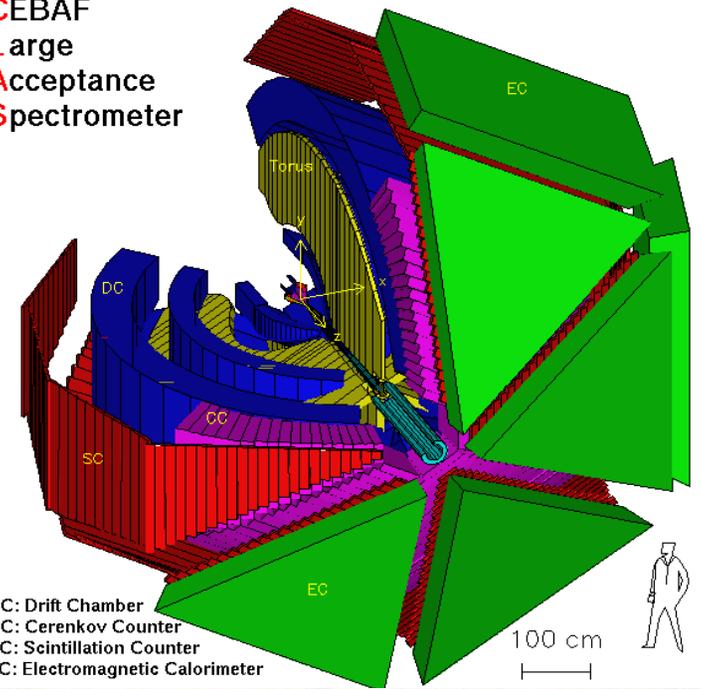
LARGE ACCEPTANCE DETECTORS



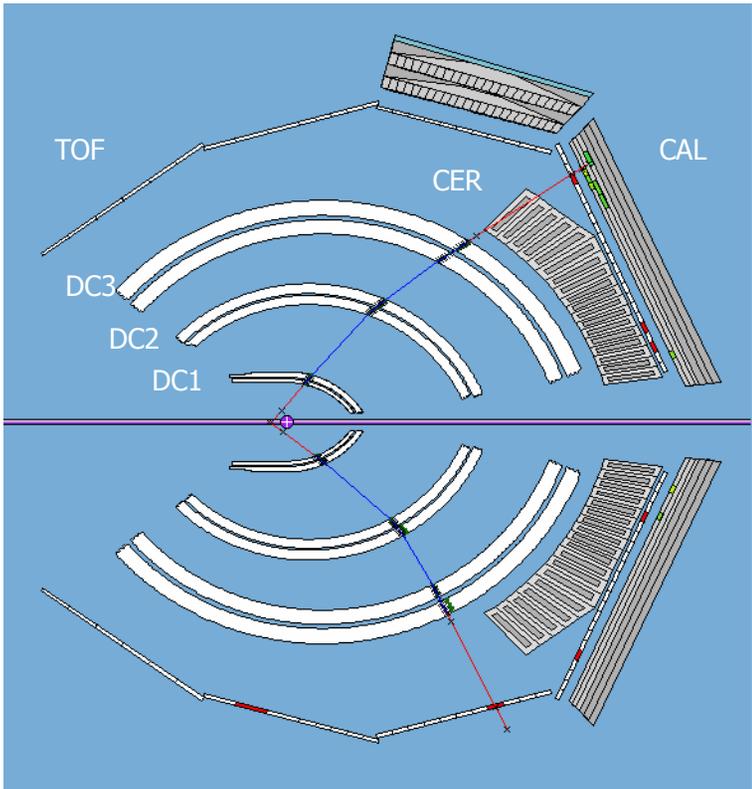
HERMES



CEBAF
Large
Acceptance
Spectrometer



DC: Drift Chamber
CC: Cerenkov Counter
SC: Scintillation Counter
EC: Electromagnetic Calorimeter



CLAS12

Under construction; expected to begin data taking in 2015 with upgraded 11 GeV beam

