An interpretation of saturation phenomena as Glauber-Gribov multiple scattering

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OVERVIEW

Minijet production in pA and AA collisions

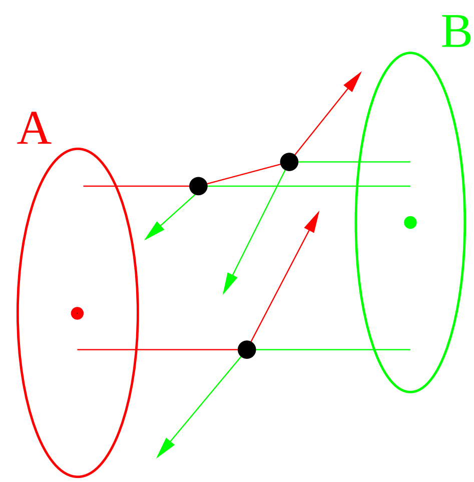
- pQCD + Glauber rescatterings
  $\rightsquigarrow$ dipole representation of qA scattering
- CGC in "Gaussian approximation"
  $\rightsquigarrow$ dipole representation of qA scattering
- CGC$_{\rm semihard}$ is pQCD+Glauber!
- What do RHIC data have to say?
- Summary and conclusions

Semihard interactions: rescatterings

Calucci, Treleani, PRD41(90)3367, PRD44(91)2746

Assumptions:

- QCD generalized factorization
  $$D_A^n = \frac{1}{n!} \underbrace{G\,\tau_A \ldots G\,\tau_A}_{n \text{ times}} e^{-\int G\,\tau_A}$$
- Factorization of the n-body x-sec.:
  $$S^{(n,m)} = \prod_{\substack{i=1,n \\ j=1,m}} S_{ij}$$
- Only partonic elastic scatterings



$N_{jet} \leq 2N_{collisions}$

Average number of minijets

Def. minijet = parton with at at least 1 semi-hard scattering
semi-hard scattering = $p_{exch} > p_0$

$$\frac{dN^A_{jet}}{dx}(b) = \int d^2r \overbrace{G(x,Q^2)\tau_A(b-r)}^{\text{density of projectiles}} \underbrace{\left(1 - e^{-\overbrace{\int_{4p_0^2/xs}^{1} dx'\hat{\sigma}_H(p_0)G(x',Q^2)\tau_B(r)}^{\langle n_B^{\text{scat}}\rangle \text{ or ``target opacity''}\,\omega_B}}\right)}_{\text{prob. of at least 1 scatt.}}$$

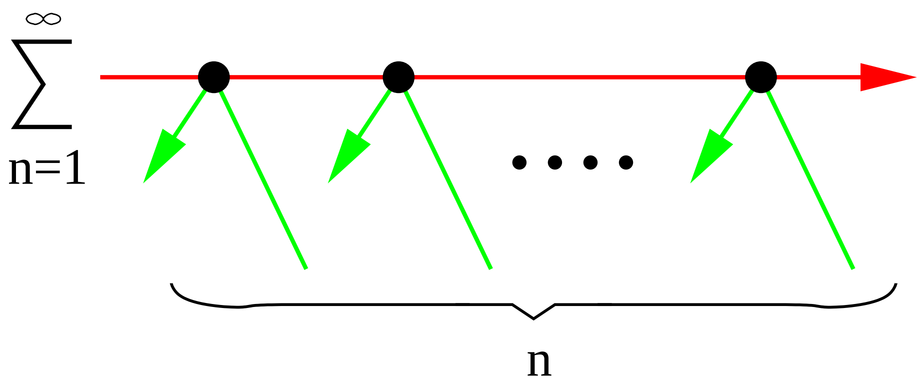
- pQCD gluon-gluon elastic x-sect.: $\hat{\sigma}_H(p_0) = \int^{k^{\max}} d^2k \frac{9/2\,\alpha_S^2(Q^2)}{(k^2+p_0^2)^2} \propto \frac{1}{p_0^2}$
- Infrared cutoff: p_0 • $k^{\max 2} = xx's/4 - p_0^2$

⇝ Glauber multiple scattering formula at parton level

Expansion in the no. of scatterings

$$\frac{dN^A_{jet}}{dx}(b) = \int d^2r\, G(x,Q^2)\tau_A(b-r)$$

$$\times \sum_{n=1}^{\infty} \frac{1}{n!} \underbrace{\left[\int_{4p_0^2/xs} dx' \sigma_H(p_0) G(x',Q^2)\tau_B(r) \right]^n \overbrace{e^{-\int dx' \sigma_H(p_0) G(x',Q^2)\tau_B(r)}}^{\text{absorption factor (prob. conservation)}}}_{\text{PROBABILITY of } n \text{ scatterings}}$$



