Resonances and QCD

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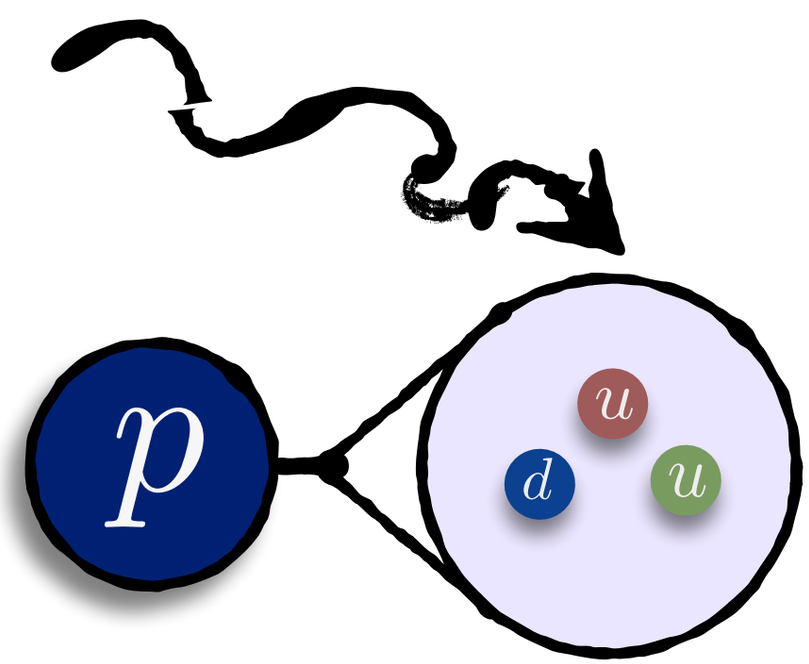
Jefferson Lab

February 2016

Resonances in experiments

(e.g., meson photo/electro-production)



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Resonances in experiments

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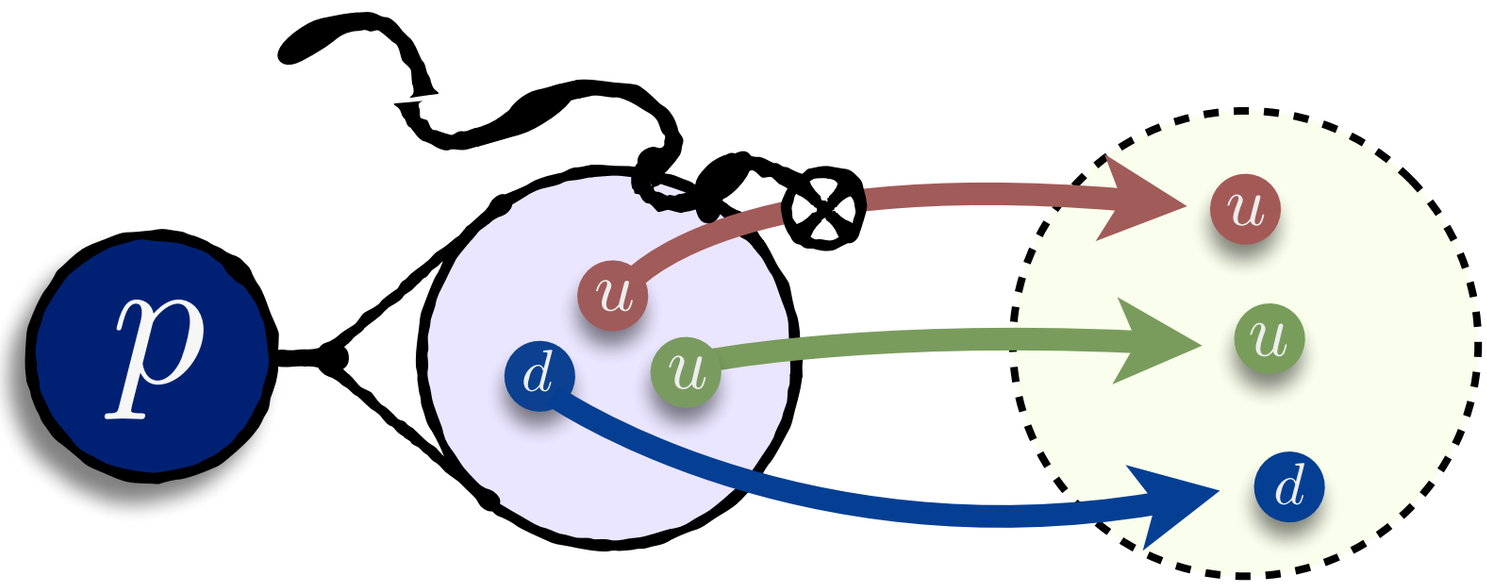

short-lived state, decays in ~10^{-23}s

$N^{\star}(1440)$

the Roper: Γ~350 MeV

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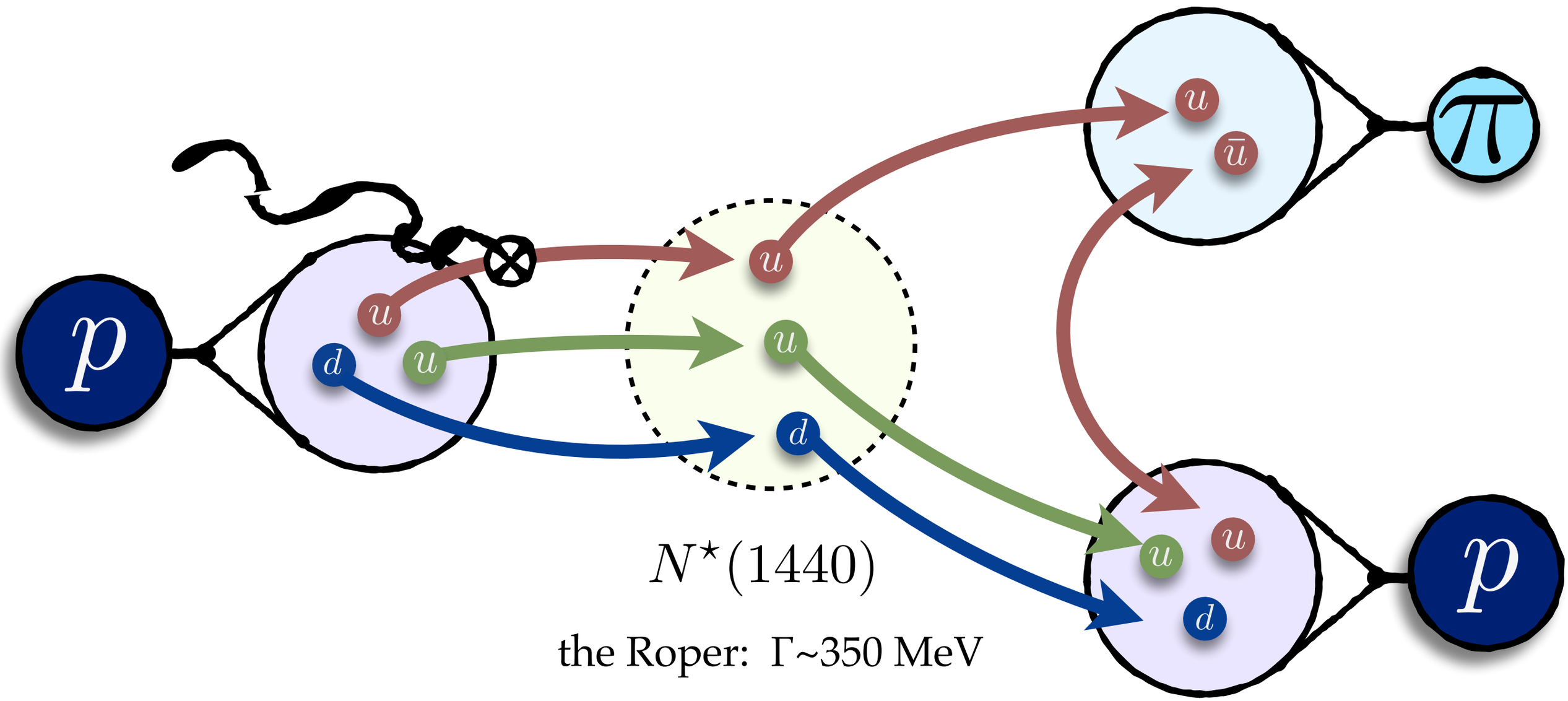
Resonances in experiments

(e.g., meson photo/electro-production)



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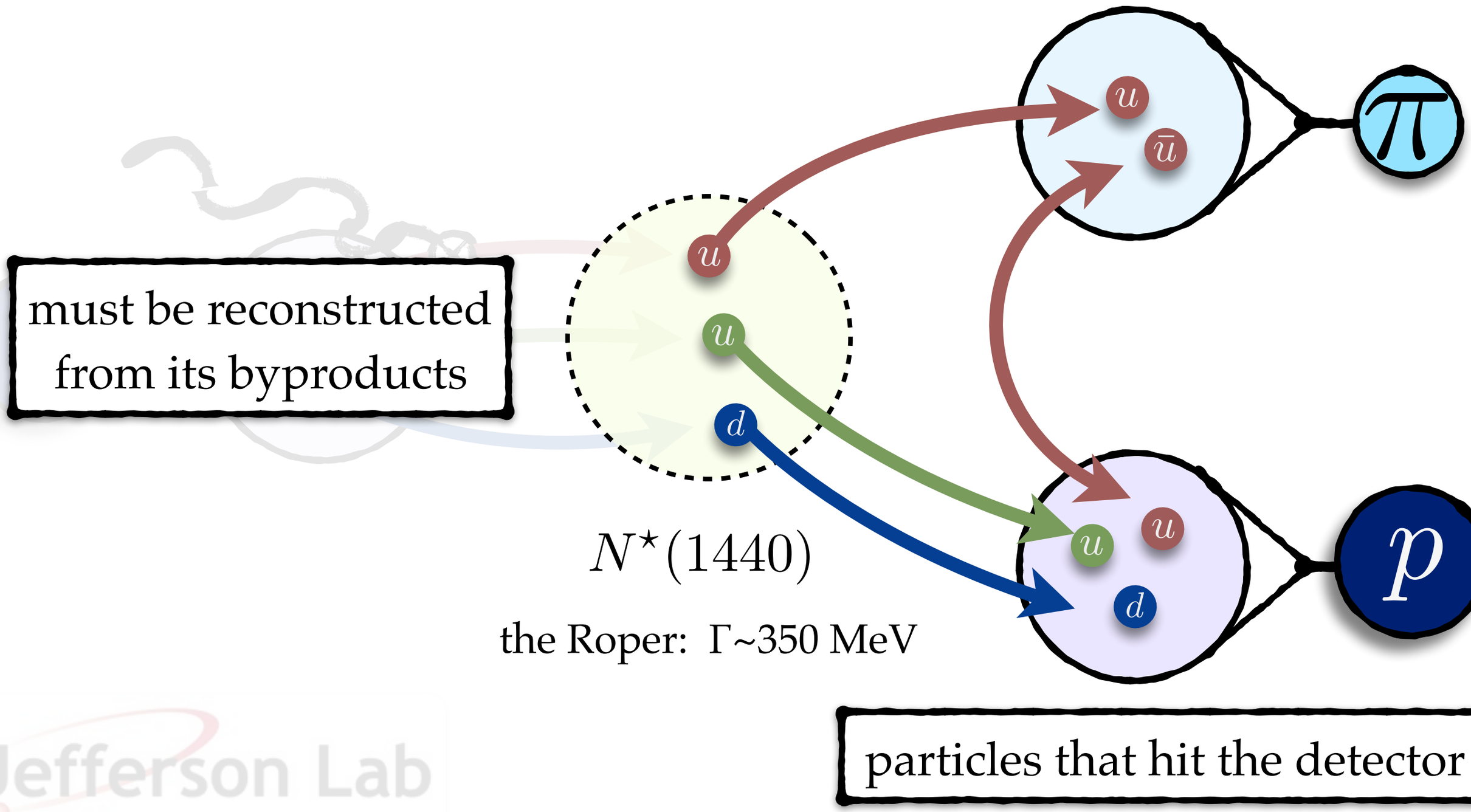
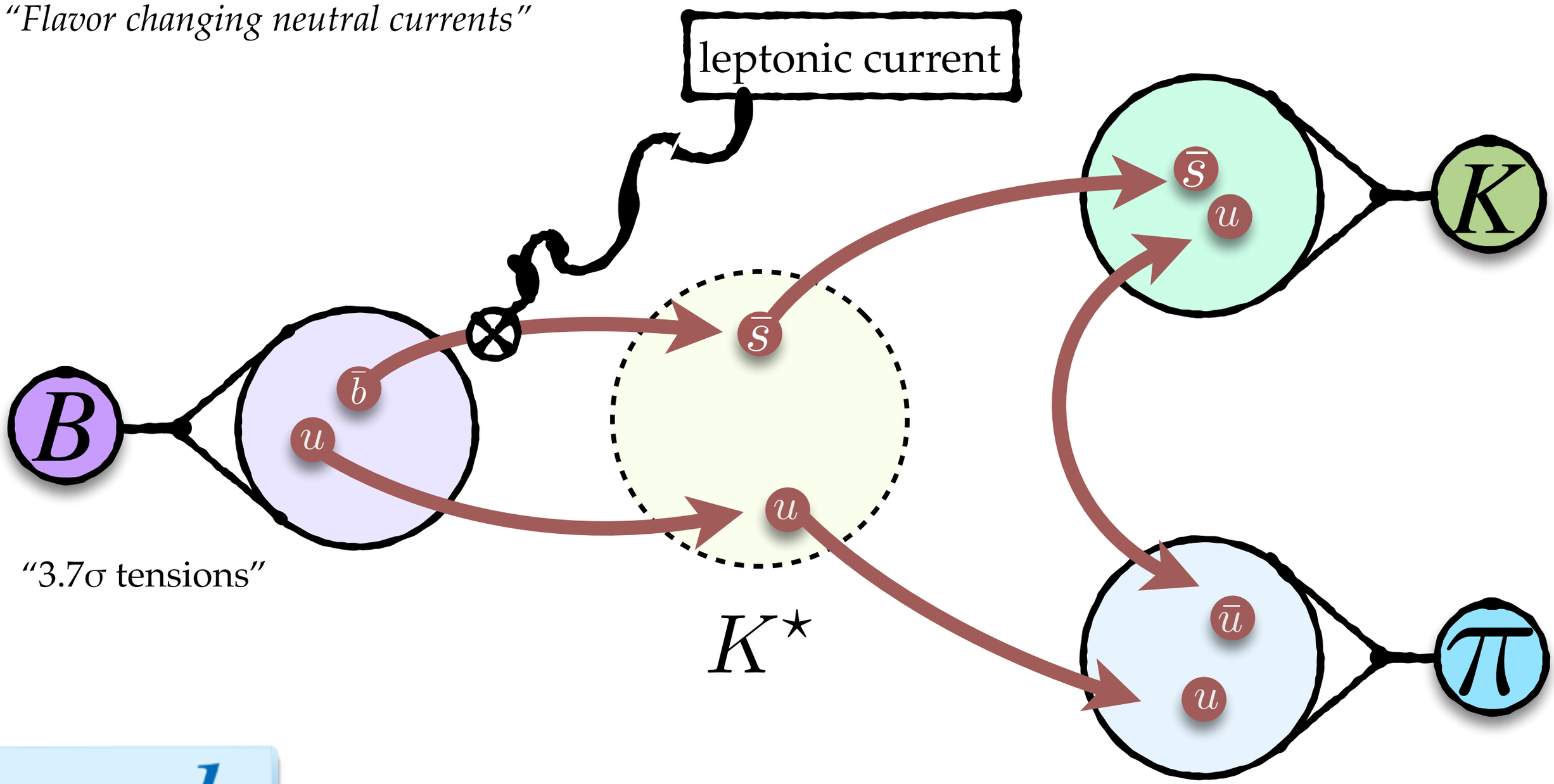


Resonances in experiments

(e.g., rare weak decays)

"Flavor changing neutral currents"

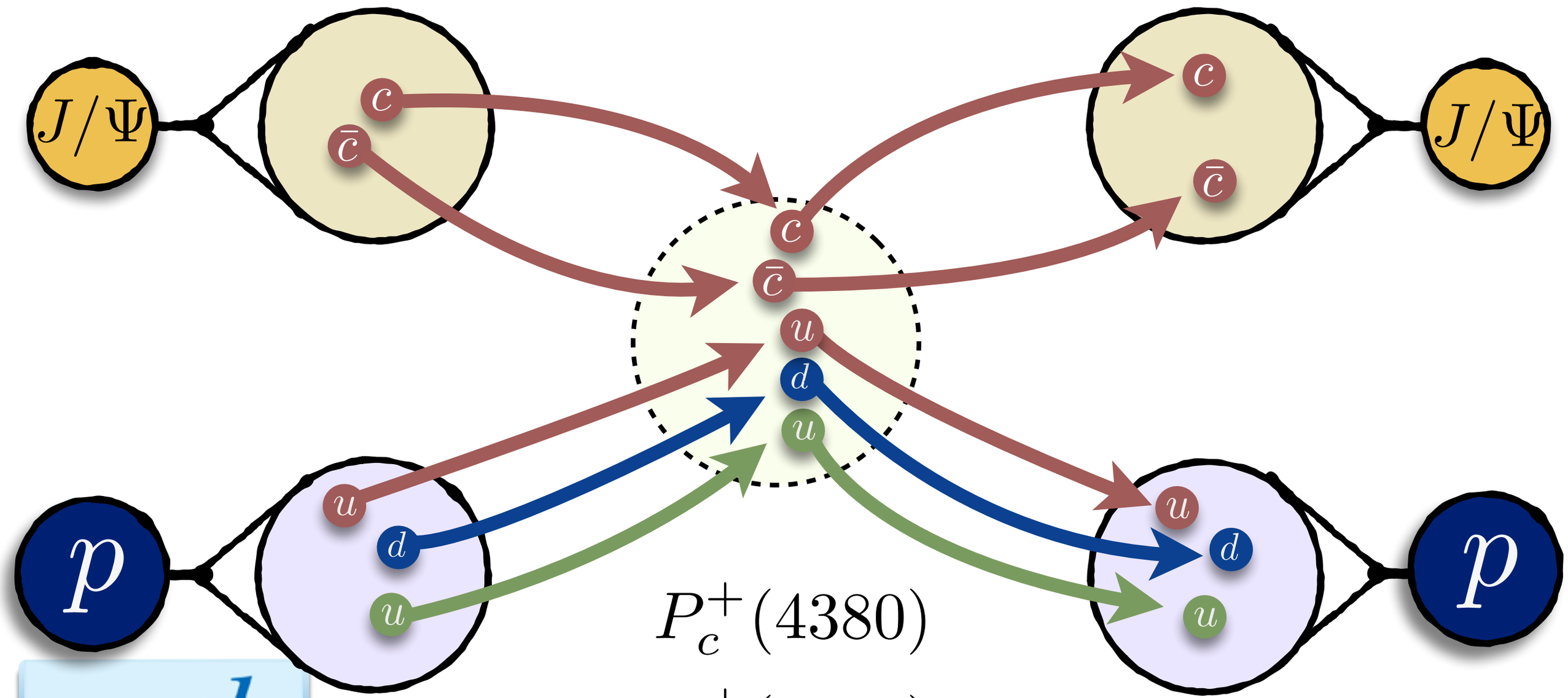


"3.7σ tensions"

2013

Resonances in experiments

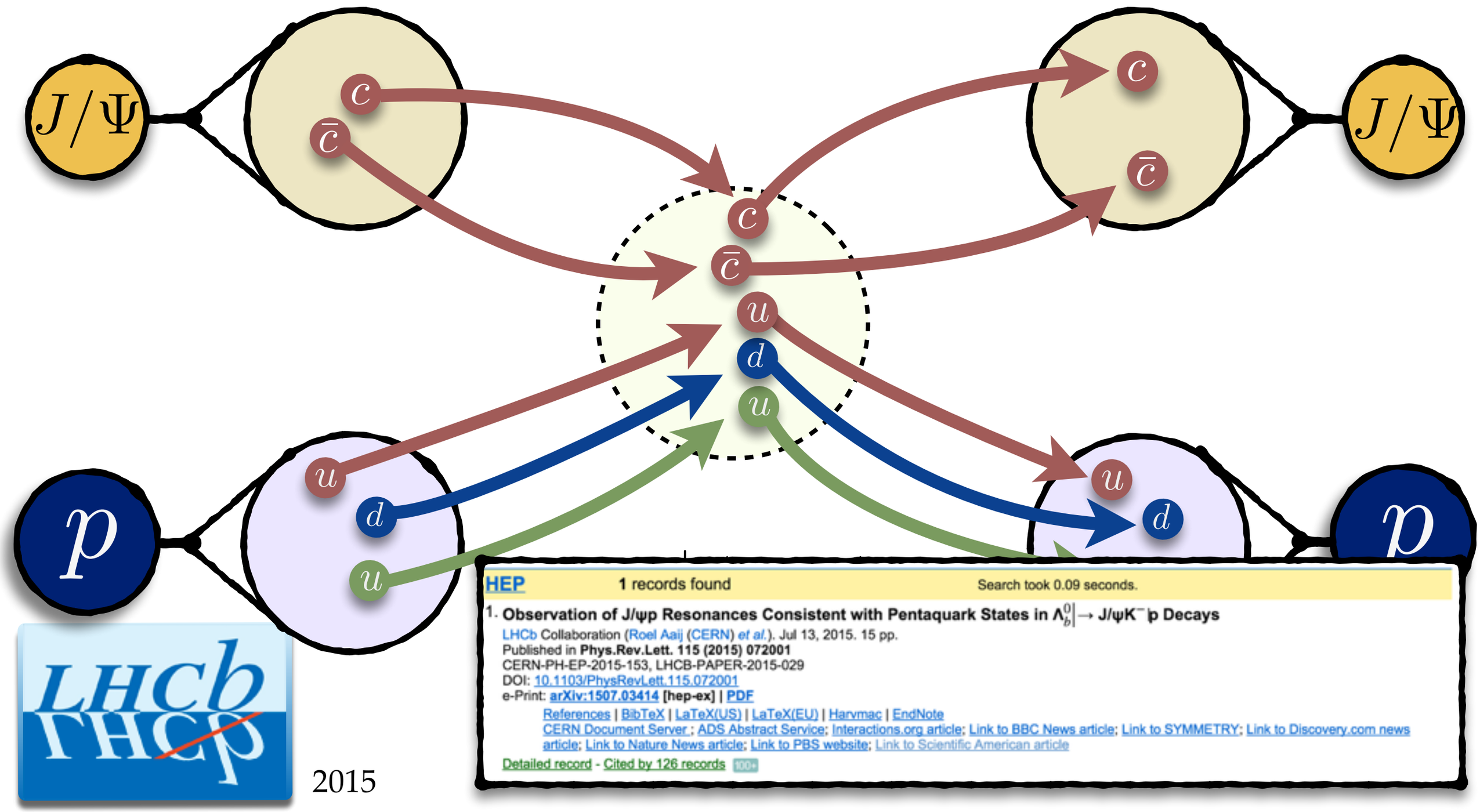
(e.g., exotic states)



2015

Resonances in experiments

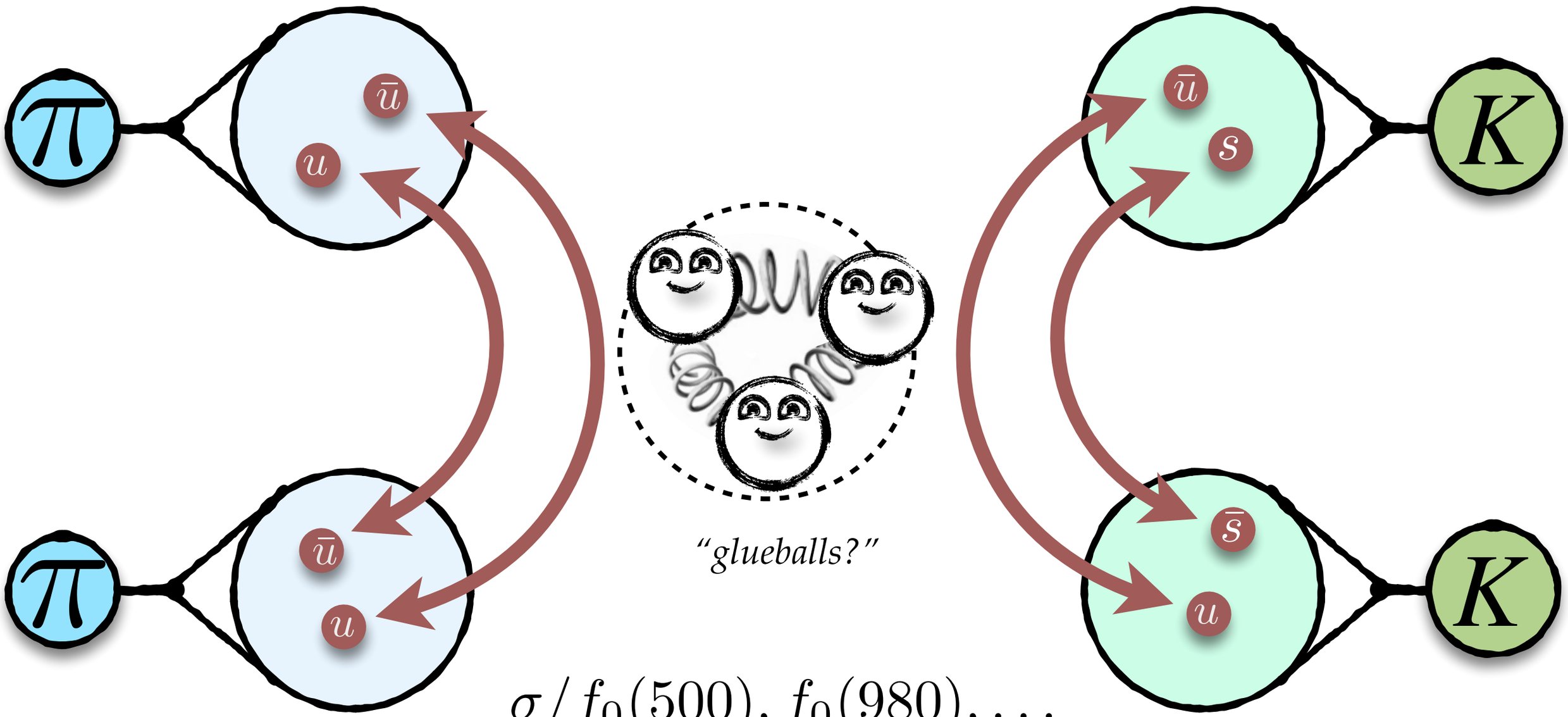
(e.g., exotic states)



2015

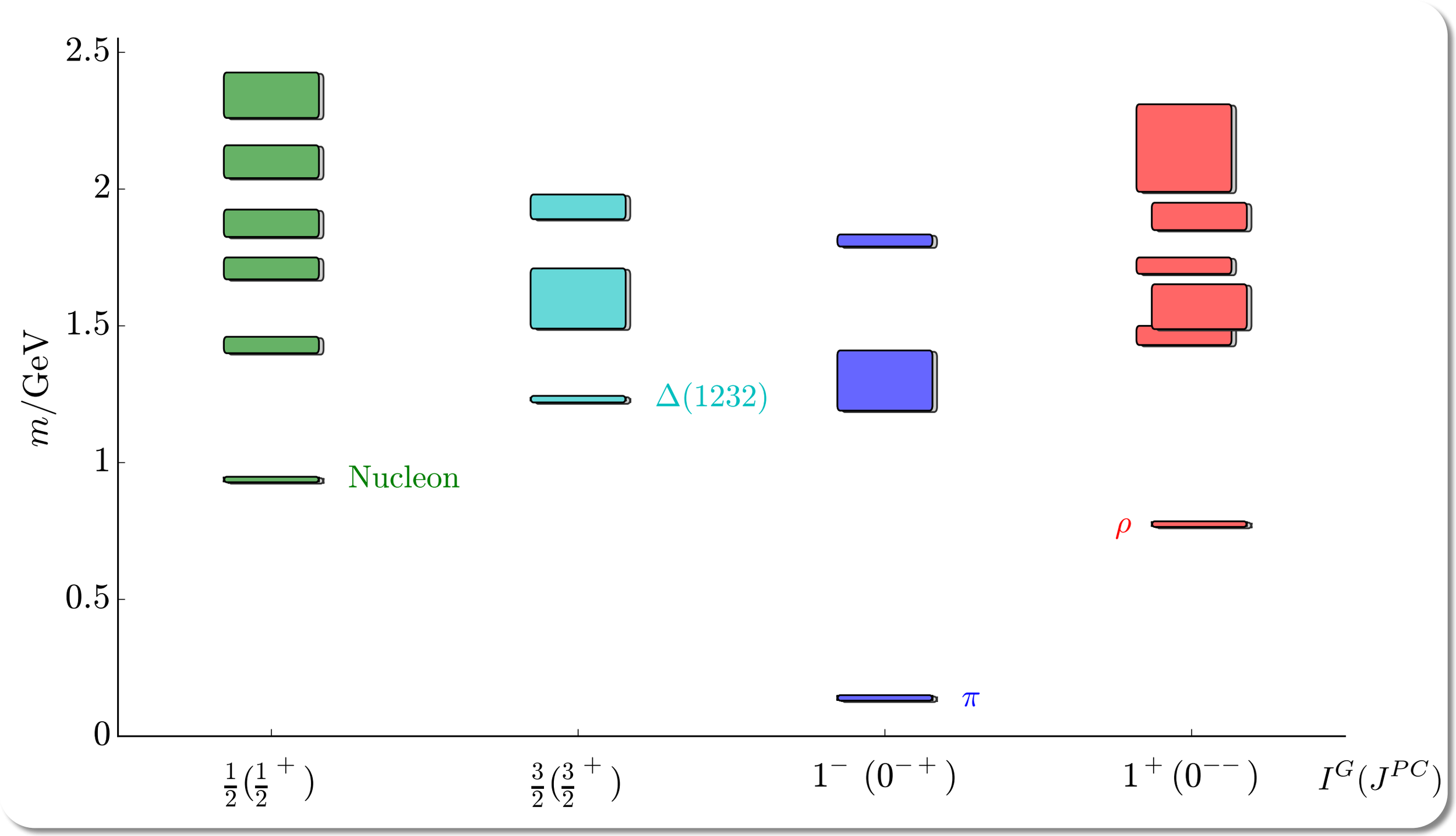
Resonant matrix elements

(e.g., enigmatic states)