Base equipment

■ Forward Detector

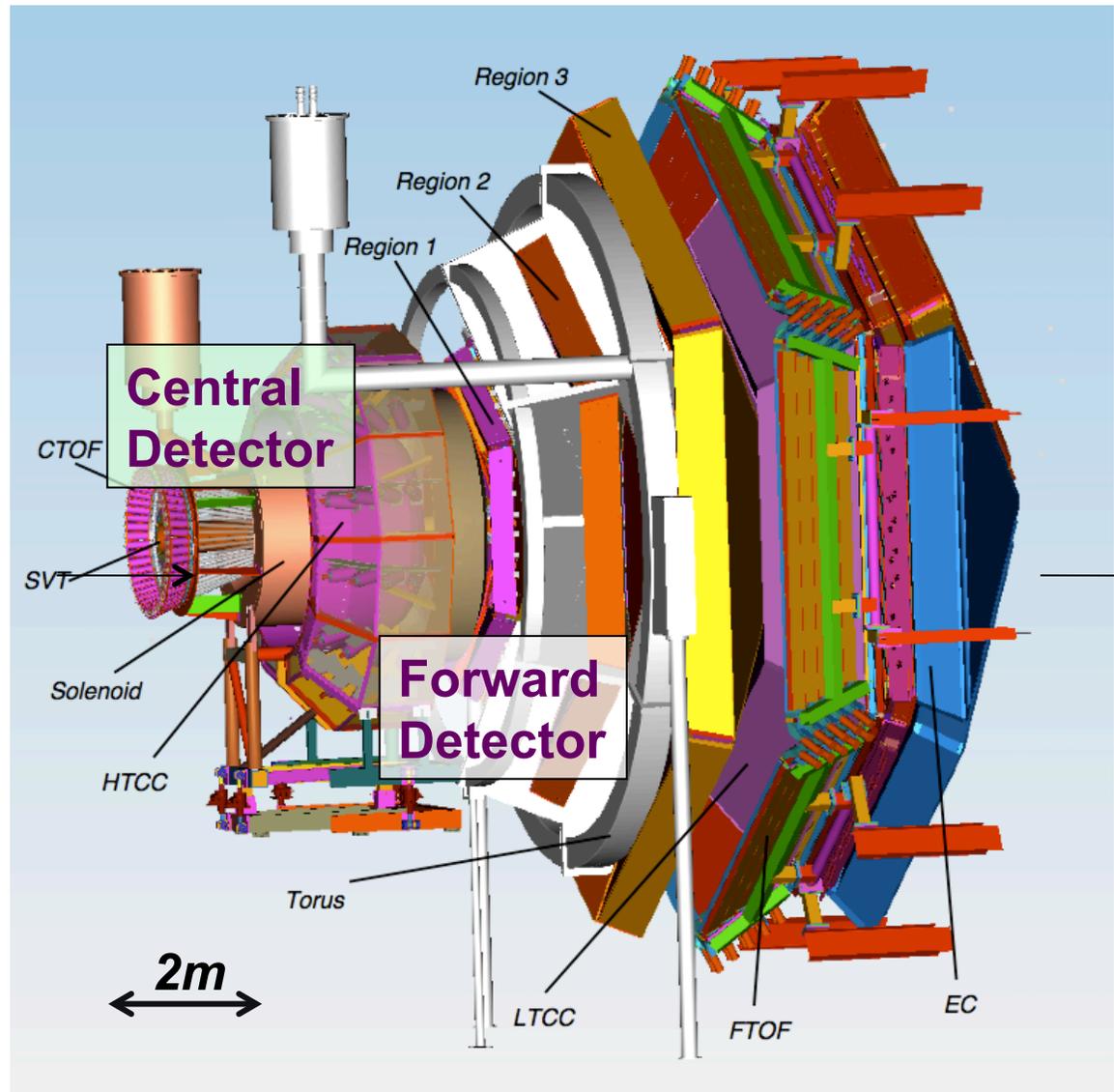
- TORUS magnet
- Forward vertex tracker
- HT Cherenkov Counter
- Drift chamber system
- LT Cherenkov Counter
- Forward ToF System
- Preshower calorimeter
- E.M. calorimeter

■ Central Detector

- SOLENOID magnet
- Barrel Silicon Tracker
- Central Time-of-Flight

Proposed equipment

- Micromegas (CD & FD)
- RICH counter (FD)
- Neutron detector (CD)
- Small angle tagger (FD)



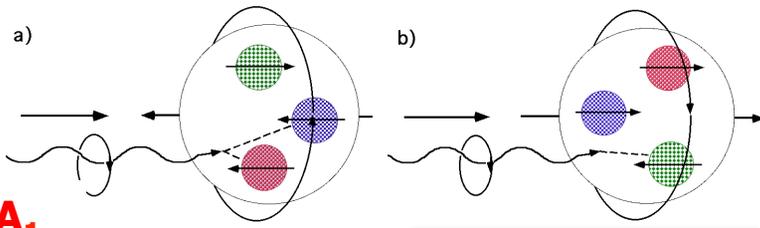
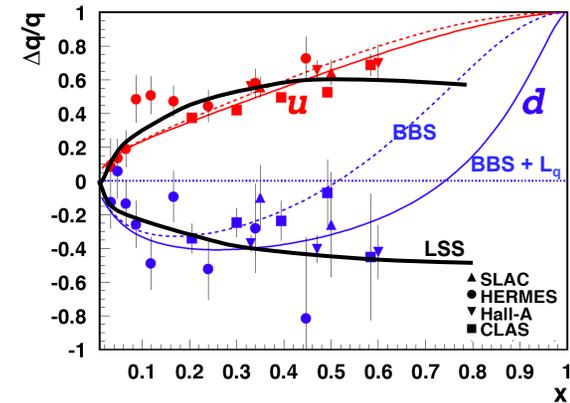
EXAMPLE DIS ON SPIN-1/2 TARGET

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4} \left(\frac{F_2(x)}{\nu} \cos^2 \frac{\theta_e}{2} + 2 \frac{F_1(x)}{M} \sin^2 \frac{\theta_e}{2} \right) (1 + h P_T A)$$

$A_{\parallel} = D(A_1 + \eta A_2)$, target polarization along beam direction

$A_{\perp} = d(A_1 + \zeta A_2)$, target polarization perpendicular to beam direction, in scattering plane

