Hyperon Production Background for the Moller Experiment

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June 20, 2013

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Abstract

Using the bremsstrahlung data from Wiser's thesis[1] and the virtual photon approximations from Tiator and Wright[4] we develop the files needed to account for hyperon weak decay backgrounds in the Moller experiment from the following reactions.

$$\gamma + p \rightarrow \Lambda + K^{+} + X,\ \gamma + p \rightarrow \Sigma^{+} + K^{0} + X,\ \gamma + p \rightarrow \Sigma^{0} + K^{+} + X$$

The hyperon decays in the rest frame have a distribution of decay product momenta which depends on the angle between the rest frame polarization of the hyperon and the vector momentum of the outgoing particles.

$$\frac{dN}{d\Omega} = \frac{N}{4\pi}(1 + h\alpha \vec{S_R} \cdot \hat{k_R}).$$

Here h is the electron helicity, α is the hyperon weak decay asymmetry parameter, $\vec{S_R}$ is the hyperon rest frame polarization and $\hat{k_R}$ is the momentum direction of the decay particle measured in the hyperon rest frame.

Scripts, spectra, particle distribution files and this report are found in

`https://userweb.jlab.org/~aniol/moller/`

0.1 Using Wiser's Bremsstrahlung Data

0.1.1 Kaon Production

David Wiser's thesis[1] reports on kaon photoproduction using bremsstrahlung beams for several values of end point energy, k_0. His data show that the variation of lambda production is exponentially sensitive to the transverse momentum of the produced kaons. His empirical cross section formula assumes that there is only a modest dependence of the invariant cross section $siginv = E\frac{d\sigma^3}{dp^3}$ on the photon energy in the bremsstrahlung beam. The cross section formula is based on the end point energy(k_0), i.e. 11 GeV for the 11 GeV beam. The important parameters in the formula include the value of s(cm energy)^2, the longitudinal moment P_L^* in the center of mass(assuming an energy $=k_0$), the value of xR=(pcm*/pcmax*), the transverse momentum P_{trans} and the lab momentum, pLab, of the kaon.

Factoring out the average siginv from the integration of the cross section over the bremsstrahlung spectrum leaves the fraction of the bremsstrahlung spectrum that contributes to his tables and plots. This requires knowing kmin, the minimum photon energy to produce the kaon+lambda. I use slac-pub-0184.pdf[2] from Tsai and Van Whitis to calculate this fraction from the first order photon spectrum for a given radiation length, t. According to these two authors for the 16% radiation length target for Moller the calculated photon spectrum should be better than 10%.
The procedure to use his [1] data are described in his analysis section (p.62 of the thesis)

$$dN_K = N_t dN_\gamma(k)\frac{d\sigma}{dpd\Omega}\Delta p\Delta\Omega \cdot eff \tag{1}$$

Where dN_K are number of detected kaons, $dN_\gamma(k)$, number of incident photons, and eff is efficiency factors.
Using the relation $E\frac{d\sigma^3}{dp^3} = \frac{E}{p^2}\frac{d\sigma}{dpd\Omega}$ eqn 1 can be rewritten as

$$dN_K = N_t(EQ\frac{\alpha(k,k_0)\Delta k}{k}) \cdot \frac{p^2}{E}(E\frac{d\sigma^3}{dp^3})\Delta p\Delta\Omega \cdot eff \tag{2}$$

In eqn 2 $dN_\gamma(k) = (EQ\frac{\alpha(k,k_0)\Delta k}{k})$, where EQ = number of equivalent quanta described below, and k is the photon energy. $\alpha(k,k_0)$ is the bremsstrahlung factor which describes the deviation of the photon spectrum from a 1/k dependence. There is a minimum photon momentum, kmin, needed to produce the detected kaon of momentum p into the solid angle $\Delta\Omega$. In principle the invariant cross section, $(E\frac{d\sigma^3}{dp^3})$ depends on photon momentum so one would need to integrate from kmin to k_0 to determine the number of detected particles N_d.

$$N_K = N_t \frac{p^2}{E} \cdot \Delta p \Delta \Omega \cdot eff \cdot EQ \int_{kmin}^{k_0} (E \frac{d\sigma^3}{dp^3}) \frac{\alpha(k, k_0) dk}{k} \tag{3}$$

The integrated cross section which Wiser shows in his tables and graphs is defined from eqn 3

$$\int_{kmin}^{k_0} (E \frac{d\sigma^3}{dp^3}) \frac{\alpha(k, k_0) dk}{k} = N_K / [N_t \frac{p^2}{E} \cdot \Delta p \Delta \Omega \cdot eff \cdot EQ] \tag{4}$$

Wiser uses (Appendix B, p194) eqn $< B-1-1 >$ to describe the number of photons as a function of energy. $dn_\gamma/dk = b\alpha(k, k_0)/k$, see his eqn $< B{-}1{-}3 >$.

The function $\alpha(k, k_0)$ for the first order photon bremsstrahlung spectrum [2] from Tsai and Van Whitis is

$$\alpha(k, k_0) = (r^{z1} - exp(-z2))/(\frac{7}{9} + \frac{4ln(r)}{3}) \tag{5}$$

Where $z1 = 4t/3$, $z2 = 7t/9$, $r = 1 - k/k_0$, and t is the fractional radiation length.

b is a normalization factor chosen so that

$$\int_0^{k_0} b\alpha(k, k_0) dk = k_0. \tag{6}$$

On p. 195 he defines equivalent quanta, $EQ = U/k_0$, where U is the total energy contained in the bremsstrahlung beam integrated over the duration of the run.

$$U = \int_0^{k_0} kb\alpha(k, k_0) dk/k \tag{7}$$

Since the total energy in the bremsstrahlung beam is $U = N_e dU$, where $dU = \int_0^{k_0} k\alpha(k, k_0) dk/k$, then the equivalent number of photons per electron is $dEQ = dU/k_0$. Note that dU depends on the radiation length. Then eqn 3 can be written in the form of detected kaons per electron as

$$\frac{dN_K}{dN_e} = N_t \frac{p^2}{E} \cdot \Delta p \Delta \Omega \cdot eff \cdot dEQ \int_{kmin}^{k_0} (E \frac{d\sigma^3}{dp^3}) \frac{\alpha(k, k_0) dk}{k} \tag{8}$$

Examples of the fits to the cross section are shown in table 1 for a fixed transverse kaon momentum and in table 2 for varying transverse kaon momenta.

The quantity "sigfit" in the tables is the integral in equation 8.

There is an effective cross section per electron for kaon production from equation 8.

$$\sigma_{eff}(pLab, P_{trans}, P_L^*, t) = dEQ \int_{kmin}^{k_0} (E \frac{d\sigma^3}{dp^3}) \frac{\alpha(k, k_0) dk}{k} \tag{9}$$

Here t is the fractional radiation thickness. The value of t affects both the dEQ and the integral in equation 9. Since the Moller target will be an extended target the rate of hyperon production from bremsstrahlung will be a function of depth into the target. Table 3 shows $\sigma_{eff}(pLab, P_{trans}, P_L^*, t)$ as a function of radiation thickness for selected values of $pLab, P_{trans}, P_L^*$. The procedure for determining the parameters, P_L^*, the longitudinal momentum and pcm^* in the center of mass is given by

$$P_L^* = \gamma(P_z - \beta E_k), \quad pcm^* = \sqrt{(P_L^*)^2 + (P_{trans})^2} \tag{10}$$

Here β and γ are determined by the end point energy, k_0 of the photon, $\beta = k_0/(k_0 + m_p)$. The kaon lab energy is $E_k = \sqrt{(pLab * pLab + m_k * m_k)}$, and the kaon's longitudinal lab momentum is obtained by $P_z = pLab * cos(\theta)$, and $\theta = asin(P_{trans}/pLab)$.

0.1.2 Procedure for Hyperon Production

Wiser measured an integrated yield of kaons, $\gamma + p \to K^+ + X$. Although there is not a breakdown of the type of hyperon or what other types of products X are produced, strangeness conservation ensures that there is a hyperon produced for each kaon produced.

$$N_k = N_\Lambda + N_{\Sigma^0} + N_{\Sigma^+}, \quad \textit{in a ratio } 3/2/1. \tag{11}$$

We calculate the number of kaons per electron in a kaon momentum bite Δp and solid angle $\Delta\Omega$ from equations 8, 9.

$$\frac{dN_K}{dN_e} = eff \cdot N_t \frac{p^2}{E} \cdot \Delta p \Delta\Omega \cdot \sigma_{eff}. \tag{12}$$

Let $pLab = p_k,\ P_{trans} = p_t = p_k \cdot sin(\theta)$ we see that since $d\Omega = 2\pi sin(\theta) d\theta$ we can write

$$d\Omega = 2\pi \frac{p_t}{p_k} d\theta, \quad d\theta = \frac{dp_t}{p_k \cdot cos(\theta)}. \tag{13}$$

We substitute equation 13 into equation 12 and find

$$\frac{dN_K}{dN_e} = eff \cdot N_t \frac{p_k^2}{E_k} \cdot \Delta p_k \cdot 2\pi \frac{p_t \Delta p_t}{p_k^2 cos(\theta)} \sigma_{eff}. \tag{14}$$

Rearranging terms we find

$$\frac{dN_K}{dN_e} = eff \cdot N_t \Delta p_k \Delta p_t \cdot (2\pi \frac{p_t}{E_k cos(\theta)} \sigma_{eff}). \tag{15}$$

In equation 15 the term N_t is the number of protons per unit area in a bite of length Δt along the LH2 target. $N_t \equiv \rho \Delta t$. Since the units for σ_{eff} are

$\mu b/GeV^2$ the ratio $\frac{dN_K}{dN_e}$ is dimensionless. We must add up the contributions to kaon production along the length of the LH2 target. We recall that σ_{eff} is itself also a function of the total length of the target up to the element of width Δt.

