Color confinement and the origin of mass in pure Yang-Mills theory

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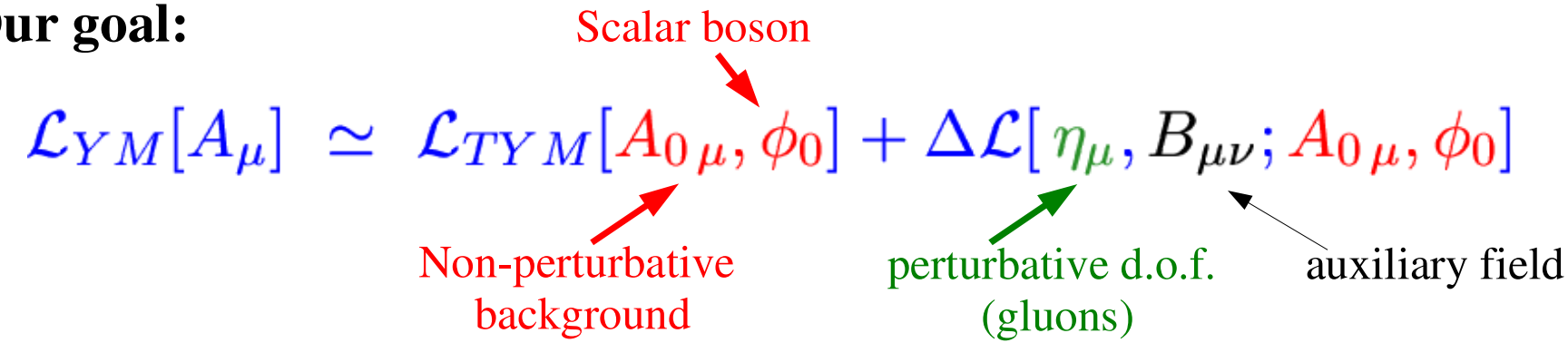
Based on: AA, Belli, Martellini, Zeni, PLB 431 (1998) 127, NPB 505 (1997) 540
Fucito, Martellini, Zeni, NPB 496 (1997) 259

Our goal:



Our path:

- Intro: BRST quantization of YM theory
- The BFYM theory - 1
  - First order formulation of YM
  - Why 1st order?
  - A problem is found...
- The BFYM theory - 2
  - ...and solved
  - Gluons as a perturbation over a topological background
- Dynamical mechanism for confinement and mass generation

Introduction

BRST quantization of YM theory

- Classical YM lagrangian:

$$\mathcal{L}^{cl}_{YM} = \frac{1}{4g^2} F_A * F_A \qquad \left(\frac{1}{4g^2}\sum_a F^a_{A\,\mu\nu} F^{a\,\mu\nu}_A\right)$$

NB: wedge products always understood

- Gauge symmetry:

$$\delta_g A = \mathrm{d}_A c = \mathrm{d}c + [A, c] \qquad \left(\partial_\mu c^a + f^{abc} A^b_\mu c^c\right)$$

- To gauge-fix it, introduce:

anti-commuting ghosts $c,\ \bar{c}$

Lagrange multiplier h_A

- BRST operator (generalization of gauge symmetry):

$$sA = \mathrm{d}_A c \qquad s\bar{c} = h_A$$

$$sc = -\frac{1}{2}[c, c] \qquad sh_A = 0$$

$$s^2 = 0$$

$\left(f^{abc} c^b c^c\right)$

BRST quantization of YM theory

- Choose a gauge fixing condition:

$$\mathrm{d}^\dagger A = 0 \qquad \left(\partial_\mu A^a_\mu = 0\right)$$

- Define the quantum YM Lagrangian:

$$\mathcal{L}_{YM} = \mathcal{L}^{cl}_{YM} + s\big(\overbrace{\bar{c} * \mathrm{d}^\dagger A + \alpha\, \bar{c} * h_A}^{\text{gauge-fixing fermion}}\big)$$

$$= \frac{1}{4g^2} F_A * F_A + \bar{c} * \mathrm{d}^\dagger \mathrm{d}_A c + h_A * \mathrm{d}^\dagger A + \alpha\, h_A * h_A$$