D, η, d, ζ are functions of Q^2, E', E_0 and F_2 / F_1



A_1

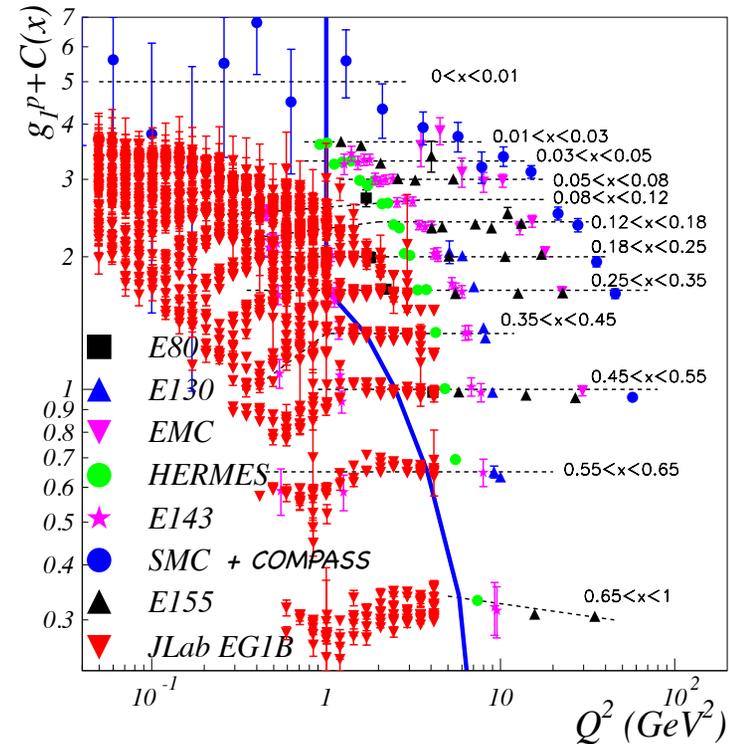
$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_T}$$

$$A_2 = \frac{\sigma_{LT'}}{\sigma_T}$$

Spin
Structure
Functions

$$g_1(x, Q^2) = \frac{\tau}{1 + \tau} \left(A_1 + \frac{1}{\sqrt{\tau}} A_2 \right) F_1$$

$$g_2(x, Q^2) = \frac{\tau}{1 + \tau} (\sqrt{\tau} A_2 - A_1) F_1 \quad \tau = \frac{\nu^2}{Q^2}$$



ANALYSIS STEPS – RAW ASYMMETRIES

Define $n^{++} = \frac{N^{++}}{\int L dt^{++}} = d \left[\sigma_{all} + \sigma_H P_b^{++} P_t^{++} (\alpha A_{LL} + \beta A_{LT}) \right]$ etc.

beam target

↑ Integrated luminosity (charge) ↑ potentially time-dependent detector acceptance, efficiency, uncorrected luminosity

$$A_{raw} = \frac{n^{++} - n^{--}}{n^{++} + n^{--}} \quad \text{or} \quad A_{raw} = \frac{n^{+-} - n^{-+}}{n^{+-} + n^{-+}}$$

Note: it is very important to properly treat integrated luminosity (beam charge). Watch out for dead time, truncation and pile-up.

For typical electron DIS experiments, lepton spin flips frequently, and any changes of acceptance and efficiency tend to cancel out if one calculates asymmetries separately for each target polarization (take average to cancel false and PV asymmetries)
 μ beams have fixed polarization, so one has to be more careful about systematic uncertainties from variations of acceptance and efficiency that vary on the time scale of target spin flips (or over the length of a target).

ANALYSIS STEPS – CORRECT FOR DILUTION

$$DF = \frac{n_H}{n_H + n_{Rest}}$$

“Rest” contains nitrogen, liquid helium and foils for cryogenic DNP targets; Al wires for heat removal in HD ice targets, and contributions from N₂ and glass windows for ³He targets. Note: Some of the above may be slightly polarized! (e.g. nitrogen)

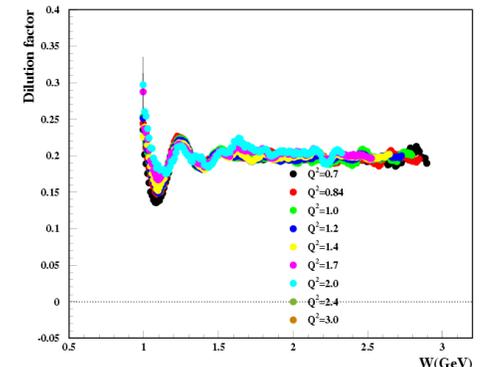
Typical procedure:

- ① Measure distances, thickness/density and composition of all target components as precisely as possible
- ② Develop accurate models for (radiated) cross sections from all elements found in the target
- ③ Use auxiliary measurements on well-defined targets (e.g., “empty” cell and a Carbon slab) to calibrate and to fit unknowns (e.g., the packing fraction of ammonia beads inside the cell)
- ④ Get smooth representation of n_{Rest} as function of kinematic parameters (E , E' , θ or x and Q^2)
- ⑤ Divide measured asymmetry by DF



Example: EG1 in JLab’s CLAS

Note: Plan ahead to have enough auxiliary runs!