Two remarkable limits

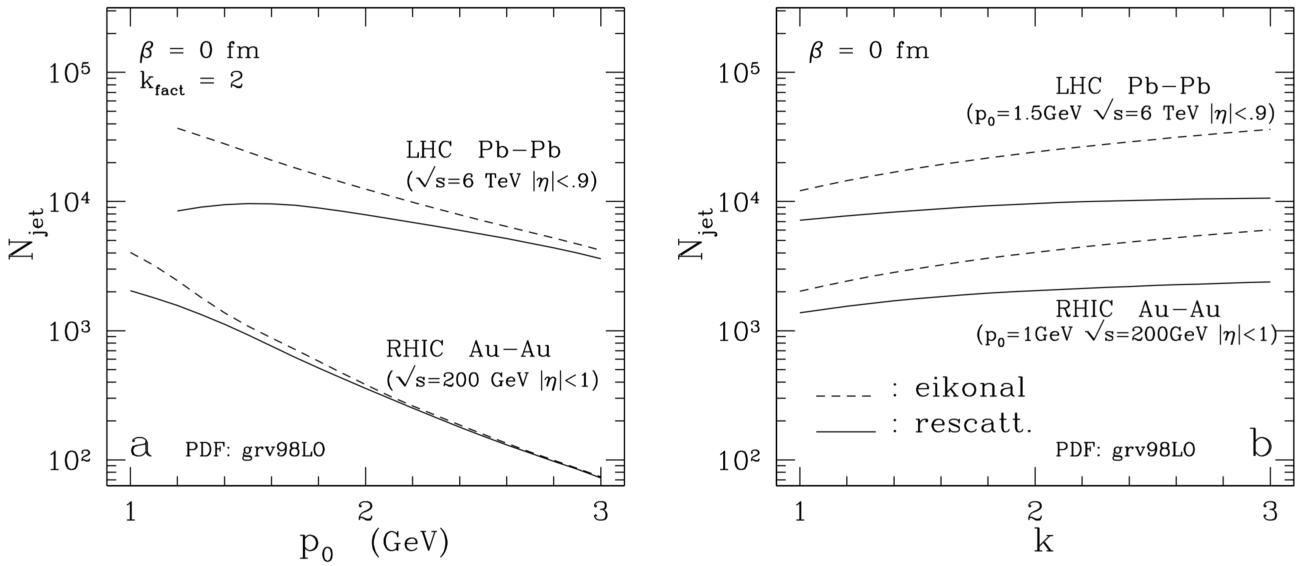
$$\frac{dN_{jet}}{dx}(b) \longrightarrow \begin{cases} 2\int G\,\tau_A\,\sigma_H\,G\,\tau_B = \dfrac{dN^{(1)}_{jet}}{dx}(b) & \dfrac{p_0}{\sqrt{s}} \to 1 \\ \int G\,\tau_A + \int G\,\tau_B \overset{def.}{=} \dfrac{dN_{lim}}{dx}(b) & \dfrac{p_0}{\sqrt{s}} \to 0 \end{cases}$$

"black-disc limit"

- at high cutoff: single parton-parton scattering
- finite limit at low cutoff:
  "Elastic semihard collisions cannot free more partons than those inside the incoming nucleus"

Initial conditions A.A., D.Treleani, Phys.Rev.D 63(2001)116002

NOTE: in all computations we set the scale $Q = p_0$.



Rescatterings vs. single-scattering:

- Less sensitive to both p_0 and the k-factor
- Minijet multiplicity tends to saturate at low p_0: the black-disc regime is setting in

Choosing typically $p_0 = 1\text{GeV}$ at RHIC
$p_0 = 2\text{GeV}$ at LHC

$$\left.\frac{dN}{dy}\right|_{y=0} \approx \; 1000 \text{ (RHIC)}, \; 5000 \text{ (LHC)}$$