- BRST (gauge) invariance:

$$\boxed{s\mathcal{L}_{YM} = 0}$$

The BFYM theory - 1

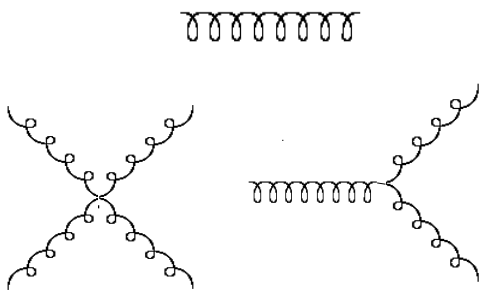
1st order formulation of YM

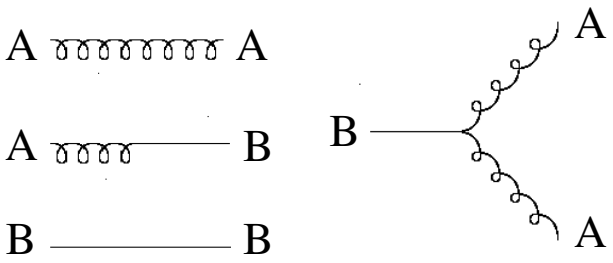
Fucito, Martellini, Zeni NPB 496 (1997) 259

- Introduce auxiliary field $B_{(2)} = B^a_{\mu\nu}$ then:

$$\mathcal{L}^{cl}_{YM} = \frac{1}{4g^2} F_A * F_A \longrightarrow \mathcal{L}^{cl}_{BFYM} = i B_{(2)} F + g^2 B_{(2)} * B_{(2)}$$

$$\left(= i\varepsilon^{\mu\nu\rho\sigma} B^a_{\mu\nu} F^{a\,\rho\sigma}_A + g^2 B^a_{\mu\nu} B^{a\,\mu\nu} \right)$$





- Equivalent to YM at classical level:

  use B eqn. of motion $B_{(2)} = -\frac{i}{2g^2} * F_A$ in $\mathcal{L}_{BFYM}$

- Equivalent at renormalized level:
  - only one coupling constant ***g***
  - same β-function at 1-loop

(3D) AA, Belli, Martellini, Zeni '97
(4D) AA, Belli '97 ; Fucito et al. '97
(4D) Martellini, Zeni '97

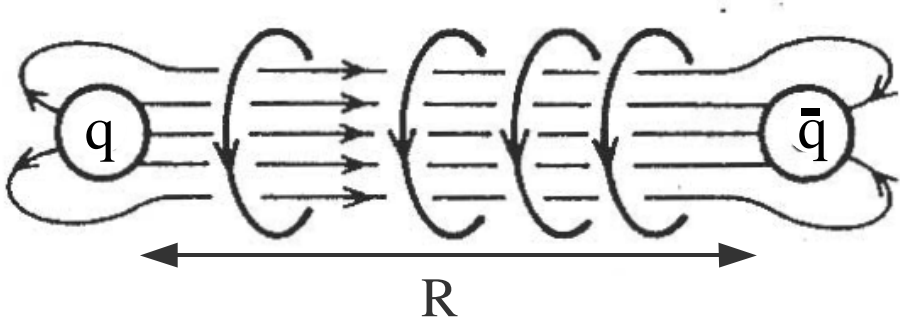
Advantages - 1 (not pursued here)

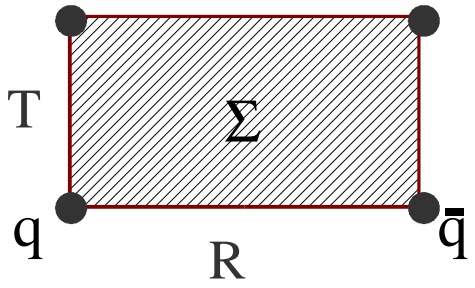
- "Dual superconductor" mechanism for color confinement ('t Hooft '78)

Wilson loop: $W(\mathcal{C}) = \mathrm{Tr}\left[\mathcal{P}\exp\left(ig\oint_{\mathcal{C}} A\cdot ds\right)\right]$

$$\langle W(\mathcal{C})\rangle \approx e^{-kTV(R)}$$

If vacuum is a ***dual type II superconductor***, quarks are confined:





$$V(R) = kR \quad \Longrightarrow \quad \langle W(C)\rangle \approx e^{-kTR} = e^{-k\Sigma}$$ **AREA LAW**