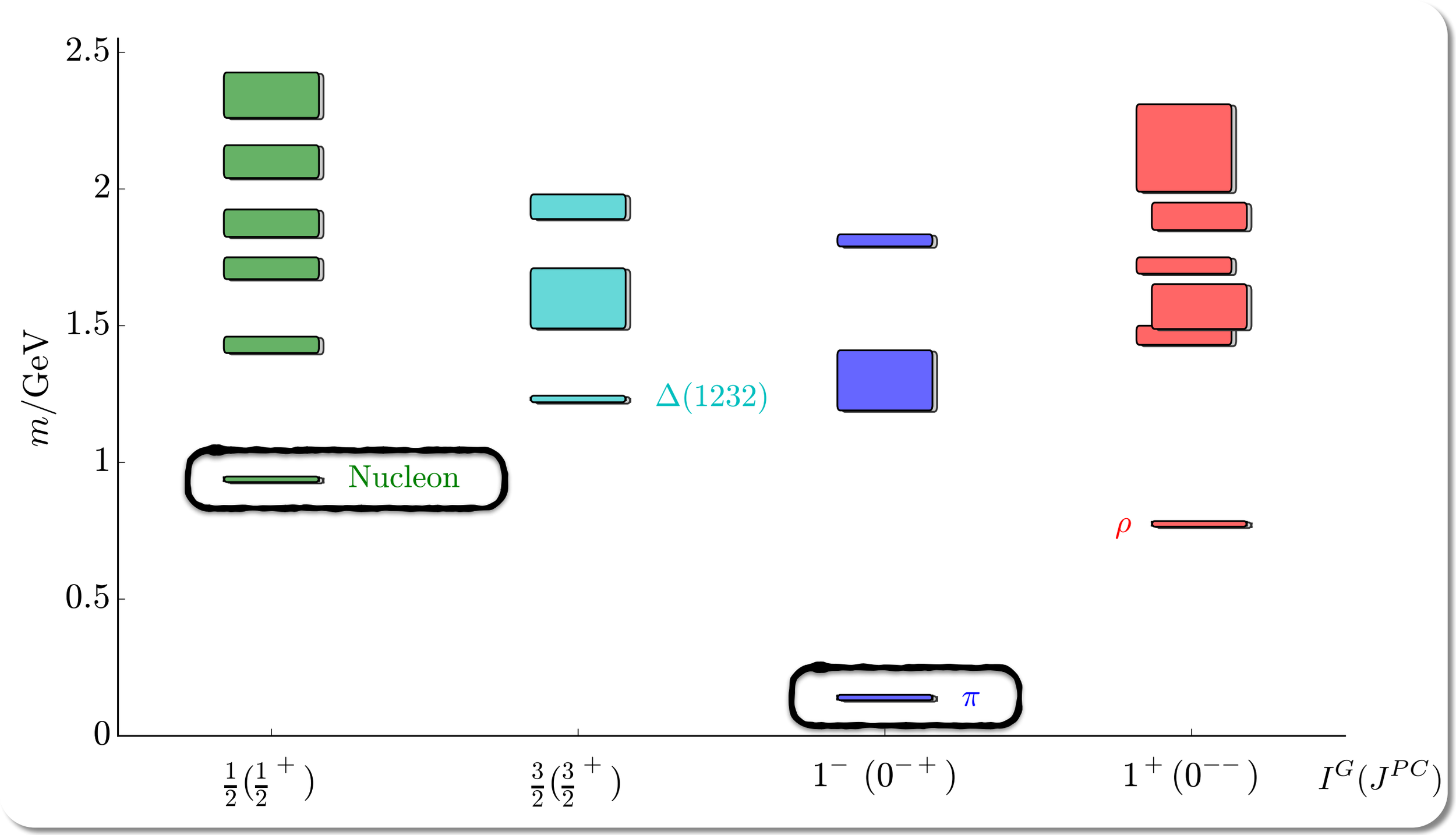


$\sigma / f_0(500), f_0(980), \ldots$

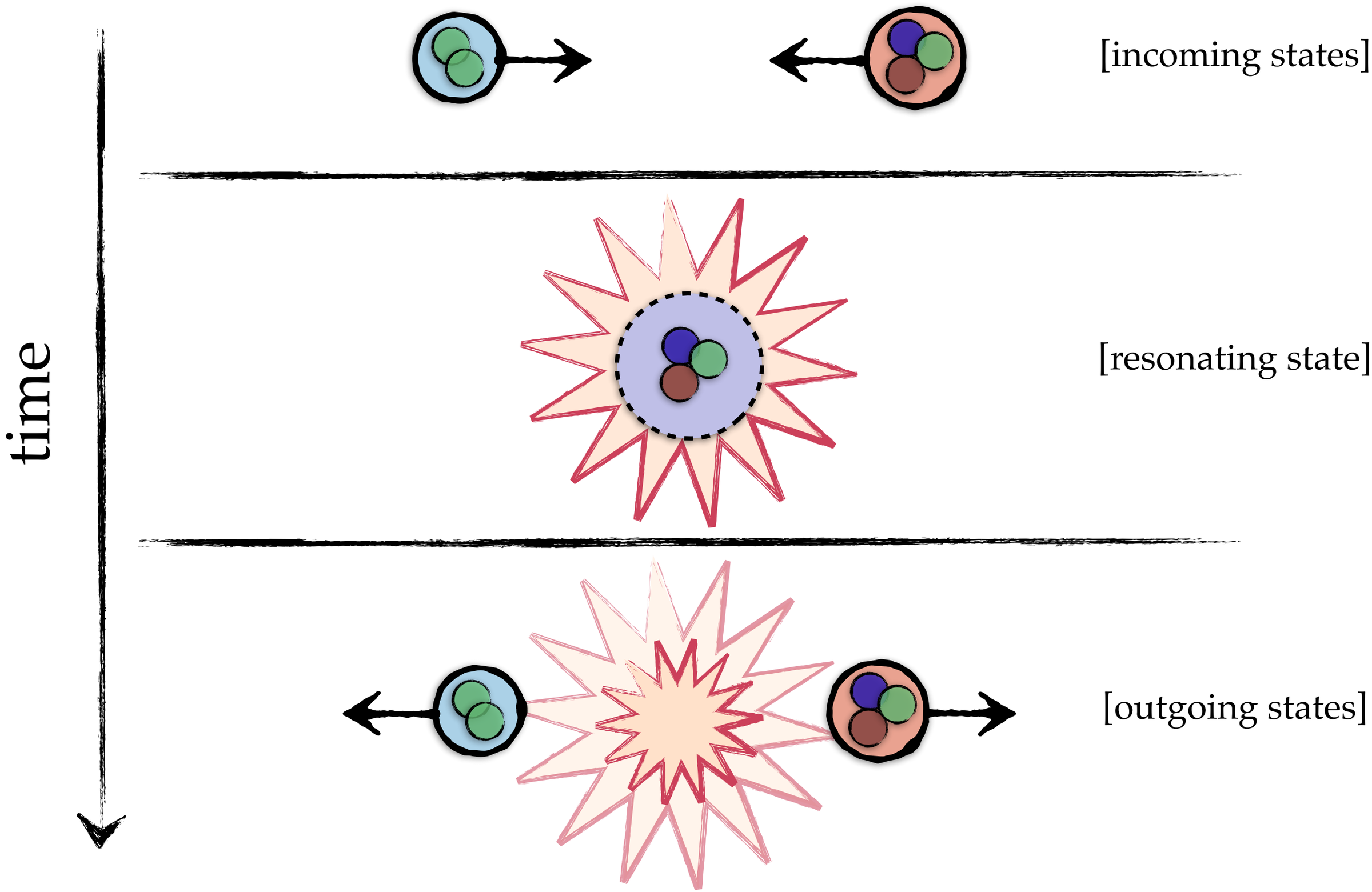
Why are resonances important?



Why are resonances important?

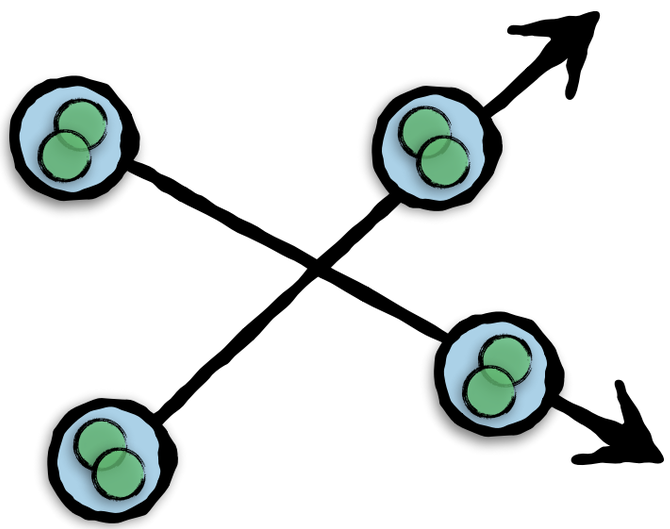


Resonances - Intuitive picture



Scattering theory 101

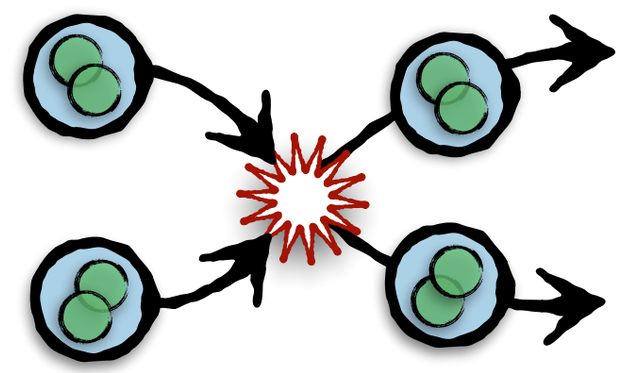
no scattering



wavefunction: $\psi \sim e^{ipz}$

$p =$ relative momentum

scattering



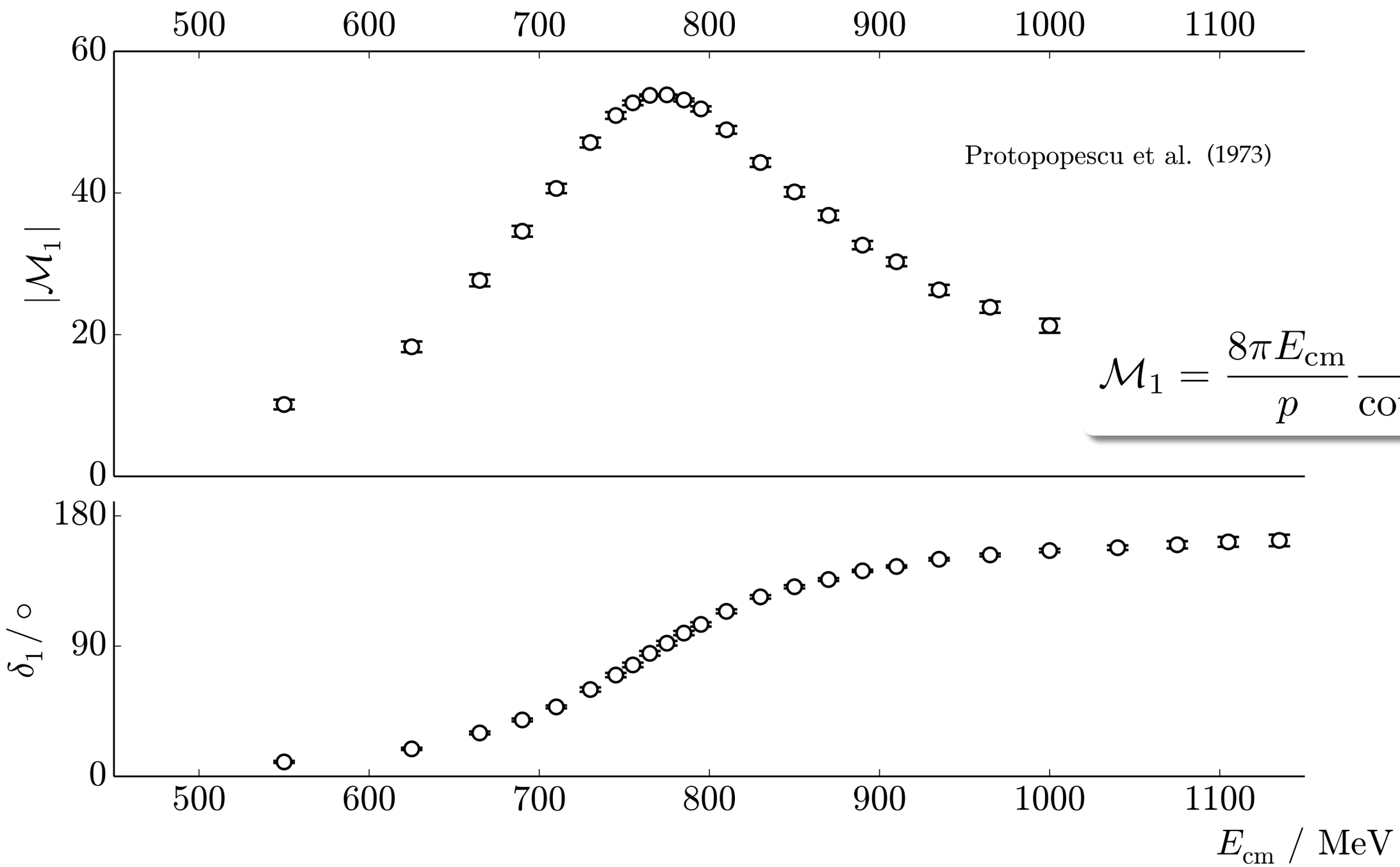
$$\psi \sim e^{ipz} + \frac{e^{i\delta}\sin\delta}{p}\frac{e^{ipr}}{r}$$

$\delta =$ scattering phase shift

"encode all physics"

A pseudo-quantitative definition

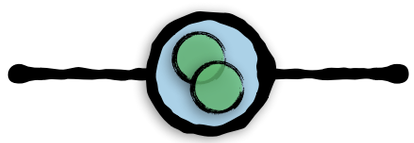
(bump in an amplitude - e.g., $\pi\pi$ scattering in ϱ-channel)



$$\mathcal{M}_1 = \frac{8\pi E_{\rm cm}}{p} \frac{1}{\cot\delta_1 - i}$$

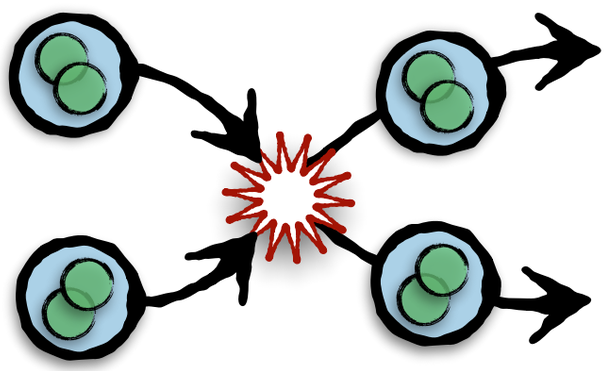
A quantitative definition

(poles in the complex plane)



$$\sim \frac{i}{p^2 - m^2}$$ [π propagator]

"poles correspond to particles"

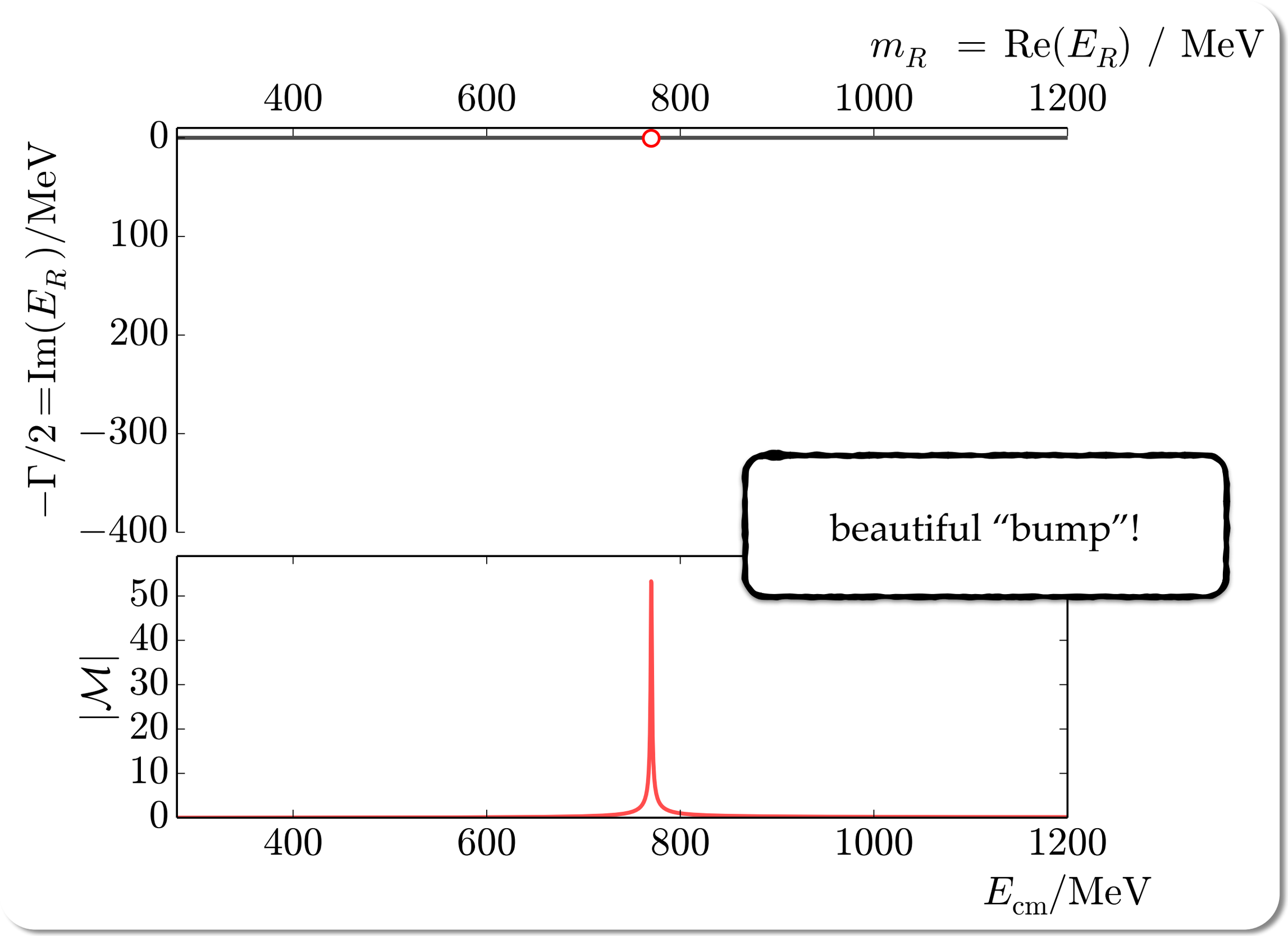


$$\sim i\mathcal{M}$$ [$\pi\pi$ scattering amplitude]

<u>poles:</u>

- below $\pi\pi$-threshold ⟶ bound state
- above $\pi\pi$-threshold ⟶ resonance ⟶ complex valued

A toy example



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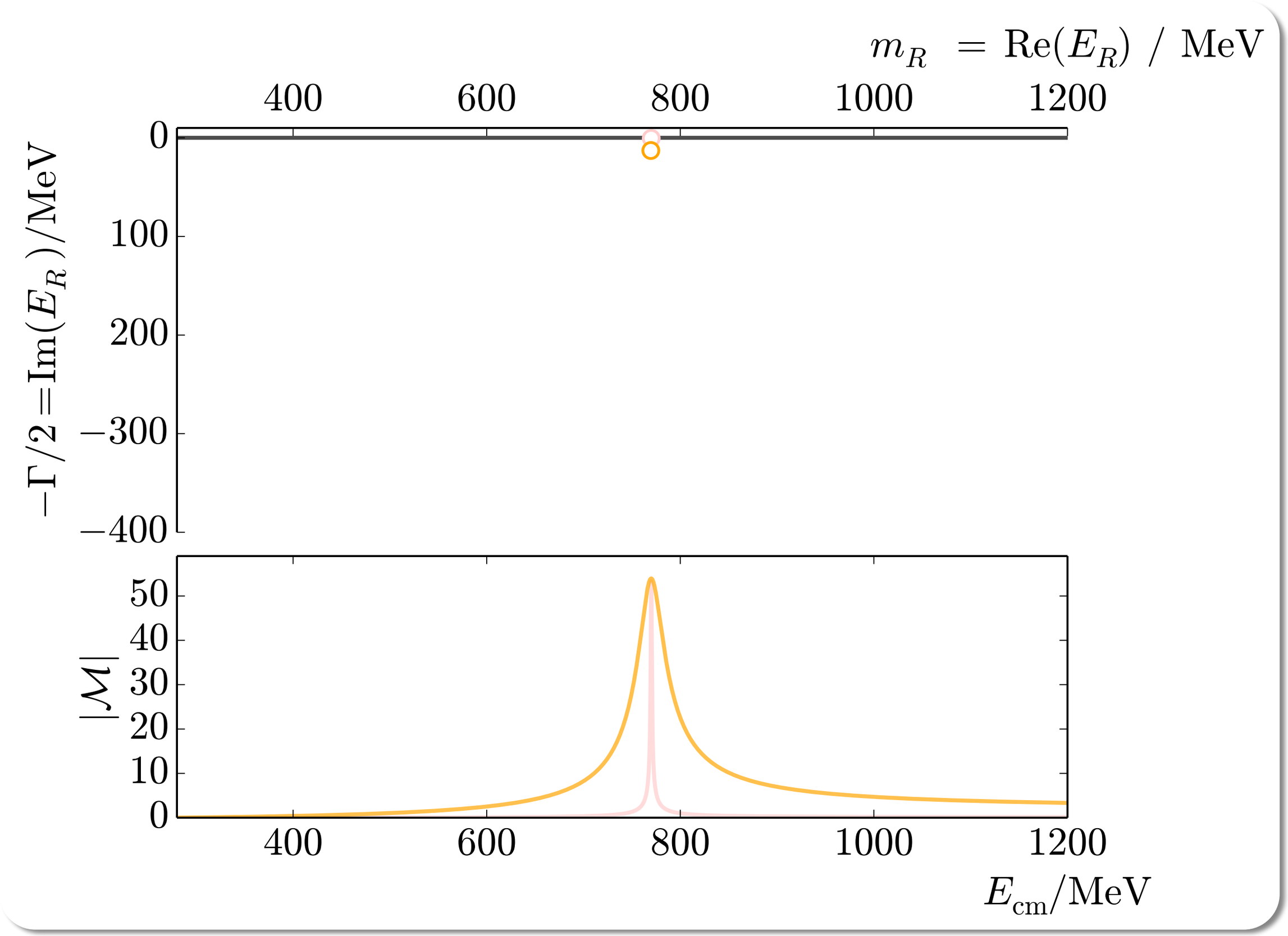

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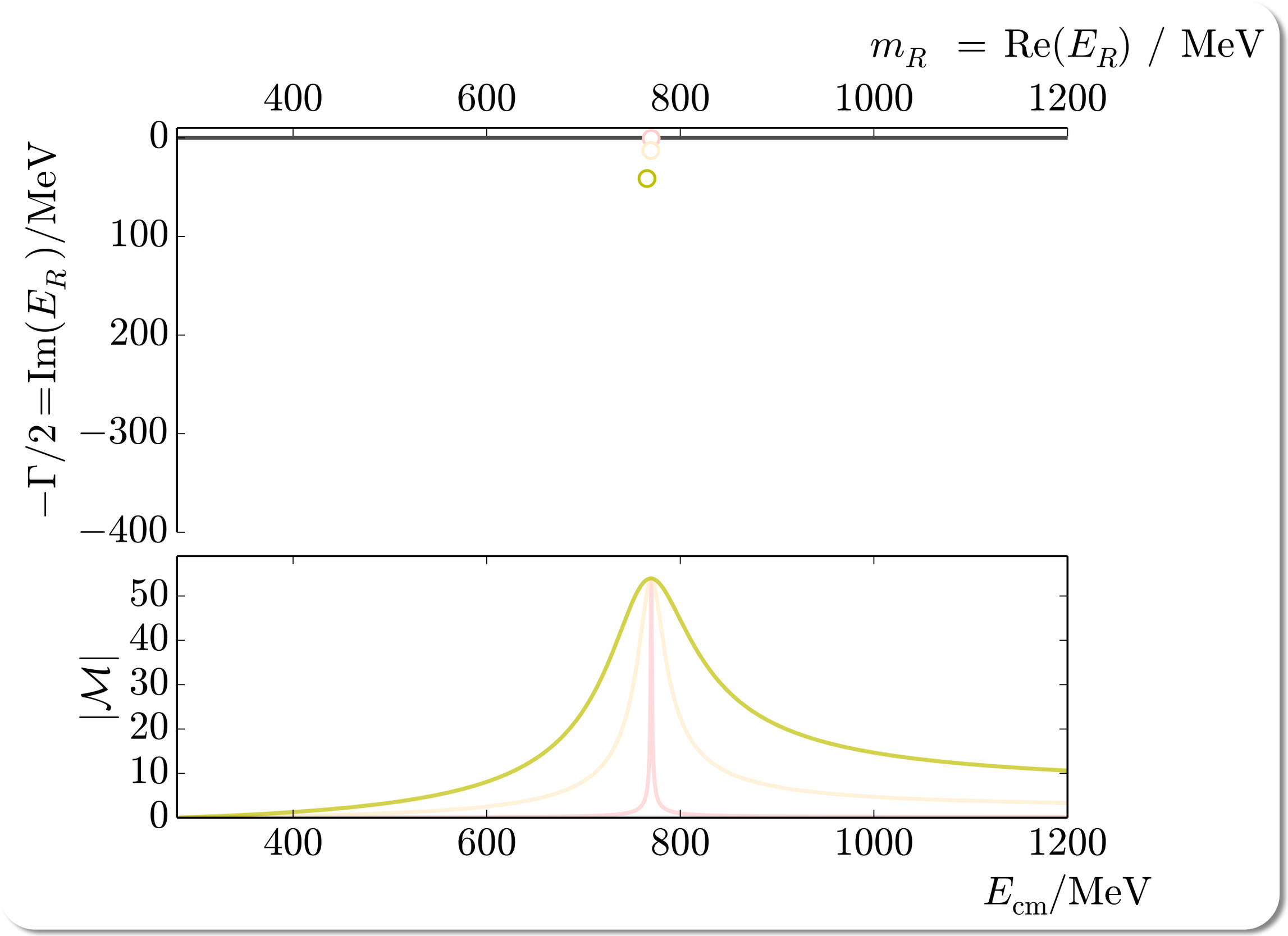

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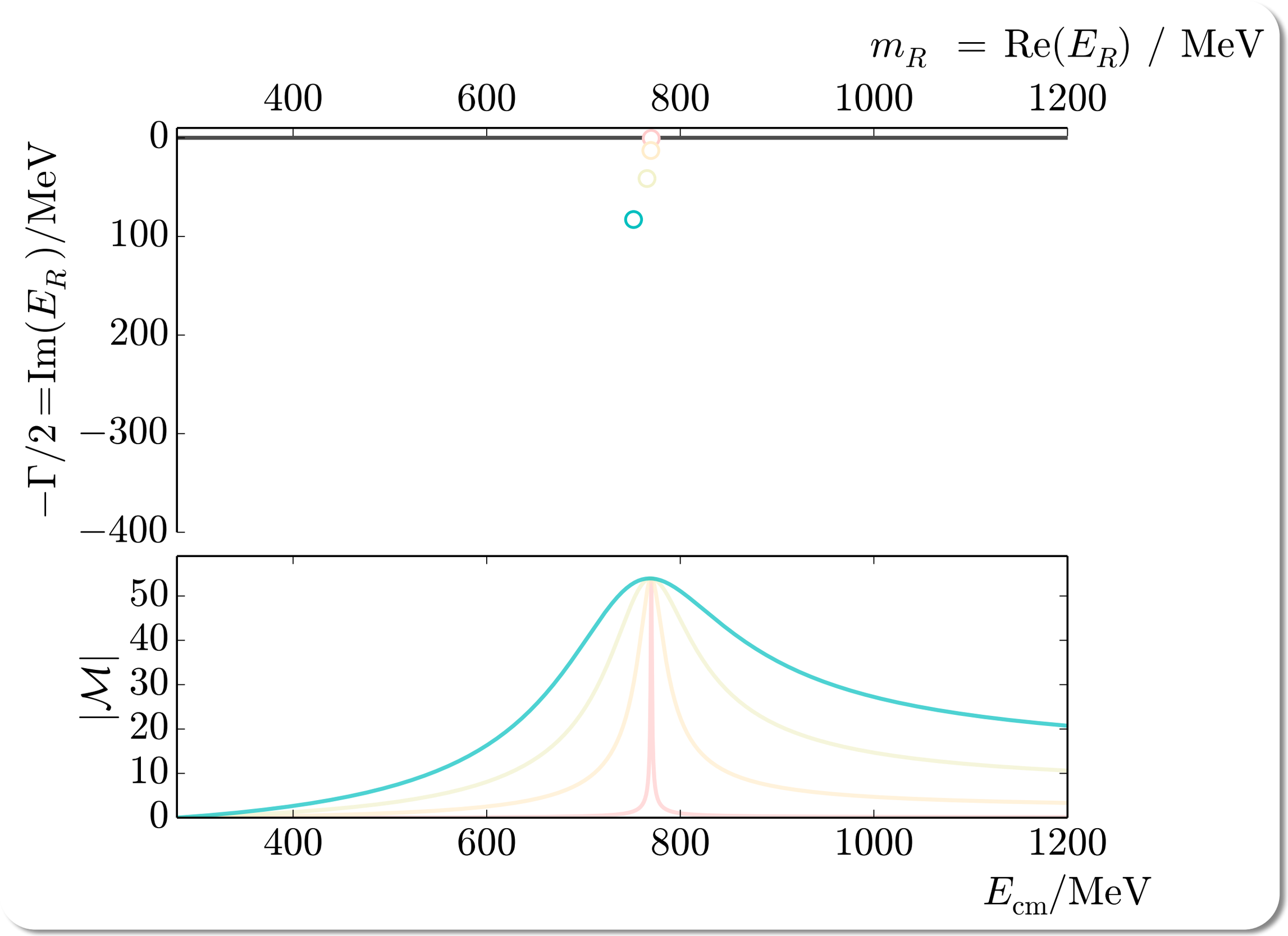