Black-disc and saturation cutoff

A.A., Phys.Rev.C 64 (2001) 064905

Let's exploit the black-disc limit

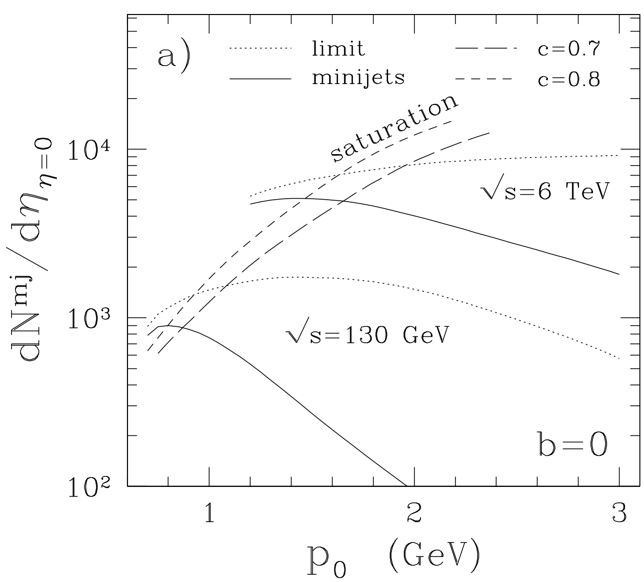
$$\frac{dN_{jet}}{dx}(b;p_0) \xrightarrow[p_0\to 0]{} \frac{dN_{lim}}{dx}(b;p_0)$$

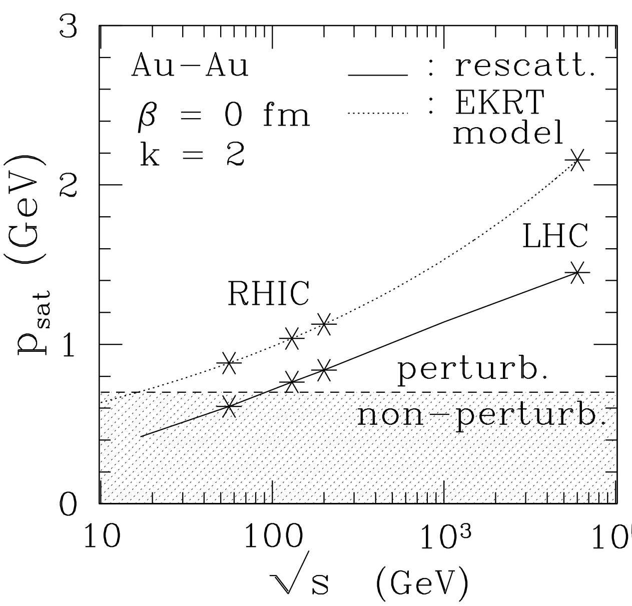
Def. Saturation cutoff: p_0 at which blackness sets in

$$N_{jet}(p_0 = p_{sat}) = 70\%\, N_{lim}(p_0 = p_{sat})$$

Def. Saturated initial conditions:

$$N_{jet}^{sat} = N_{jet}(p_{sat})$$





- 70% is a parameter. Results don't depend strongly on it.
- Def. of p_{sat} as blackness of the target equivalent to local saturation of gluon distribution per unit transverse area

Iancu,Itakura,McLerran, hep-ph hep-ph/0212123

Dipole representation

AA,Treleani PRD64(2001); Gyulassy,Vitev PRD66(2002)
AA hep-ph/0212148

- Consider a quark scattering on A at impact parameter b

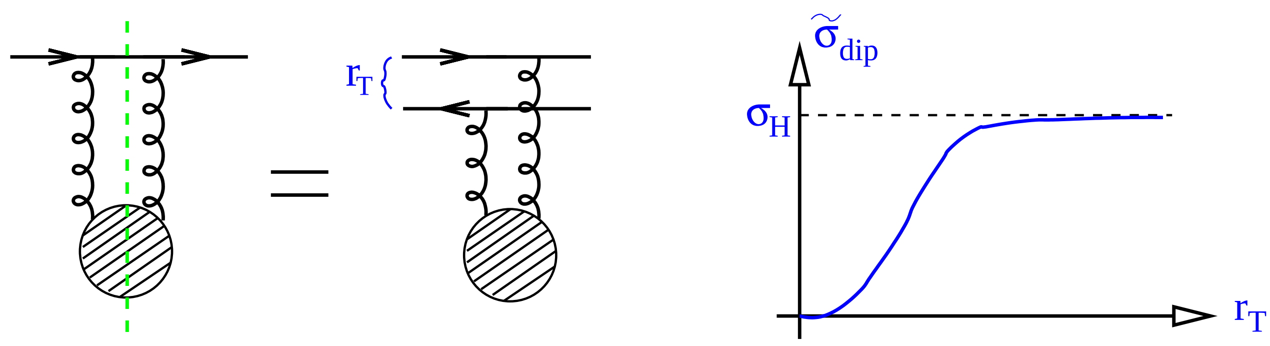
$$\frac{d\sigma^A_{qA}}{d^b dx d^2p_T} = \sum_{n=1}^{\infty} \delta^{(2)}(p_T - \sum_{j=1,n} k_{Tj}) \times \frac{[\tau_A(b)]^n}{n!} e^{-\sigma_H \int dx' G(x')\tau_A(b)}$$
$$\times \frac{d\sigma_H}{d^2k_{T1}} \int_{4p_0^2/xs} dx'_1 G(x'_1) \times \ldots \times \frac{d\sigma_H}{d^2k_{Tn}} \int_{4p_0^2/xs} dx'_n G(x'_n)$$