- Order and disorder parameter:

$W(\mathcal{C})$:= creates a color electric flux line along $\mathcal{C}$

$M(\mathcal{C}')$:= creates a color magnetic flux line along $\mathcal{C}'$

- Classification of the vacuum:

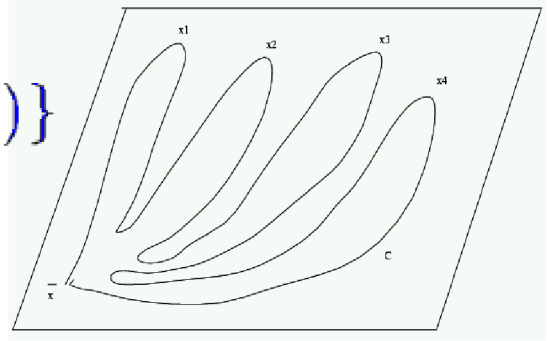
$-\log\langle W(\mathcal{C})\rangle$	$-\log\langle M(\mathcal{C})\rangle$	*phase*
$\Sigma(\mathcal{C})$	$L(\mathcal{C})$	permanent quark confinement
$\Sigma(\mathcal{C})$	$\Sigma(\mathcal{C})$	...
$L(\mathcal{C})$	$L(\mathcal{C})$	...
$L(\mathcal{C})$	$\Sigma(\mathcal{C})$	...

- In YM no explicit formula for *M(C)*

- In BFYM one may use $B_{\mu\nu}$ to construct *M(C)*:

$$M(\Sigma, C) \equiv \mathrm{Tr}\exp\{ik\int_{\Sigma} d^2y\ \mathrm{Hol}^{y}_{\bar{x}}(\gamma)B(y)\mathrm{Hol}^{\bar{x}}_{y}(\gamma')\}$$

$$\left(\mathrm{Hol}^{y}_{\bar{x}}(\gamma) \equiv P\exp\{i\int_{\bar{x}}^{y} dx^{\mu}A_{\mu}(x)\}\right)$$



- And $\begin{cases} -\log\langle W(\mathcal{C})\rangle = \Sigma(\mathcal{C}) \\ -\log\langle M(\mathcal{C})\rangle = L(\mathcal{C}) \end{cases}$ BFYM = YM is confining

Fucito, Martellini, Zeni (97)

Advantages - 2 (today's focus)

- New perturbative expansion around the "topological BF theory"

$$\mathcal{L}^{cl}_{BFYM} = iB_{(2)}F + g^2 B_{(2)} * B_{(2)} \xrightarrow[g^2 \to 0]{} iB_{(2)}F = \mathcal{L}_{BF}$$

- Definition: topological field theory

  "Energy momentum tensor is BRST exact: $T_{\mu\nu} = s\, V_{\mu\nu}$ **"**

  - Observables such that $\frac{\delta}{\delta g_{\mu\nu}} \mathcal{O} = s\, \mathcal{R}_{\mu\nu}$ have metric-independent v.e.v.
  - Large class of topological invariants (e.g. Donaldson polynomials)
  - They have no local degrees of freedom (i.e. contain no particle states)

- The BF "core" of the theory (invisible in 2nd order YM) is sensitive to
  - topology of the base manifold (i.e., manifold on which theory is defined)
  - topology of the gauge field configurations
  - nonlocal excitations: instantons, ...

- YM seen as a perturbation of its BF topological core gives access to non perturbative information

- Loosely speaking, in BFYM:

  gluons seen as fluctuations over a non-perturbative background

A problem, though...