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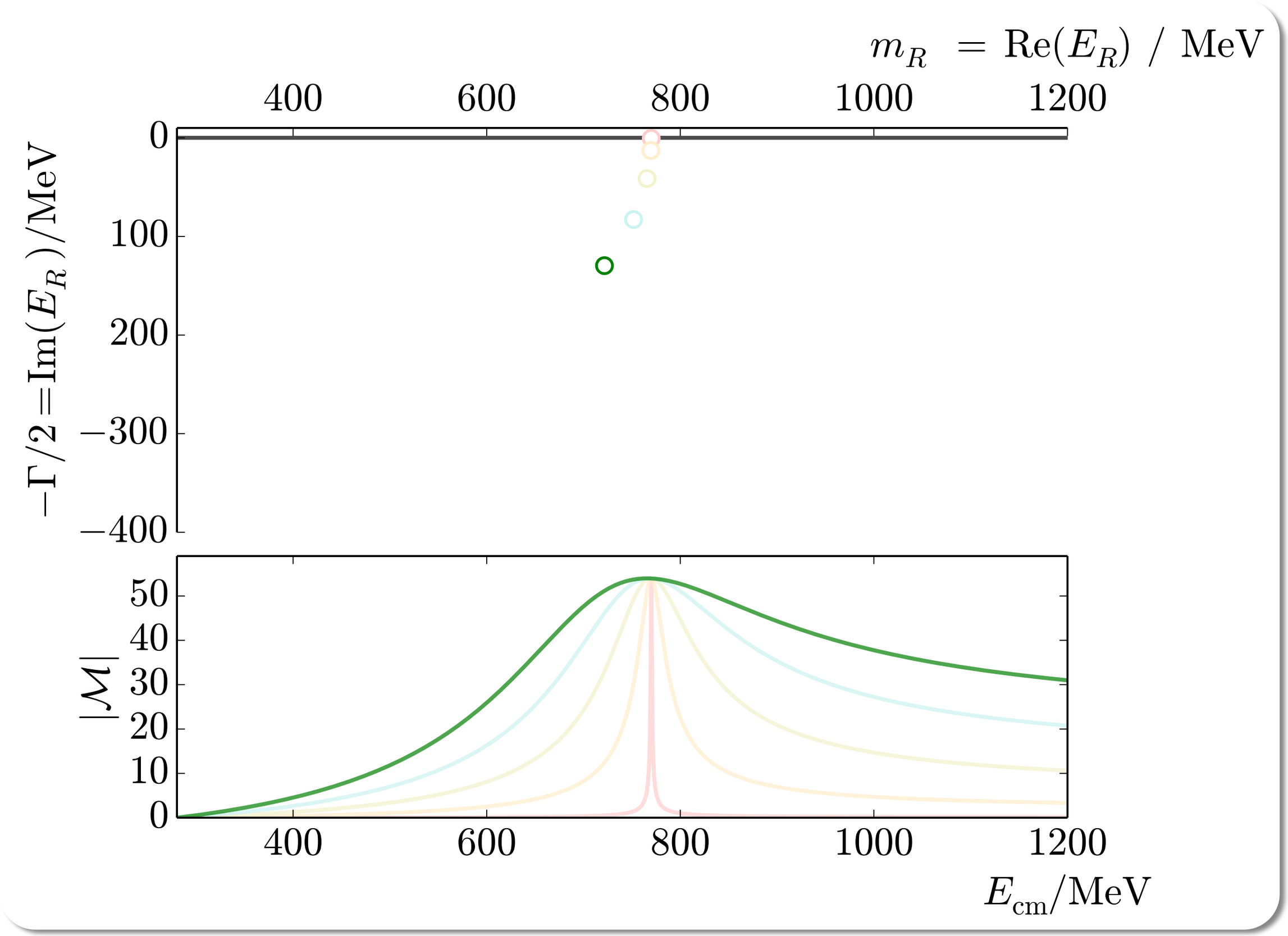

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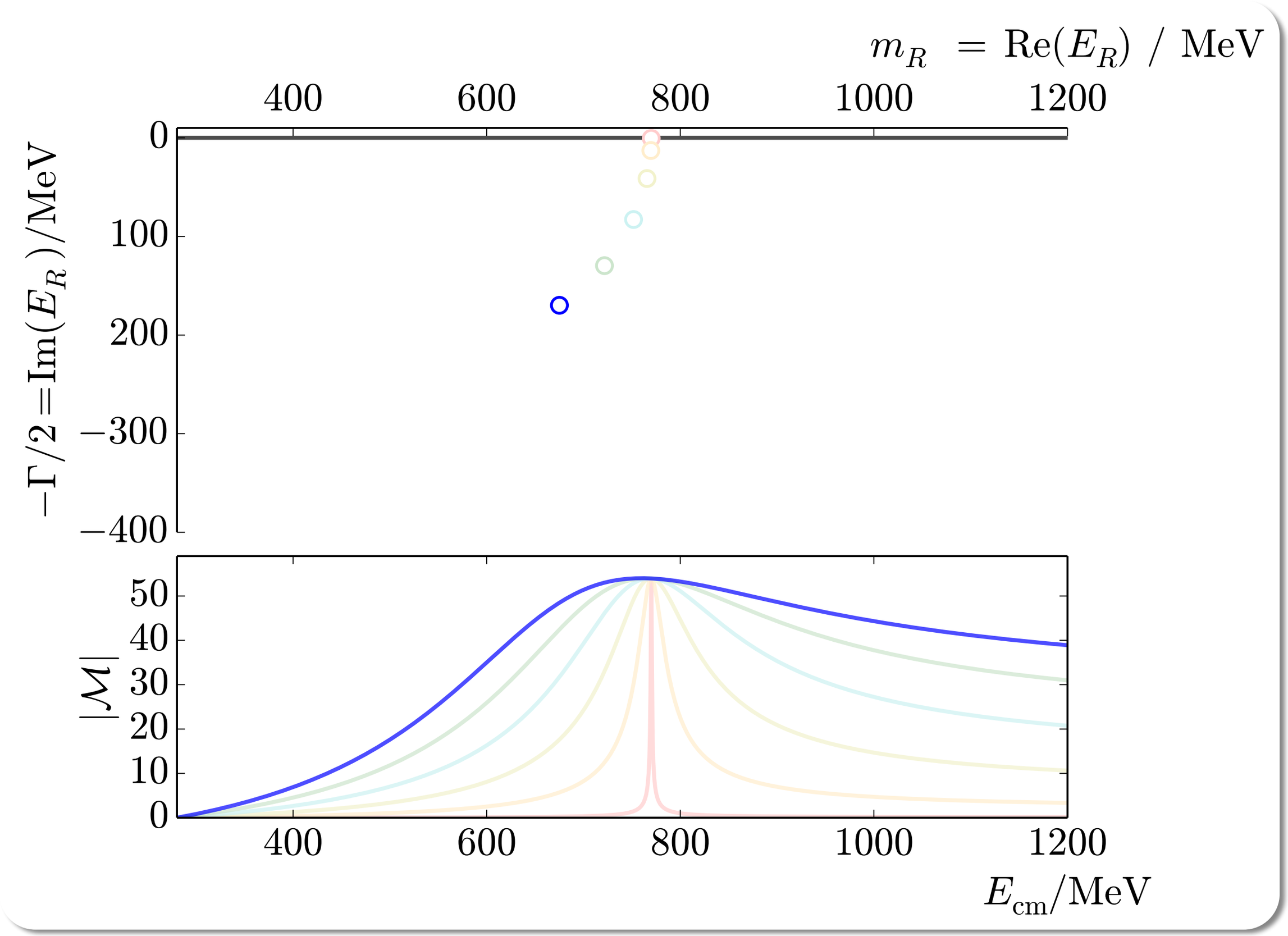

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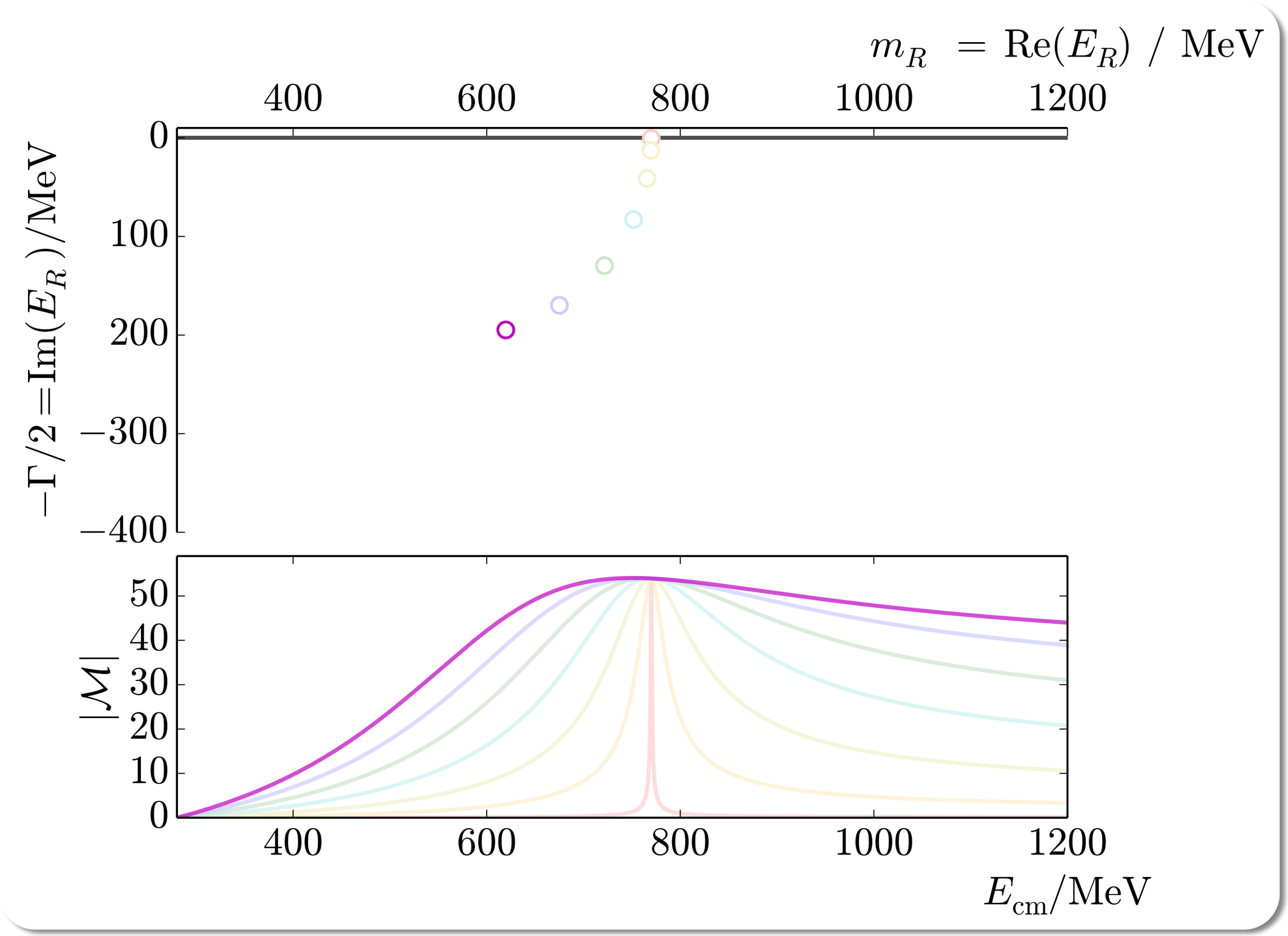
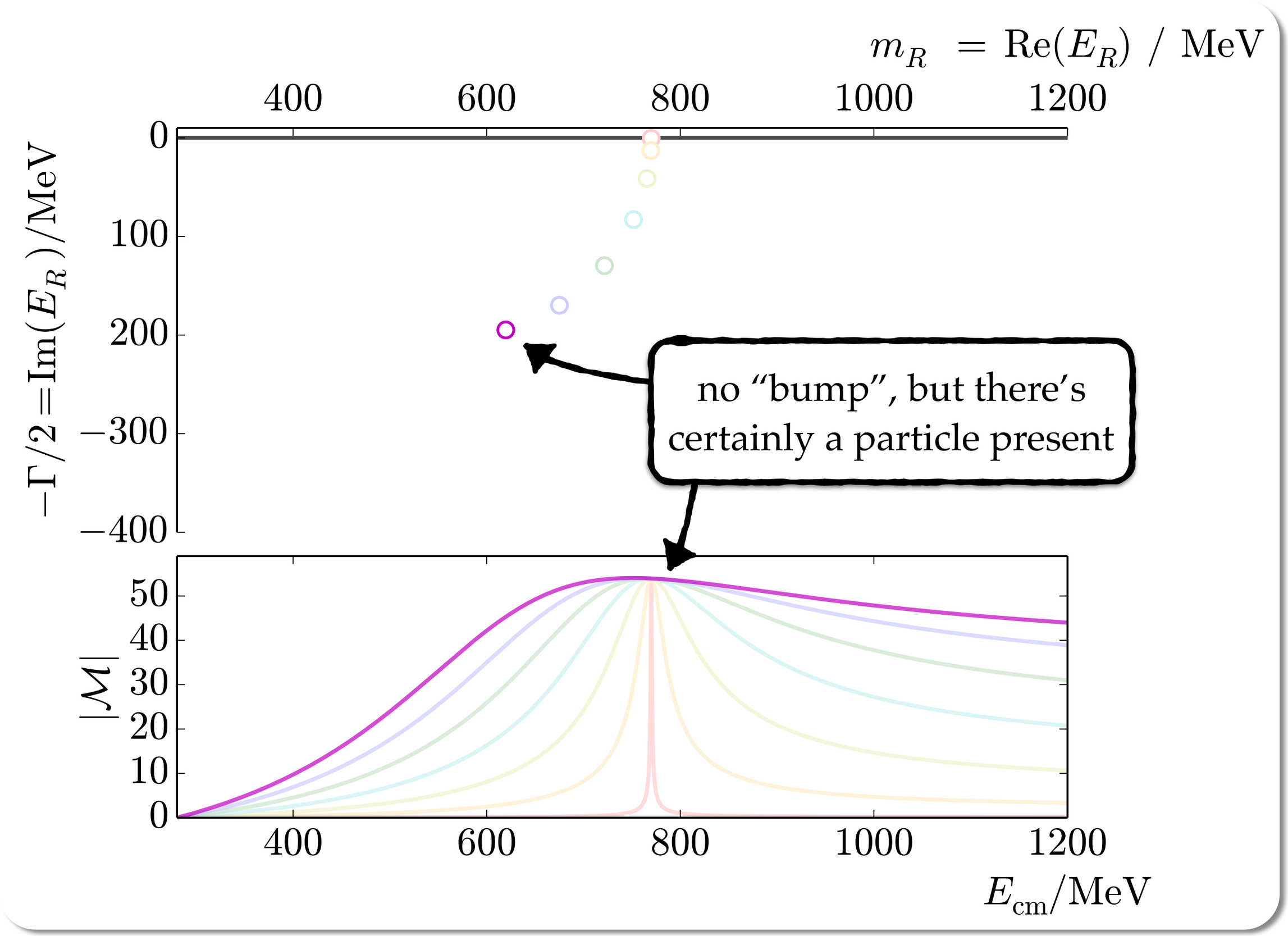


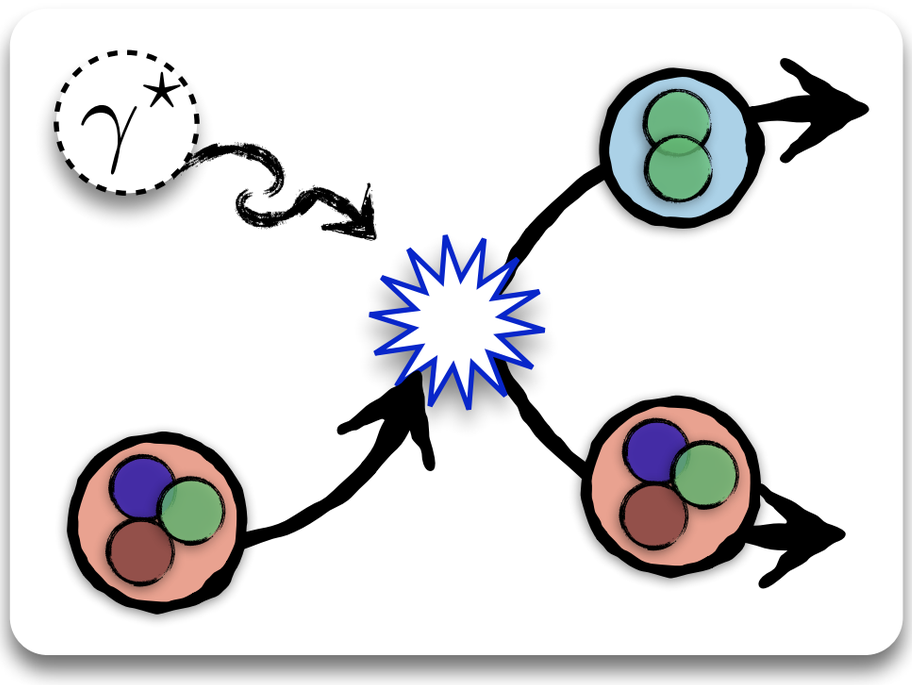
A toy example



$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{\mathbf{1}}{\mathbf{4}} \mathrm{tr} \left(\mathrm{GG} \right)$$

"Seven parameters describe ALL of nuclear physics"

$$\Lambda_{\mathrm{QCD}}, \frac{\mathrm{m_u}}{\Lambda_{\mathrm{QCD}}}, \frac{\mathrm{m_d}}{\Lambda_{\mathrm{QCD}}}, \frac{\mathrm{m_s}}{\Lambda_{\mathrm{QCD}}}, \frac{\mathrm{m_c}}{\Lambda_{\mathrm{QCD}}}, \frac{\mathrm{m_b}}{\Lambda_{\mathrm{QCD}}}, \alpha_{\mathrm{QED}}$$

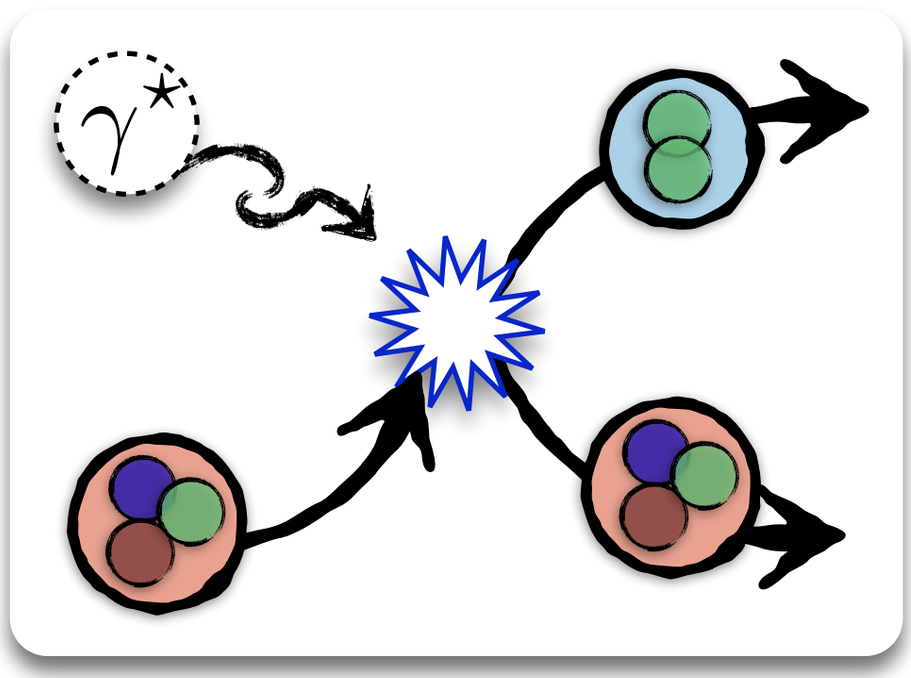


only dials at our disposal,
no *"tuning"* can be done!

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_{\mathrm{f}} \left(\mathrm{i} \not{D} - \mathrm{m_f} \right) \psi_{\mathrm{f}} - \frac{1}{4} \mathrm{tr} \left(\mathrm{GG} \right)$$

need to fit theory into computer

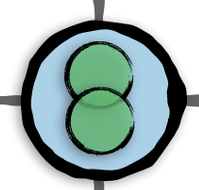
If only life were so easy!



$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{\mathbf{1}}{\mathbf{4}} \mathrm{tr} \left(\mathrm{GG} \right)$$

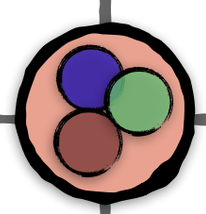
Lattice QCD in four *"easy"* steps

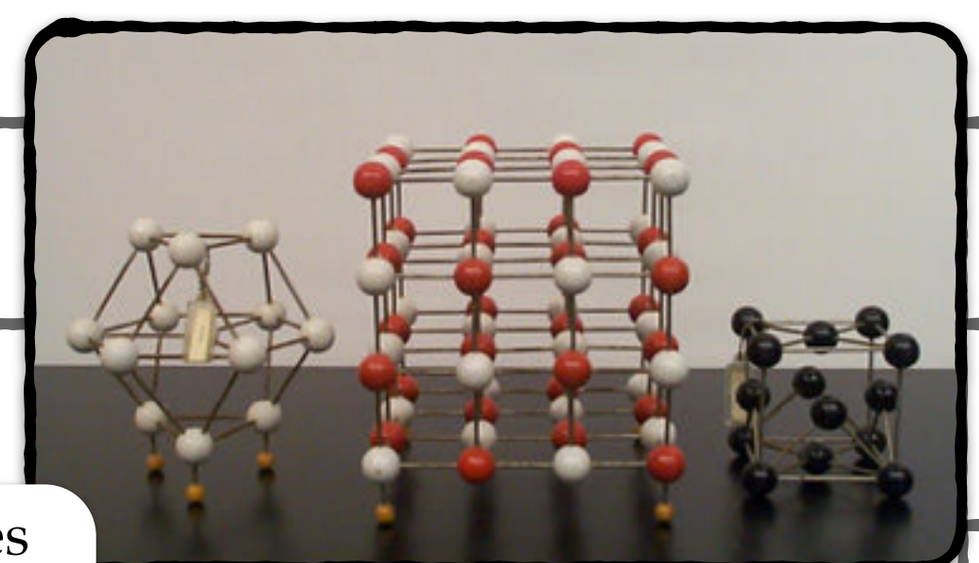
1. Discretize spacetime



- A separation scale between all points in space.

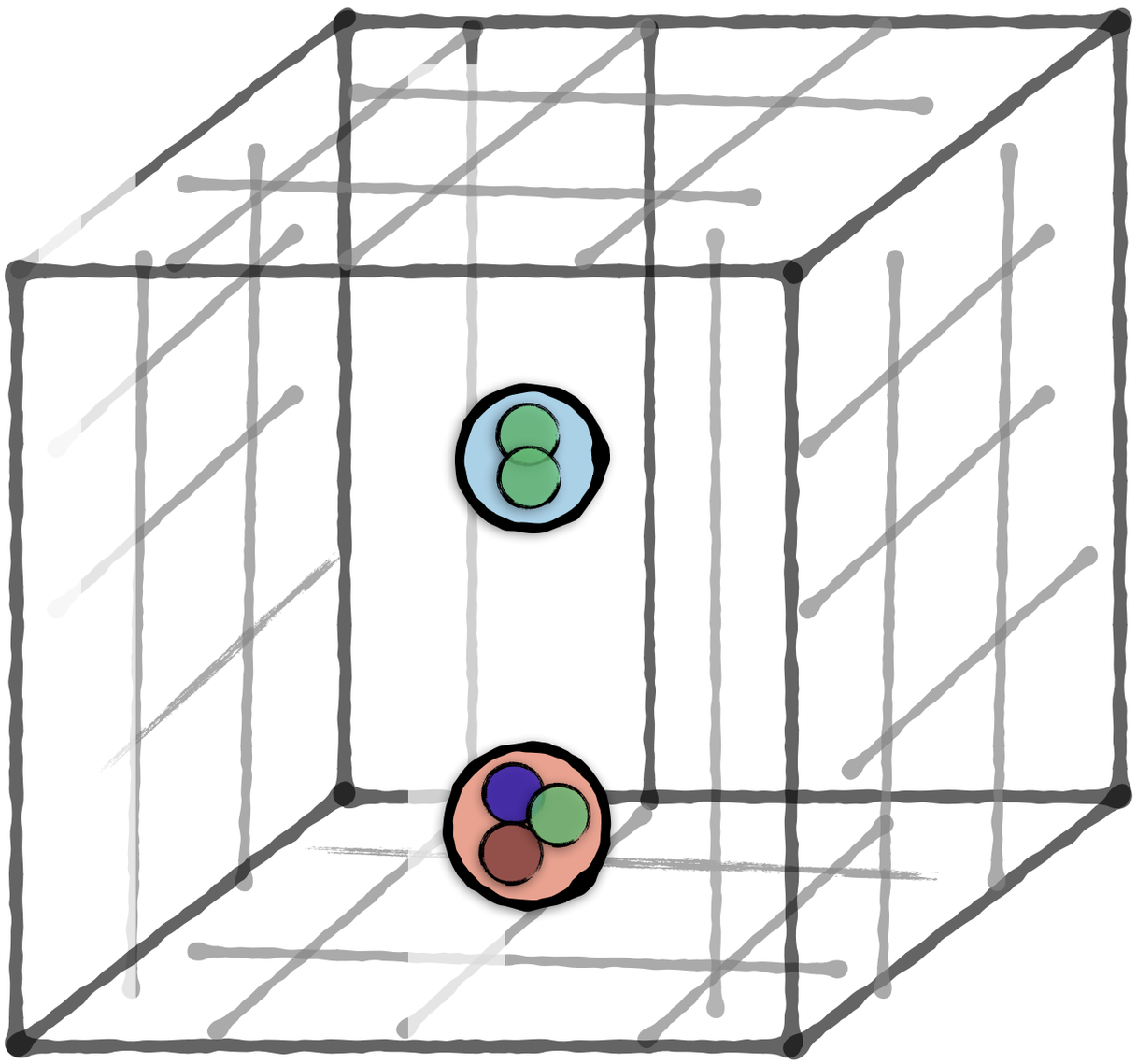
- ~0.1fm





more familiar lattices

$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{\mathbf{1}}{\mathbf{4}} \mathrm{tr} \left(\mathrm{GG} \right)$$

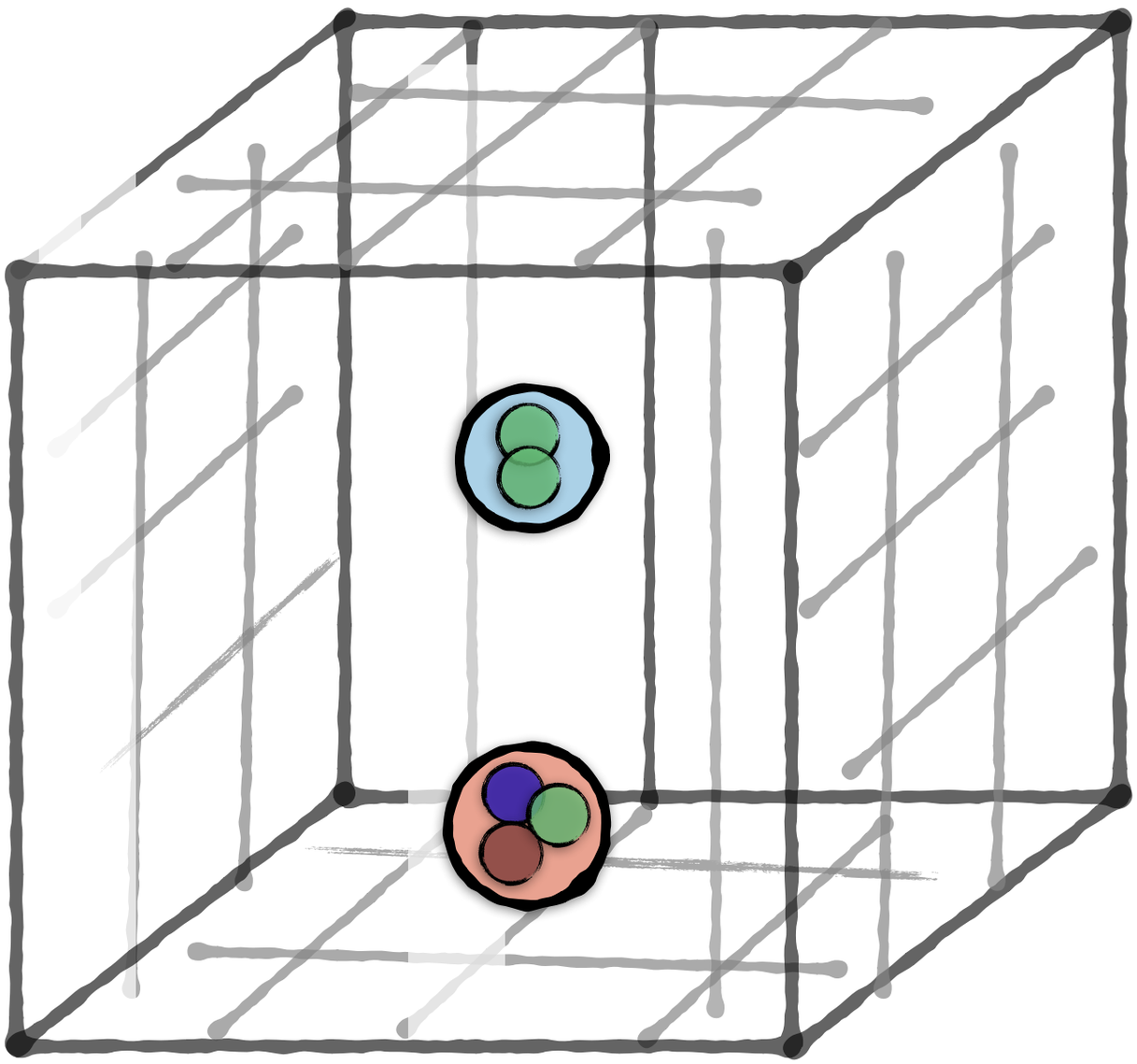


2. Truncate spacetime

- So it fits into a computer!
- ~4-6 fm

"It is a small world after all"

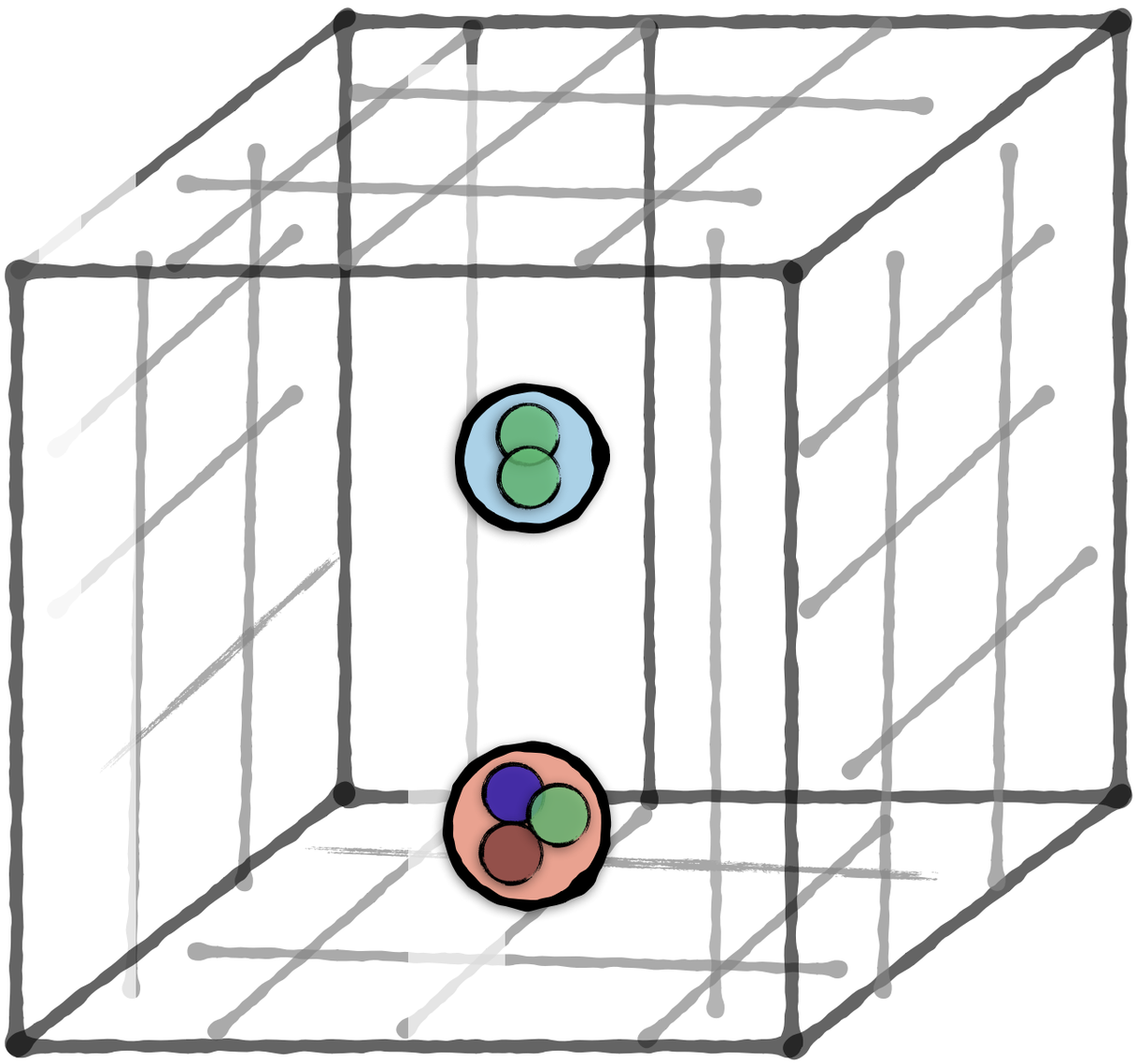
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{\mathbf{1}}{\mathbf{4}} \mathrm{tr} \left(\mathrm{GG} \right)$$