- Resummations are possible in coordinate space:

$$\frac{d\sigma^A_{qA}}{d^b dx d^2p_T} = \int \frac{d^2r}{(2\pi)^2} e^{i\vec{p}_T \cdot \vec{r}_T} \underbrace{\left[e^{-\tilde{\sigma}_{dip}(r_T,b;p_0)\tau_A(b)} - e^{-\sigma_H(b;p_0)\tau_A(b)} \right]}_{S_{pQCD}(r_T,b)_{p_0}}$$

where $\tilde{\sigma}_{dip}(r_T, p_0) = \int d^2k_T \frac{2\pi\alpha_s^2 \int dx' G(x')}{(k_T^2 + p_0^2)^2} \left[1 - e^{-\vec{k}_T \cdot \vec{r}_T} \right]$
and $\sigma_H(p_0) = \lim_{r_T \to \infty} \tilde{\sigma}_{dip}(r_T; p_0)$

- $\tilde{\sigma}_{dip}$ is interpreted as the cross-section in coordinate space for the semihard scattering of a dipole on a nucleon.

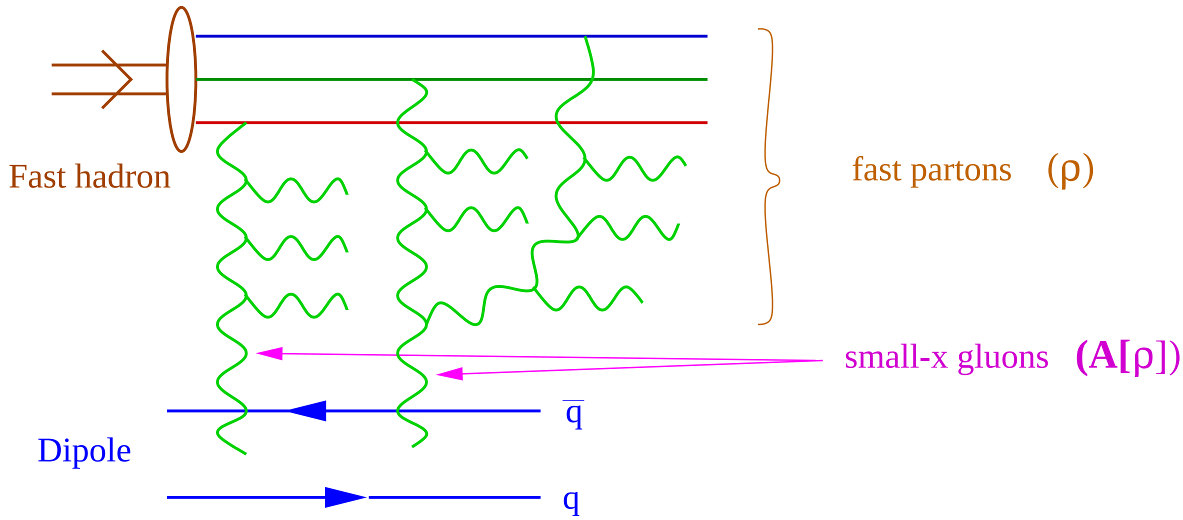


$\Longrightarrow$ qA is rewritten as a multiscattering of a $q\bar{q}$ dipole on A

The Colour Glass Condensate

review: Iancu, Leonidov, McLerran, hep-ph/0202270

An effective theory for the nucleus gluon field at small-x



- Observables $\mathcal{O}\big[A[\rho]\big]$ are averaged over ρ with weight function $W_\tau[\rho]$, where $\tau = \log(1/x_{Bj})$:

$$\langle \mathcal{O} \rangle_\tau = \int D\rho W_\tau[\rho] \mathcal{O}\big[A[\rho]\big]$$