- The "free" Lagrangian L_{BF} has 1 more symmetry than the full L_{BFYM}

$$\mathcal{L}^{cl}_{BFYM} = iB_{(2)}F + g^2 B_{(2)} * B_{(2)}$$

$$s_g A = \mathrm{d}_A c$$
$$s_g B_{(2)} = [B_{(2)}, c]$$

$$\mathcal{L}^{cl}_{BF} = iB_{(2)}F$$

$$s_g A = \mathrm{d}_A c$$
$$s_g B_{(2)} = [B_{(2)}, c]$$

$$s_T A = 0$$
$$s_T B_{(2)} = -\mathrm{d}_A \psi$$

topological symmetry

$\Longrightarrow$ zero-mode problem in the BFYM propagators

$\Longrightarrow$ problems in the perturbative expansion

The BFYM theory - 2

Solution of the problem

- Promote the zero-mode to a full field: $B_{(2)} \longrightarrow B_{(2)} - \frac{\mathrm{d}_A \eta_{(1)}}{\sqrt{2g}}$

$$\mathcal{L}^{cl}_{BFYM} \longrightarrow iB_{(2)}F + g^2\left(B_{(2)} - \frac{\mathrm{d}_A\eta_{(1)}}{\sqrt{2g}}\right) * \left(B_{(2)} - \frac{\mathrm{d}_A\eta_{(1)}}{\sqrt{2g}}\right)$$

- We gain not 2 but 3 symmetries!

$$s_g A = \mathrm{d}_A c \qquad s_g B_{(2)} = [B_{(2)}, c] \qquad s_g \eta_{(1)} = [\eta_{(1)}, c]$$

$$s_T A = 0 \qquad s_T B_{(2)} = -\mathrm{d}_A \psi_{(1)} \qquad s_T \eta_{(1)} = -\sqrt{2g}\psi_{(1)}$$

$$s' A = 0 \qquad s' B_{(2)} = [F_A, \rho]/\sqrt{2g} \qquad s' \eta_{(1)} = \mathrm{d}_A \rho$$

anticommuting scalars and vector ghosts

- The vector ghost ψ admits a guage transformation:

$$s_g \psi_{(1)} = -[\psi_{(1)}, c] \qquad s_T \psi_{(1)} = \mathrm{d}_A \phi \qquad s' \psi_{(1)} = 0$$

a scalar boson!!
(ghost for ghost)

Isolating non-perturbative d.o.f.

NOTE: I drop the suBscripts (i)

$$\mathcal{L}^{cl}_{BFYM} \longrightarrow iBF + g^2 (B - \frac{\mathrm{d}_A \eta}{\sqrt{2}g}) * (B - \frac{\mathrm{d}_A \eta}{\sqrt{2}g})$$

no local d.o.f. $\Longrightarrow$ gluons sit in η

Moreover, e.o.m. $\eta = 0$ correct for a local fluctuation

- Let's isolate the non-perturbative and perturbative piece of A:
  - non-perturbative background: $A_0 = A - \sqrt{2} g \, \eta$
  - perturbative part = gluons: η
  - Moreover, $c_0 = c - \sqrt{2} g \rho; \psi_0 = \psi - [\eta \rho]; \phi_0 = -\phi + \frac{1}{2}[\rho . \rho]$

BRST symmetry

- BRST operator: $s = s_g + s_T + s'$

$$
\begin{aligned}
sA_0 &= \mathrm{d}_{A_0} c_0 + 2g^2 \psi_0 \\
sc_0 &= -\frac{1}{2}\,[c_0, c_0] + 2g^2 \phi_0 \\
s\psi_0 &= -\,[\psi_0, c_0] - \mathrm{d}_{A_0} \phi_0 \\
s\phi_0 &= [\phi_0, c_0]
\end{aligned}
$$

fields and symmetries of Witten's Topological YM theory (TYM)

$$
\begin{aligned}
sB &= [B, c_0] - \mathrm{d}_{A_0} \psi_0 + f(B, \eta.\rho; A_0, \psi 0) \\
s\eta &= [\eta, c_0] - \sqrt{2} g \psi_0 + \mathrm{d}_{A_0} \rho + \sqrt{2} g\,[\eta, \rho] \\
s\rho &= -\,[\rho, c_0] - \sqrt{2} g \phi_0 - \frac{g}{\sqrt{2}}\,[\rho, \rho]
\end{aligned}
$$

(plus transformations of $\bar{c}, h_{A_0} \quad \bar{\chi}, h_B \quad \bar{\phi}_0, h_{\psi_0} \quad \bar{\rho}, h_\eta$)