3. Tune quark masses

- Set them to physical values
- *"Easier said than done!"*
- Dial the quark masses

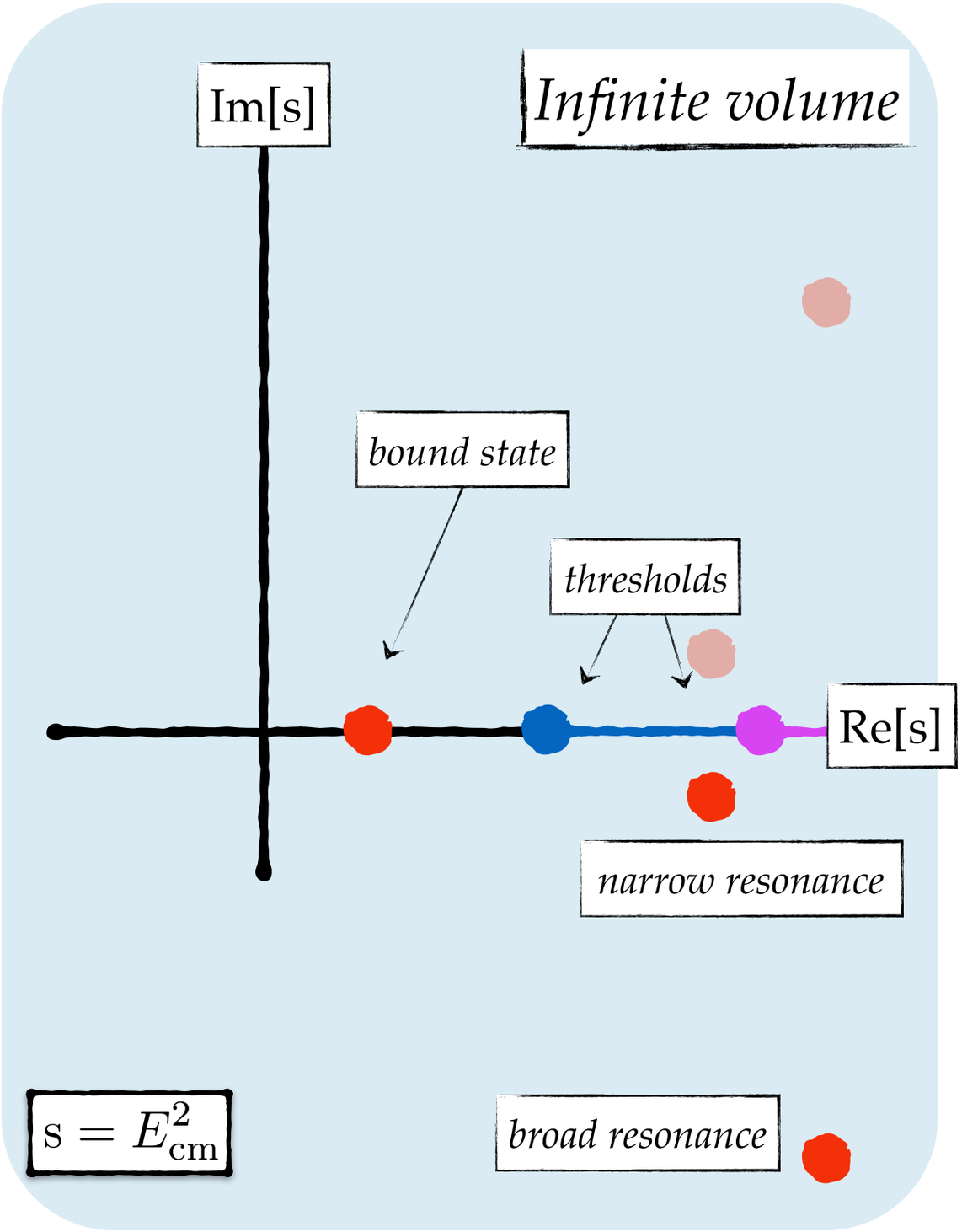
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{\mathbf{1}}{\mathbf{4}} \mathrm{tr} \left(\mathrm{GG} \right)$$



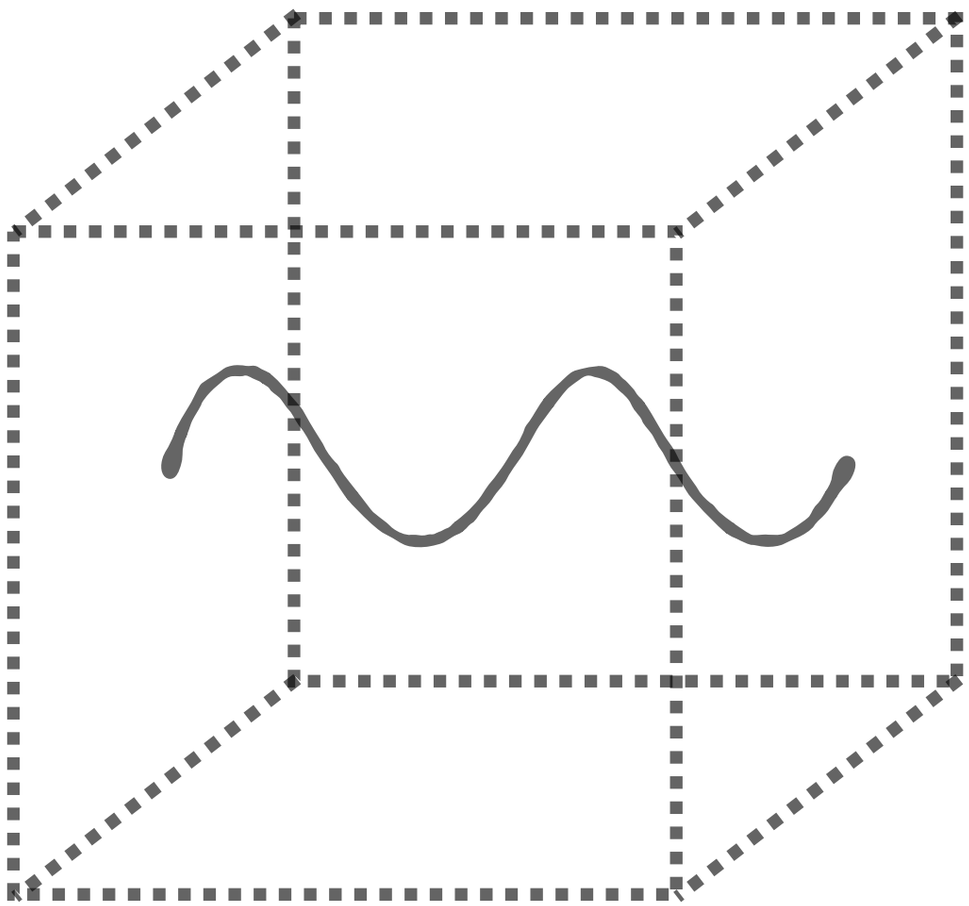
4. Perform an actual calculation

- Again, *"Easier said than done!"*

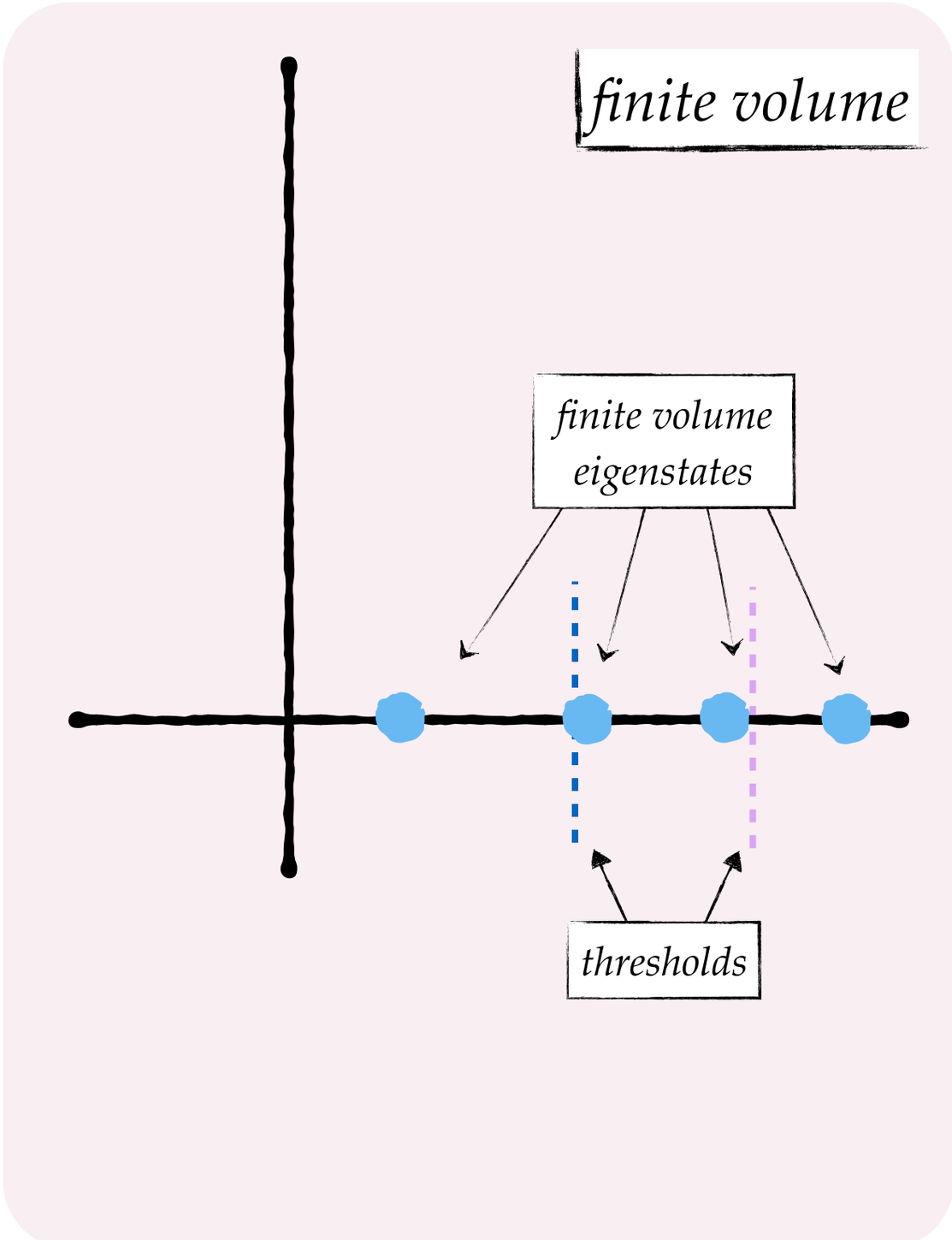
Finite vs. infinite volume spectrum



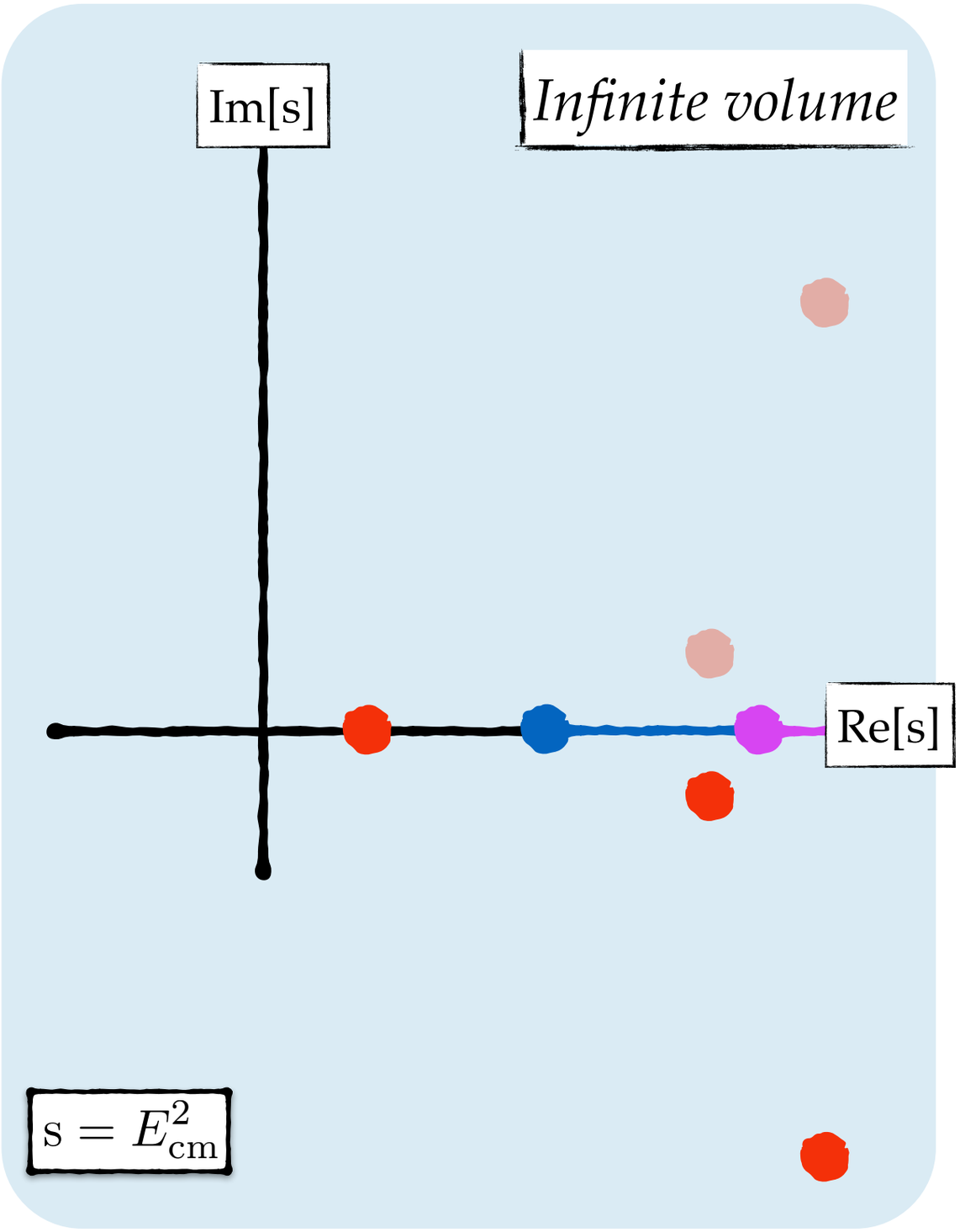
Finite vs. infinite volume spectrum

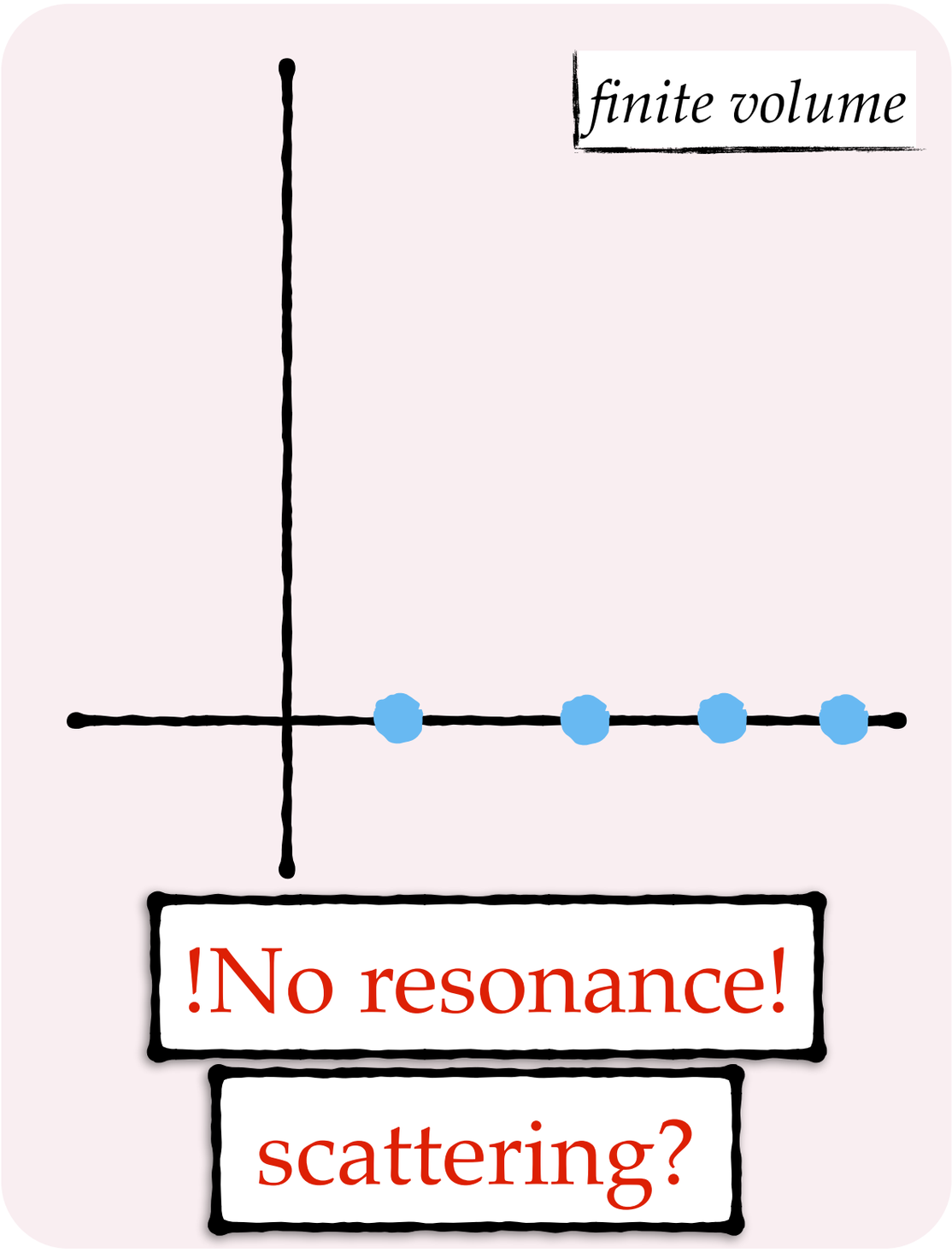


"only a finite number of modes can exist in a finite volume"



Finite vs. infinite volume spectrum



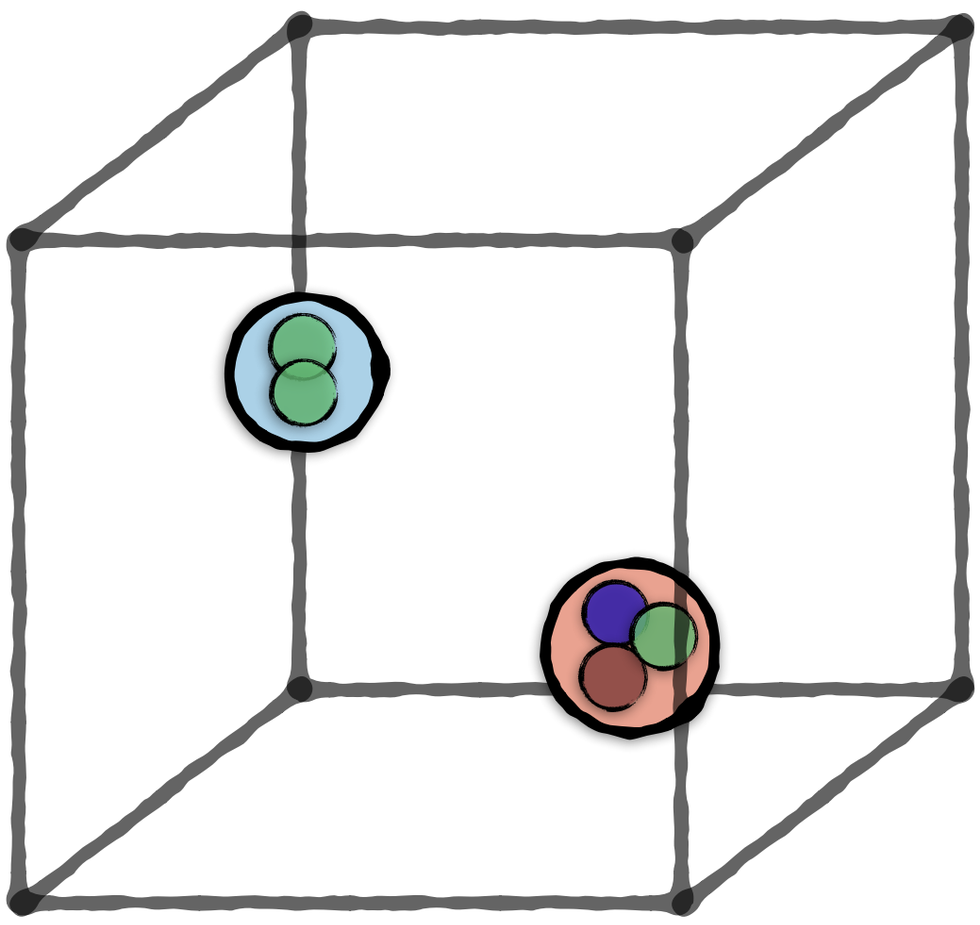


Scattering in finite volume: *impossible!*

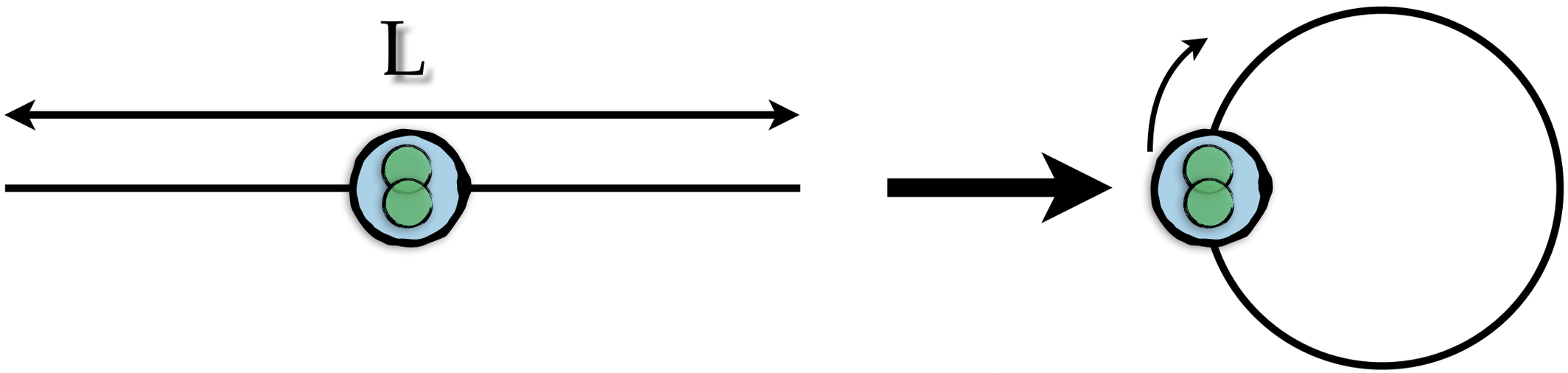
Finite volume - a necessity for lattice QCD

- No asymptotic states, i.e., no scattering, resonances, etc.
- Challenging, but ***not*** an limitation
- Finite volume effects allow us to determine amplitudes

Huang & Yang (1957)
Lüscher (1986)
Lellouch & Lüscher (2000)



Physics in a 1D-box

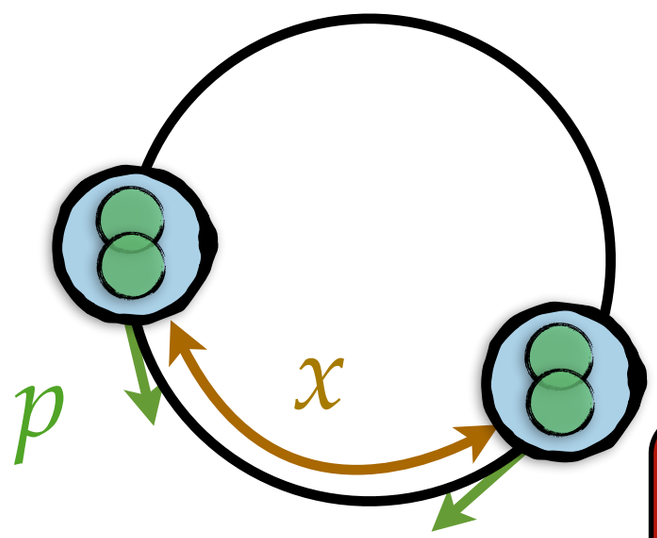


$$\phi(x) \sim e^{ipx}$$

Periodicity: $$L\ p_n = 2\pi n$$

Physics in a 1D-box

Two identical particles:



infinite volume scattering phase shift

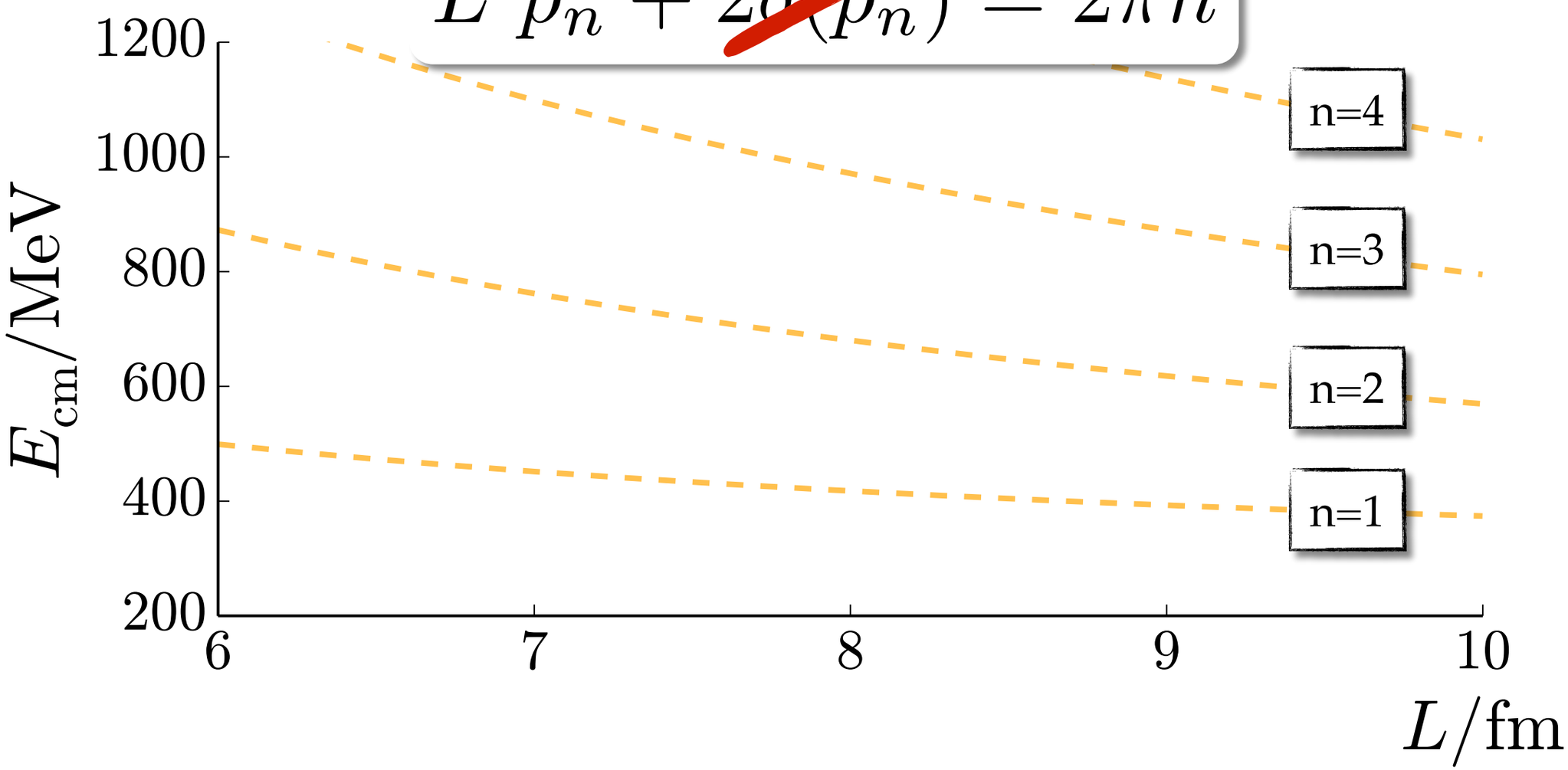
$$\psi(x) \sim \cos(p|x| + \delta(p))$$

Asymptotic wavefunction

Periodicity: $$L\ p_n + 2\delta(p_n) = 2\pi n$$

Physics in a 1D-box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



Physics in a 1D-box

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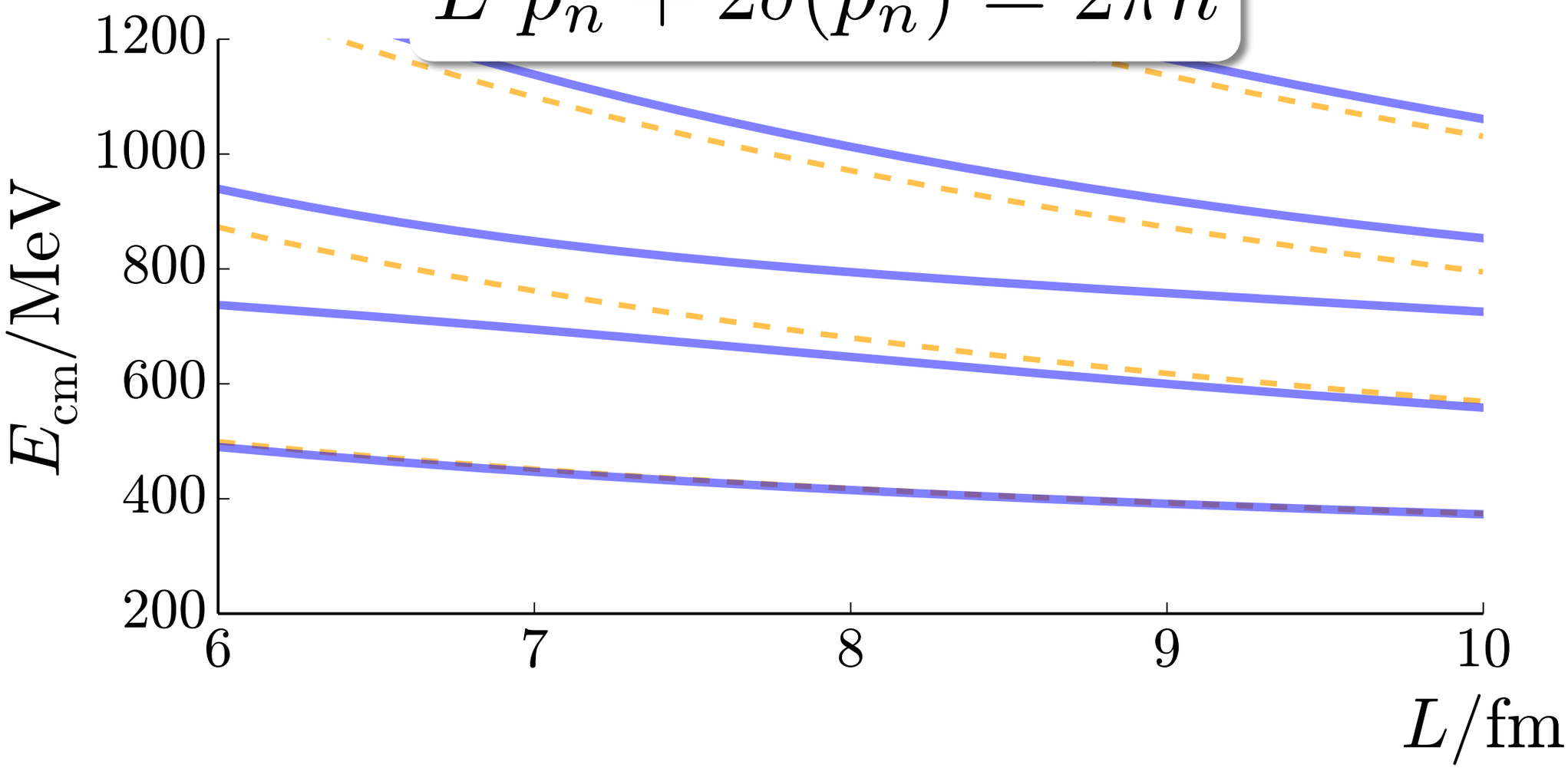