- Gluons at x' are colour sources for gluons at $x < x'$
  $\rightsquigarrow$ RGE for W_τ: schematically

$$\frac{\partial W_\tau}{\partial \tau} = \frac{1}{2} \int \delta_\rho \chi[\rho] \delta_\rho W_\tau[\rho]$$

- **Gaussian approximation (GA)** Iancu,Itakura,McLerran '02

A self consistent approximation for W_τ:

$$W_\tau[\rho] = \mathcal{N}_\tau \exp\left[-\frac{1}{2} \int_\infty^\tau dy \int dx_\perp dy_\perp \frac{\rho^a(x_\perp)\rho^a(y_\perp)}{\lambda_y(x_\perp, y_\perp)} \right]$$

- **qA scattering and the dipole**: Gelis, Jalilian-Marian '02

  Related to the $q\bar{q}$-nucleus cross-section:

$$\frac{d\sigma_{qA}^{A}}{d^b dx d^2 p_T} = \int \frac{d^2 r}{(2\pi)^2} \, e^{i\vec{p}_T\cdot\vec{r}_T} \, S(r_T;b)$$

- Assume: straight popagation, no gluon bremsstrahlung

  $\Rightarrow$ in the Gaussian Approximation:

$$S_{GA}(r_T;b) = \exp\Big\{ - \underbrace{g^2 C_R \int d^2 k_T \frac{\mu_\tau(k_T,b)}{k_T^4} \left[1 - e^{-\vec{k}_T\cdot\vec{r}_T}\right]}_{\tilde{\sigma}_{dip}(r_T)\,\tau_A(b)} \Big\}$$

  where μ_τ is interpreted as unintegrated gluon PDF, and has two limits

$$\mu_\tau(k_T,b) = \begin{cases} k_T^2 \phi_\tau(k_T,b) \propto k_t^2 \alpha_s \int_{-\infty}^{\tau} d\tau' \frac{\partial\, x' G(x',k_T^2)}{\partial k_T^2} \tau_A(b) & k_T^2 \rangle\rangle Q_s^2 \\ \delta_\tau k_T^2 \tau_A(b) & k_T^2 \langle\langle Q_s^2 \end{cases}$$

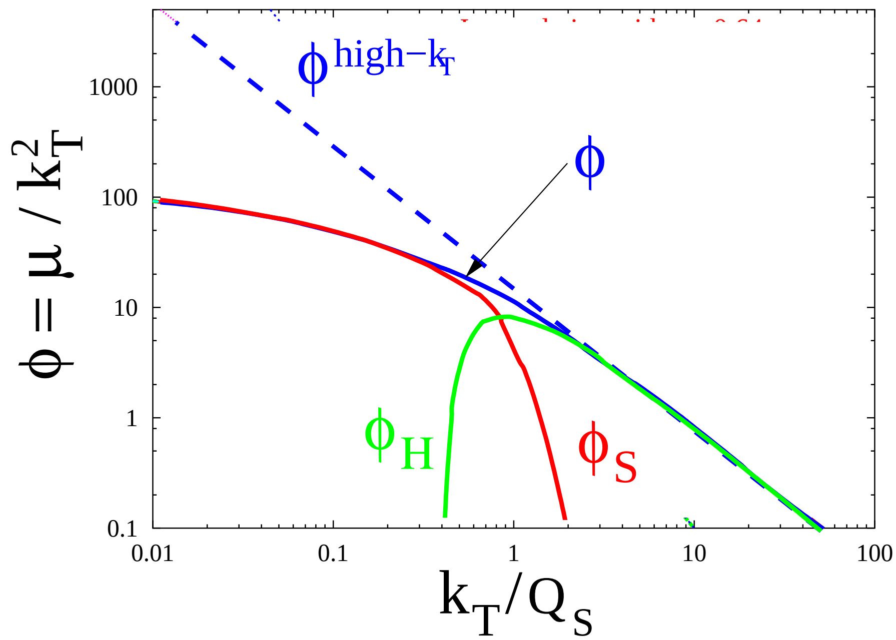
  with $Q_s^2 = Q_0^2 \, e^{c\alpha_s(\tau-\tau_0)}$ the saturation momentum.

- In the high-k_T limit S_{GA} very similar to S_{pQCD}, but self-regulates the IR divergences.