In order to calculate the total number of kaons we divide the range of possible kaon momenta between the minimum and maximum values for a given photon energy $\vec{k_\gamma}$. The center of mass momentum for $\gamma + m_p \rightarrow K^+ + \Lambda$ is

$$pcm = \sqrt{\frac{(s - m_\Lambda^2 - m_K^2)^2 - 4m_\Lambda^2 m_K^2}{4s}}, \quad s = 2k_\gamma m_p + m_p^2. \tag{16}$$

The lab values of p_{kz}, p_t are given by

$$p_t = pcm * sin(\theta_{cm}), \quad p_{kz} = \gamma(pcm * cos(\theta_{cm}) + \beta * E_{kcm}). \tag{17}$$

For the Moller experiment, $k_0 = 11\ GeV$, $0.913 \leq p_k \leq 10.7919\ GeV/c$ and $0. \leq\ p_t \leq 2.155\ GeV/c$. We divide these intervals into bins of width Δp_k, Δp_t.

Consider lambda production, that is, $X = \Lambda + Q$, where Q is whatever else is produced. Thus, $N_\Lambda = N_K$. The task is to find where the N_Λs decay in space and the lab momenta and rest frame polarizations at this location.

Wiser's data for lambda production use the lab momentum of the kaon and the transverse momentum, (p_k, p_t). From these observables one can calculate the minimum photon energy, kmin, needed to produce the $K^+ + \Lambda$ final state.

$$kmin = \frac{m_\Lambda^2 - (m_p - E_k)^2 + p_k^2}{2(mp - E_k + p_k cos(\theta))}. \tag{18}$$

The products $K^+ + \Lambda + Q$ are produced by bremsstrahlung photons from $kmin$ to k_0. Choose a photon k_γ within this range whose direction is along the lab z axis. k_γ is determined by picking a random number x and solving for k_γ.

$$x = \frac{\int_{kmin}^{k_\gamma} dk\ \alpha(k, k_0)/k}{\int_{kmin}^{k_0} dk\ \alpha(k, k_0)/k}. \tag{19}$$

Where $\alpha(k, k_0)$ is given in equation 5. Then

$$k_\gamma + m_p = E_k + E_x, \qquad \vec{k} = \vec{p_k} + \vec{p_x} \tag{20}$$

The missing energy E_x and missing momentum $\vec{p_x}$ are obtained from equations 20. We can transform into the center of mass of $X = \Lambda + Q$.

$$s_x = E_x^2 - p_x^2, \quad \beta_x = \frac{p_x}{E_x}. \tag{21}$$

The momentum in the center of mass of X is given by

$$pcmx = \sqrt{\frac{(m_\Lambda^2 + m_Q^2 - s_x)^2 - 4m_\Lambda^2 m_Q^2}{4s_x}}. \tag{22}$$

pLab	P_L^*	sigma	dsigma	sigfit
GeV/c	GeV/c	$\mu b/GeV^2$	$\mu b/GeV^2$	$\mu b/GeV^2$
3.	-0.72	3.86e-4	0.68e-4	4.77e-4
4.	-0.151	6.67e-4	0.69e-4	8.82e-4
5.	0.255	4.40e-4	0.47e-4	8.89e-4
6.	0.588	3.37e-4	0.34e-4	6.68e-4
7.	0.882	1.75e-4	0.15e-4	4.03e-4
8.	1.152	1.20e-4	0.11e-4	1.61e-4

Table 1: Measured cross sections for kaon production and the parametrized fit(sigfit) from Wiser's data for $P_{trans} = 1.625 GeV/c$ for several lab momenta. Radiation thickness = 0.025, $k_0 = 11$ GeV. P_L^* is the longitudinal momentum in the center of mass as calculated by the method described in the text.

From equation 22 we see that there is a range of possible values for m_Q

$$0 \leq m_Q \leq \sqrt{s_x} - m_\Lambda. \tag{23}$$

Now pick a mass m_Q at random within the range of equation 23. This then fixes the center of mass momentum *pcmx* from equation 22. The direction for the Λ is chosen assuming the distribution in the center of mass of X is isotropic. That is, for a random number $0 \leq x \leq 1$,

$$\phi = 2\pi x, \quad cos(\theta) = 2x - 1. \tag{24}$$

The Lorentz transformation uses equation 36, for which the axes in the X system are parallel to the lab frame axes and the boost is along $\vec{p_x}$. The lab values of $\vec{p_\Lambda}$ now give the decay point in the lab frame of the Λ.

Evaluate the cross section from his formula as per siginv2.cpp (as per an example in the Appendix) and weight the lambda event with the factor from equation 15.

$$weight(p_t, E_k, t) = (Frac_\Lambda)\Delta t \Delta p_k \Delta p_t \cdot (2\pi \frac{p_t}{E_k cos(\theta)} \sigma_{eff}). \tag{25}$$

Here $Frac_\Lambda = 0.5$, while $Frac_{\Sigma^+} = 0.333$ and $Frac_{\Sigma^0} = 0.167$. We can generate lambdas as a function of depth into the target from t = 0% to t = 16%. From the kinematics there should be a cloud of decaying lambdas extending quite far, about 0.5 meters, from the LH2 target. The number of kaons per electron can then be determined by

$$\frac{dN_k}{dN_e} = eff * \sum_t \Delta t \sum_{p_t} \Delta p_t \sum_{p_k} \Delta p_k \cdot (2\pi\rho \frac{p_t}{E_k cos(\theta)} \sigma_{eff}). \tag{26}$$

P_{trans} GeV/c	P_L^* GeV/c	sigma $\mu b/GeV^2$	dsigma $\mu b/GeV^2$	sigfit $\mu b/GeV^2$
0.875	0.488	2.82e-1	0.16e-1	4.00e-1
1.001	0.410	1.17e-1	0.07e-1	1.66e-1
1.125	0.321	3.63e-2	0.12e-1	6.68e-2
1.250	0.221	1.53e-2	0.07e-2	2.52e-2
1.375	0.110	6.77e-3	0.52e-3	9.00e-3
1.500	-0.014	2.04e-3	0.16e-3	2.96e-3
1.625	-0.151	6.67e-4	0.69e-4	8.82e-4
1.750	-0.300	1.43e-4	0.21e-4	2.27e-4
1.875	-0.464	2.24e-5	0.64e-5	4.38e-5

Table 2: Measured cross sections for kaon production and the parametrized fit(sigfit) from Wiser's data for $P_L = 4GeV/c$ for several transverse momenta, P_{trans}. Radiation thickness = 0.025, k_0=11 GeV. P_L^* is the longitudinal momentum in the center of mass as calculated by the method described in the text.

t rad. thickness	sigfit $\mu b/GeV^2$	dEQ equivalent photons/electron	dEQ*sigfit $\mu b/GeV^2$
0.04	1.57e-4	0.0377	5.92e-6
0.08	1.48e-4	0.0726	1.07e-5
0.12	1.39e-4	0.105	1.46e-5
0.16	1.31e-4	0.1347	1.76e-5

Table 3: Effective cross section per electron, $\sigma_{eff}(pLab, P_{trans}, P_L^*, t)$ for $pLab = 8GeV/c, P_{trans} = 1.625GeV/c, P_L^* = 1.152GeV/c$, for different radiation thicknesses, t and incident electron $=11GeV$.

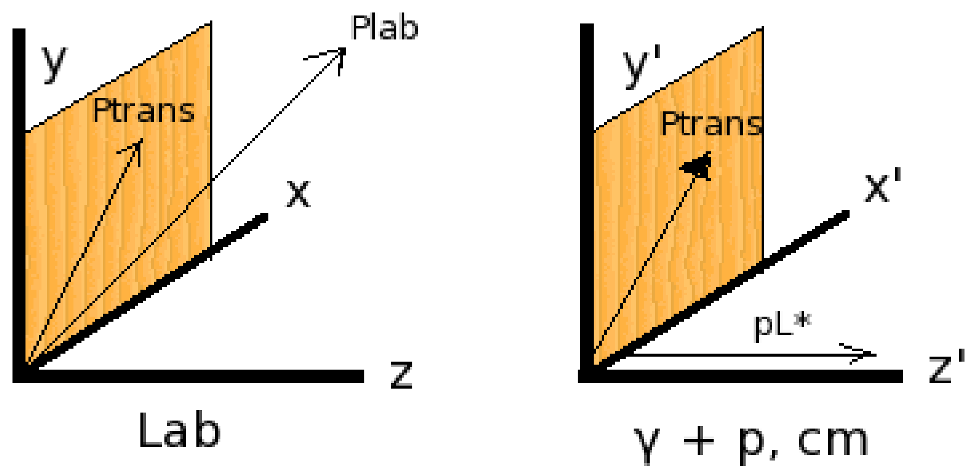

Figure 1: Lab to center of mass transformation for $\gamma + p \rightarrow K^{+} + X$. The kaon momenta Plab, Ptrans and pL* are needed in calculating Wiser's fitted cross sections. The beam direction is along the z axis

0.2 Input files for geant4 use of hyperon data

I suggest we create a file which has as its parameters for each event
(i) x_H,y_H,z_H (decay point in lab of hyperon)
(ii) p_H(x),p_H(y),p_H(z) (lab momenta of hyperon at decay point)
(iii) rest frame polarization of hyperon, S_{xR}, S_{yR}, S_{zR}
(iv) cross section weight for hyperon creation from equation 25.

$$weight(p_t, E_k, t) = (Frac_H)\Delta t \Delta p_k \Delta p_t \cdot (2\pi \frac{p_t}{E_k cos(\theta)} \sigma_{eff}). \tag{27}$$

Here p_t and E_k are the transverse momentum and lab energy of the kaon.