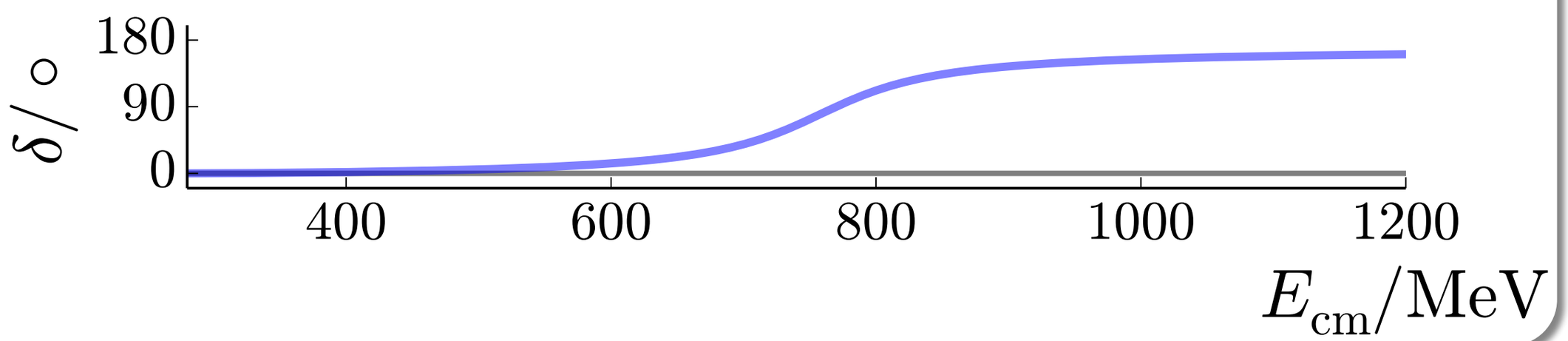

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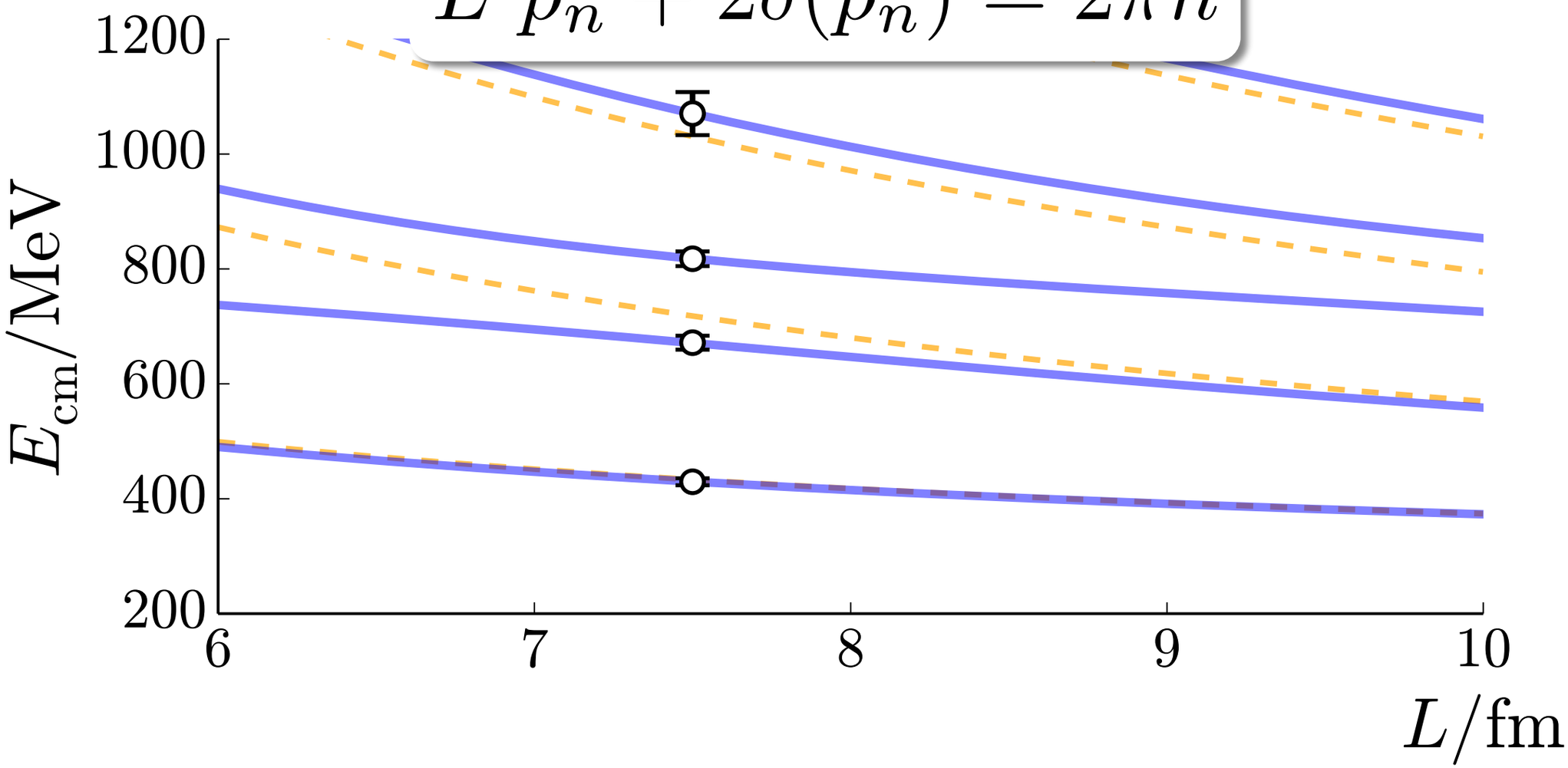
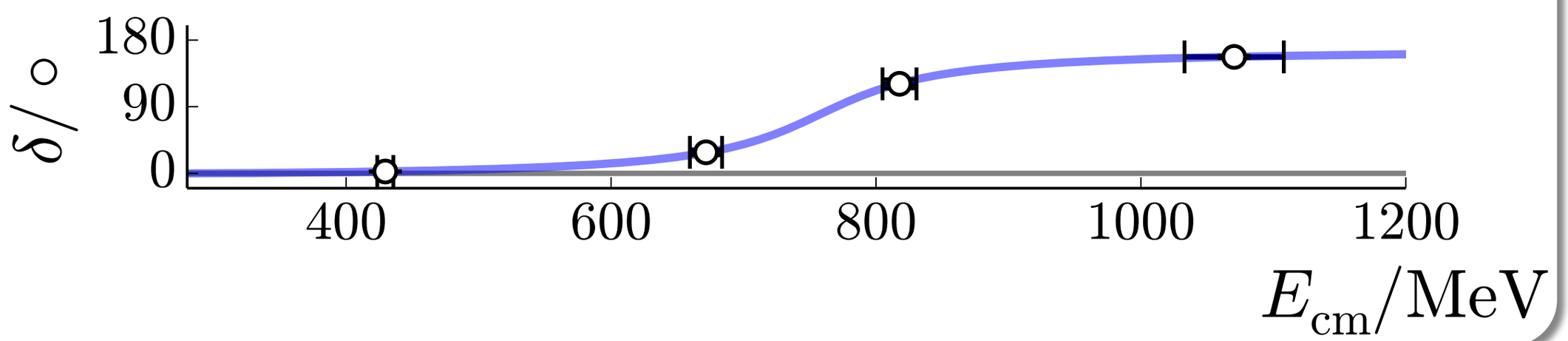
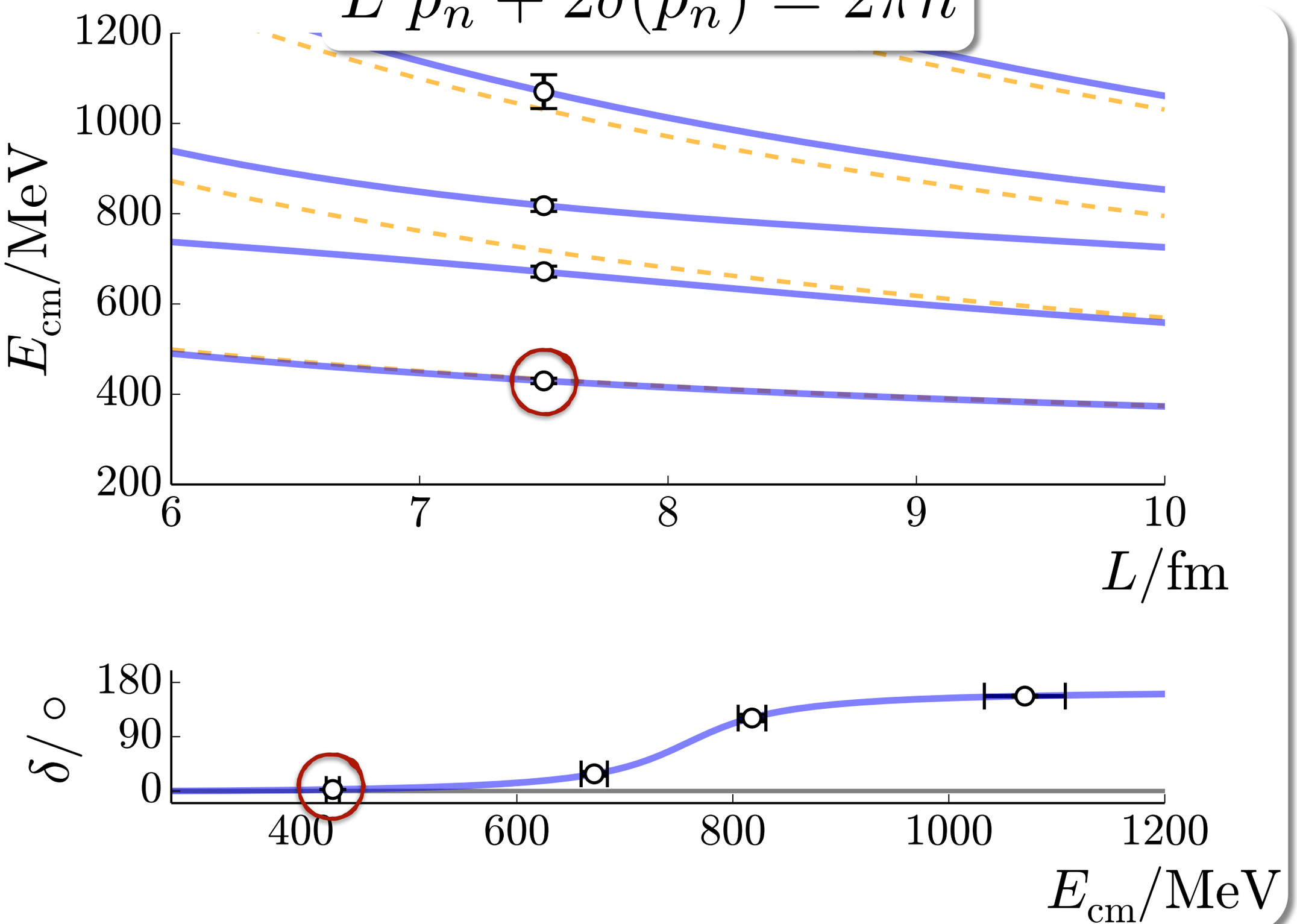
$$L\ p_n + 2\delta(p_n) = 2\pi n$$





Physics in a 1D-box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



Physics in a 1D-box

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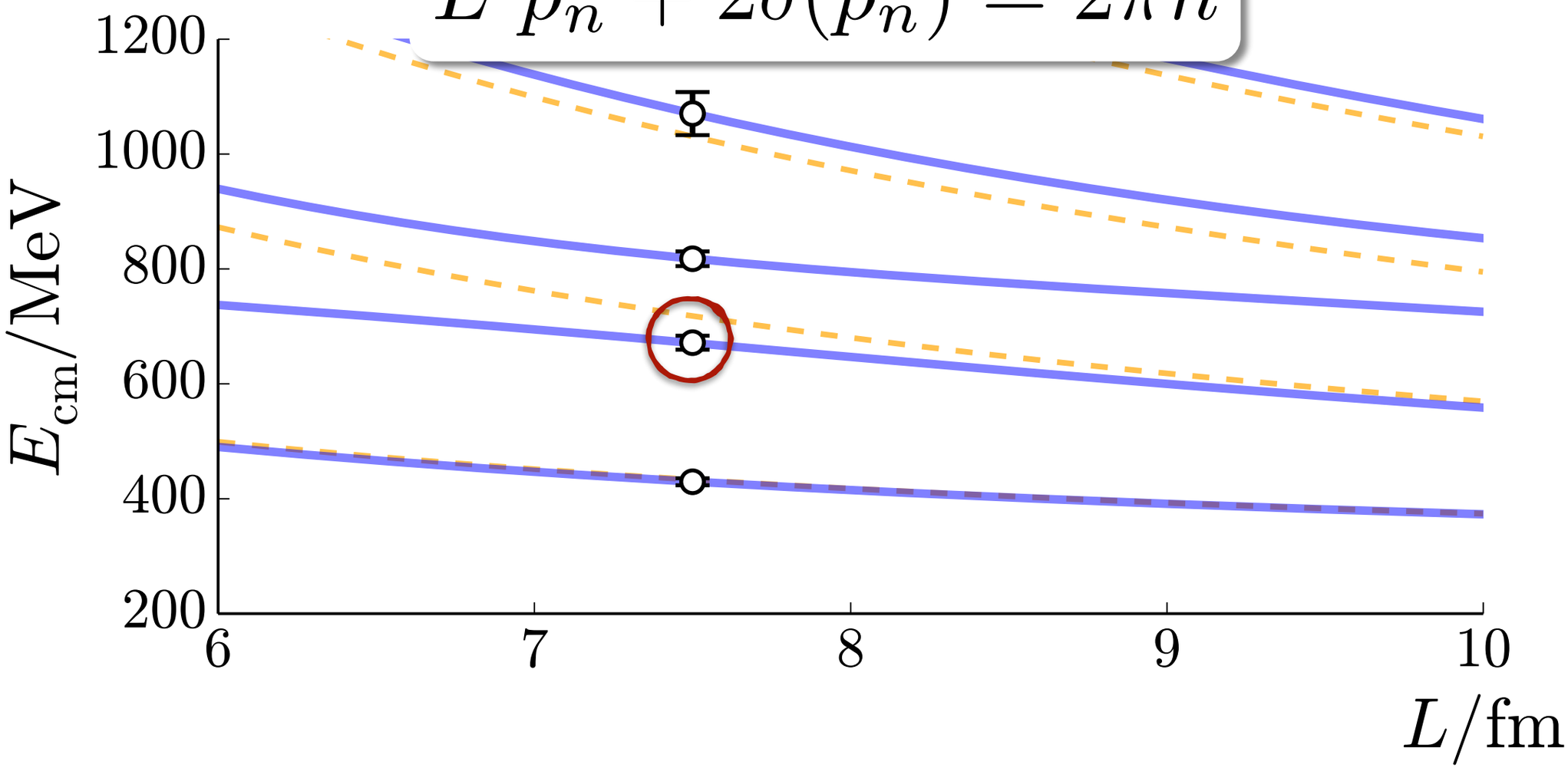

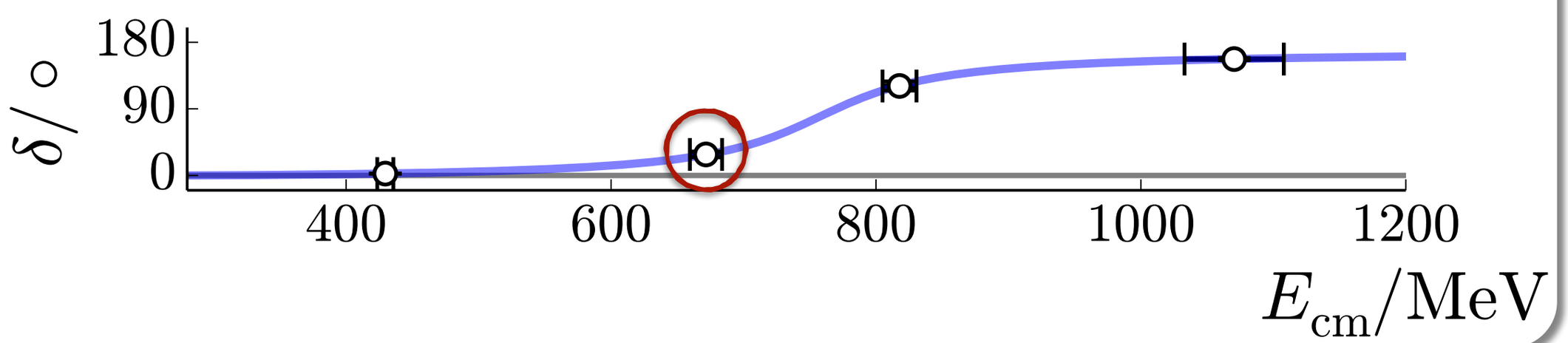

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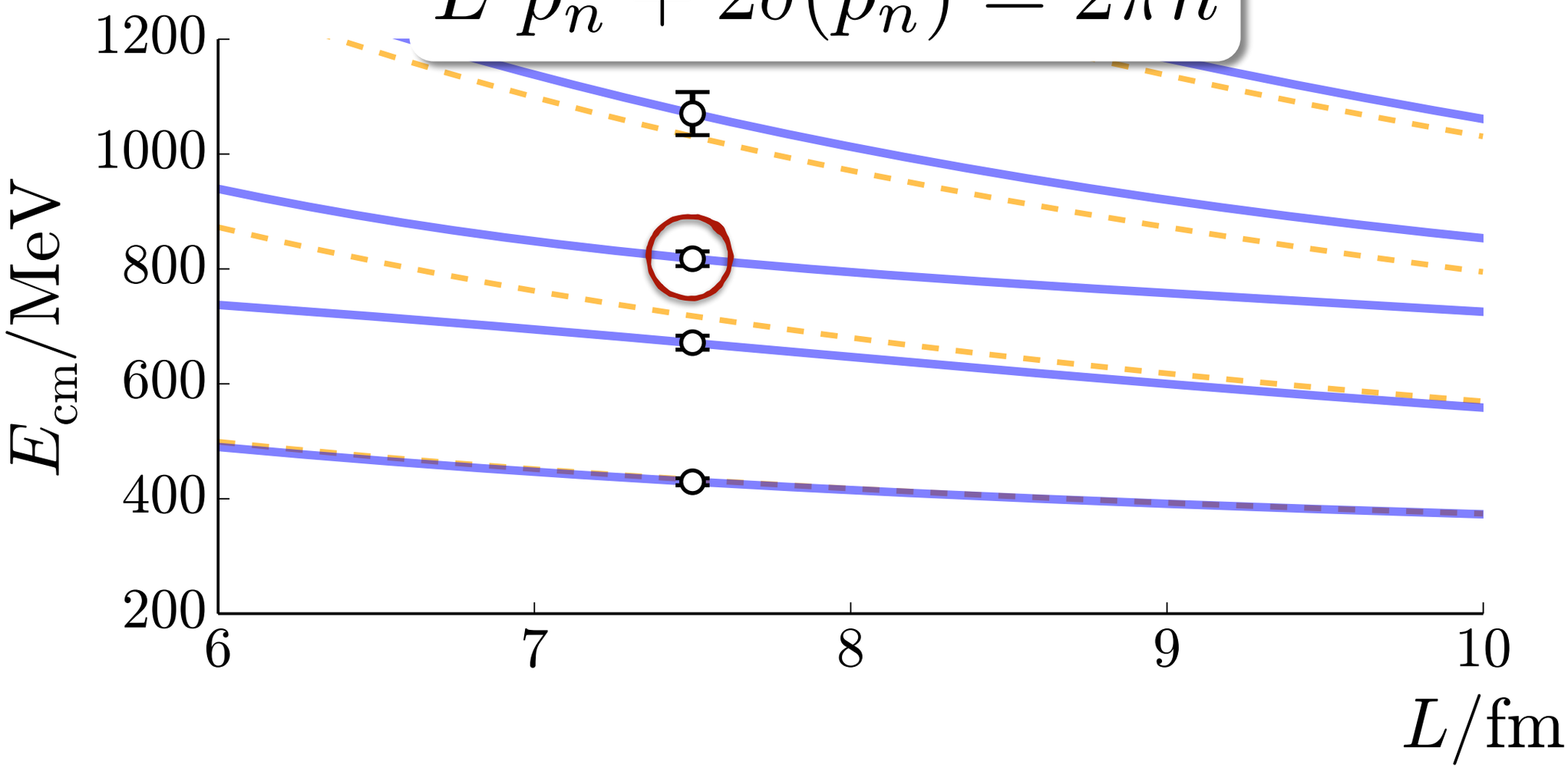
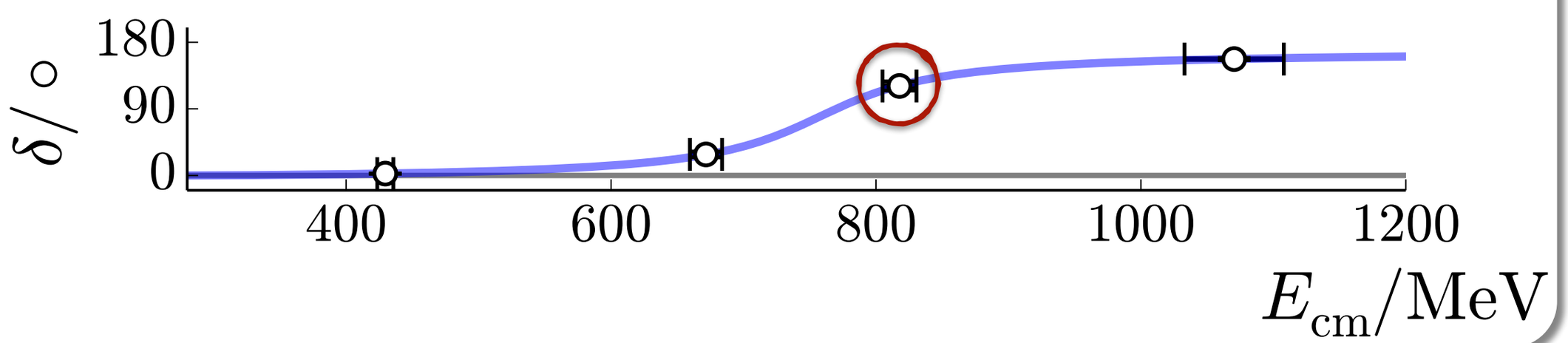
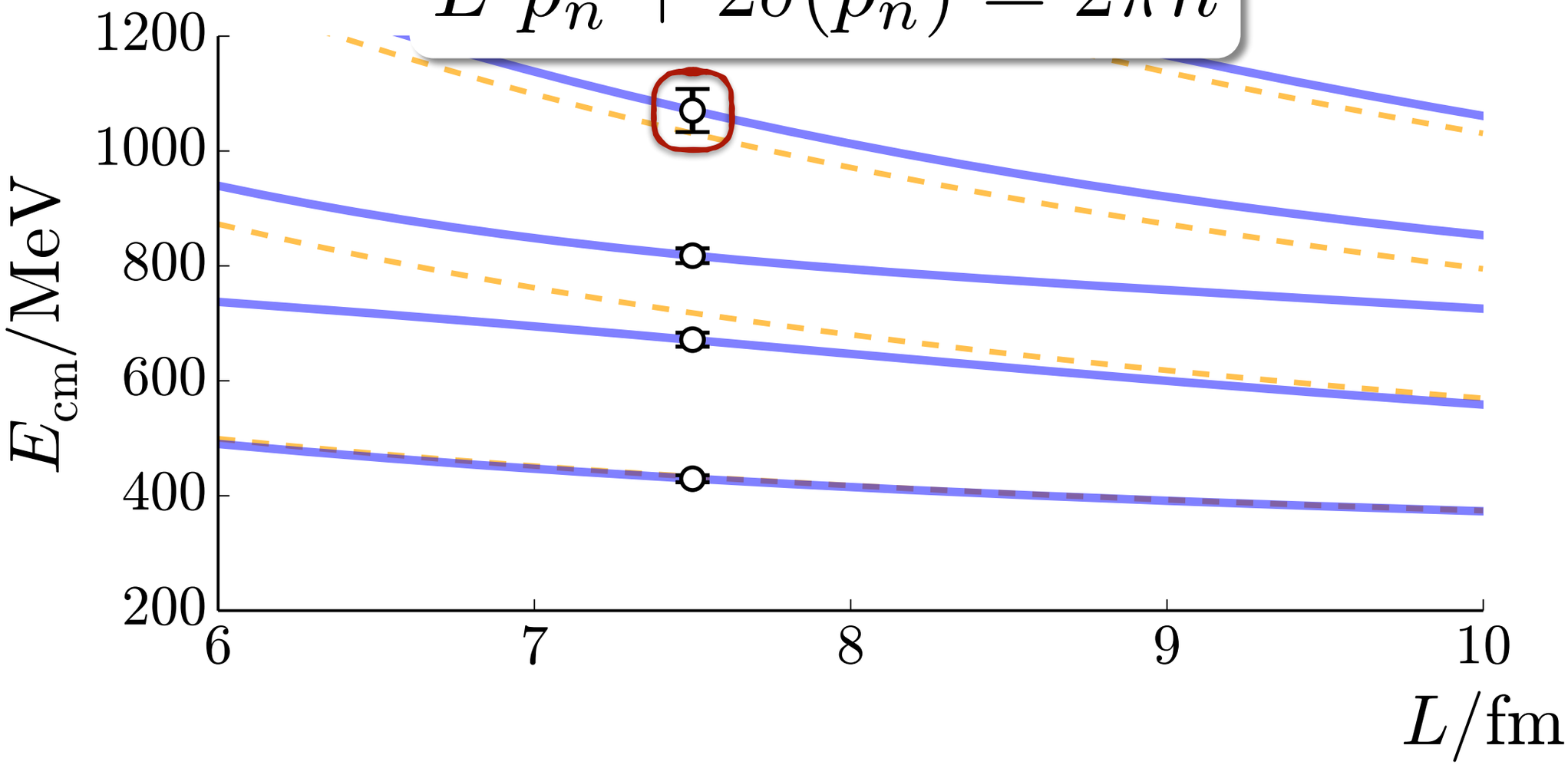
$$L\ p_n + 2\delta(p_n) = 2\pi n$$