CGC vs. pQCD+rescatterings

Step 1 In CGC separate "hard" and "soft" interactions:

Def. $\mu_\tau = \mu_\tau^S + \mu_\tau^H$ with $\begin{cases} \mu_\tau^H = \mu_\tau^{\text{high}-k_T} \times \dfrac{k_T^4}{(k_T^2 + Q_s^2)^2} \\ \mu_\tau^S = \mu_\tau - \mu_\tau^H \end{cases}$



Accordingly: $\tilde{\sigma}_{dip}(r_T) = \tilde{\sigma}_S(r_T) + \tilde{\sigma}_H(r_T)$

- The "hard" dipole-nucleon cross-section is then

$$\tilde{\sigma}_H \propto \int d^2k_T \frac{\mu_\tau^{\text{high}-k_T}(k_T, b)}{(k_T^2 + Q_s^2)^2} \left[1 - e^{-\vec{k}_T \cdot \vec{r}_T}\right]$$

Step 2 Aproximate unintegrated PDF with integrated ones:

$$\mu_\tau^{\text{high}-k_T} \propto k_T^2 \int_{-\infty}^{\tau} d\tau' \, \frac{\partial x' G(x', k_T^2)}{\partial k_T^2} \tau_A(b) \approx \int_{-\infty}^{\tau} d\tau' \; x' G(x', k_T^2)$$

and take $G(x, Q^2)$ from, e.g., the GRV98 parametrization.

$\Longrightarrow$ We obtain the same as in pQCD but with $p_0 = Q_s$:

$$\tilde{\sigma}_H \, \tau_A \propto \int d^2 k_T \frac{\int_{-\infty}^{\tau} d\tau' x' G(x', k_T^2) \tau_A(b)}{(k_T^2 + Q_s^2)^2} \left[1 - e^{-\vec{k}_T \cdot \vec{r}_T} \right]$$

Step 3 Assume dominance of hard scatterings

For integrated multiplicities:

$$\frac{d\sigma_q}{dx \, d^2 b} = \underbrace{e^{-\sigma_H \tau_A} \left[1 - e^{-\sigma_S \, \tau_A} \right]}_{\text{soft part}} + \underbrace{\left[1 - e^{-\sigma_H \tau_A} \right]}_{\text{hard part}} \approx \left[1 - e^{-\sigma_H \tau_A} \right]$$

$\Longrightarrow$ The semihard limit of CGC is pQCD+rescatterings

Step 4 Use pQCD+rescattering to

QUANTITATIVELY COMPUTE!

Charged multiplicities in pQCD+rescatt.

A.A., Phys.Rev.C 64 (2001) 064905

Semihard part: from our model + saturation cutoff

$$\left.\frac{dN^{ch}}{d\eta}\right|_{s.h.}(b) = 0.9\,\frac{2}{3}\,\frac{dN^{sat}_{jet}}{d\eta}(b)$$

- Isentropic expansion & d.o.f conversion $\rightsquigarrow$ 0.9
- Parton-hadron duality $\rightsquigarrow$ 2/3

Soft part (= non minijet-like):

from the wounded nucleon model

$$\left.\frac{dN^{ch}}{d\eta}\right|_{soft}(b) = x(\sqrt{s})\; n_{pp}(\sqrt{s})\; N_{part}(b)$$

$x(\sqrt{s})$ = relative weight of soft and hard
$n_{pp}(\sqrt{s})$ = ch. part. pseudorap. density in *pp* scattering
$N_{part}(b)$ = Number of participant nucleons

Two-component model (soft + semihard)

$$\frac{2}{N_{part}(b)}\frac{dN^{ch}}{d\eta}(b) = 2\,x(\sqrt{s})\; n_{pp}(\sqrt{s}) + \frac{2}{N_{part}(b)}\frac{3}{5}\frac{dN^{sat}_{jet}}{d\eta}(b)$$