This file could be created once producing a cloud of decaying hyperons which could be read into the geant4 simulation. There may be more than one attempt to adjust shielding to shield the detectors from the decays of the hyperon. The cloud of hyperons is independent of any other geometry in the Moller set up. It depends only on the LH2 target and the beam energy.

An example of the predicted lambda momentum distribution for two different values of Δp_t is shown in figure 2. An example of the lambda distribution in the z direction is shown in figure 3. The spatial distribution of the lambdas is quite extensive as shown in figure 4.

With the variables in the file we can make the necessary transformation to the rest frame to produce the pions from

$$\Lambda \to p + \pi^- \quad (64\%) \qquad \Lambda \to n + \pi^0 \quad (36\%) \tag{28}$$

We may need to shield the detectors from the photons from π^0 decay. The lambdas will frequently decay outside of the LH2 target.

0.2.1 Hyperon decay momentum spectra

Consider the decay of the Λ as per an example in equation 28. The Λ decay distribution file gives us the position, vector momentum and $\frac{dN_\Lambda}{dN_e}$. In the center of mass of the Λ the momenta and angular distribution of the decay products is known, $\Lambda \to a + b$. From equation 22 we find the center of mass momentum k,

$$k = \sqrt{\frac{(m_a^2 + m_b^2 - m_\Lambda^2)^2 - 4m_a^2 m_b^2}{4m_\Lambda^2}}. \tag{29}$$

The angular distribution of the particle a in the center of mass is given by

$$\frac{dP_a}{d\Omega} = c(1 + hp_e \alpha S_R cos(\theta_a)), \quad d\Omega = 2\pi dcos(\theta_a). \tag{30}$$

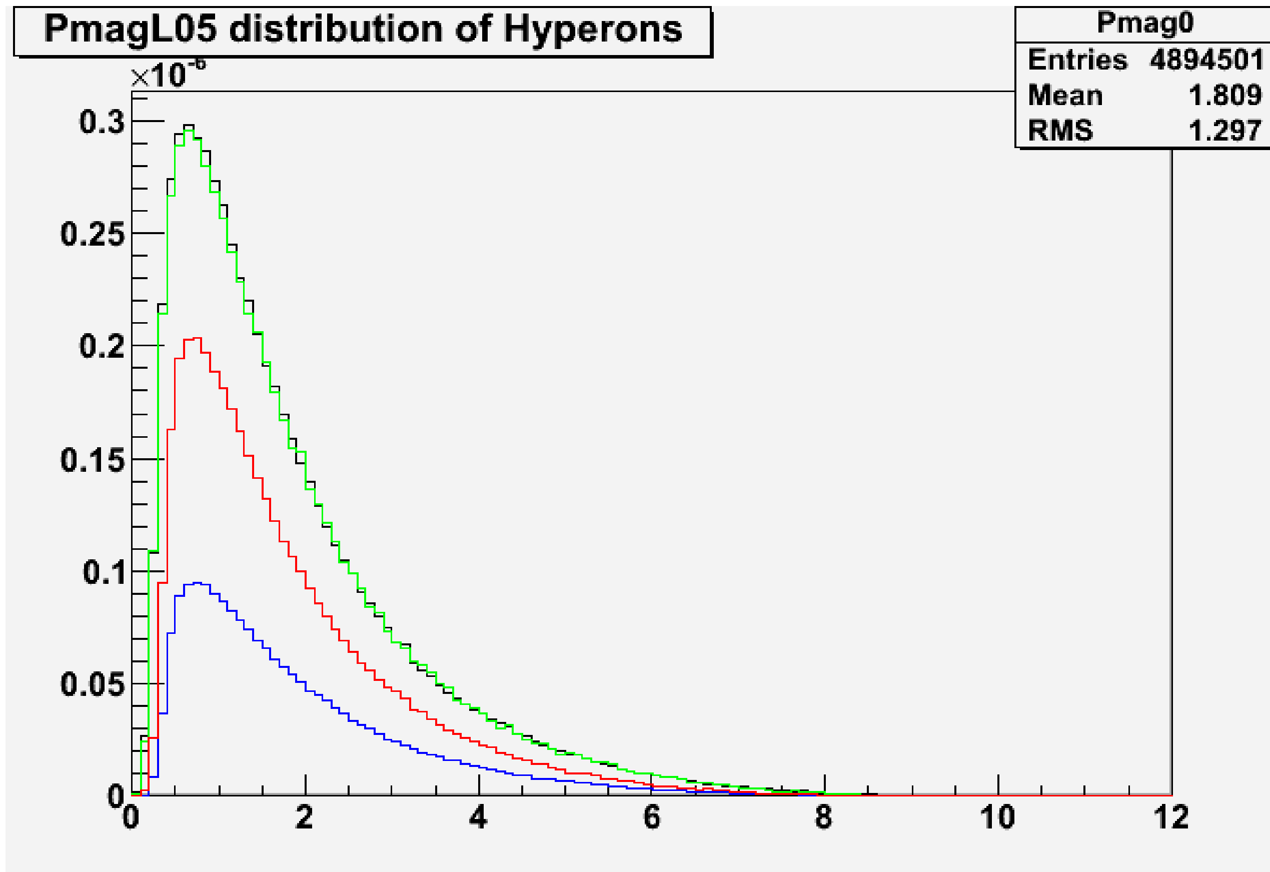

Figure 2: Predicted Hyperon momentum distributions, $\frac{dN_\Lambda}{dN_e}$. Λ for two separate runs using $\Delta p_t = 0.05 GeV/c$(black) compared to $\Delta p_t = 0.1 GeV/c$(light green). Σ^0(red), Σ^+(blue). In all cases $\Delta t = 0.1 cm$, $\Delta p_k = 0.1 GeV/c$. Hyperon cross sections are in the ratio 3/2/1.

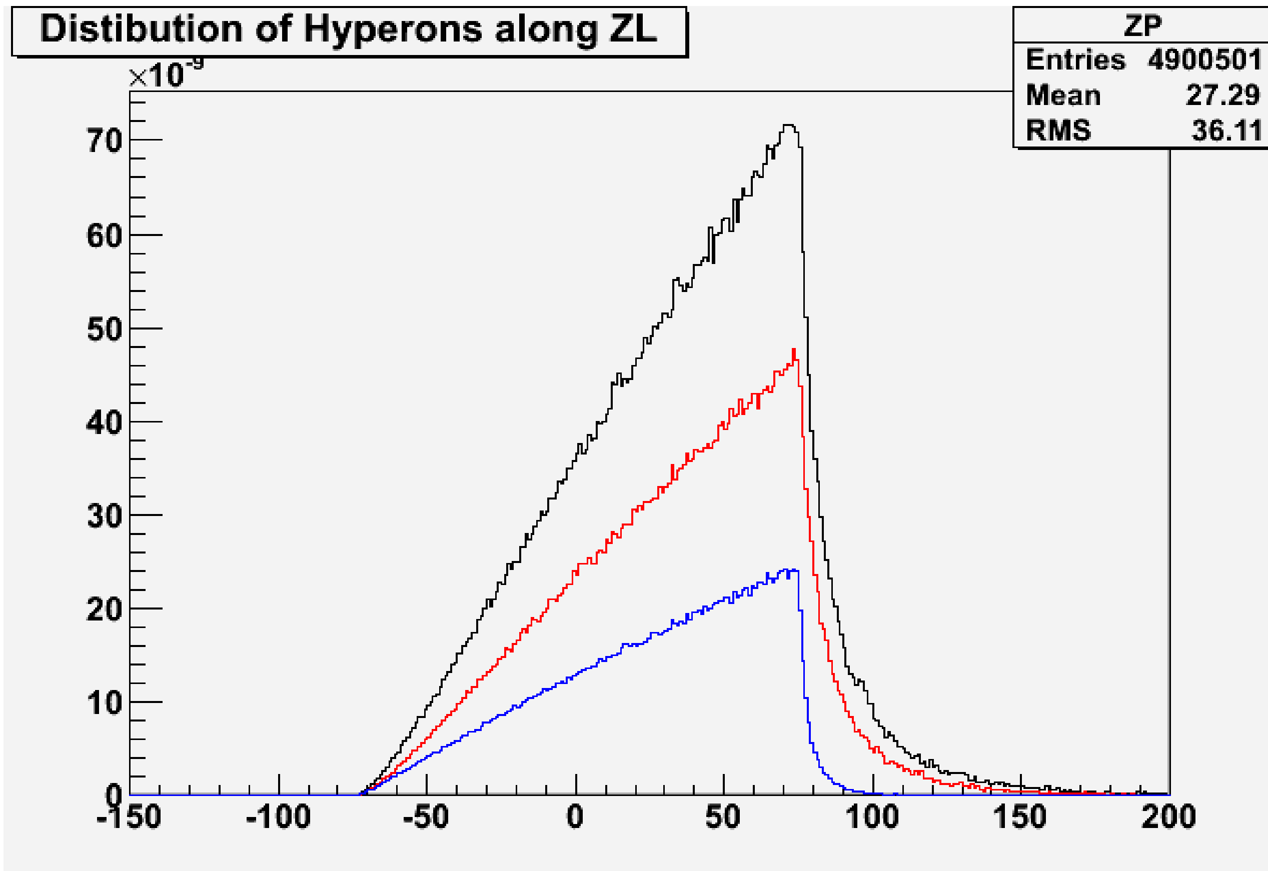

Figure 3: Predicted Hyperon Z distribution $\frac{dN_\Lambda}{dN_e}$ for $\Delta t = 0.1cm$, $\Delta p_k = 0.1GeV/c$, $\Delta p_t = 0.05GeV/c$. The three distributions are for Λ (black), Σ^0 (red), Σ^+ (blue) with cross sections in the relation 3/2/1.