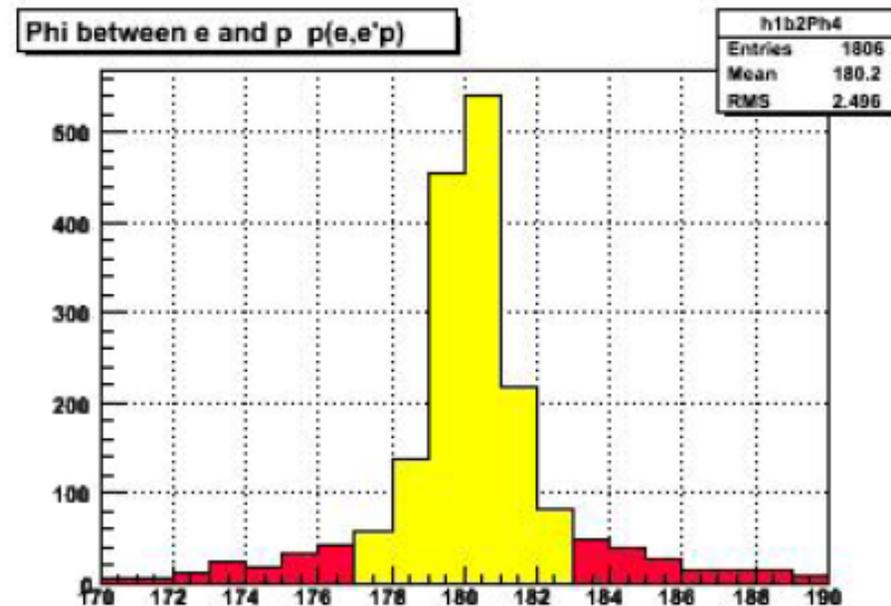
ANALYSIS STEPS – NORMALIZE FOR POLARIZATION

Polarizations must be either measured directly – e.g., beam polarization via Møller or Compton polarimeter, and target polarization via NMR – or inferred from auxiliary measurements.

One example: Exclusive H(e,e'p) reaction has well known double spin asymmetry – measuring it gives product $P_b P_t$:

$$P_b P_t = \frac{A_{raw}}{DF \cdot A_{||}^{th}}$$

Making use of the fact that the exclusive reaction is 4-fold overdetermined, one can apply tight cuts that reduces the unpolarized (nuclear) background and makes DF close to 1 (and very well-known).

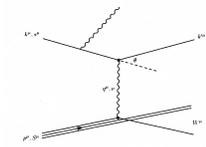


ANALYSIS STEPS – CORRECT FOR BACKGROUND

Possible sources of background:

- Particles misidentified as electrons
 - Most likely π^-
 - Need to carefully evaluate acceptance and rejection efficiency of cuts on EC and CC signals as function of kinematics
- Misidentified hadrons
 - Need to understand performance of ToF, CC and other PID detectors; most likely problem: separating K^+ 's from both π^+ and p
- Electrons that come from other processes than scattering
 - Photons converting into e^+e^- pairs; Dalitz decay $\pi^0 \rightarrow \gamma e^+e^-$
 - Charge symmetric \rightarrow correct by measuring e^+ rates
- Polarized nucleons in the non-H target material (e.g., N in NH_3 is slightly polarized)

ANALYSIS STEPS – CORRECT FOR HIGHER ORDER QED CONTRIBUTIONS



Bremsstrahlung radiation emitted from incoming electron

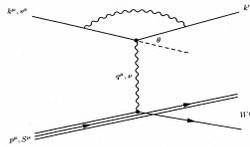
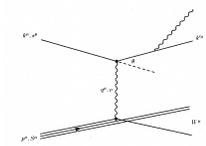