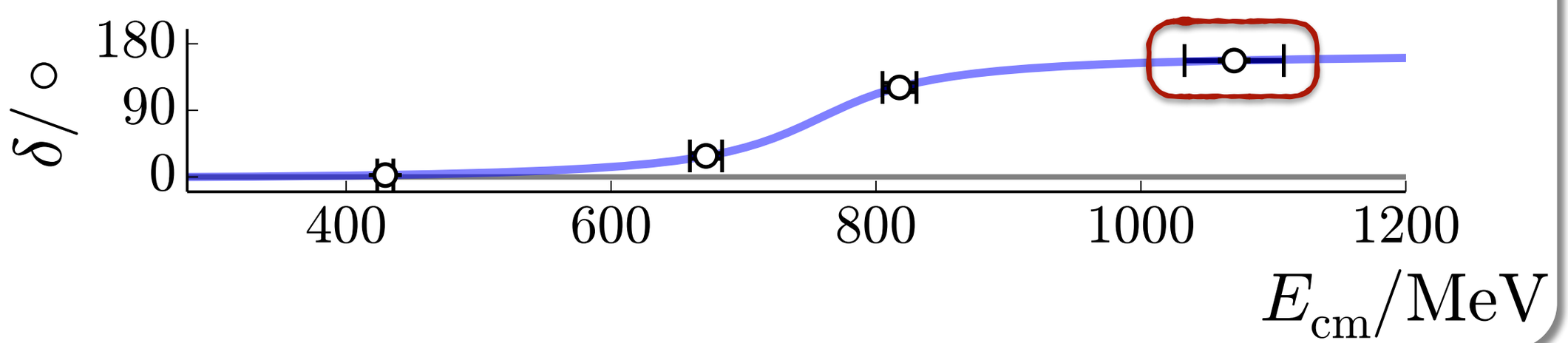




Physics in a 1D-box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$





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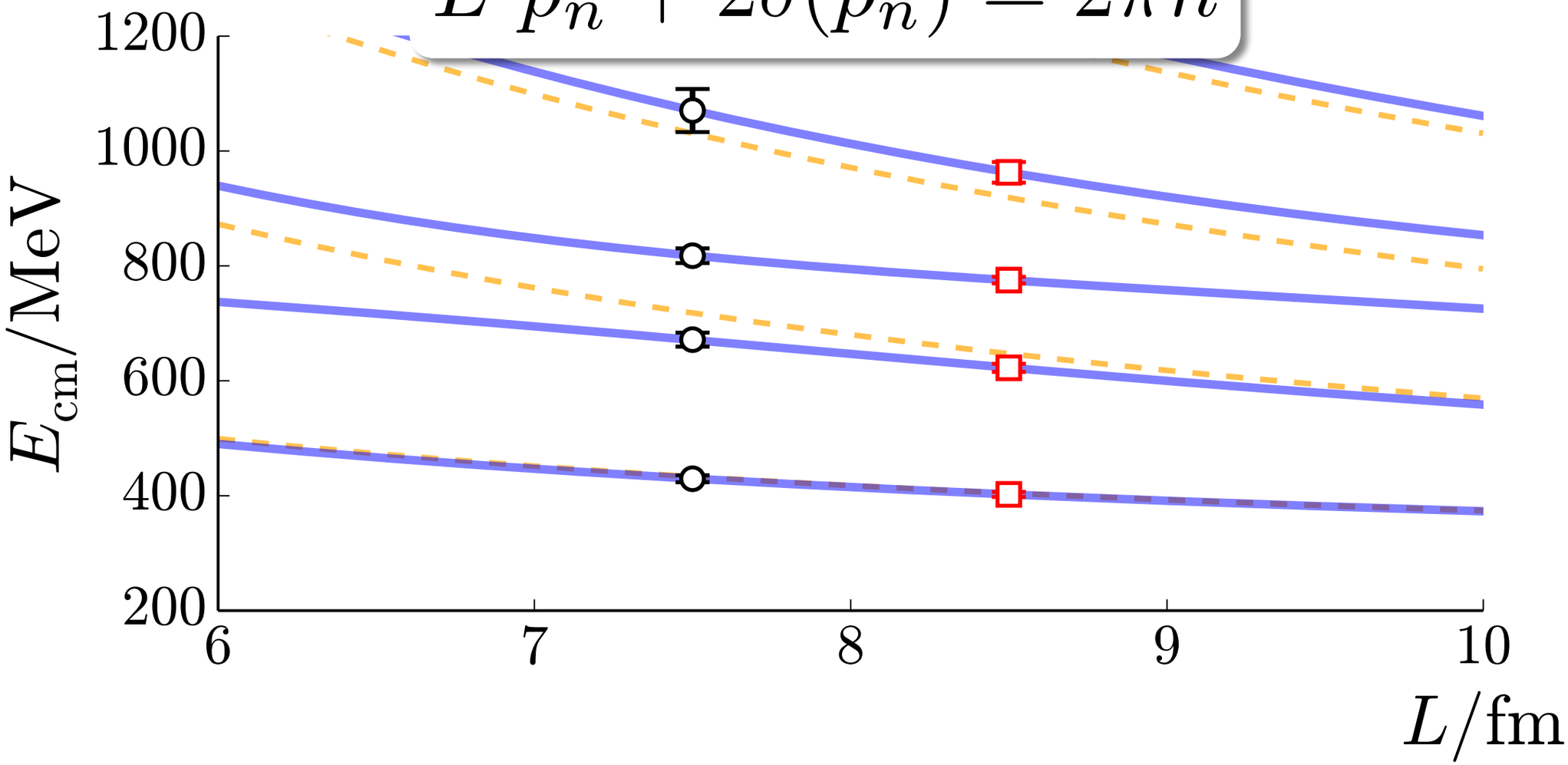
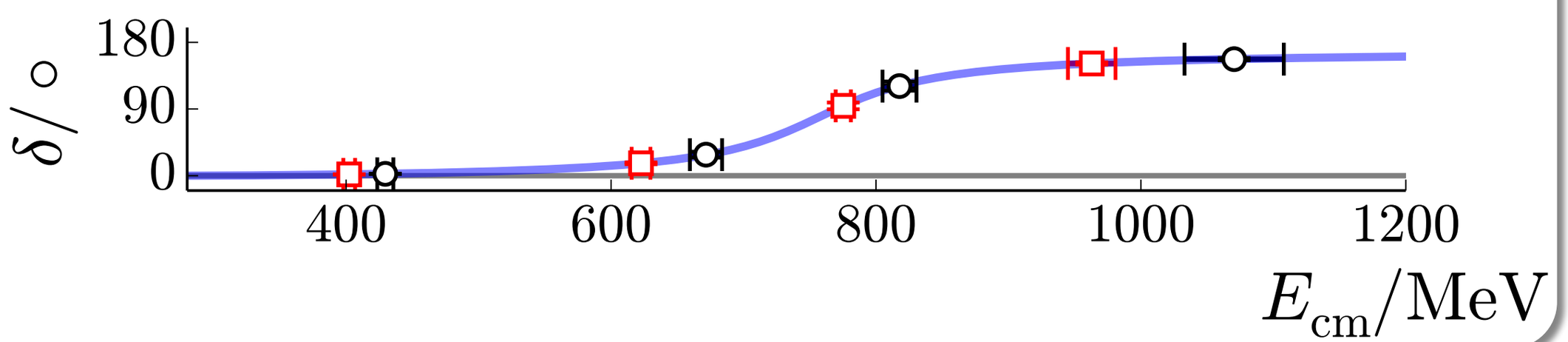




3D Quantization condition

Lüscher formalism

- Lüscher (1986), (1991)
- Rummukainen and Gottlieb (1995)
- Bedaque (2004)
- Li and Liu (2004)
- Feng, Li, and Liu (2004)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and. Sharpe (2005)

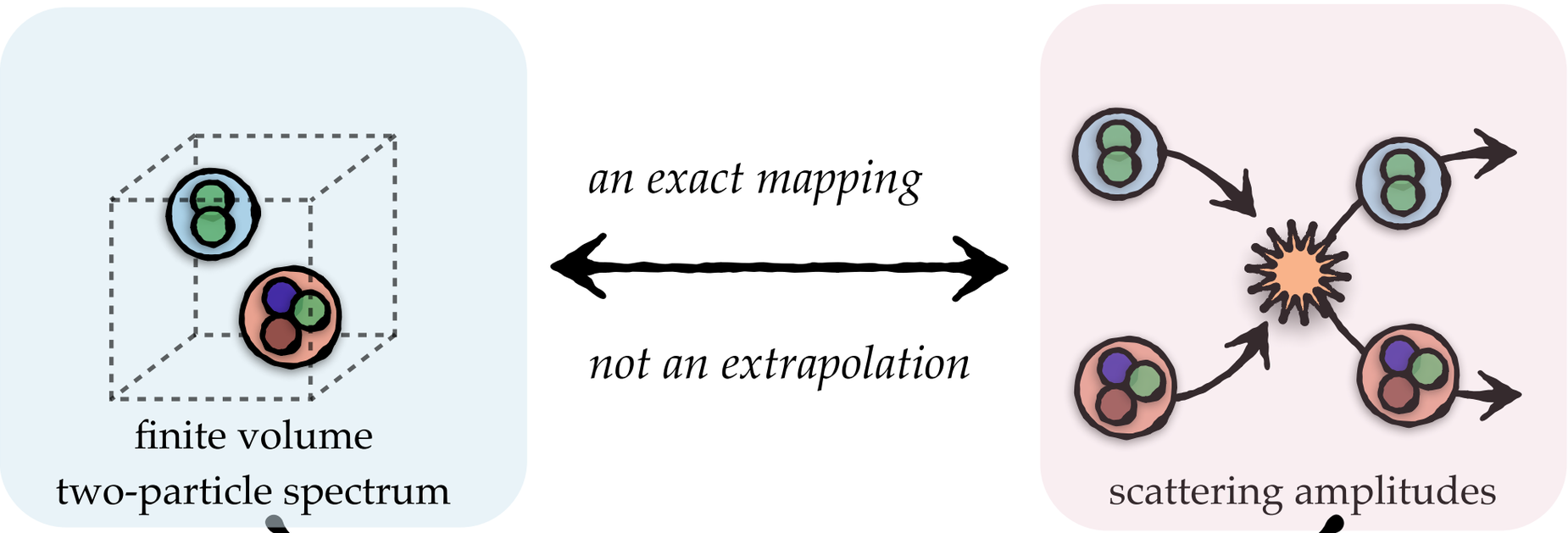
- He, Feng, and Liu (2005)
- Bernard, Lage, Meissner, and Rusetsky (2008)
- Hansen and Sharpe (2012)
- RB and Davoudi (2012)
- Li and Liu (2013)
- RB, Davoudi, and Luu (2013)
- RB, Davoudi, Luu and Savage (2013)
- ...

summarizes everything previously done and more!

$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

the most general two-particle quantization condition in a finite volume **[RB (2014)]** hold for any number of two-particle states with any spin!

3D Quantization condition



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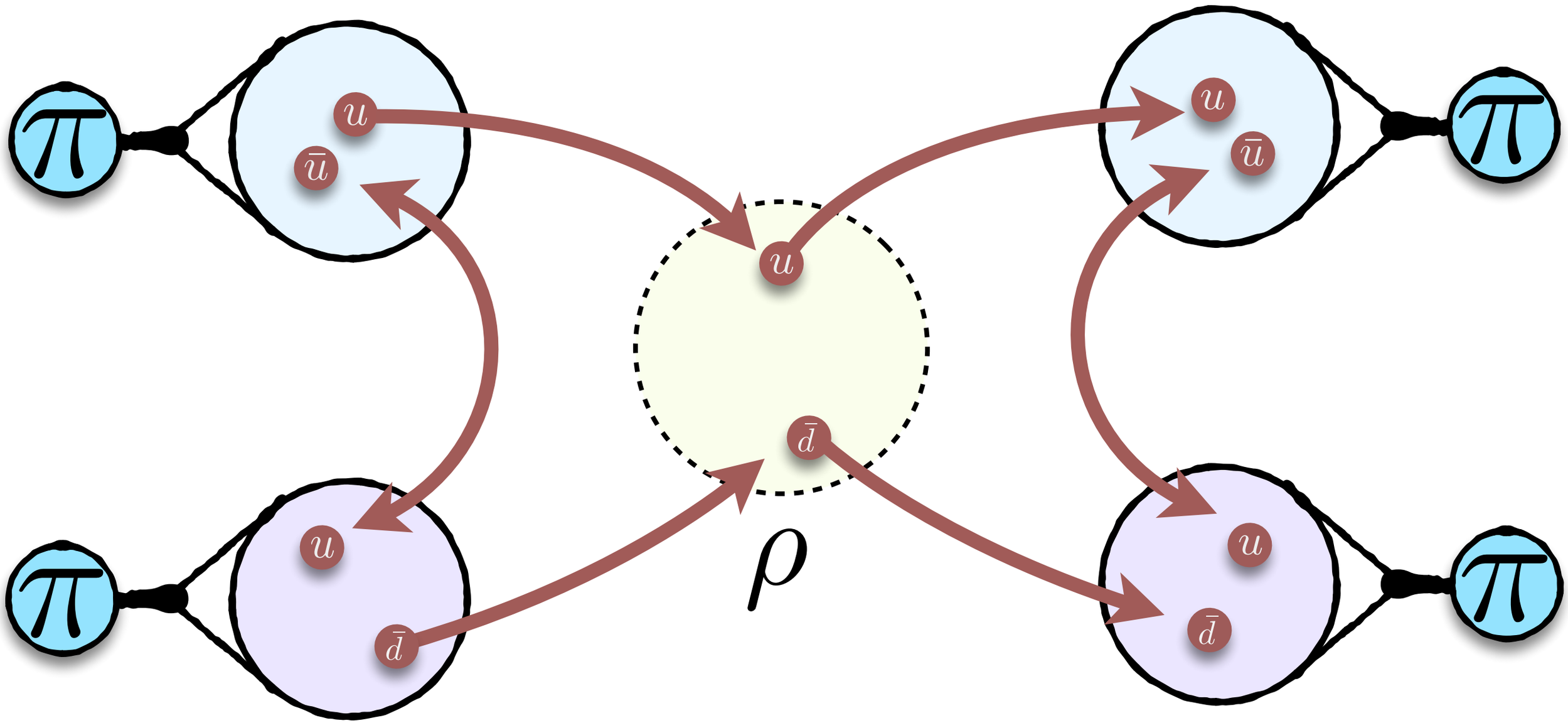
the most general two-particle quantization condition in a finite volume **[RB (2014)]** hold for any number of two-particle states with any spin!

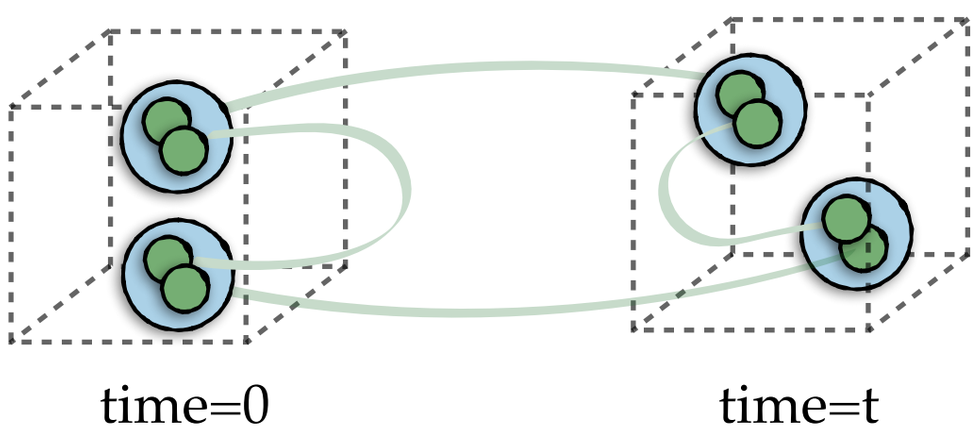
Resonances in hadronic scattering



Extracting the spectrum

Two-point correlation functions:

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t, \mathbf{P})\mathcal{O}_a^\dagger(0, -\mathbf{P})|0\rangle$$



Extracting the spectrum

Two-point correlation functions:

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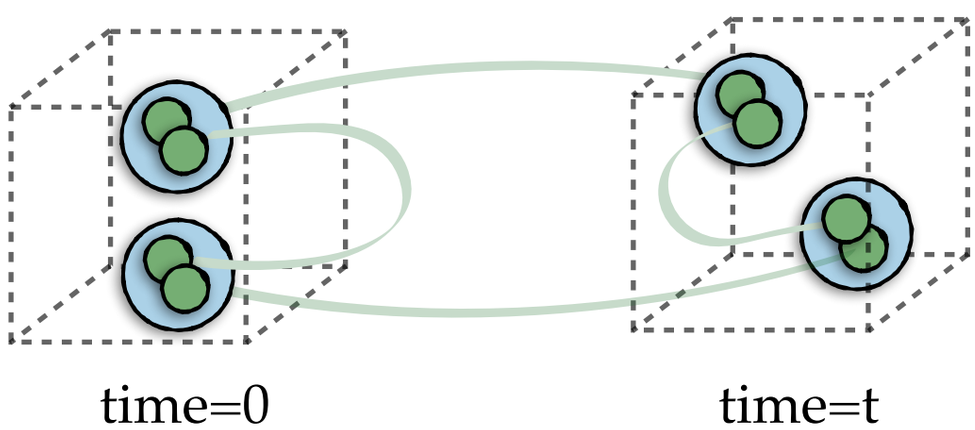
$$= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle$$

insert complete set of states

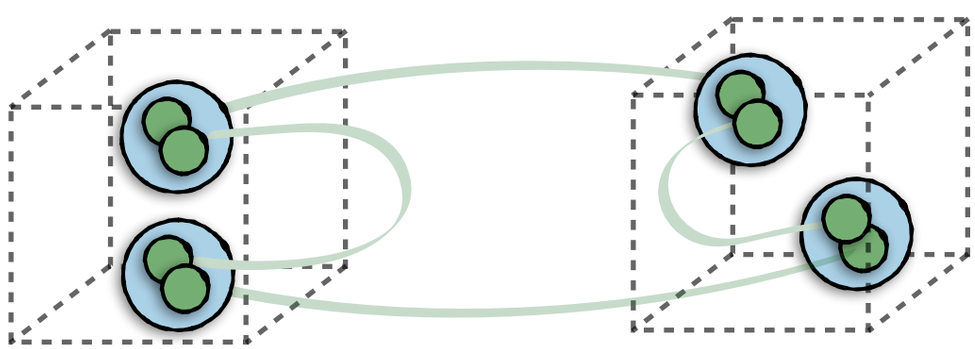


Extracting the spectrum

Two-point correlation functions:

$$
\begin{aligned}
C_{ab}^{2pt.}(t,\mathbf{P}) &\equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle
\end{aligned}
$$

remember Heisenberg operators? in Euclidean spacetime?



time=0 time=t

Extracting the spectrum

Two-point correlation functions:

$$
\begin{aligned}
C_{ab}^{2pt.}(t,\mathbf{P}) &\equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
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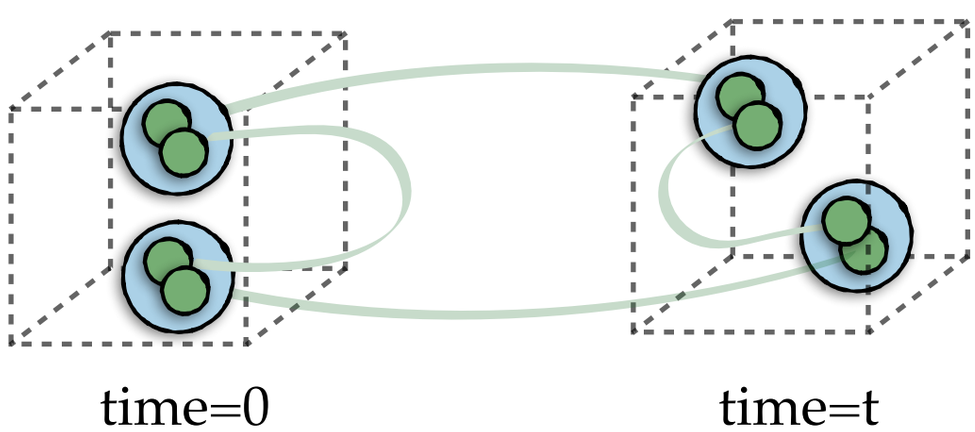
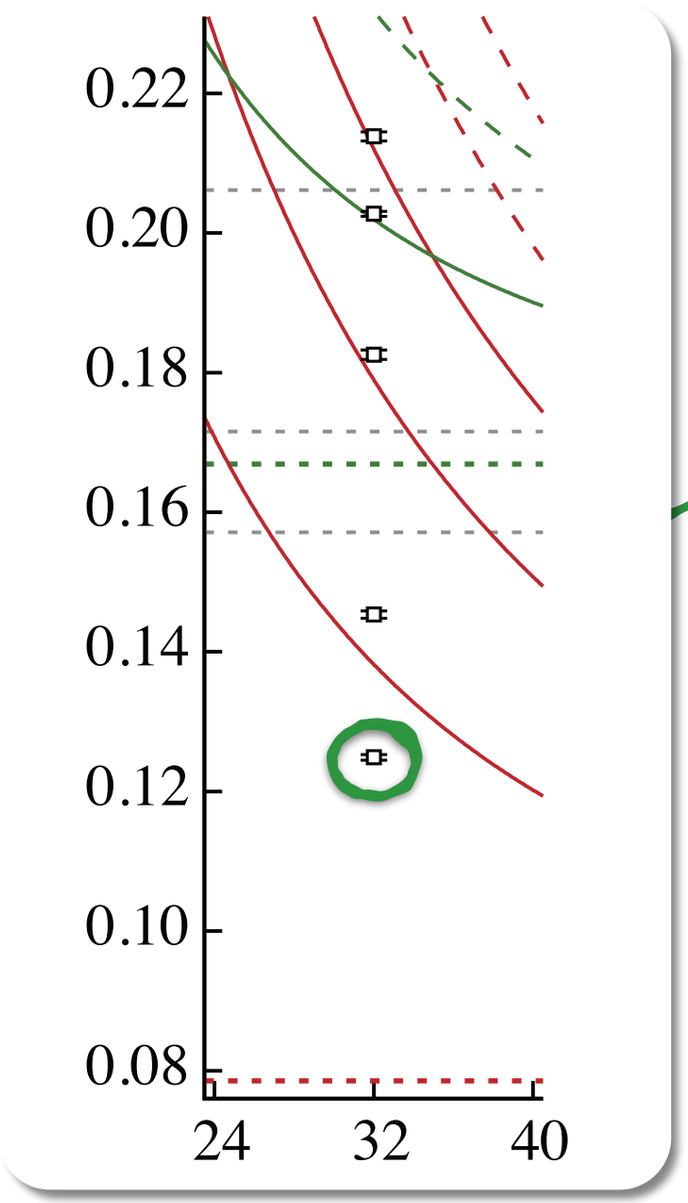
&= \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}
\end{aligned}
$$

spectrum



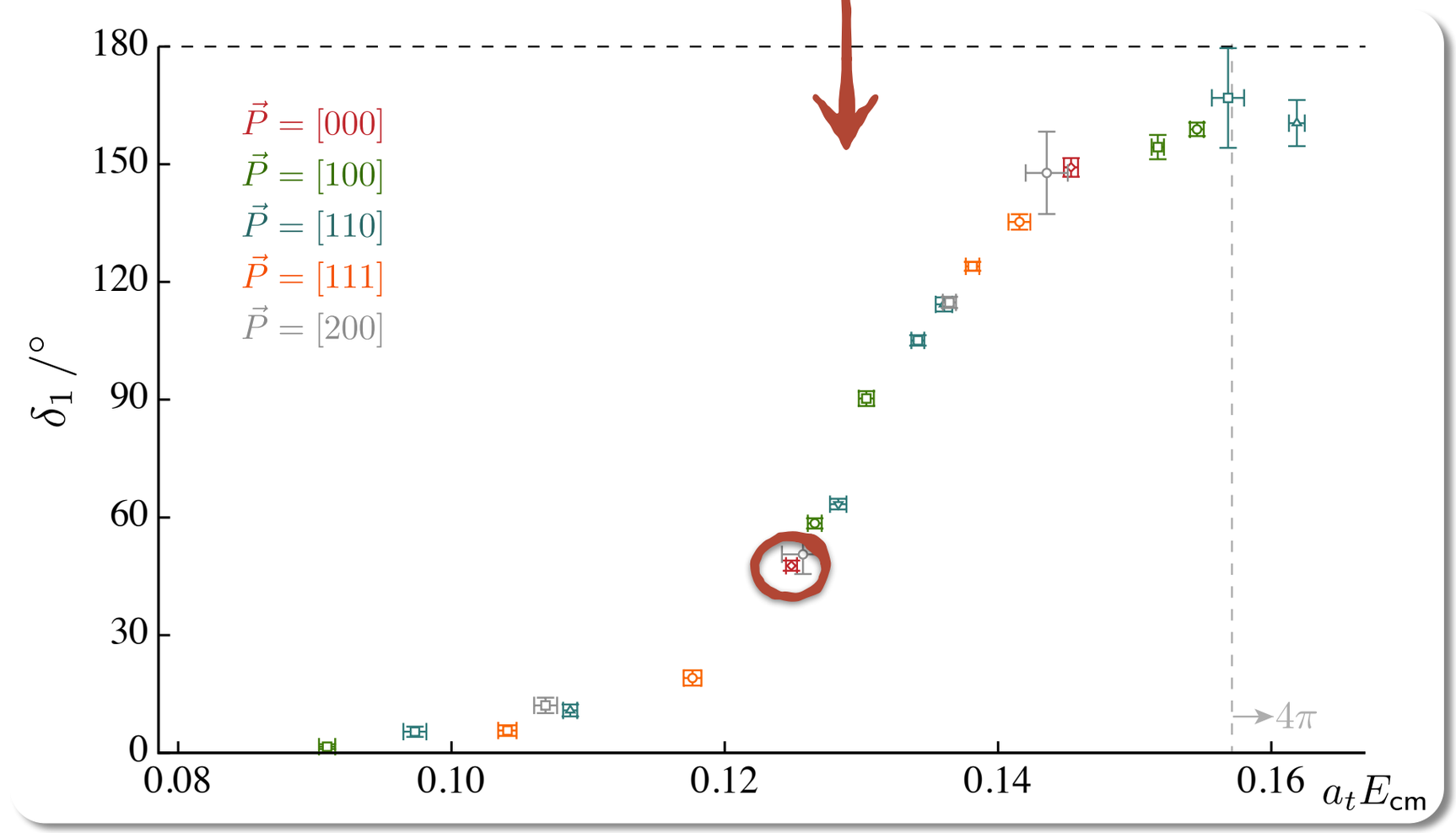
$\pi\pi$ scattering

(spectrum and interpretation)



Spectrum at rest

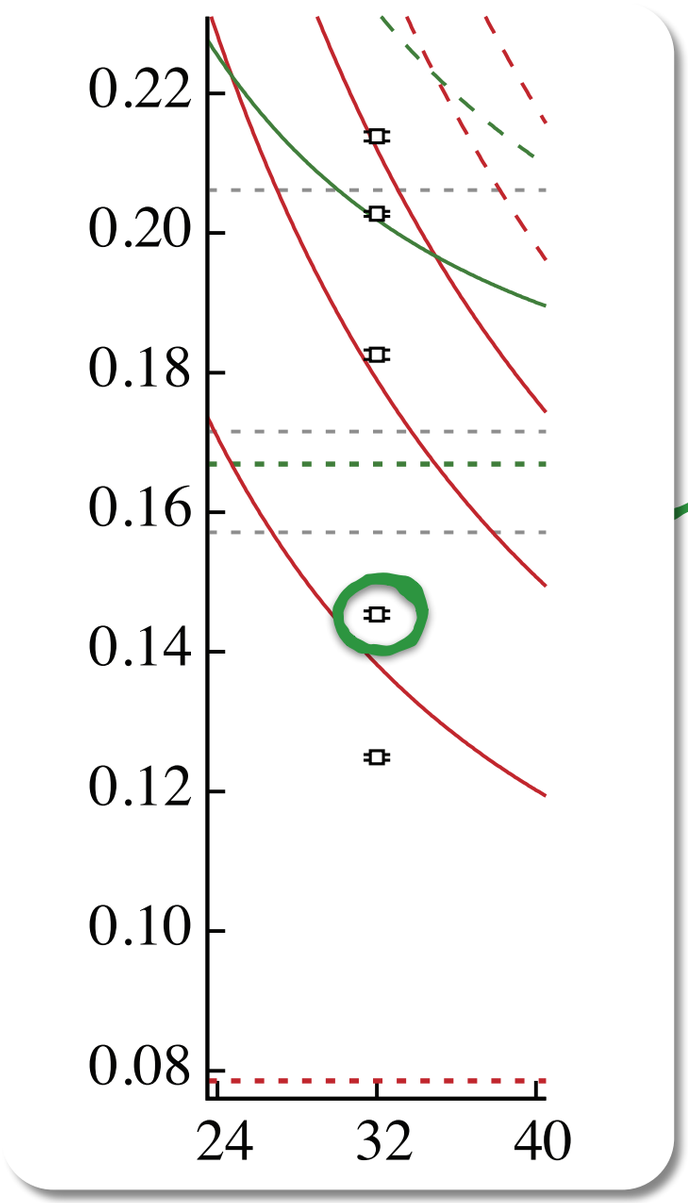
$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$



Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

(spectrum and interpretation)