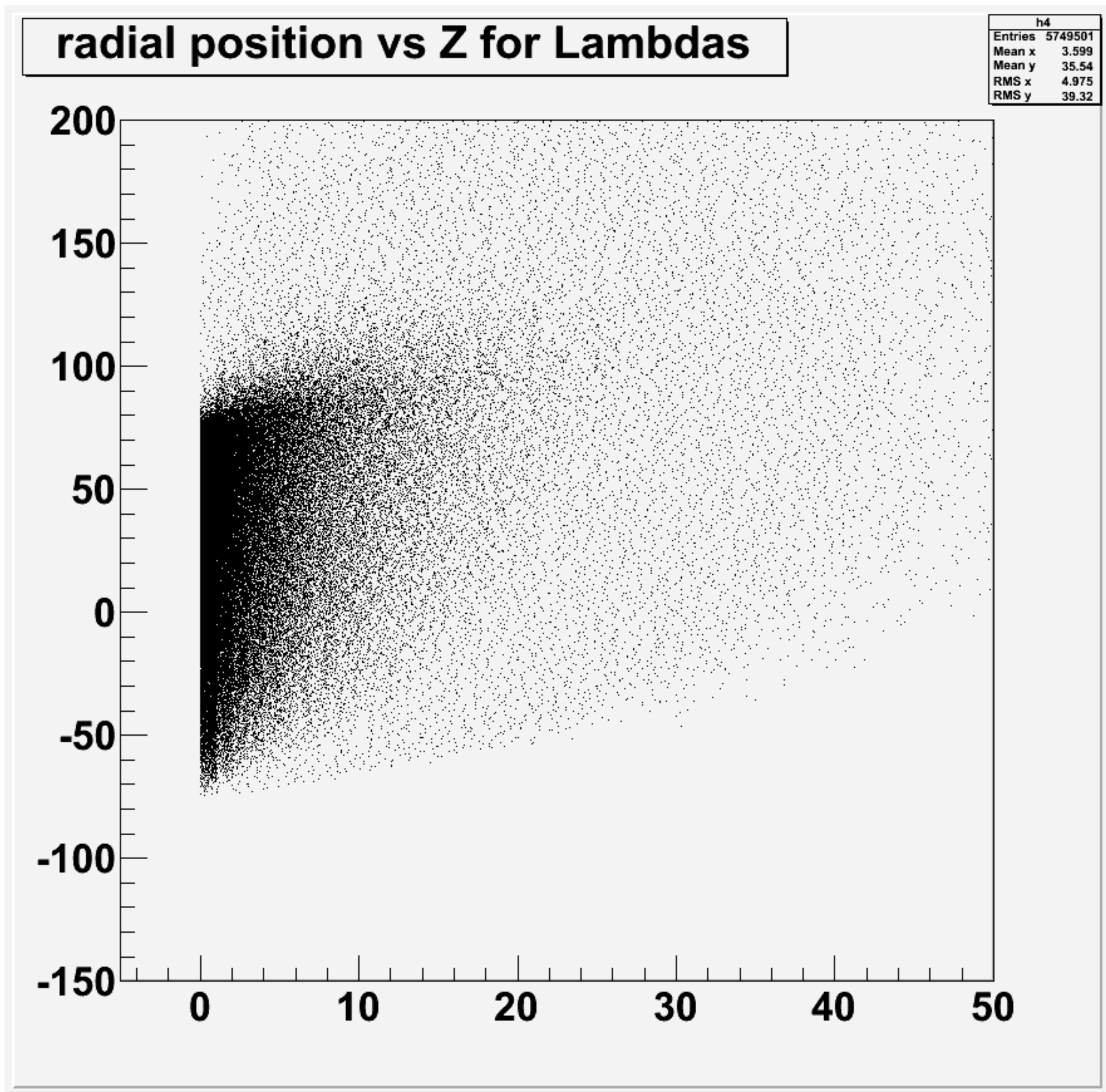

Figure 4: Predicted Lambda radial vs Z distribution $\frac{dN_\Lambda}{dN_e}$ for $\Delta t = 0.1cm$, $\Delta p_k = 0.1GeV/c$, $\Delta p_t = 0.05GeV/c$. The axes are in cm.

In equation 30 h is the beam helicity, p_e is beam polarization, S_R is the hyperon polarization in the rest frame, α is the hyperon decay parameter, $c = \frac{1}{4\pi}$ is the normalization to give the integral of the probabilty $\frac{dP_a}{d\Omega}$ equal to 1 and θ_a is the angle of momentum $\vec{k_a}$ with respect to $\vec{S_R}$. We select a random number r and solve for $x = cos(\theta_a)$ using

$$r = \frac{1}{2}\int_{-1}^{x}(1. + hp_e\alpha S_R x')dx'. \tag{31}$$

The solution for $x = cos(\theta_a)$ is

$$x = h\sqrt{\frac{2}{b}(2r + \frac{b}{2} - 1) + \frac{1}{b^2}} - \frac{1}{b}, \quad b = hp_e\alpha S_R. \tag{32}$$

The angle $\phi_a = 2\pi r'$, r' is a random number. The momenta of particle a in the Lambda restframe is

$$p_{az} = kcos(\theta_a), \quad p_{ax} = ksin(\theta_a)cos(\phi_a), \quad p_{ay} = ksin(\theta_a)sin(\phi_a). \tag{33}$$

If the decay is, for example, $\Lambda \to p + \pi^-$, the four momentum of the pion in the lab frame is given by the Lorentz transformation from the Λ center of mass, $p_\pi^\mu = (\sqrt{m_\pi^2 + k^2}, \vec{p_\pi})$. If the decay is $\Lambda \to n + \pi^0$, one first determines the photon momenta in the π^0 rest frame. Since the π^0 Lab momentum is known the photon momentum in the Lab frame can be calculated. Examples of Lab pion momentum distributions for $\Lambda \to p + \pi^-$ are shown in figures 5, 6, and 10. Examples of Lab photon momenta from $\Lambda \to n + \pi^0$ are shown in figures 7, 8, and 9. Examples of Lab neutron momenta from $\Lambda \to n + \pi^0$ are shown in figures 11, 12

0.2.2 Lorentz transformations

The Lorentz transformations for the kinematics, see figure 13, and for the 4 vector polarization codes have been written as per the procedures in Jackson's book and the book he coauthored with Hagedorn [6]. The lab frame coordinate system uses the beam direction for the z-axis, the y-axis is vertical for a right handed coordinate system. The code treats the rest frame axes of the lambda as parallel to the lab frame with the boost along the direction of the lambda momentum. For reference purpose I include here the transformation for two frames $K \to K'$ when the axes are parallel, they share a common origin in space-time, and K' is moving with velocity $\vec{\beta}$ away from K.

$(x01, x1', x2', x3')$ in K' is calculated from $(x0, x1, x2, x3)$ in K, with $x0 = ct, \quad x0' = ct'$

$$x0' = \gamma(x0 - \vec{\beta}\cdot\vec{r}). \tag{34}$$

and

$$\vec{r'} = \vec{r} + (\gamma - 1)(\vec{\beta}\cdot\vec{r})\vec{\beta}/\beta^2 - \gamma x0\vec{\beta}. \tag{35}$$