Figure 4.18: Vertex Correction



Bremsstrahlung radiation emitted from scattered electron

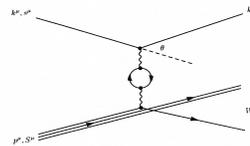


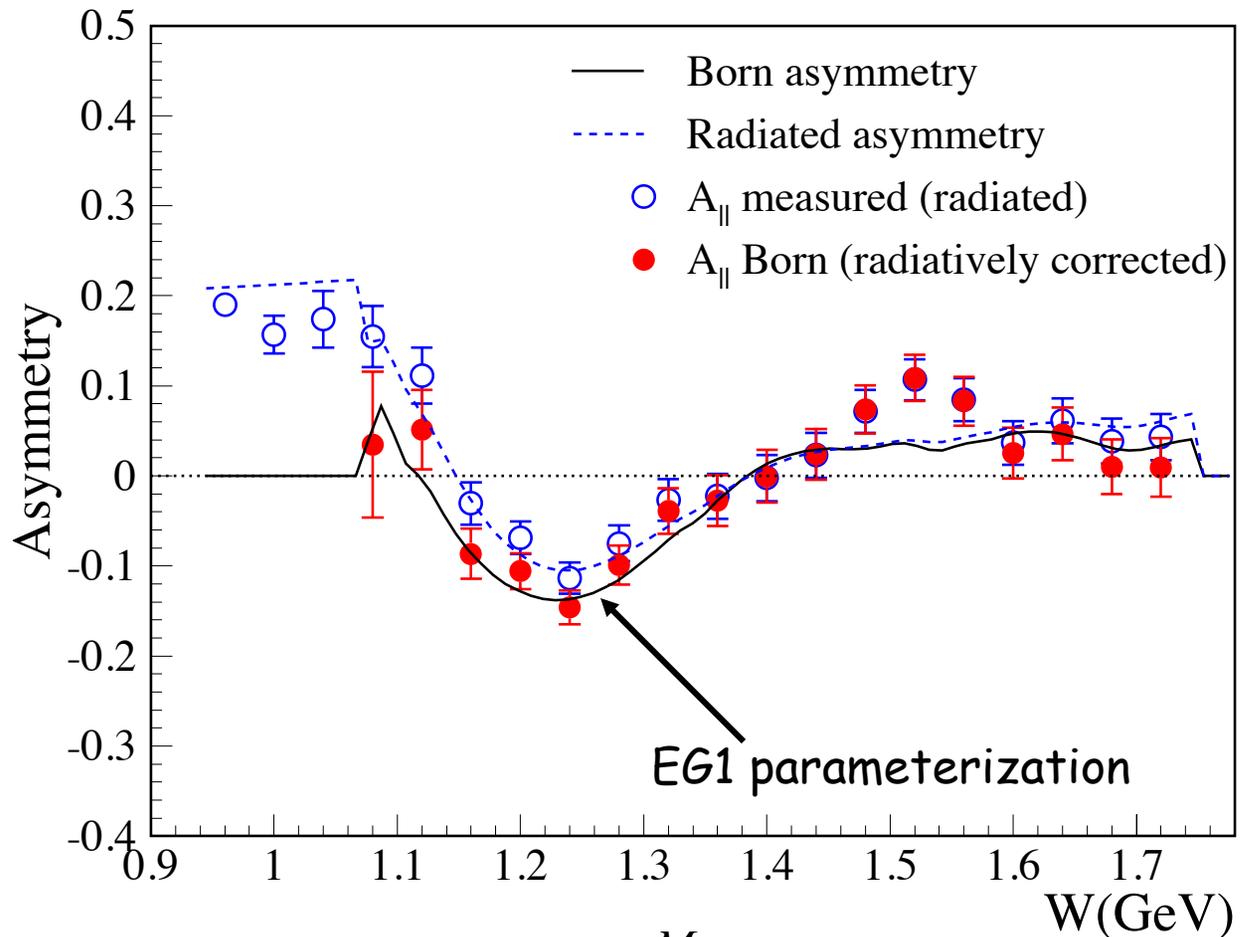
Figure 4.19: Vacuum Polarization

Various codes:

“**RC**SLACPOL” developed at SLAC or POLRAD

- Internal radiative corrections - Kuchto & Shumeika
- External radiative corrections - Tsai

The radiative correction includes an additive piece and a dilution.

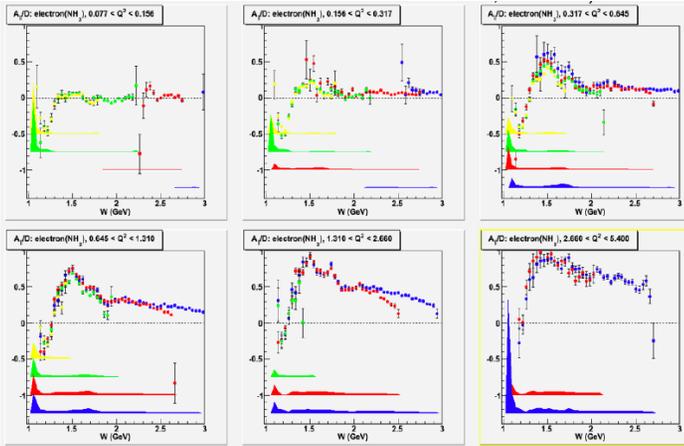


$$A_{\parallel}^{Born} = \frac{A_{\parallel}^{Meas}}{F_{RC}} + A_{RC}$$

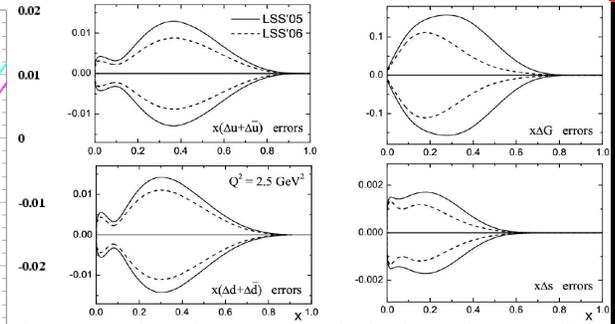
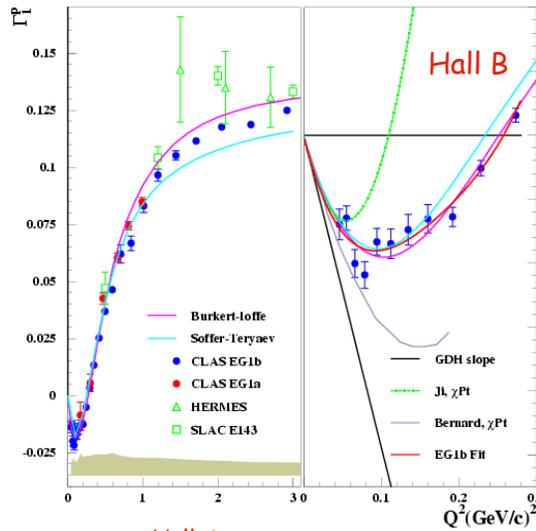
ANALYSIS STEPS – MODEL INPUT