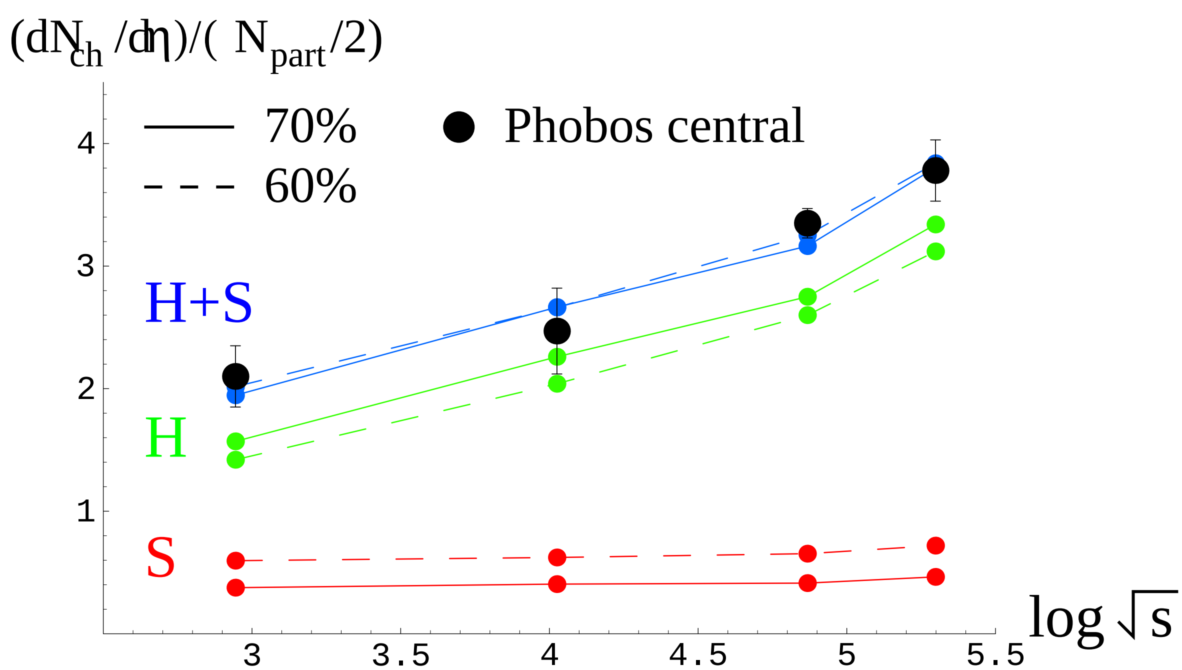
Note: soft part independent of impact parameter

Soft component at $b = 0$

- Naive modeling of the fraction of soft processes

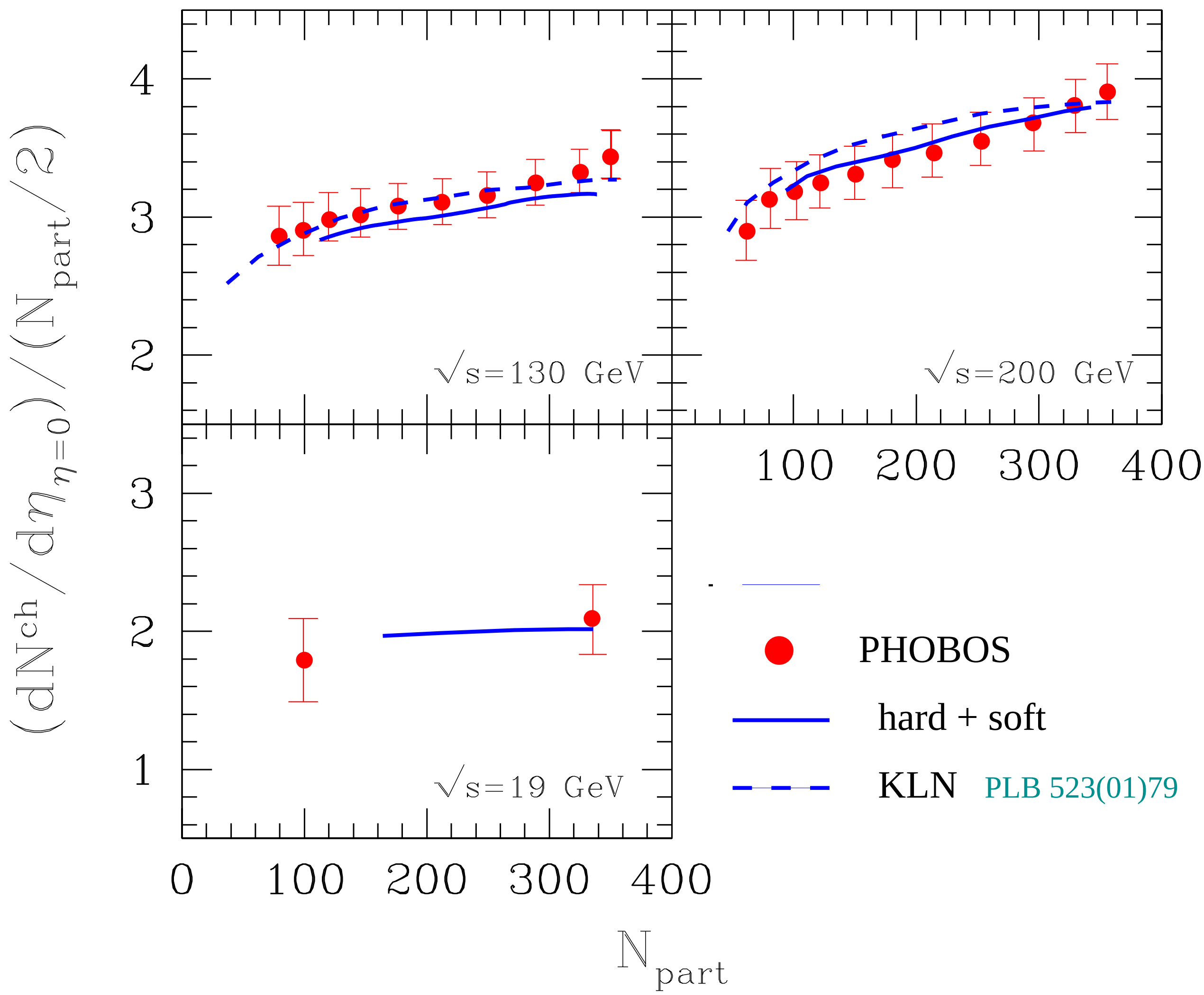
$$\frac{dN^{ch}_{soft}}{d\eta}(s) \Big/ \frac{dN^{ch}}{d\eta}(s) \;=\; \frac{1}{1 + a\log(\sqrt{s})^b} \;\xrightarrow[\sqrt{s}\to\infty]{}\; 0$$