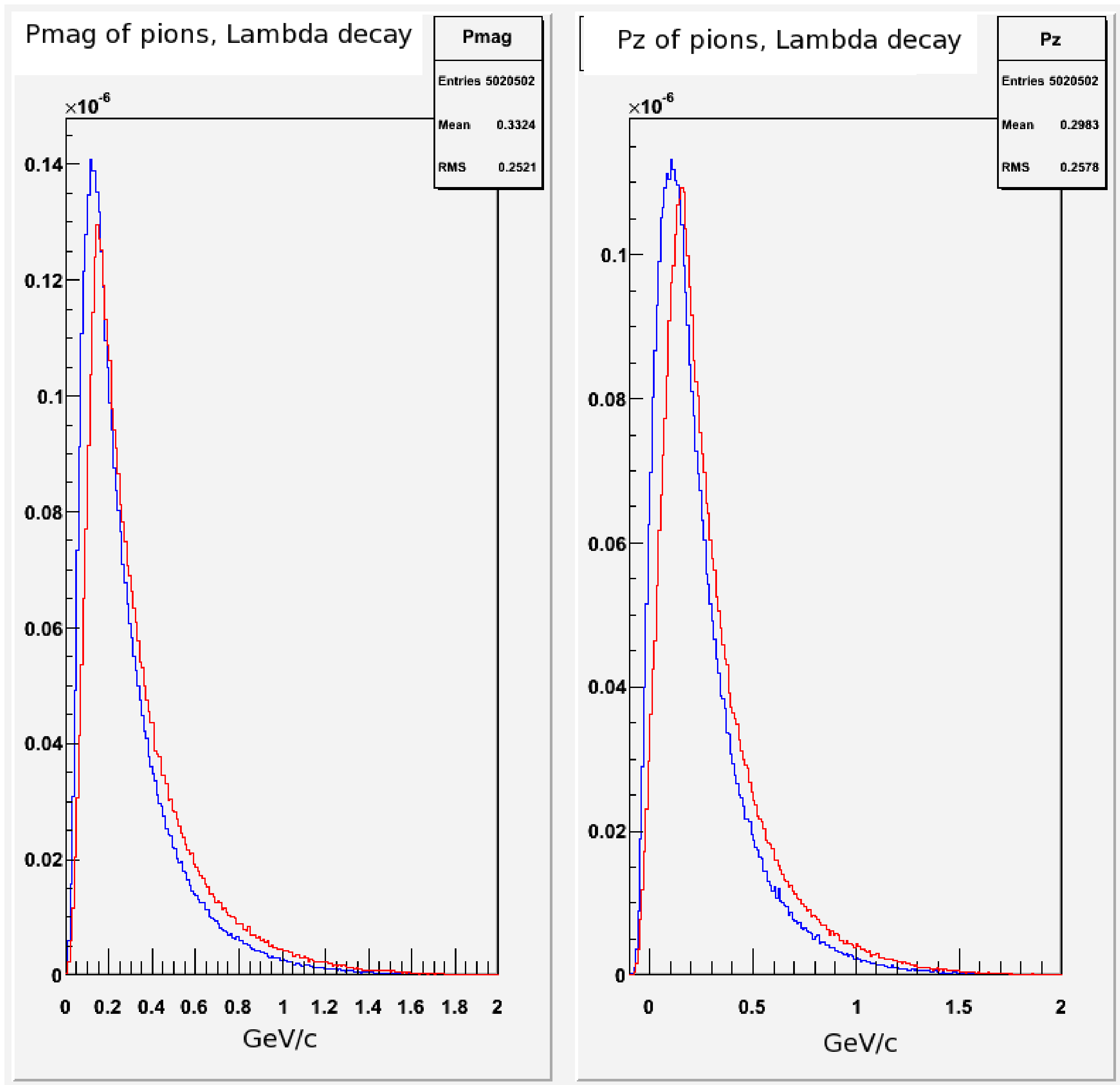

Figure 5: Pion total momentum and Pz distribution for $\Lambda \to p + \pi^-$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{N_\pi}{N_e}$ for the Moller target.

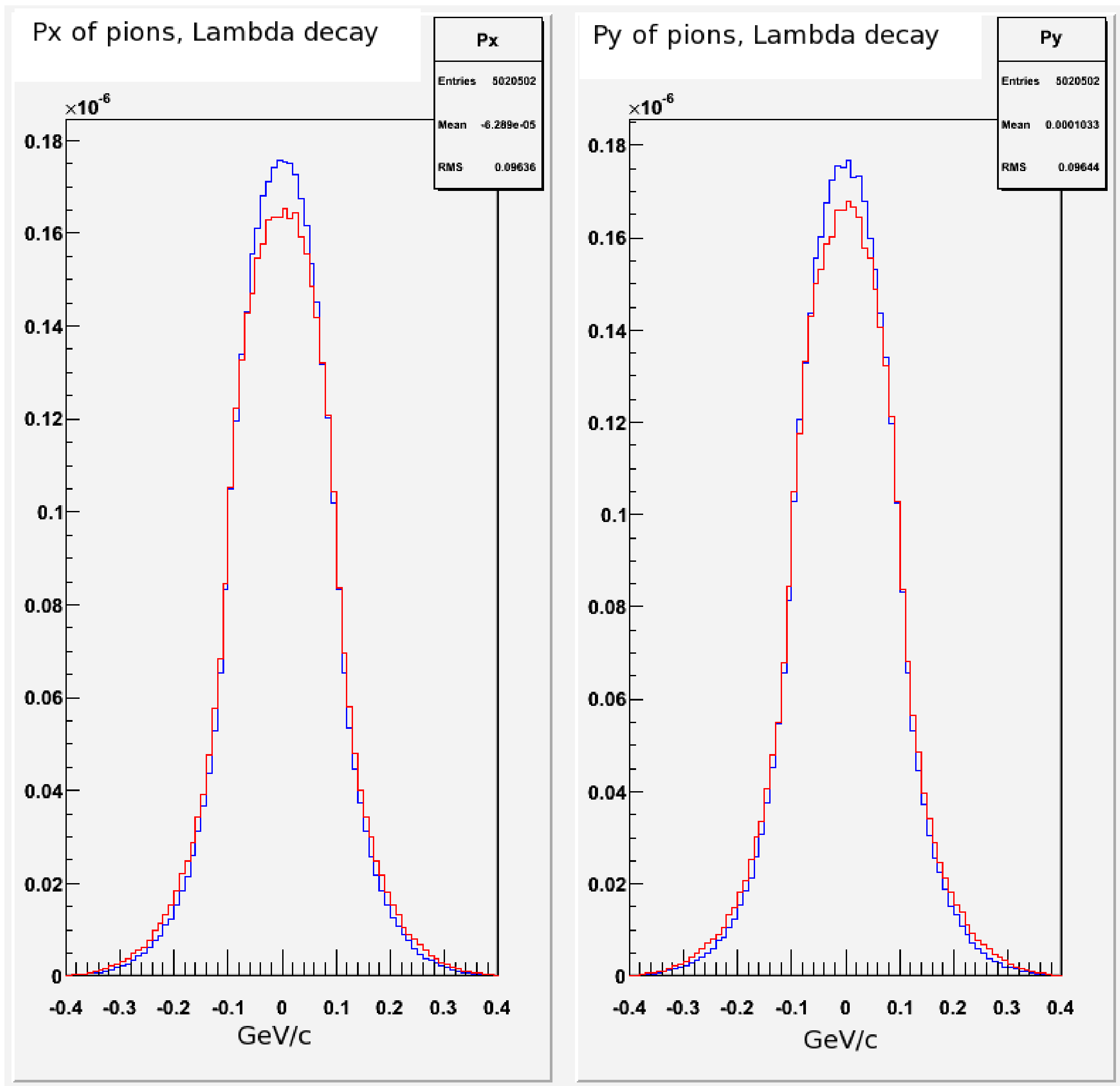

Figure 6: Pion Px and Py distribution for $\Lambda \to p + \pi^-$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{N_\pi}{N_e}$ for the Moller target.

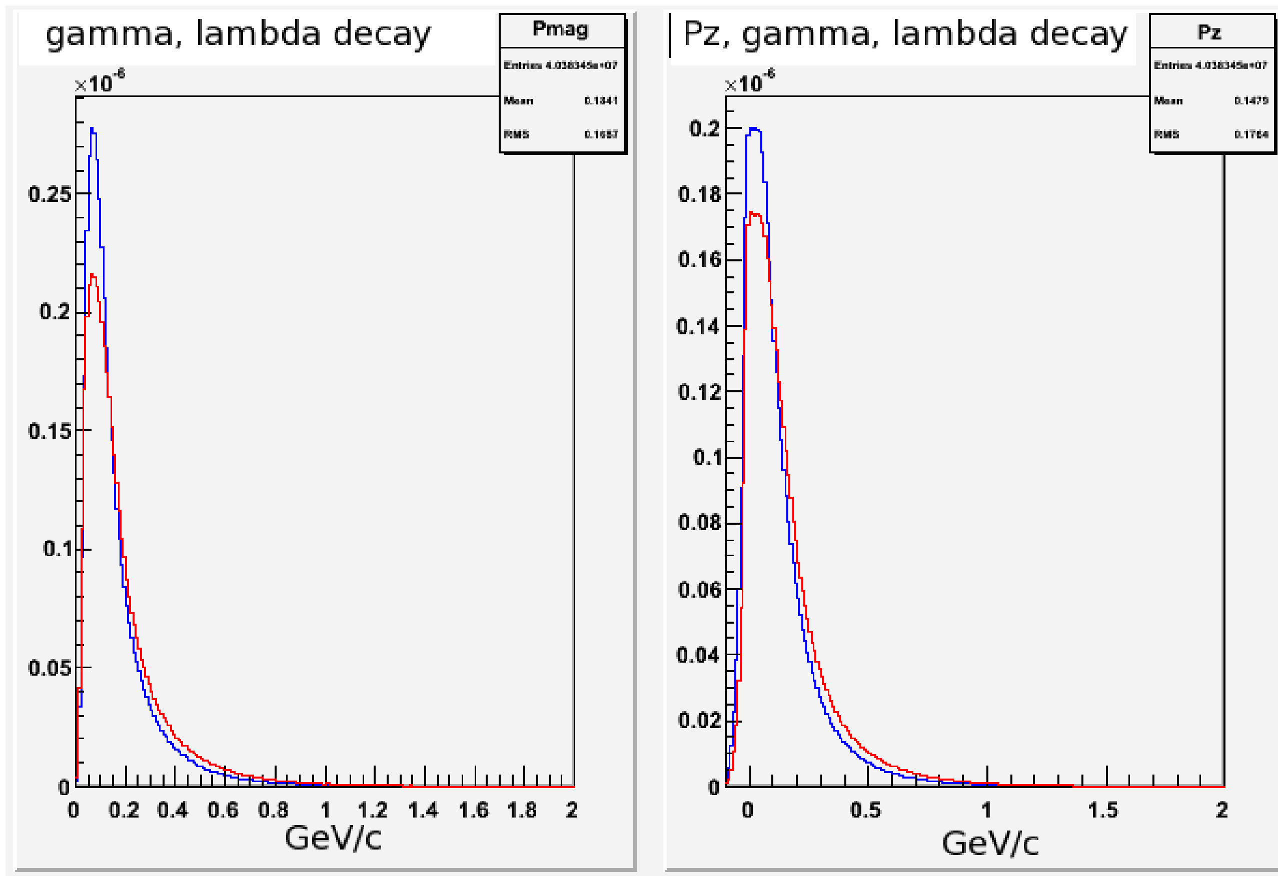

Figure 7: Gamma total momentum and Pz distribution for $\Lambda \to n + \pi^0$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{N_\gamma}{N_e}$ for the Moller target.