- Need parametrizations of cross sections on various nuclear species for dilution factor
- Need parametrizations of unpolarized structure functions to convert asymmetries into cross section differences and polarized structure functions
- Need model for all types of structure functions for Monte Carlo simulations and radiative corrections (often iterative)

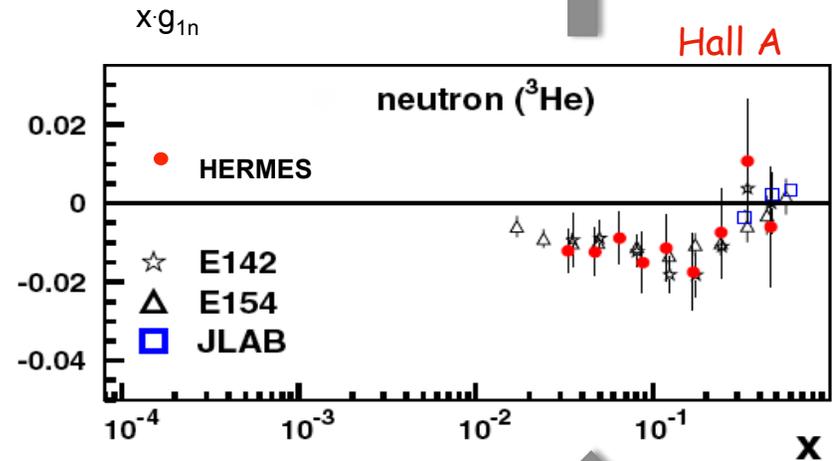
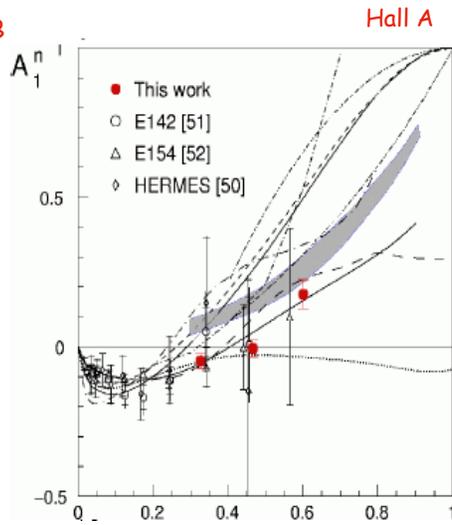
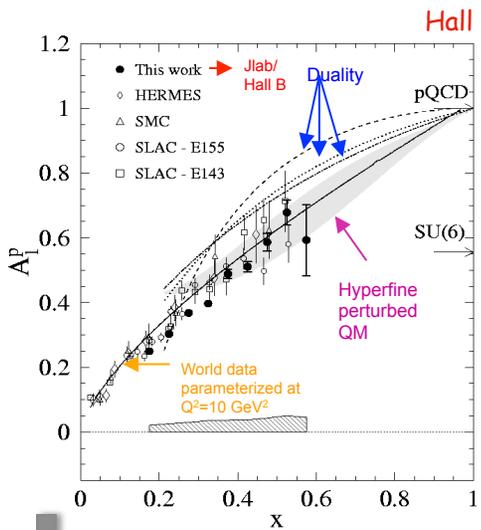
ANALYSIS STEPS – PHYSICS RESULTS



Combine asymmetries from different E_{beam}



...to parton distributions and integrals



From asymmetries to structure functions...

UNTANGLING DOUBLE AND SINGLE SPIN ASYMMETRIES

beam target

$$\bar{n}^{++} = \frac{N^{++}}{\int L dt^{++}} = d^{++} \sigma_H \left(\frac{1}{DF^+} + P_b^{++} A_{LU} + P_t^{++} A_{UL} + P_b^{++} P_t^{++} A_{LL} \right) \quad \text{etc.}$$

time-dependent detector acceptance, efficiency, uncorrected luminosity

In electron SIDIS, beam helicity can be flipped so rapidly that conditions can only change on the longer time scale of target polarization reversal \Rightarrow