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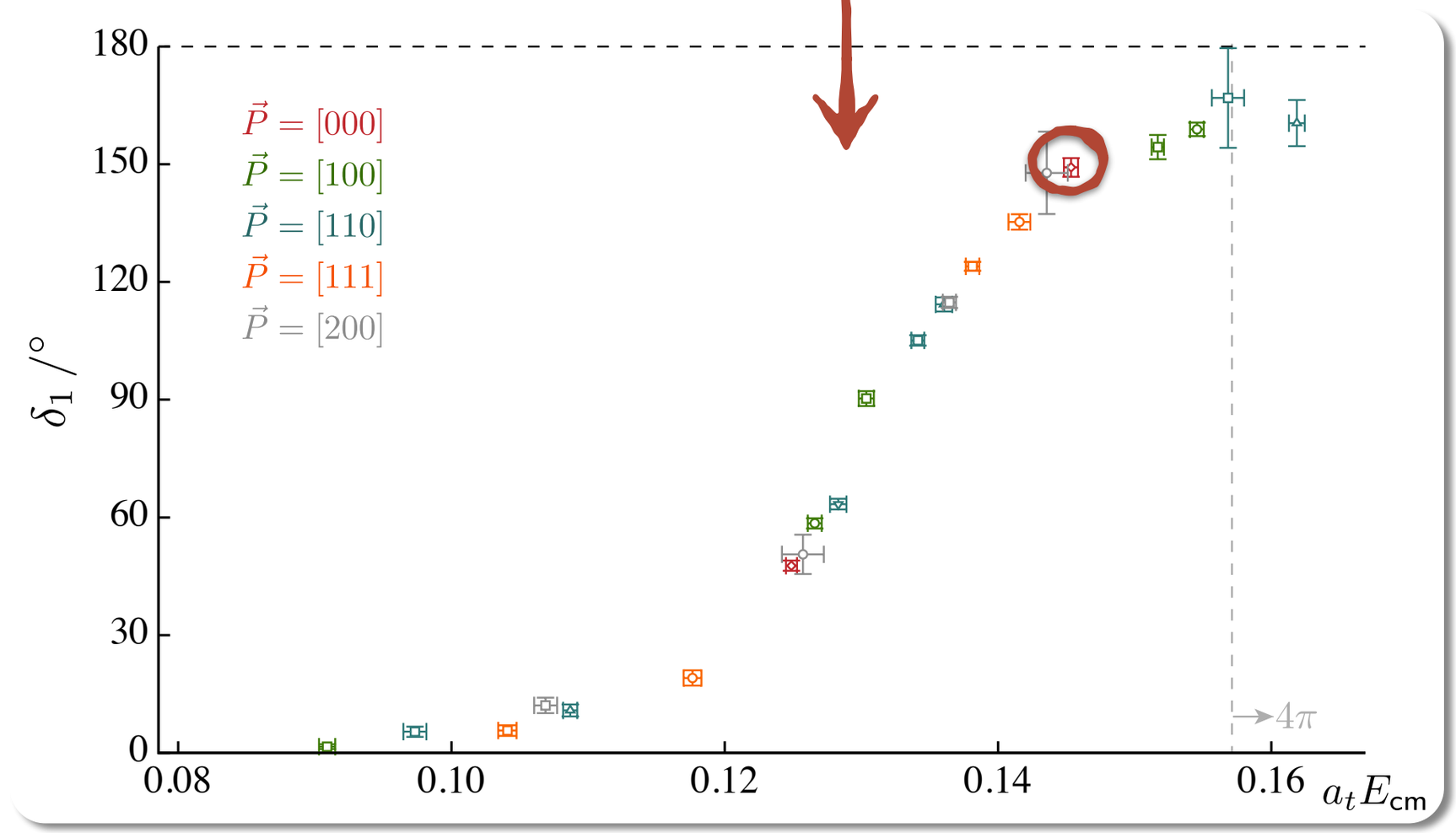
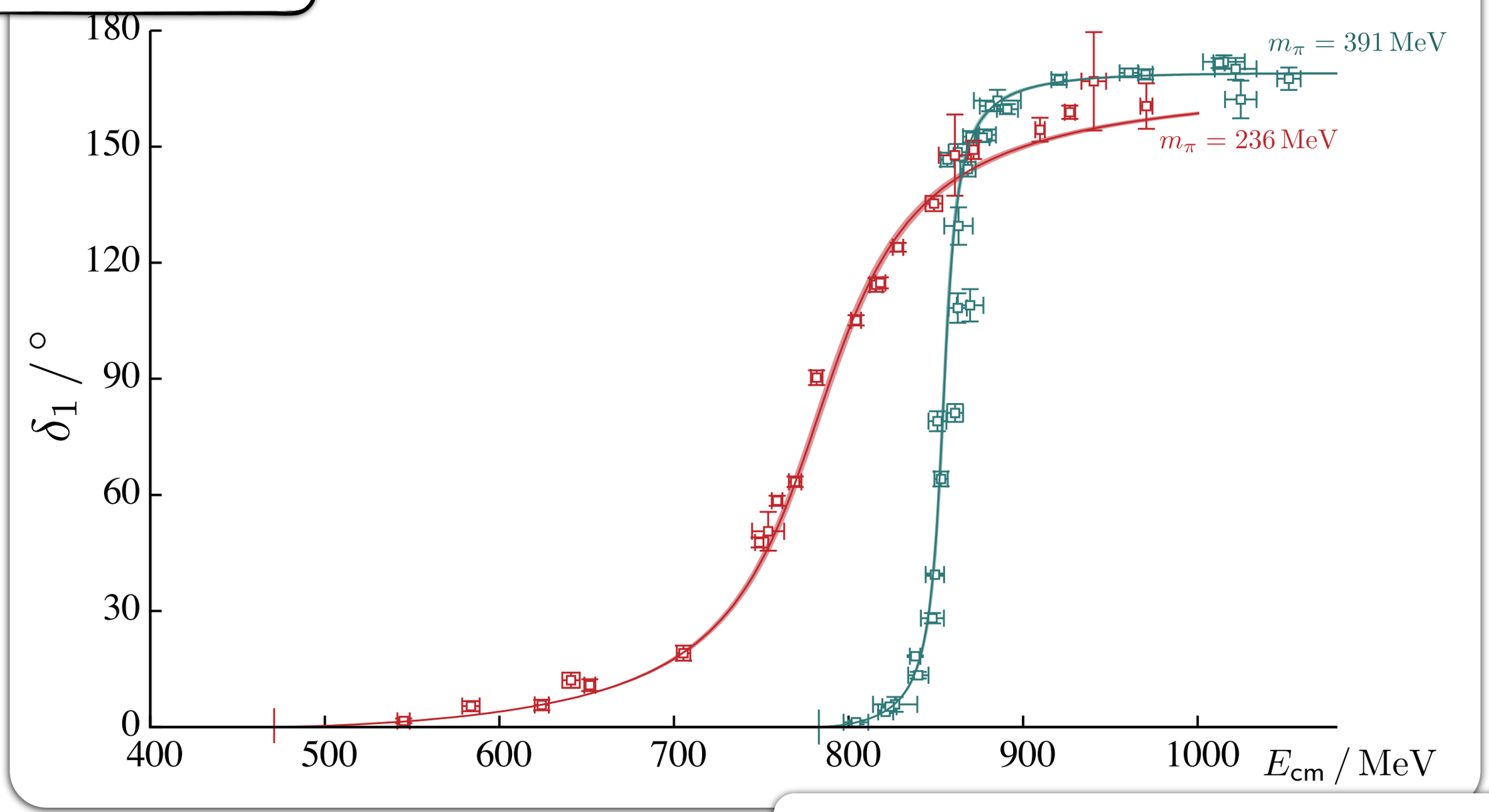


Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

(spectrum and interpretation)

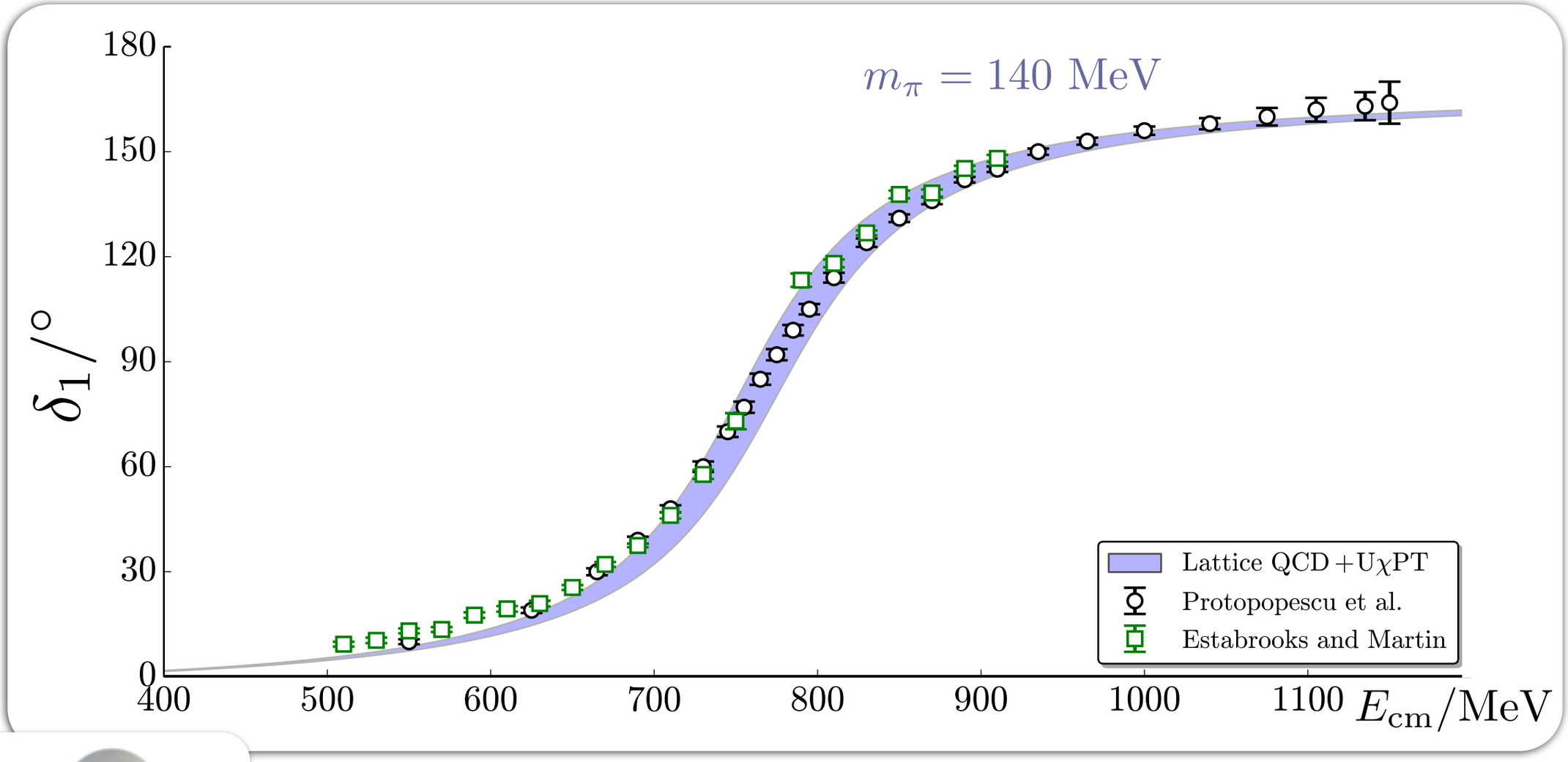
HadSpec Collaboration



$\pi\pi$ in I=1 channel

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparing with experiment



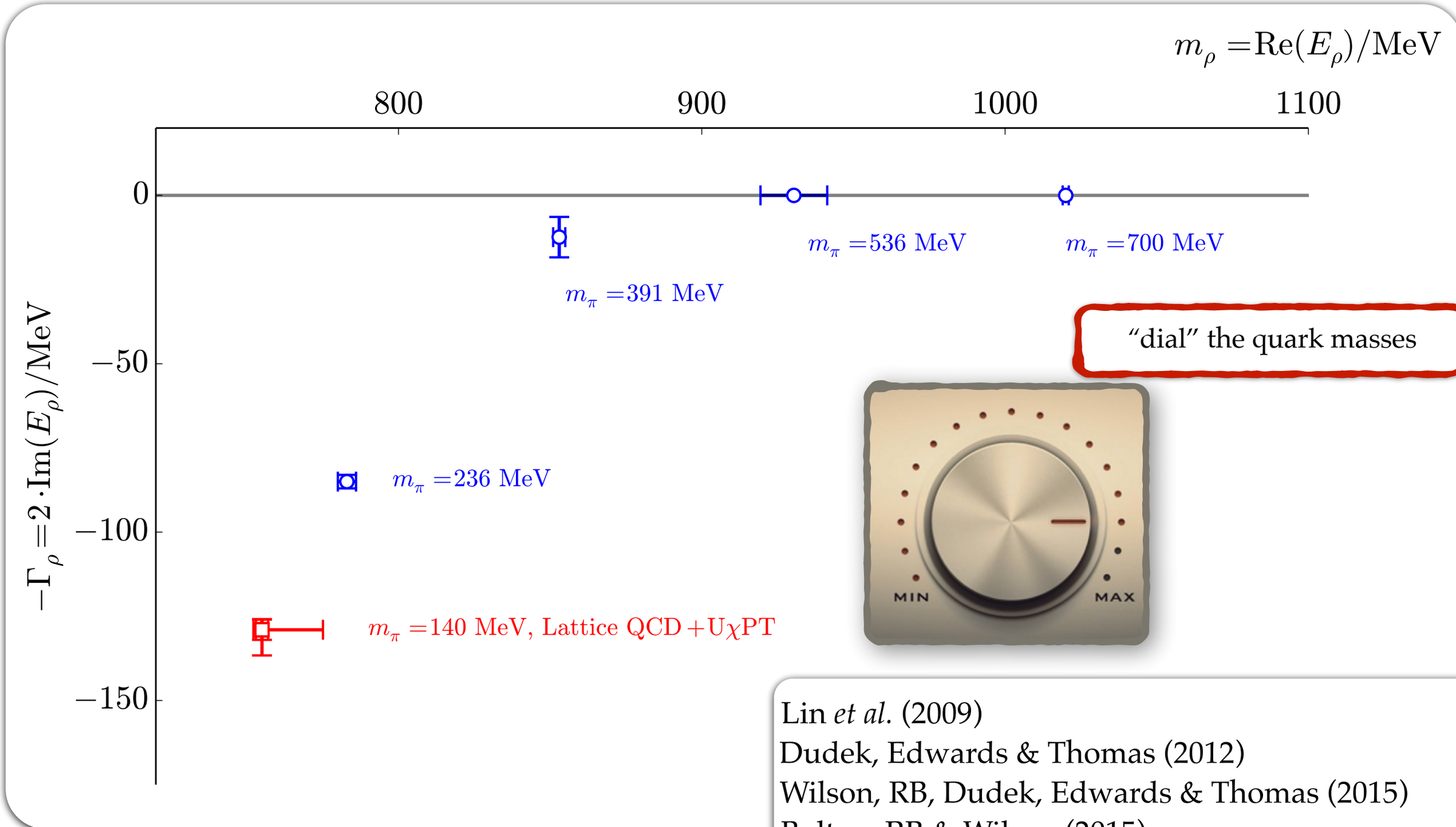
cute, but aren't experiments performed using m_π=140 MeV?

Bolton, RB & Wilson (2015)

Bolton

Wilson

Quark-mass dependence of poles



Lin *et al.* (2009)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

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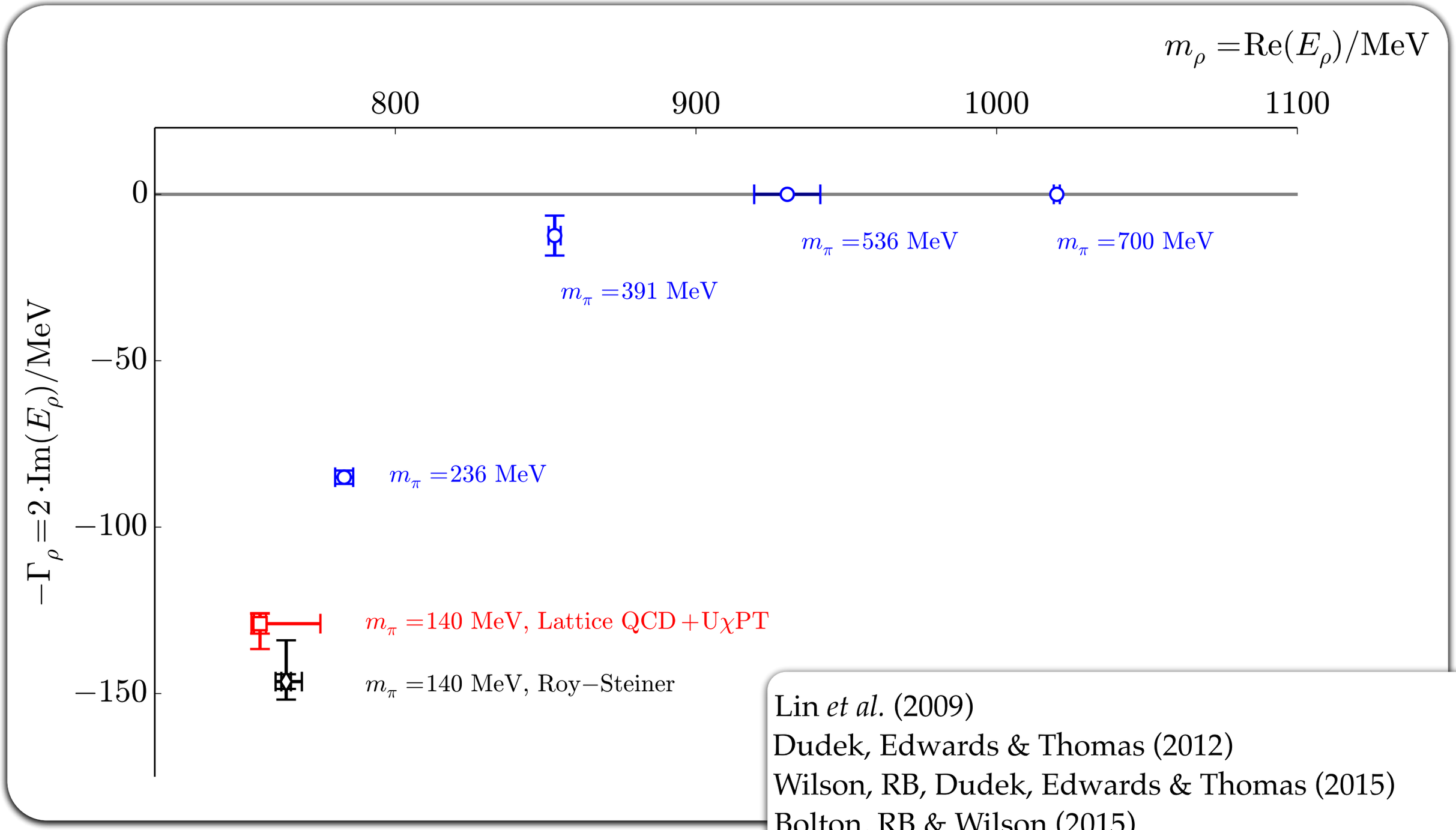

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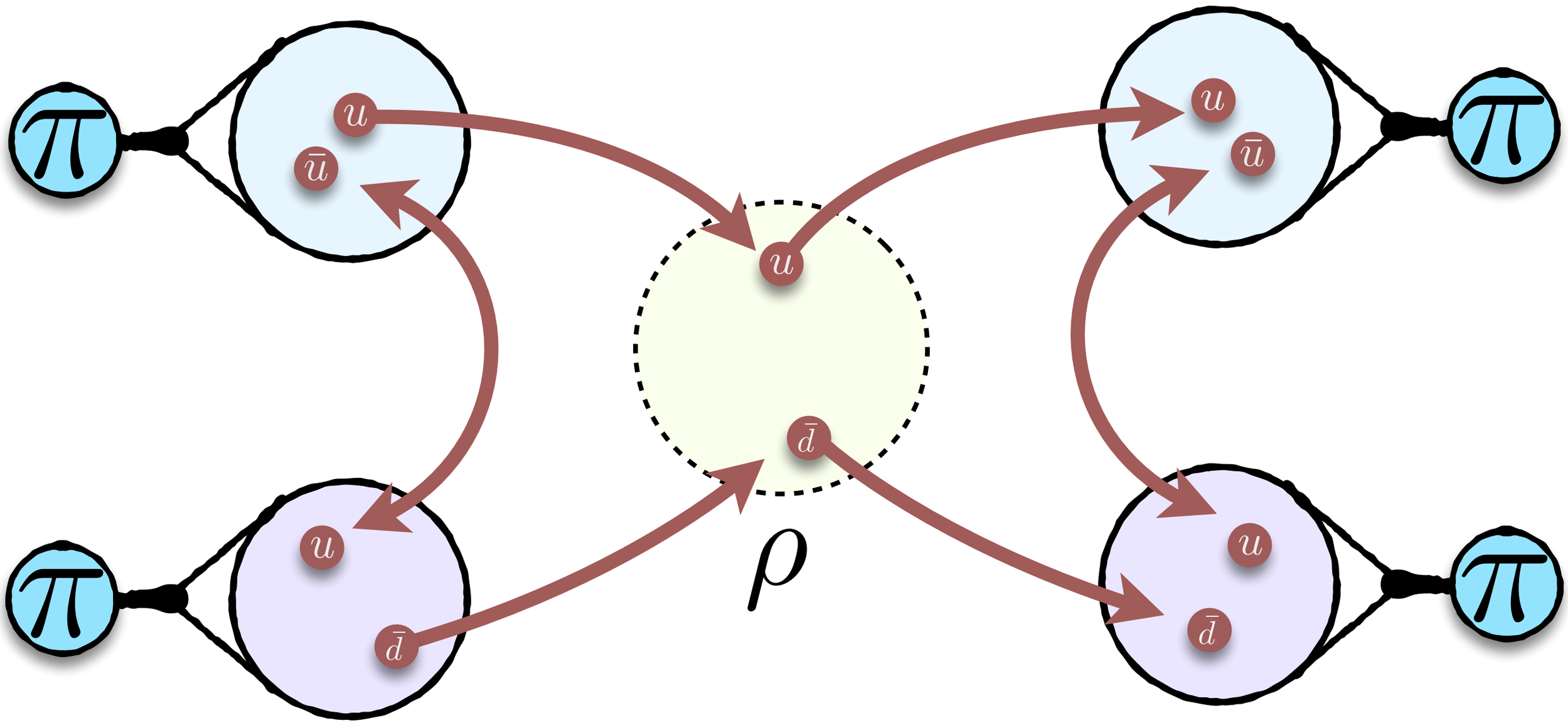
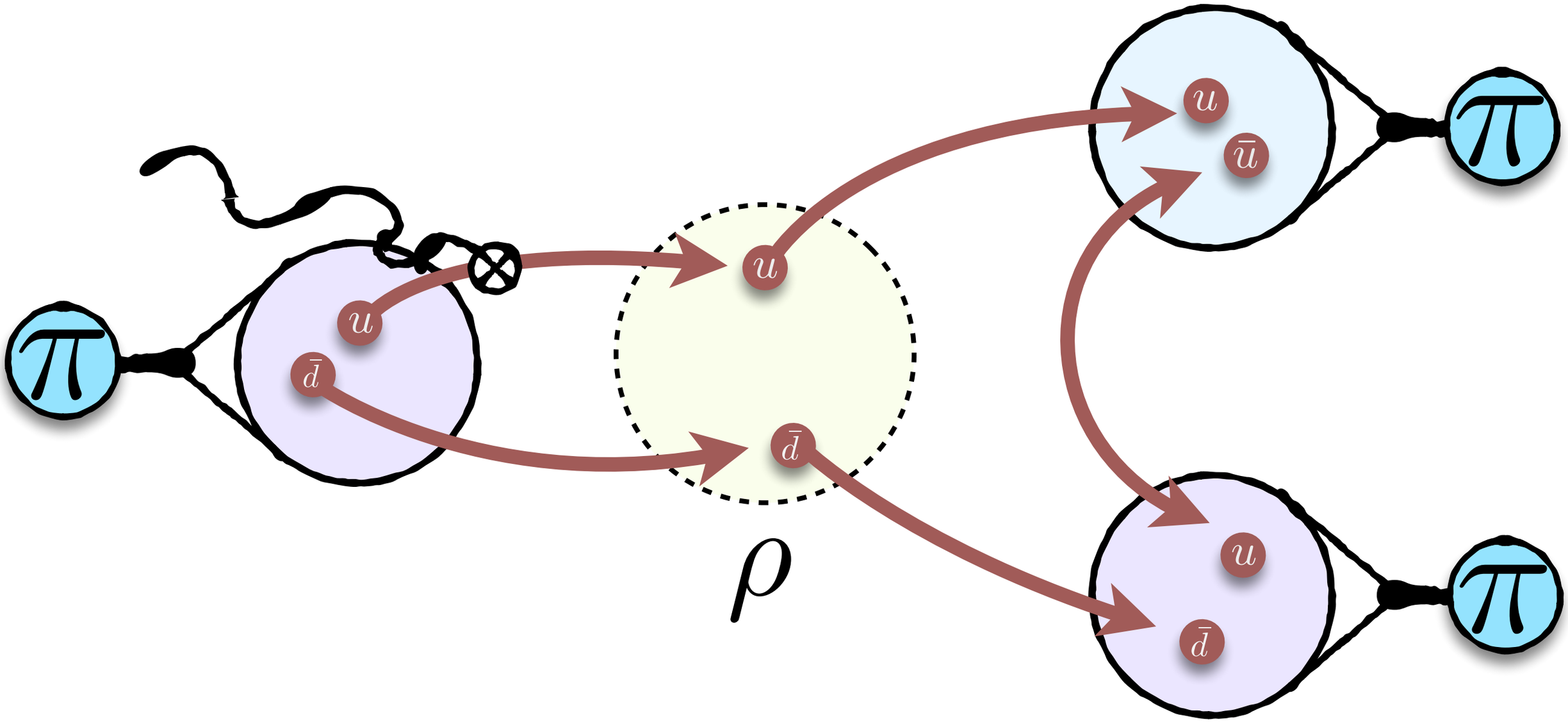
Resonances in hadronic scattering



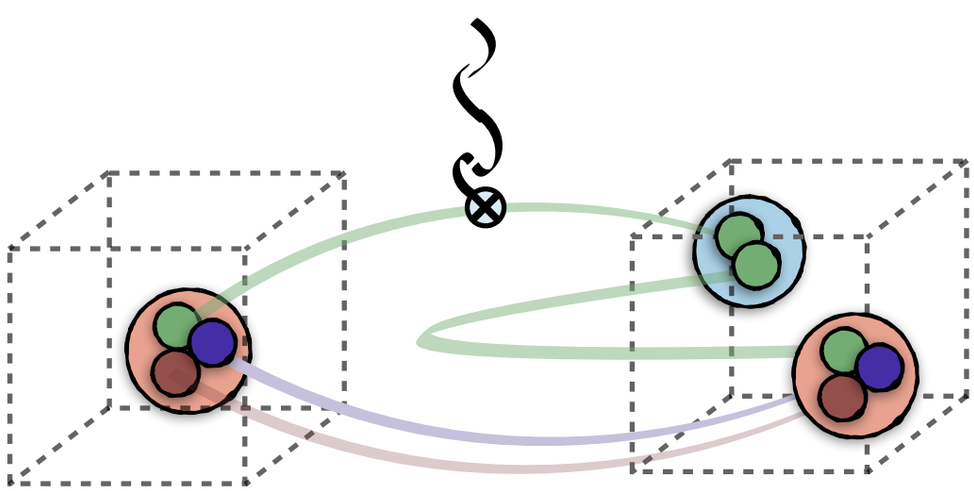
Resonances in electroweak processes



Sketch of formalism

1) Access matrix elements:

$$C^{3pt.}_{\mathbf{2}\to\mathbf{1}\mathcal{J}} = \langle \mathcal{O}_1(\delta t)\mathcal{J}(t)\mathcal{O}_2^\dagger(0)\rangle \longrightarrow \langle \mathbf{1}\big|\mathcal{J}\big|\mathbf{2}\rangle_L Z_1 Z_2^* e^{-(\delta t - t)E_1} e^{-tE_2} + \cdots$$



RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

Hansen

Walker-Loud

Sketch of formalism

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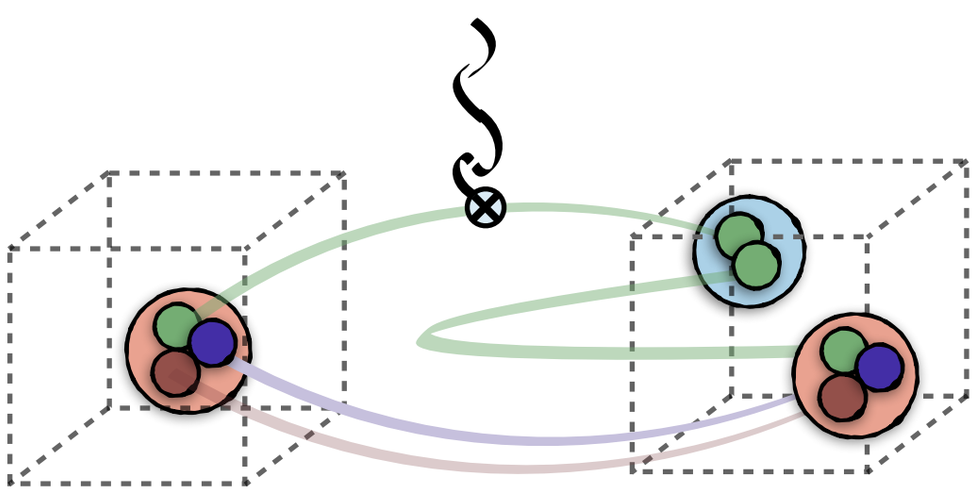
2) Interpret matrix elements:

$$\boxed{\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\ \mathcal{R}\ \mathcal{H}}$$



RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

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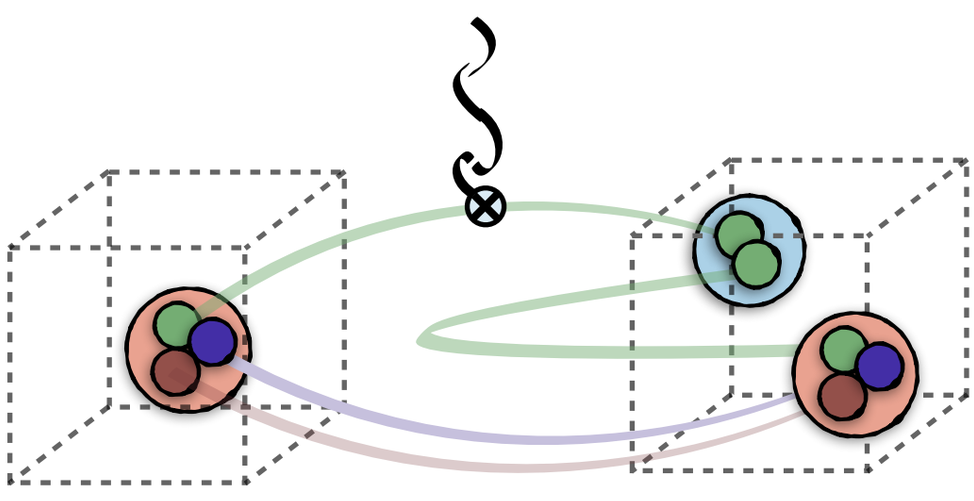
$$\boxed{\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\underline{\mathcal{R}}\,\mathcal{H}}$$

known finite volume function

$$\mathcal{R}\left(E_{\mathbf{2}}, L, \delta, \frac{\partial\delta}{\partial E_{\mathbf{2}}}\right)$$



RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

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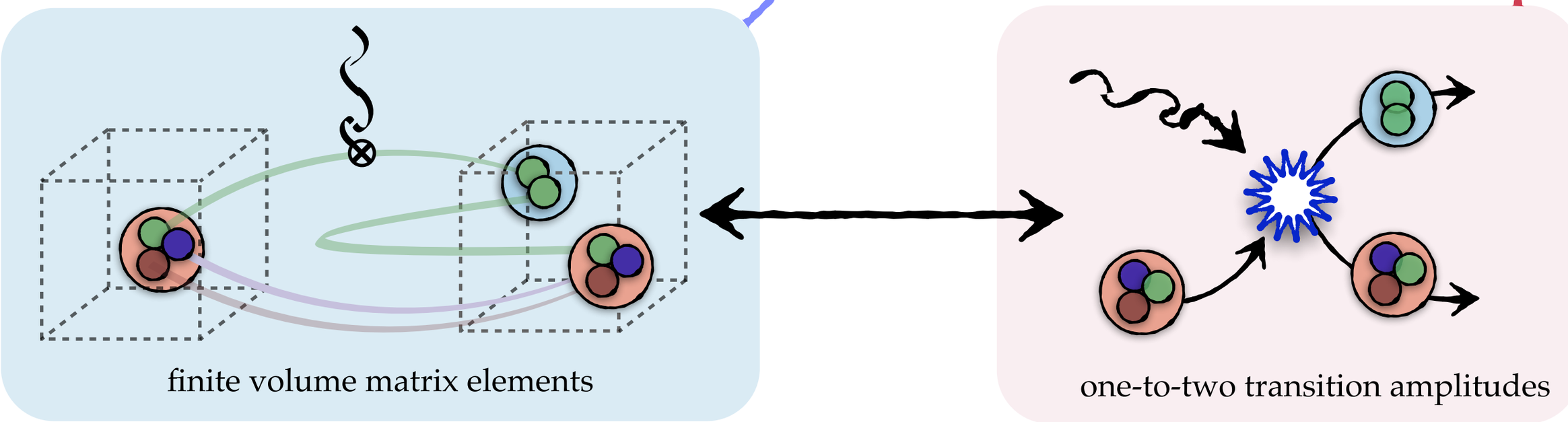
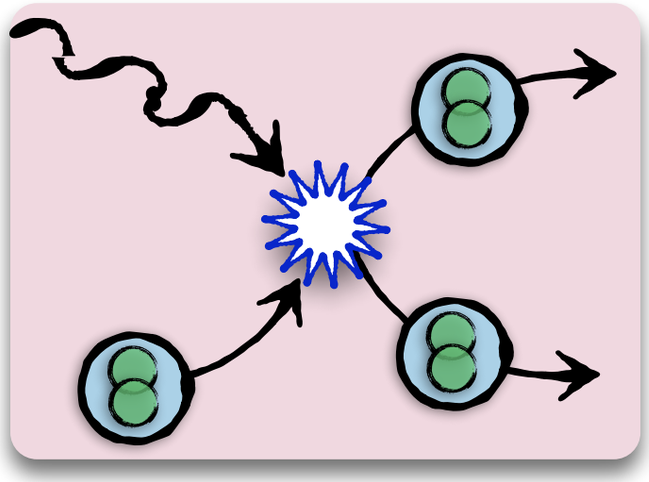