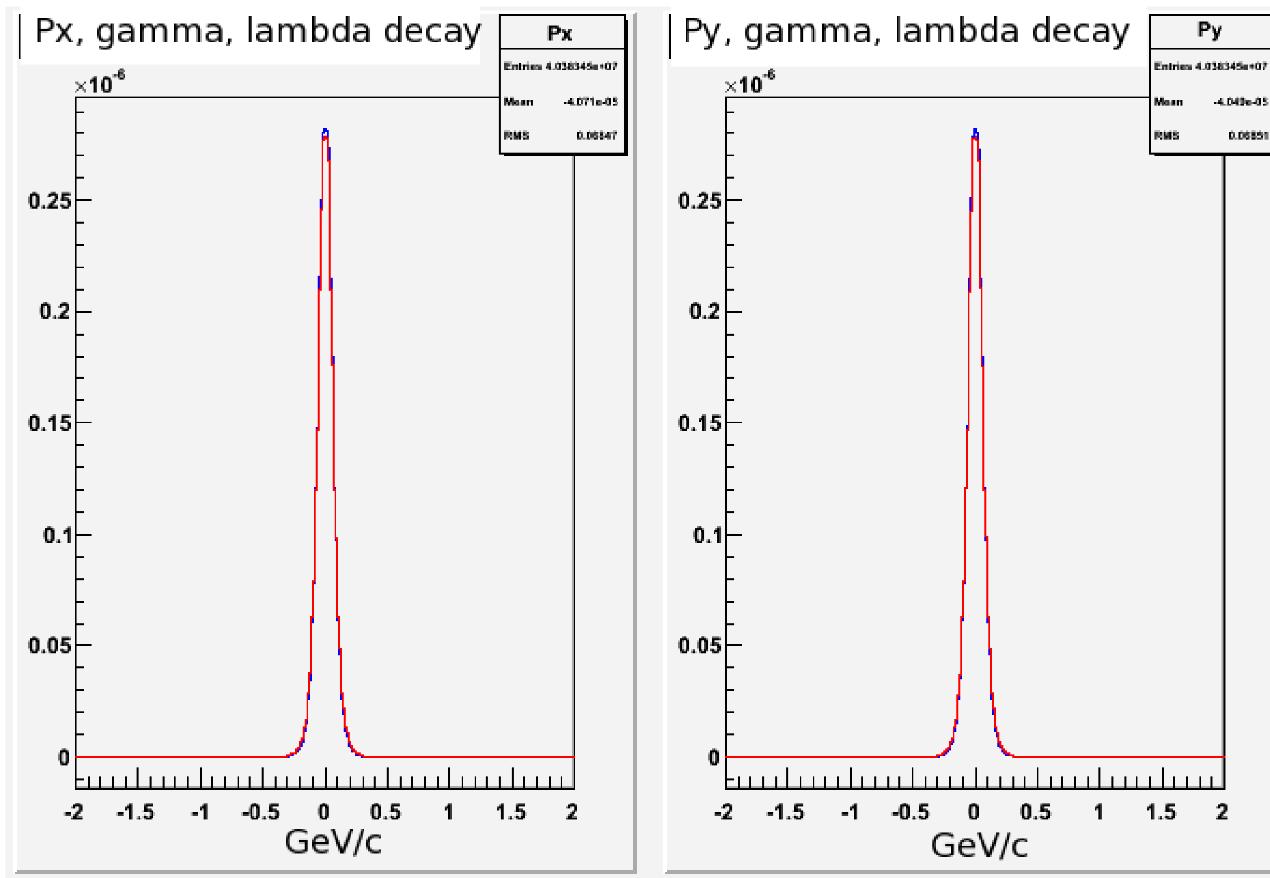

Figure 8: Gamma Px and Py distribution for $\Lambda \rightarrow n + \pi^0$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{N_\gamma}{N_e}$ for the Moller target.

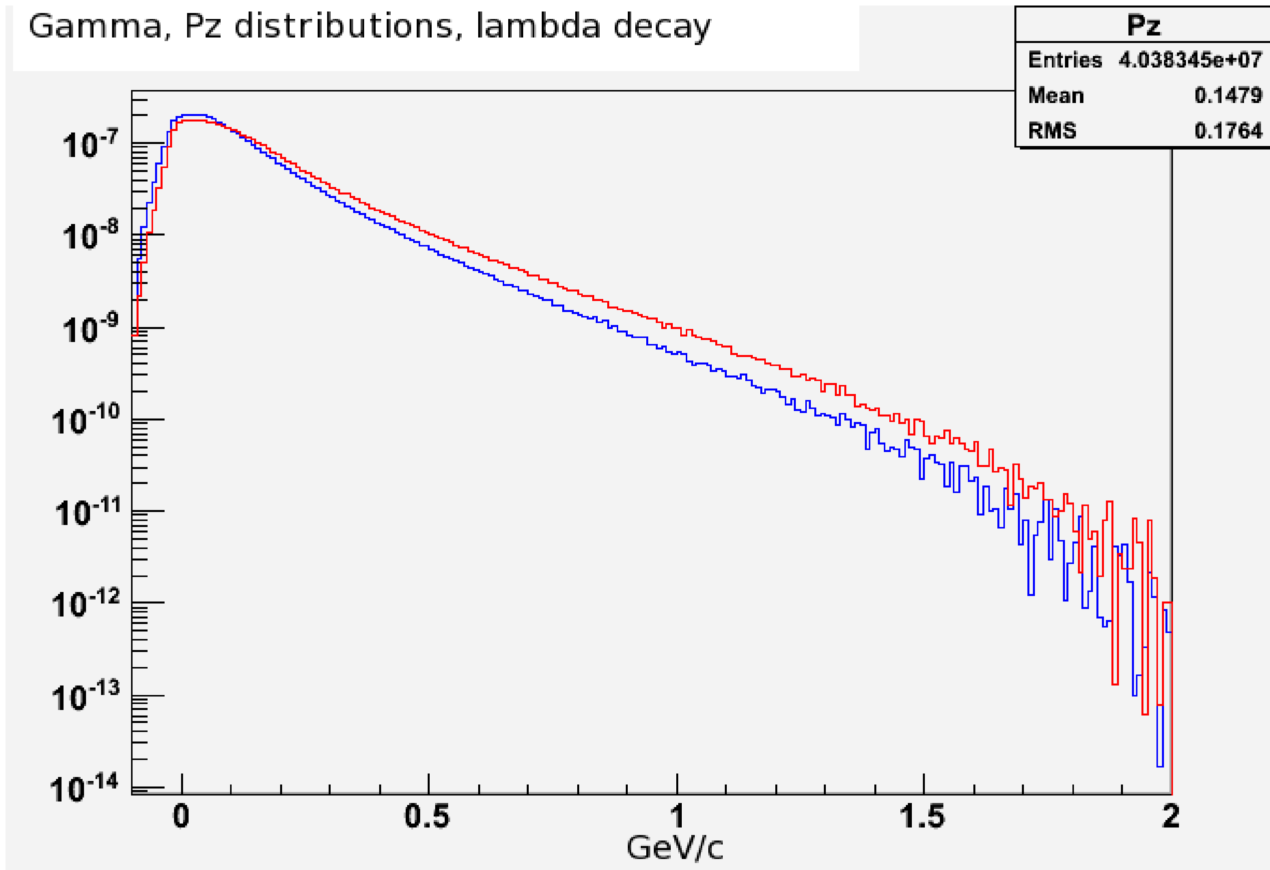

Figure 9: Gamma momentum Pz distribution for $\Lambda \rightarrow n+\pi^0$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{N_\gamma}{N_e}$ for the Moller target.