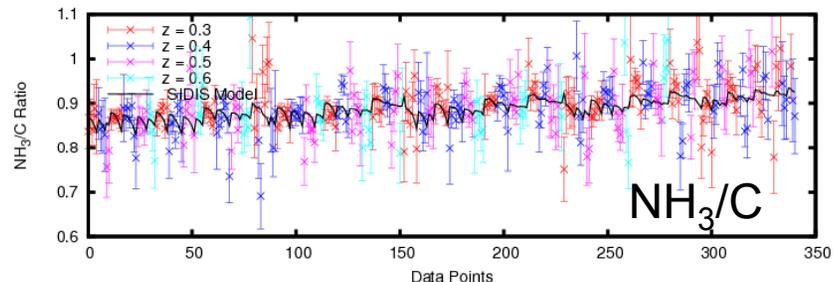
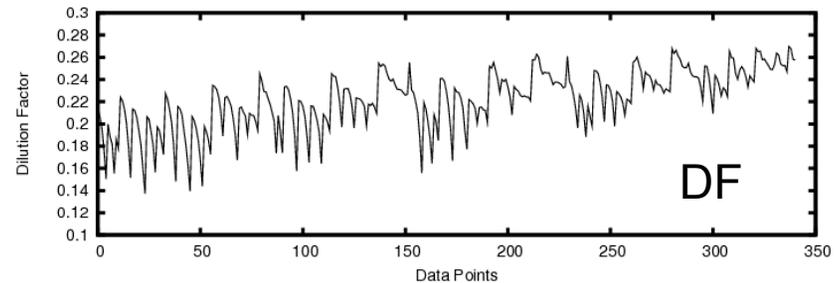
$$A_{LL} = - \frac{1}{|P_b^+ P_t^+| |P_b^-| + |P_b^- P_t^-| |P_b^+|} \left(\frac{|P_t^-|}{DF^+} + \frac{|P_t^+|}{DF^-} \right) \frac{|P_b^-| (n^{++} - n^{+-}) + \frac{d^+}{d^-} |P_b^+| (n^{--} - n^{+-})}{|P_t^-| (n^{++} + n^{+-}) + |P_t^+| \frac{d^+}{d^-} (n^{--} + n^{+-})}$$

$$\approx - \frac{1}{\langle DF \rangle} \frac{|P_b^-| (n^{++} - n^{+-}) + \frac{d^+}{d^-} |P_b^+| (n^{--} - n^{+-})}{|P_b^+| |P_b^- P_t^-| (n^{++} + n^{+-}) + |P_b^-| |P_b^+ P_t^+| \frac{d^+}{d^-} (n^{--} + n^{+-})}$$

$$A_{UL} = \frac{\left(\frac{|P_t^-|}{DF^+} + \frac{|P_t^+|}{DF^-} \right)}{(|P_t^+| + |P_t^-|)} \frac{n^{++} + n^{+-} - \frac{d^+}{d^-} (n^{--} - n^{+-})}{|P_t^-| (n^{++} + n^{+-}) + |P_t^+| \frac{d^+}{d^-} (n^{--} + n^{+-})} \approx \frac{1}{\langle DF \rangle} \frac{n^{++} + n^{+-} - \frac{d^+}{d^-} (n^{--} - n^{+-})}{|P_t^-| (n^{++} + n^{+-}) + |P_t^+| \frac{d^+}{d^-} (n^{--} + n^{+-})}$$

DILUTION: ADDITIONAL CONCERNS

- Dilution Factor depends not only on x and Q^2 , but also on z , P_{hT} and possibly even ϕ_h
 - “Smearing” of all kinematic variables due to scattering on a moving nucleon inside nucleus; nucleon off-shell (“EMC”) effects
 - In addition to EMC effect on parton distribution, nuclear environment has strong effect on hadronization: v and z -dependent suppression (transparency), P_{hT} broadening, secondary interactions...
 - Much less well-developed models (but important data from HERMES and Jefferson Lab)
- Requires fit to ratios of rates on physics/reference targets with several parameters that are reasonably well constrained.



DILUTION: ADDITIONAL CONCERNS

$$\text{Ratio}_{\pi+} = \frac{\rho_{NH_3} * pf * rpp * \frac{3}{17} + \rho_{He} * atthe * (bl - pf) * rdp + \rho_{Al} * attal * al * ralp + \rho_{NH_3} * attn * pf * rdp * \frac{14}{17}}{\rho_{C_{12}} * attc * cl * rdp + \rho_{He} * atthe * (bl - cl) * rdp + \rho_{Al} * attal * al * ralp}$$

$$\text{Dilution}_{\pi+} = \frac{\rho_{NH_3} * pf * rpp * \frac{3}{17}}{\rho_{NH_3} * attn * pf * rpp * \frac{3}{17} + \rho_{He} * atthe * (bl - pf) * rdp + \rho_{Al} * attal * al * ralp + \rho_{NH_3} * attn * pf * rdp * \frac{14}{17}}$$

$$rpp(Q^2, xb, z) = (4 * u + Ds(Q^2, xb)) + (4 * Us(Q^2, xb) + d) * \frac{1 + F}{(1 + G * z)^2}$$

$$rnp(Q^2, xb, z) = (4 * d + Us(Q^2, xb)) + (4 * Ds(Q^2, xb) + u) * \frac{1 + F}{(1 + G * z)^2}$$