$\pi\gamma^*$-to-$\pi\pi$

Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ calculation:

- proof of principle / demonstration
- $m_\pi = 391$ MeV



HadSpec Collaboration

Wilson

Shultz

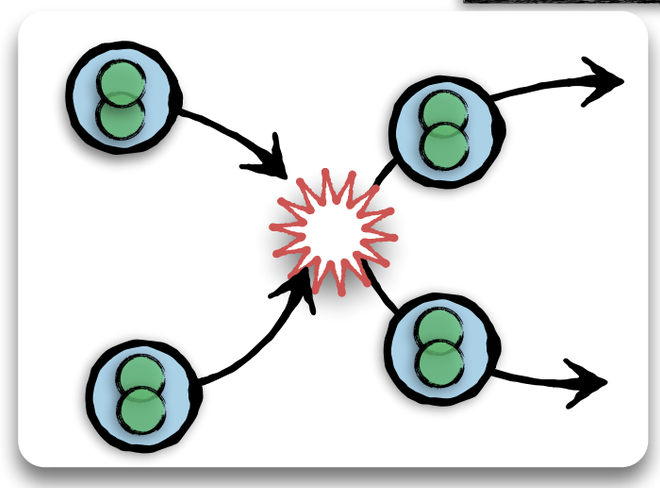
Thomas

Dudek

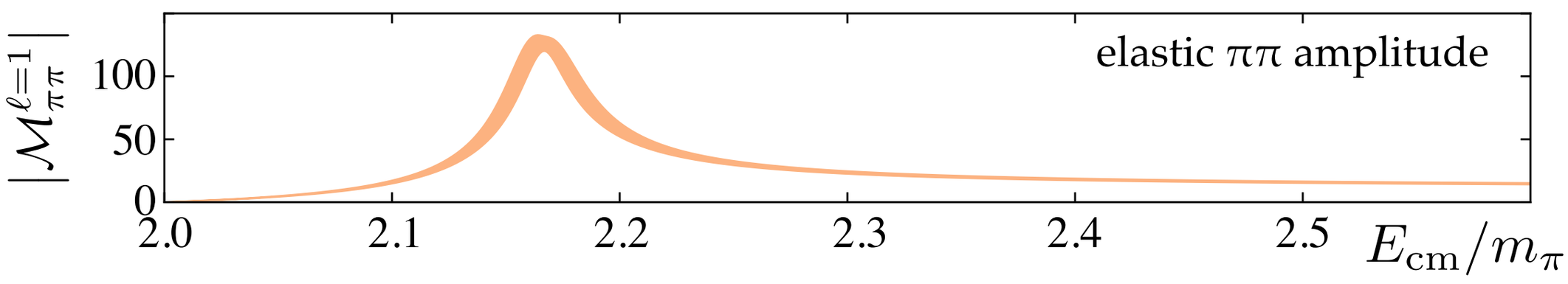
Edwards

$\pi\gamma^*$-to-$\pi\pi$ amplitude

$m_\pi = 391$ MeV



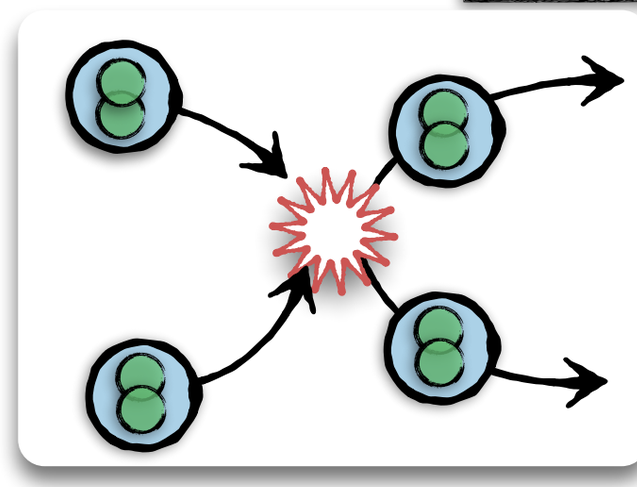
- elastic $\pi\pi$ amplitude as function of energy [ϱ-channel]



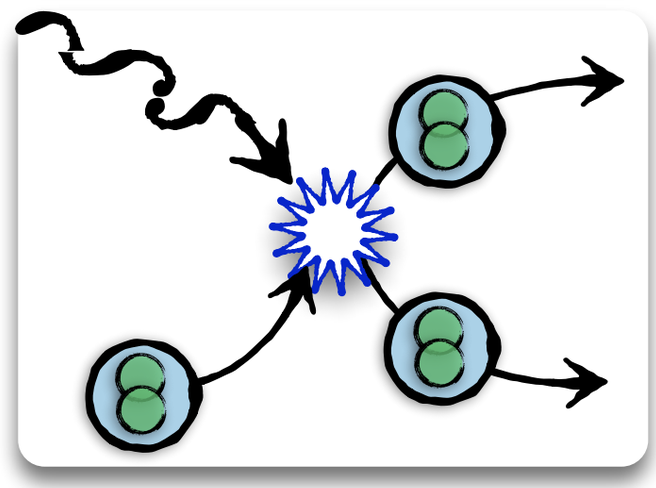
$\pi\gamma^*$-to-$\pi\pi$ amplitude

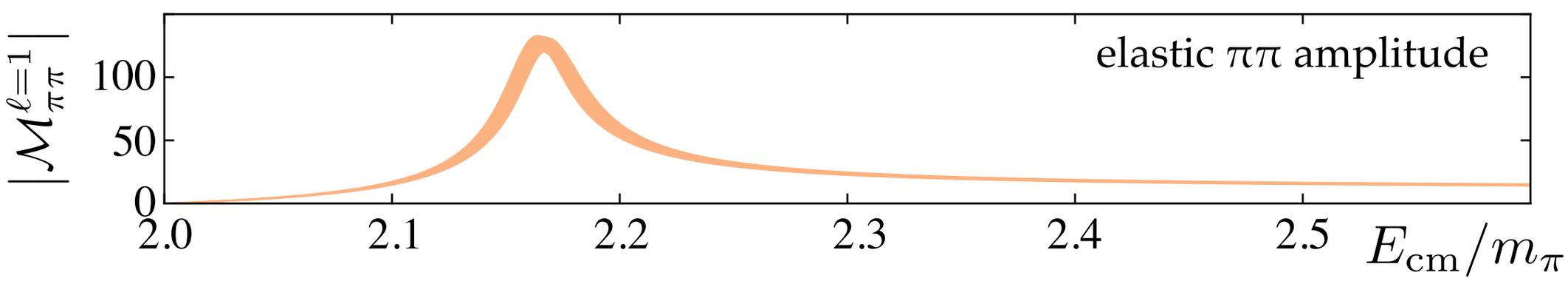
$m_\pi = 391$ MeV

- elastic $\pi\pi$ amplitude as function of energy [ρ-channel]

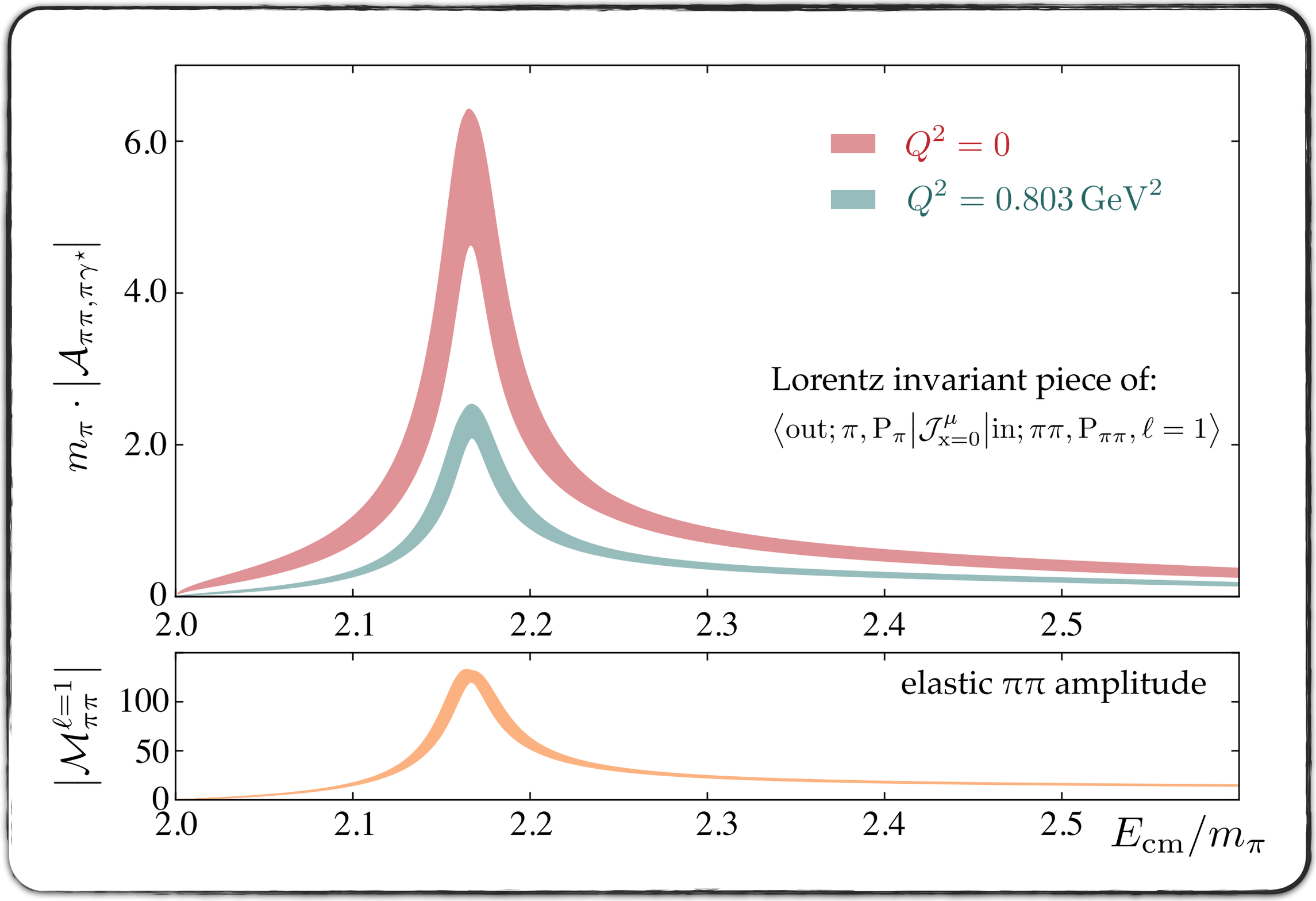


- how does the $\pi\gamma^*$-to-$\pi\pi$ look like?



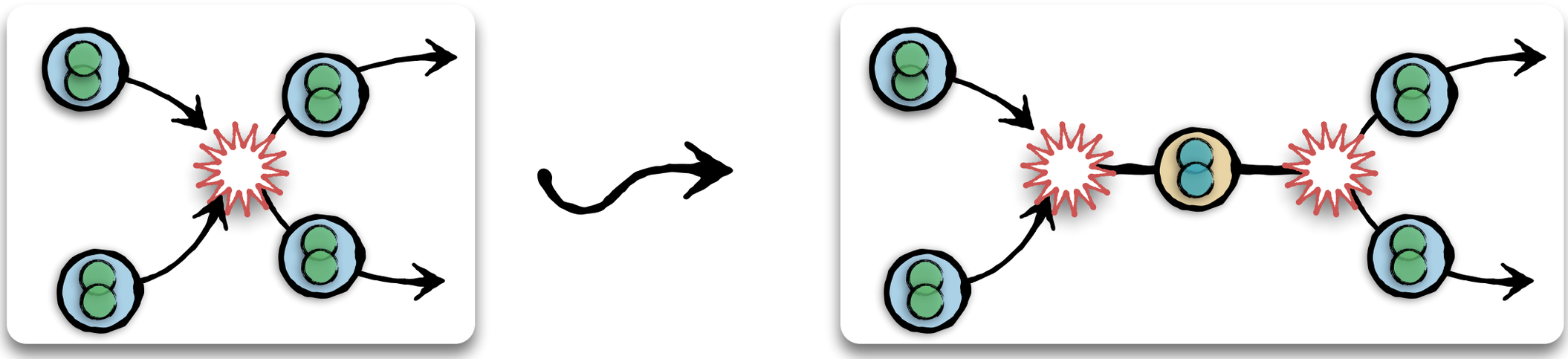


$\pi\gamma^*$-to-$\pi\pi$ amplitude

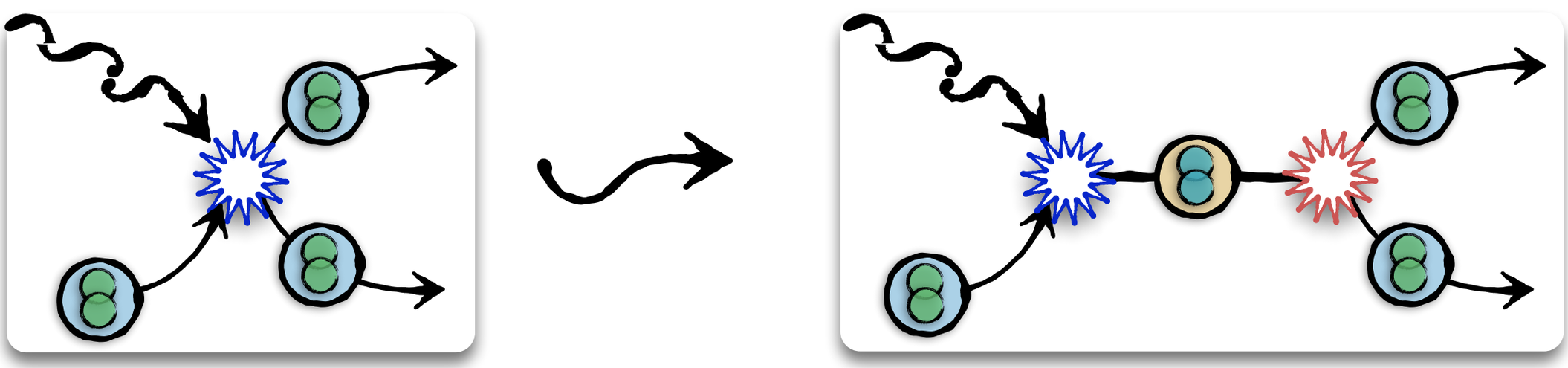


Intuitive explanation

- the elastic $\pi\pi$ amplitude is dynamically enhanced by the presence of the ϱ-meson

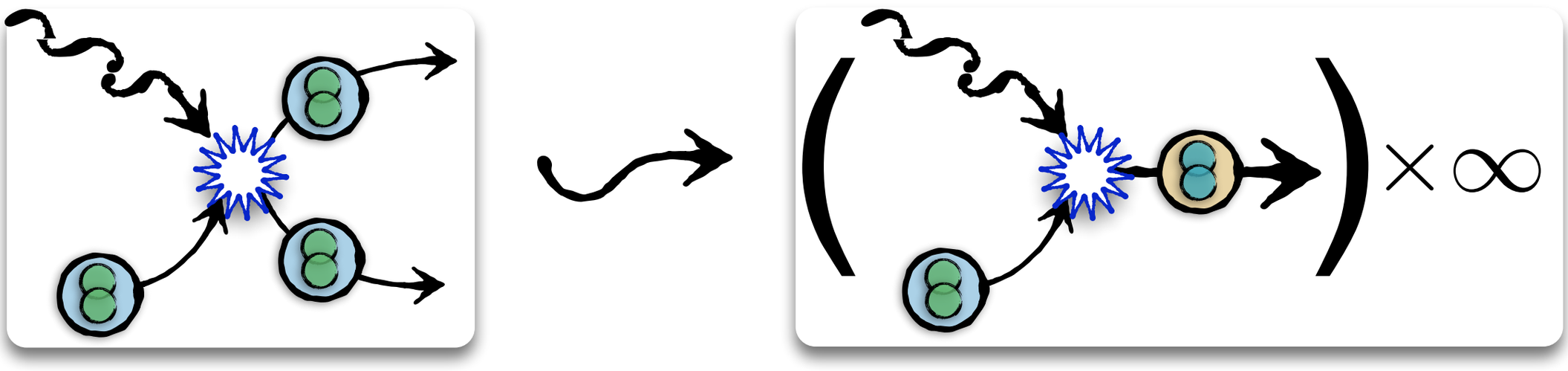


- Similarly, the $\pi\gamma^*$-to-$\pi\pi$ amplitude is enhanced by the ϱ-meson

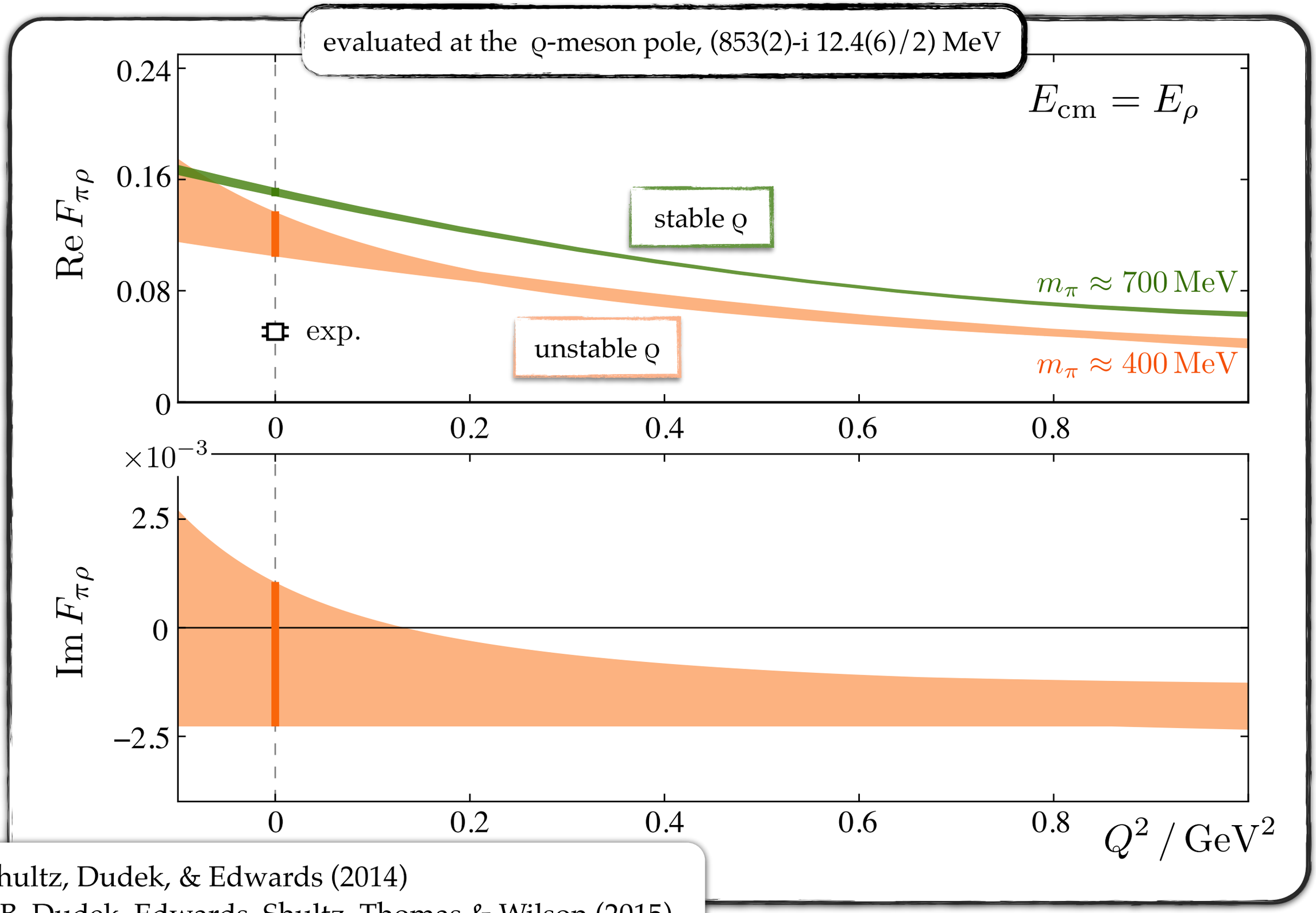


Form factor at ϱ pole

- Near the ϱ-pole, the $\pi\gamma^*$-to-$\pi\pi$ diverges
- The residue is the $\pi\gamma^*$-to-ϱ form factor

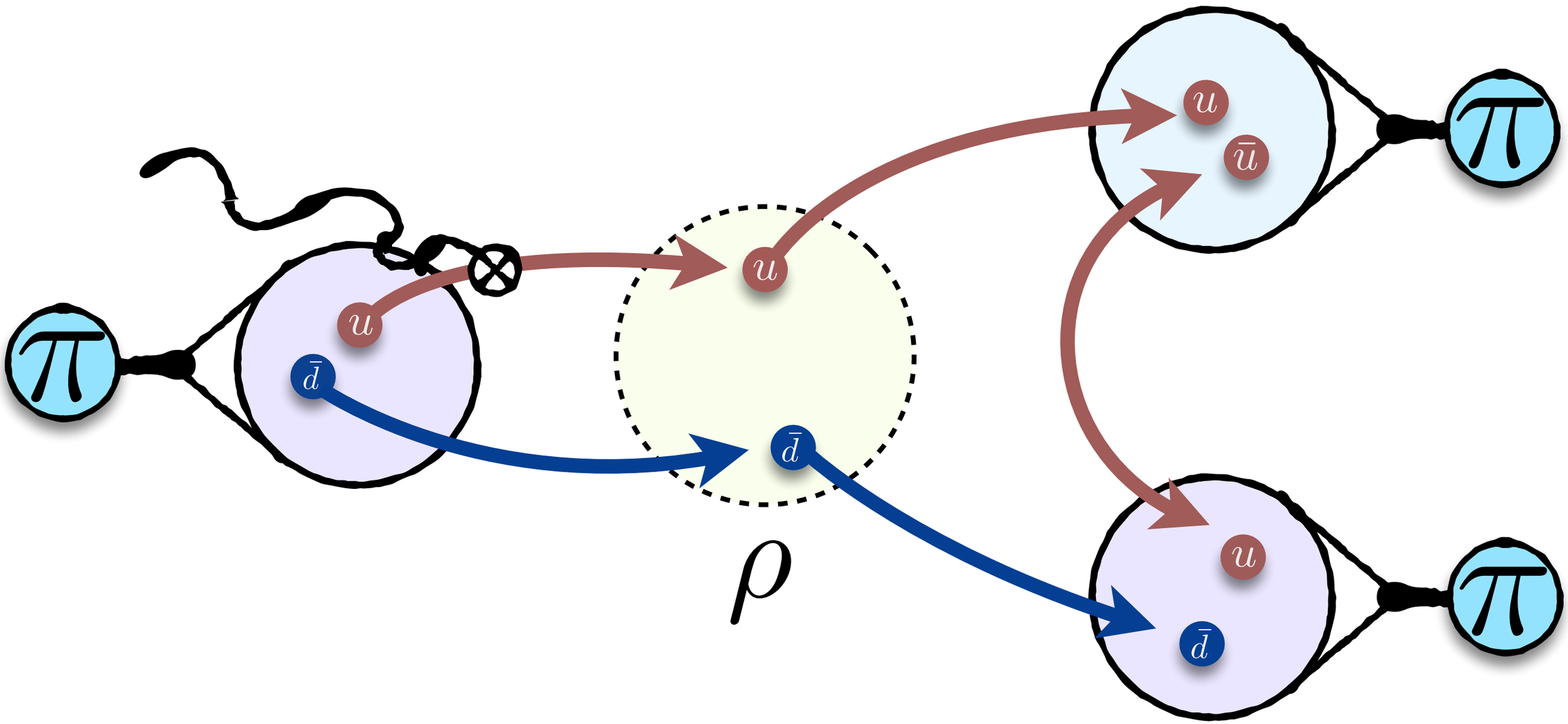


Form factor at ϱ pole



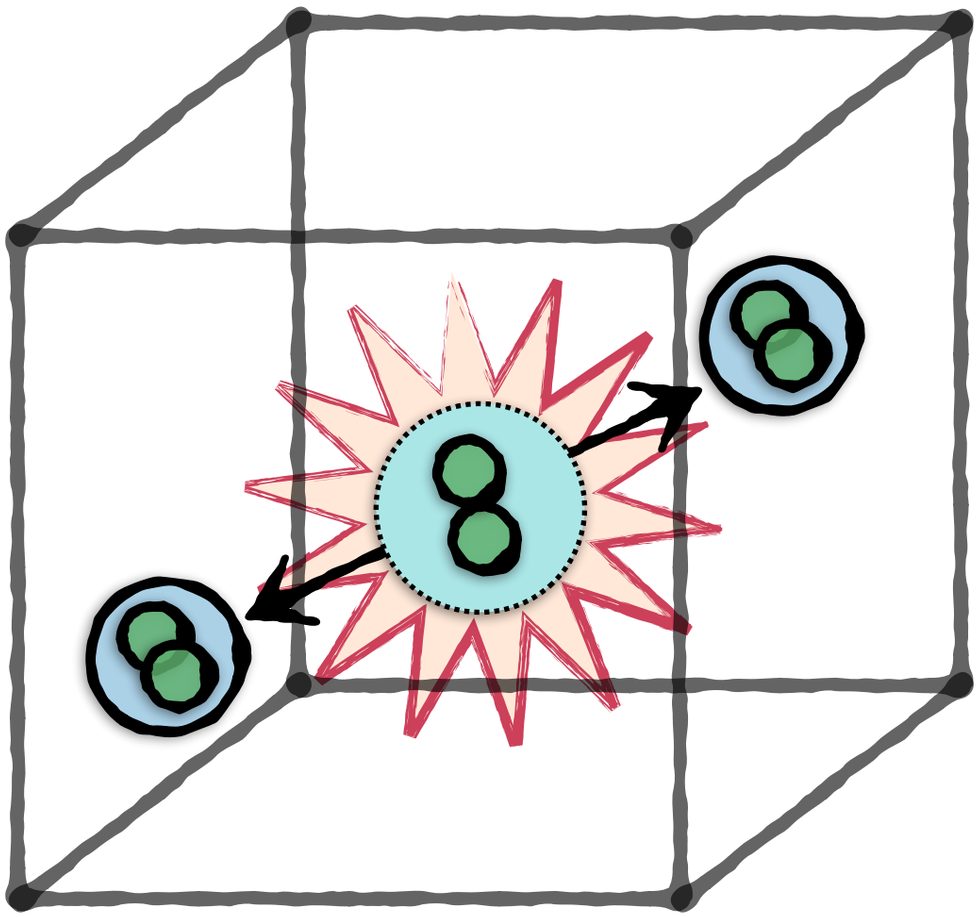
Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

Resonances in electroweak processes

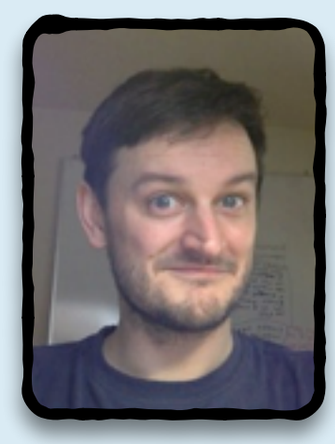


The immediate future

- Scalars: $\sigma/f_0(500), f_0(980), \ldots$



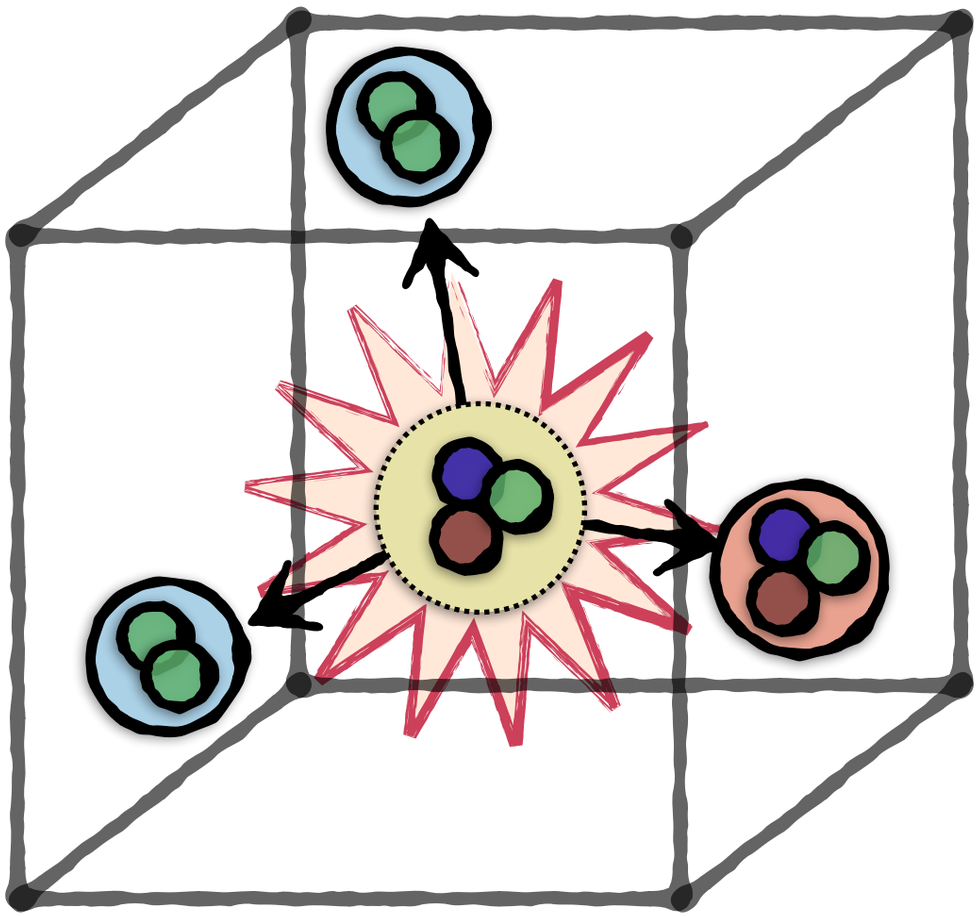
Wilson



Dudek

Edwards

The immediate future



- Scalars: $\sigma / f_0(500), f_0(980), \ldots$
- Resonances that decay to three-particles or more?

Hansen

Sharpe

Collaborators

formalism

Hansen

Sharpe

Walker-Loud

numerical

Wilson

Shultz

Thomas

Dudek

Edwards

Bolton

HadSpec Collaboration