- By fitting RHIC data for central collision only at $\sqrt{s} = 56, 130, 200$ GeV we obtain



- at RHIC $\sqrt{s} = 130 - 200$ GeV $\Rightarrow$ 15-20% soft contribution
- extrapolation: at $\sqrt{s} = 19$ GeV $\Rightarrow$ 25-40% soft
  at $\sqrt{s} = 5500$ GeV $\Rightarrow$ 5-10% soft

Centrality dependence



- At $\sqrt{s} = 130,\ 200$ GeV, we used k=2 and 70% saturation
- At $\sqrt{s} = 19$ GeV, we used k=5 and 60% saturation

We uesed standard DGLAP evolved PDF's (GRV98)

$\Longrightarrow$ **no sign of saturation!**

CONCLUSIONS

- Based on the data for N_{ch} and the presented phenomenological analysis
  – no clear sign of saturation effects at RHIC
- We have two approaches: pQCD and CGC
  – we should fully exploit these two handles, theoretically & numerically
- To asses or disprove the above conclusions:
  – study more exclusive observables and kinematic regions sensitive to saturation effects
  – e.g.: Cronin in dA, back-to-back correlations

Remark I

Average minijet energy

$$E_{jet}(b) = \int d^2r dx \left(x\frac{\sqrt{s}}{2} \right) G(x) \tau_A(b-r) \left(1 - e^{-\int dx' \sigma_H(xx') G(x') \tau_B(x',r)} \right) + A \leftrightarrow B$$

- Performing the integral over d^2r we have an upper limit for the minijet total energy:

$$E_{jet}(b) \lesssim N_{part}(b) \frac{\sqrt{s}}{2} \int dx\, xG(x, p^2_{sat})$$
$$= N_{part}(b) \frac{\sqrt{s}}{2}$$

Energy conservation has been implemented
(consequence of the unitarization)

- The inequality is saturated in the black disk limit, only geometry survives:

Participant scaling

Remark II

Minijet multiplicity and transverse energy

$$N_{jet}(b) = \int d^2r dx \, G(x) \, \tau_A(b-r) \left(1 - e^{-\int dx' \sigma_H(xx') G(x') \tau_B(x',r)}\right) + A \leftrightarrow B$$

- In the **black disk limit** not only geometry survives:

$$N_{jet}(b) \approx N_{part}(b) \times \int dx \, G(x, p_{sat}^2(b))$$

$$E_{T\,jet}(b) \approx N_{part}(b) \times p_{sat}(b) \int dx \, G(x, p_{sat}^2(b))$$

(p_{sat} = scale at which the black disk limit is reached)

Participant scaling violations
(due to pQCD evolution)

- **Charged multiplicitity**:

Naively $N_{ch} \propto N_{jet}$ $\Longrightarrow$ **Scaling violations are observable in the final state**