$$rpm(Q^2, xb, z) = (4 * u + Ds(Q^2, xb)) * \frac{1 + F}{(1 + G * z)^2} + (4 * Us(Q^2, xb) + d)$$

$$rnm(Q^2, xb, z) = (4 * d + Us(Q^2, xb)) * \frac{1 + F}{(1 + G * z)^2} + (4 * Ds(Q^2, xb) + u)$$

$$rdp = \frac{rpp + rnp}{2}$$

$$ralp = \frac{13 * rpp + 14 * rnp}{27}$$

$$rdm = \frac{rpm + rnm}{2}$$

$$ralm = \frac{13 * rpm + 14 * rnm}{27}$$

$$u = Uv(Q^2, xb) + Us(Q^2, xb);$$

$$d = Dv(Q^2, xb) + Ds(Q^2, xb);$$

$$attal = 1 + \frac{B}{(\frac{\nu}{2.5})^A * (0.45 + z)} \left(\frac{27}{12}\right)^{\frac{1}{3}} \left[\left(\frac{pt}{C + \frac{1}{2}(z - 0.4)} \right)^D - 1 \right]$$

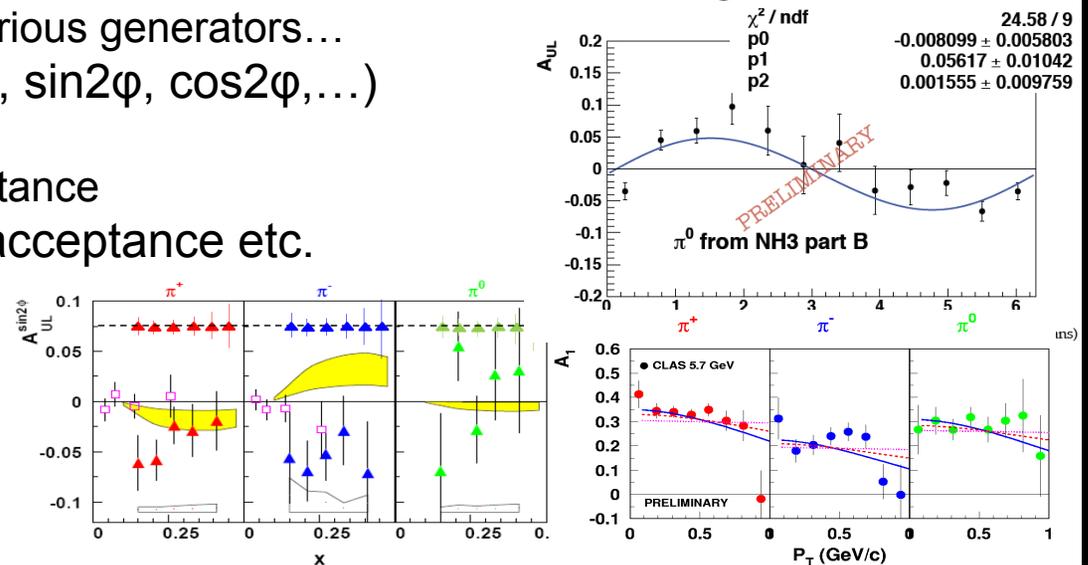
$$atthe = 1 + \frac{B}{(\frac{\nu}{2.5})^A * (0.45 + z)} \left(\frac{4}{12}\right)^{\frac{1}{3}} \left[\left(\frac{pt}{C + \frac{1}{2}(z - 0.4)} \right)^D - 1 \right]$$

$$attc = 1 + \frac{B}{(\frac{\nu}{2.5})^A * (0.45 + z)} \left(\frac{12}{12}\right)^{\frac{1}{3}} \left[\left(\frac{pt}{C + \frac{1}{2}(z - 0.4)} \right)^D - 1 \right]$$

$$attn = 1 + \frac{B}{(\frac{\nu}{2.5})^A * (0.45 + z)} \left(\frac{14}{12}\right)^{\frac{1}{3}} \left[\left(\frac{pt}{C + \frac{1}{2}(z - 0.4)} \right)^D - 1 \right]$$

WHAT ELSE DO WE NEED?

- Target and Beam Polarization
 - Can use similar methods (Møller/Compton scattering, NMR, inclusive and exclusive scattering)
- Backgrounds
 - Same as DIS...
 - + misidentified hadrons (π^+ vs. K^+ vs. p)
 - + combinatorial background ($\pi^0 \rightarrow \gamma\gamma$)
 - + accidental coincidences
- Radiative Corrections
 - More complicated but less pronounced than for inclusive scattering
 - POLRAD, HAPRAD and various generators...
- Extract moments ($\sin\phi$, $\cos\phi$, $\sin 2\phi$, $\cos 2\phi$, ...)
 - Fine binning and fit
 - Moments; correct for acceptance
- Monte Carlo simulations for acceptance etc.
- Models \Rightarrow TMDs, GPDs, ...
- Final results, publications, talks, ...



SUMMARY

Polarized beams

+ Polarized targets

+ Detector Elements

= **Data**

- Backgrounds

/ Polarization*Dilution

apply radiative corrections etc.

⇒ **Observables**

Use models to interpret

⇒ **FINAL RESULTS**

Fame