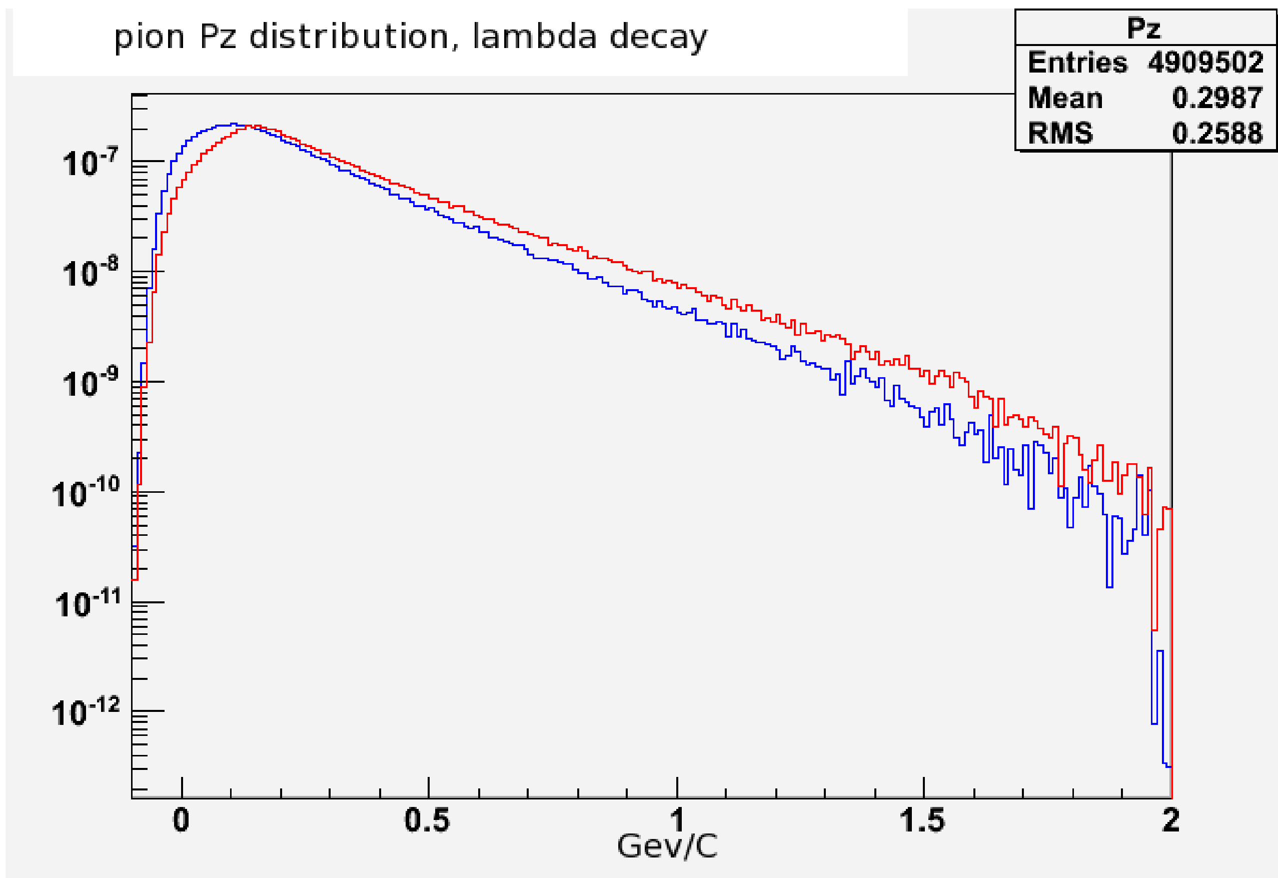

Figure 10: Pion Pz distributions for $\Lambda \rightarrow p + \pi^-$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{N_\pi}{N_e}$ for the Moller target.

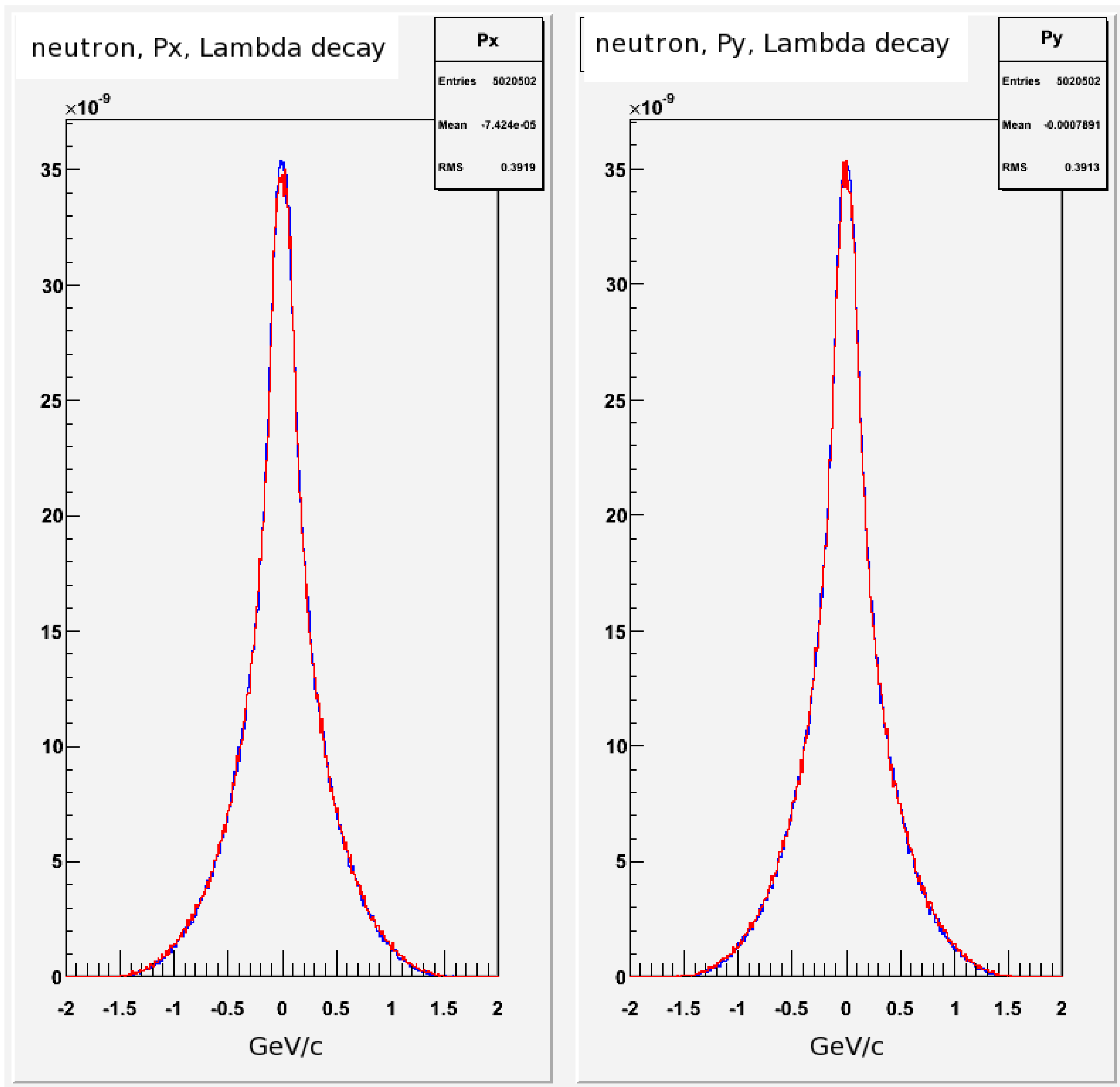

Figure 11: Neutron Px and Py distribution for $\Lambda \to n + \pi^0$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{Nn}{Ne}$ for the Moller target.

