NJL jet model for quark fragmentation functions

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Quark distribution and fragmentation functions are the basic nonperturbative ingredients for a QCD-based analysis of hard scattering processes. In this paper we show the results of recent calculations of fragmentation functions in the NJL-jet model [1].

The spin-independent fragmentation function for the process $q \to h$ is defined by

$$D_q^h(z) \;=\; \frac{z}{12}\int \frac{d\omega^-}{2\pi} e^{ip_-\omega^-/z} \hat{\sum}_n \langle p(h), p_n|\overline{\psi}(0)|0\rangle\, \gamma^+ \,\langle 0|\psi(\omega^-)|p(h), p_n\rangle .$$

The field operators refer to a quark of flavor q, the symbol $p(h)$ refers to a hadron h with momentum p, and p_n labels the spectator state. The light-cone components of a 4-vector a^μ are defined by $a^\pm = a_\mp = (a^0 \pm a^3)/\sqrt{2}$. From this definition one can derive the expression

$$D_q^h(z)\, dz = \frac{1}{6} dp_- \int d^2p_\perp \sum_\alpha \frac{\langle k(\alpha)|a_h^\dagger(p) a_h(p)|k(\alpha)\rangle}{\langle k(\alpha)|k(\alpha)\rangle}\,,$$

where the creation and annihilation operators refer to the hadron h, $k(\alpha)$ labels a quark state of flavor q with momentum k and spin-color α, and $p_- = zk_-$ for some fixed $k_- > 0$. The above result can be interpreted as the light-cone momentum distribution of the hadron h in the quark q.

The momentum and isospin sum rules obtained from the above formula are

$$\sum_h \int_0^1 dz\, z\, D_q^h(z) = 1\,, \qquad \sum_h \int_0^1 dz\, t_h\, D_q^h(z) = t_q\,.$$

The condition which lies at the basis of these sum rules is that the initial quark state is an eigenstate of the momentum and isospin operators, expressed solely in terms of hadrons. Their physical content is that 100% of the initial quark light-cone momentum (k_-) and isospin (t_q) are transferred to the hadrons. (Note that the definition of the fragmentation function implies an average over the isospin of the soft quark remainder of a fragmentation chain.)

In order to satisfy the momentum and isospin sum rules, it is necessary to take into account the possibility that the fragmenting quark produces a cascade of mesons. In order to describe these multi-fragmentation processes, we use the ideas of the quark jet model of Field and Feynman[2]. Assuming that the fragmenting quark can produce a maximum of N mesons, we make a product ansatz to express the total fragmentation function as a product of N elementary splitting functions. Because only in the limit $N \to \infty$ it becomes

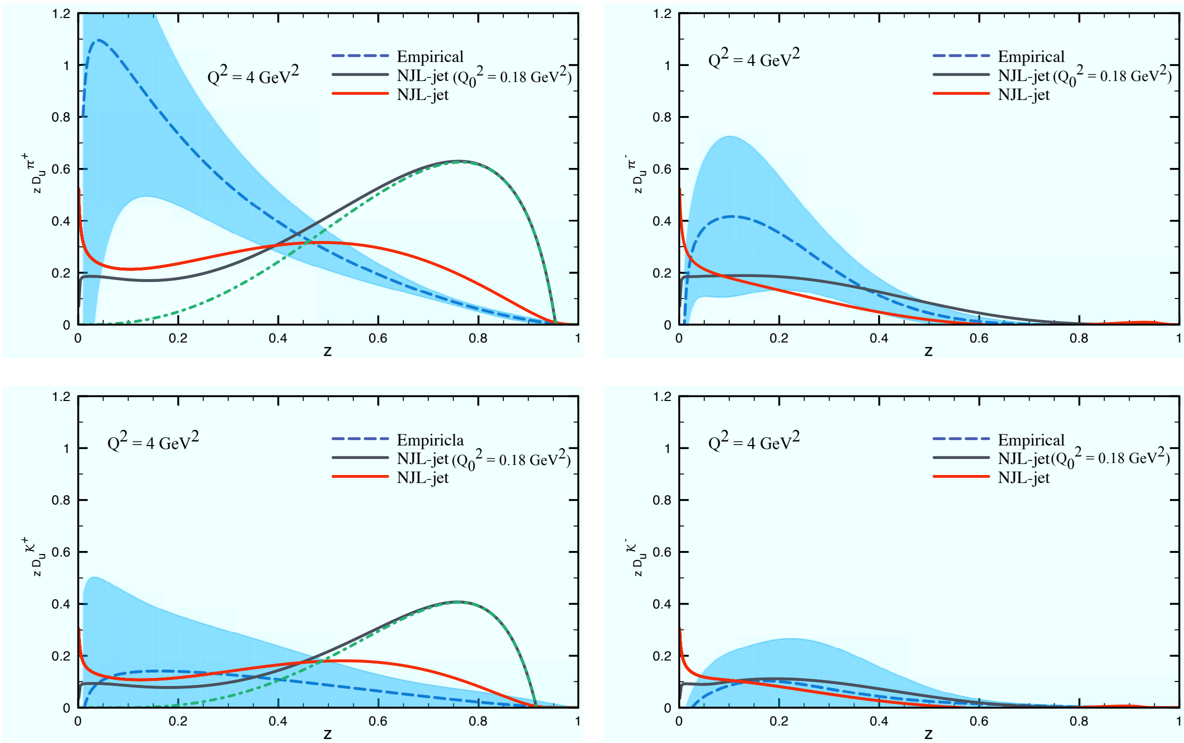

Figure 1: Fragmentation functions for $u \to \pi^{\pm}$ and $u \to K^{\pm}$. The black solid line is the result of the NJL-jet model at the model scale ($Q_0^2 = 0.18$ GeV^2), the red solid line is the result after LO evolution to $Q^2 = 4$ GeV^2, and the blue dashed line is the empirical NLO result of Ref. [3], evolved to $Q^2 = 4$ GeV^2. We also indicate the uncertainties of the empirical functions by the shadowed areas. The green dash-dotted line shows the renormalized function for the elementary fragmentation process.

possible to transfer 100% of the initial quark momentum and isospin to the mesons, we take this limit in the final results. The details of this product ansatz can be found in Ref.[1].

In the numerical calculations we take into account the fragmentation to pions and kaons only. To describe these mesons, we use the Nambu-Jona-Lasinio (NJL) model as an effective quark theory. The results for the "favored" fragmentation processes $u \to \pi^+$ and $u \to K^+$ are shown in the left panels of Fig. 1, and those for the "unfavored" processes $u \to \pi^-$ and $u \to K^-$ are shown in the right panels. Approximately 70% (80%) of the initial u quark light cone momentum (isospin) goes to the pions, and the rest to the kaons. In order to improve the agreement with the empirical fragmentation functions, one should include the fragmentation processes to intermediate vector mesons (ρ, ω), and perform the Q^2 evolution in next-to-leading order. These points, as well as the fragmentation to $N\overline{N}$ pairs, will be discussed in a forthcoming paper [4].

Because the NJL model has been shown to provide a powerful basis to describe processes at finite baryon density [5], it would be very interesting to apply this model to to the description of fragmentation processes on nuclear targets.

References

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