Gauge fixing

- Gauge fixing conditions:
  - $F^-_{A_0} = 0$ selects instanton sector
  - $\mathrm{d}^\dagger A_0 = 0$ covariant gauge for background A_0
  - $\mathrm{d}^\dagger_{A_0} \psi_0 = 0$ background gauge for vector ghost ψ_0
  - $\mathrm{d}^\dagger_{A_0} \eta = 0$ background gauge for gluons η

- Lagrangian:

$$\mathcal{L} = \mathcal{L}^{cl}_{BFYM} + s\left(\bar{c}_0 * \mathrm{d}^\dagger A_0 + \bar{\chi}_0 * F^-_{A_0} + \bar{\phi}_0 * \mathrm{d}^\dagger_{A_0}\psi_0 + \bar{\rho} * \mathrm{d}^\dagger_{A_0}\eta\right)$$

A scalar boson! gluons non-pert.background

$$= \mathcal{L}_{TYM}[A_{0\,\mu}, c_0, \psi_0, \phi_0] + \Delta\mathcal{L}[\eta_\mu, B_{\mu\nu}; A_{0\,\mu}, c_0, \psi_0, \phi_0]$$

Witten's TYM - no local d.o.f

auxiliary field

- Perturbatively equivalent to YM (AA,Belli '97; Fucito et al. '97)

Summary up to now

- A careful analysis of 1st order Yang-Mills allows to:
  - display a topological sector governing non perturbative dynamics
  - interpret gluons (η) as fluctuations over non-pert. background (A_0)
  - dynamics of background is governed by the TYM lagrangian
  - a scalar boson (ϕ_0) naturally appears in non-pert. sector

$$\mathcal{L}_{YM}[A_\mu] \simeq \mathcal{L}_{TYM}[A_{0\,\mu}, \phi_0] + \Delta\mathcal{L}[\eta_\mu, B_{\mu\nu}; A_{0\,\mu}, \phi_0]$$

Non-perturbative background

A scalar boson!

perturbative d.o.f. (gluons)

auxiliary field

- The new Lagrangian L_{BFYM} is equivalent to the original L_{YM}, but explicitly displays non perturbative d.o.f. which were hidden in the 2nd order formalism

A dynamical mechanism for confinement and mass generation

Idea: confinement is a property of the non-perturbative fields A_0, c_0, ψ_0, ϕ_0

$\Longrightarrow$ **consider BFYM as a perturbation of TYM**

Step 1: freeze local d.o.f. $\eta = \rho = (B) = 0$

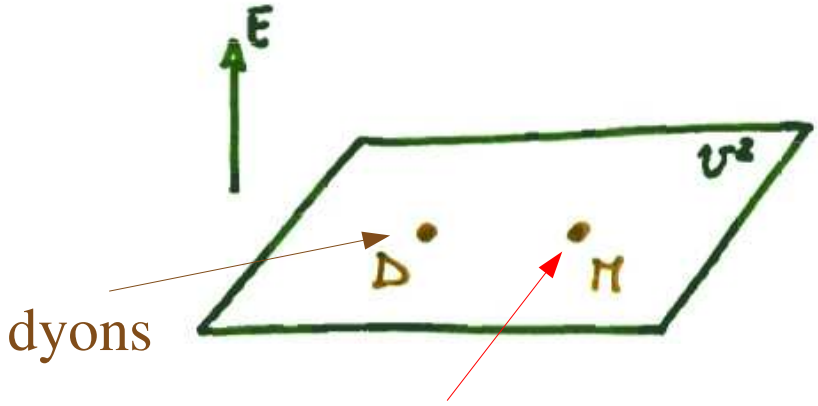
- When $\eta=\rho=B=0$, the BFYM reduces to the Topological Yang-Mills, which is a "twisted" version of N=2 SYM: (Witten '89)

$$\mathcal{L}_{BFYM}(\text{frozen}) = \mathcal{L}_{TYM} \xleftrightarrow[\text{“twist”}]{} \mathcal{L}_{\text{SYM N=2}}$$

- Seiberg-Witten analysis of N=2 SYM: (Seiberg,Witten '94)
  - Vacua of the theory ("moduli space") are classified by

$$v = \langle Tr\phi_0^2 \rangle$$

  - All vacua are degenerate
  - 3 singularities: $v = -u,\ u,\ \infty$



dyons

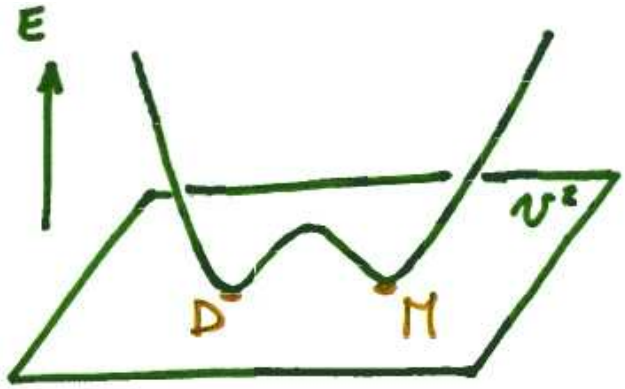
magnetic monopoles
+
confinement as dual superconductor

Step 2: break N=2 to N=1 manually

- If one adds to the N=2 lagrangian terms that break SUSY to N=1

$$\mathcal{L}_{N=2} + \Delta_1 = \mathcal{L}_{N=1}$$

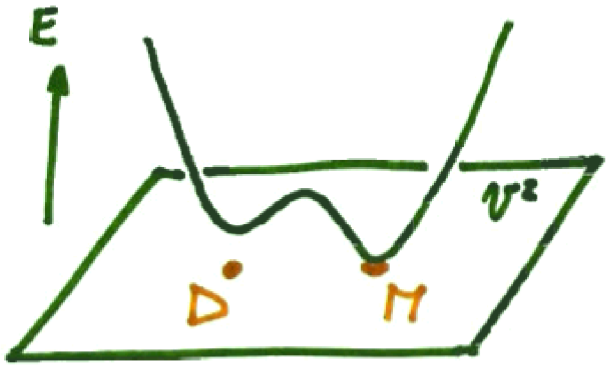
$\Rightarrow$ **degeneracy of the vacua is lifted**



- However, in pure N=2 SYM, the 2 supersymmetries do not allow the breaking term Δ_1 to be generated by quantum correction to the effective potential
  - Δ_1 must be added manually to lift the degeneracy

Step 3: unfreeze local d.o.f. η, ρ, B

- The full BFYM lagrangian has only "N=1/2" susy: the topological s_T
  - corresponds via twist to the scalar combination of N=2 super-charges
- General rule "All that is not forbidden is allowed":
  quantum corrections to the effective action will generate
  - the breaking term Δ_1 from N=2 to N=1
  - also a breaking term $\Delta_{1/2}$ to N=1/2
- The vacua degeneracy is completely lifted
  - **only 1 vacuum survives**



confinement as dual superconductor

$$v = \langle Tr\phi^2 \rangle \neq 0$$

dynamically generated mass scale

Step 4: check this scenario

- Compute the effective action
- Find the minimum of the quantum potential
- Mechanism is valid for any gauge group:
  why $SU(3)_{color}$ is confining but SU(2)xU(1) not?
- ...

if you wish to join the effort, you are welcome!

Conclusions

- YM can be explicitly written to display an hidden topological sector (TYM) describing non-local degrees of freedom

$$\mathcal{L}_{YM}[A_\mu] \simeq \mathcal{L}_{TYM}[A_{0\,\mu}, \phi_0] + \Delta\mathcal{L}[\eta_\mu, B_{\mu\nu}; A_{0\,\mu}, \phi_0]$$

  - reformulation is perturbatively equivalent to YM (when A_0=0)

- a scalar boson ϕ_0 naturally appears, which was hidden in 2nd order YM
  - it's v.e.v. $v = \langle Tr\phi_0^2 \rangle$ may give a mass parameter

- Similar to N=2 SYM, but has only a N=1/2 supercharge:
  - quantum fluctuations due to local d.o.f. in BFYM may select a non-trivial vacuum such that

    a) pure Yang-Mills is confining as a dual superconductor

    b) $v = \langle Tr\phi_0^2 \rangle \neq 0$ gives a dynamically generated mass scale