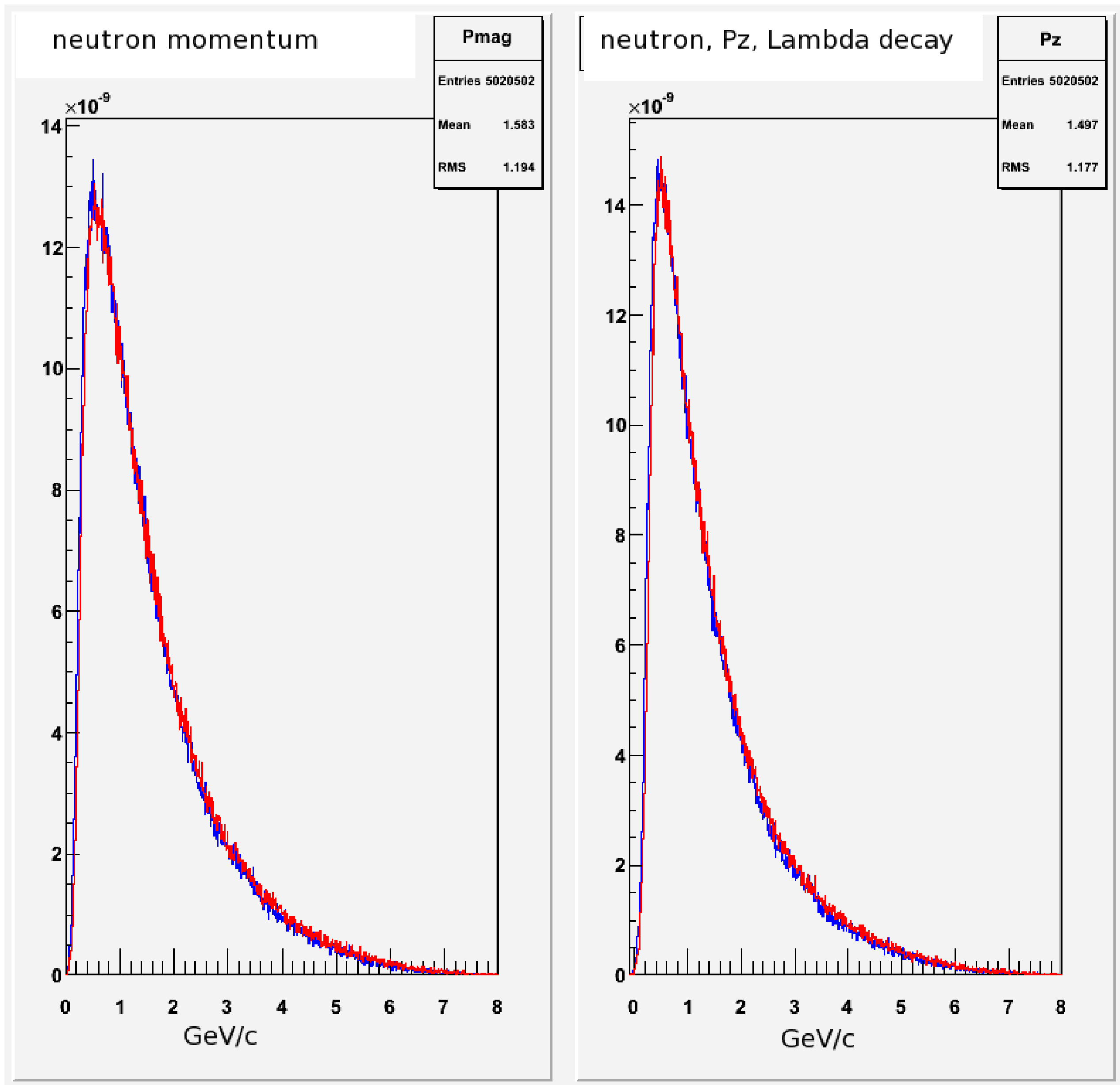

Figure 12: Neutron Pmag and Pz distribution for $\Lambda \rightarrow n + \pi^0$ for the two helicity states; h = -1, blue and h = +1, red. The vertical axes is $\frac{Nn}{Ne}$ for the Moller target.

The explicit matrix elements from equations 34 and 35 are

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ 1 & -\gamma\beta_x & (1+(\gamma-1)\beta_x^2/\beta^2) & (\gamma-1)\beta_x\beta_y/\beta^2 & (\gamma-1)\beta_x\beta_z/\beta^2 \\ 2 & -\gamma\beta_y & (\gamma-1)\beta_x\beta_y/\beta^2 & (1+(\gamma-1)\beta_y^2/\beta^2) & (\gamma-1)\beta_y\beta_z/\beta^2 \\ 3 & -\gamma\beta_z & (\gamma-1)\beta_x\beta_z/\beta^2 & (\gamma-1)\beta_y\beta_z/\beta^2 & (1+(\gamma-1)\beta_z^2/\beta^2) \end{array} \tag{36}$$

Polarization 4 vector

Hagedorn and Jackson [6] discuss the polarization 4 vector in chapter 11 of *Classical Electrodynamics* and in chapter 9 of *Relativistic Kinematics*. They assert that setting the time component of the polarization 4 vector to zero in the particle's rest frame, figure 14, leads to consistent equations. Thus the ansatz is for $S_R^\mu = (0, \vec{S_R})$ in the rest frame. Since $S_R^\mu S_{\mu R} = S_R^2$ is invariant this condition requires a certain relationship between the time and space components in the lab frame. From equation 34 this fixes the relationship between the lab components, S_L^μ,

$$S_L^0 = \vec{\beta} \cdot \vec{S_L} = \gamma\beta \cdot \vec{S_R} \tag{37}$$

$$\vec{S_L} = \vec{S_R} + \frac{\gamma^2}{\gamma+1}(\beta \cdot \vec{S_R})\vec{\beta} \tag{38}$$

The utility of equations 37 and 38 lies in the fact that the Λ transferred polarization has been measured in Class data [5] to be about 75% and lies along the virtual photon direction, which is along the Lab frame z direction for this analysis. These properties of the transferred polarization were also cited by Liu [3] when describing the Λ decay background in the G0 analysis. Since the virtual photon direction is along the Lab z-axis, and we will use Lambda's rest frame axes parallel to the Lab axes, the polarization in the Lambda's rest frame is, in fact, about 0.75 along the rest frame z-axis.

Liu also considered the other hyperon channels,

$$\begin{array}{ll} \gamma + p \to \Lambda + K^+ & (a) \\ \gamma + p \to \Sigma^0 + K^+ & (b) \\ \gamma + p \to \Sigma^+ + K^0 & (c) \end{array} \tag{39}$$

The production ratios for reactions in equation 39 are $\sim 3:2:1$. The Σ^+ decays weakly into $p+\pi^0$(52%) and $n+\pi^+$(48%). The Σ^0 decays electromagnetically into a Λ and γ with the Λ carrying a polarization of about $-\frac{1}{3}P_{\Sigma^0}$. These hyperons also contribute to the asymmetry. See Liu's thesis [3] for references.

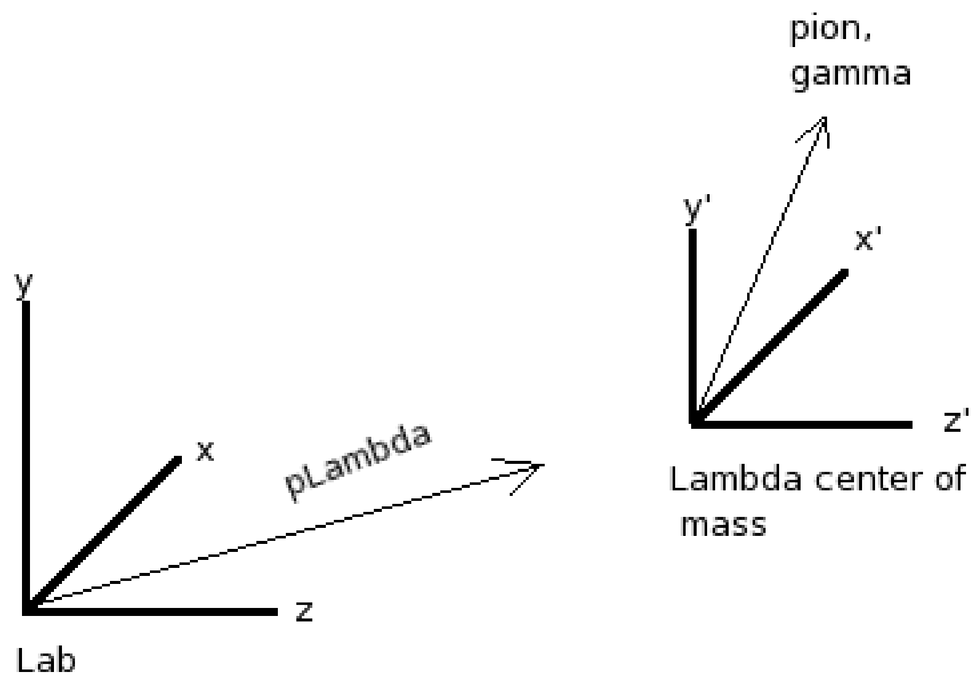

Figure 13: Lab to Λ center of mass transformation. Pions or gammas from π^0 decay show a helicity dependence in the momenta of the decay particles. The coordinate axes of the Lab and Λ center of mass frames are parallel and boosted along the direction of the Λ momentum.

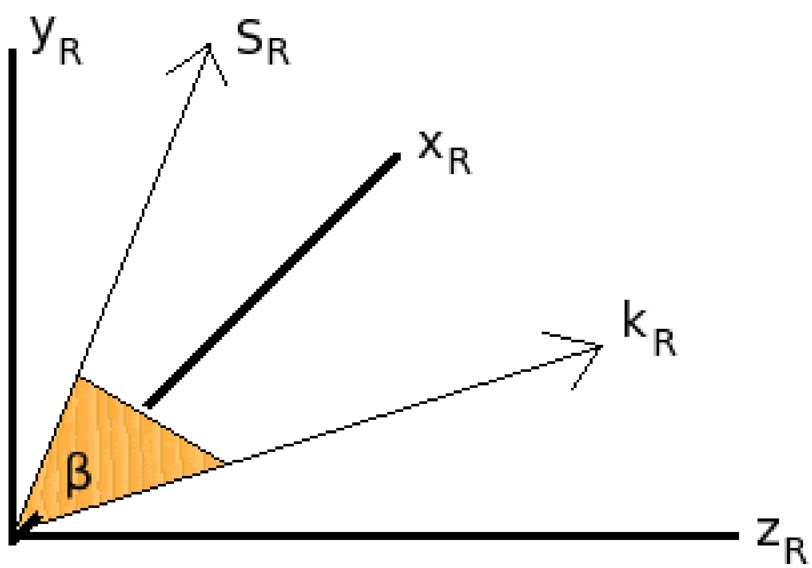

Figure 14: $\vec{S_R}$ is the polarization vector in the Λ rest frame. $\vec{k_R}$ is the momentum of a decay particle. The angle β governs the angular distribution of the weak decay products in the hyperon rest frame.

0.2.3 Order of the Calculation

The functions and steps used for a particular hyperon are:

(1) Vertex() - determine the kaon production vertex in lab frame

(2) KaonMomentum() - select kaon lab momentum and transverse momentum including sums over all possible lab and transverse momenta.

(3) SigInv() - determine the invariant kaon production cross section from Wiser's paramterization and determine the weight per equation 25.

(4) Select $k\gamma$, $kmin \leq k\gamma \leq k_0$

(5) HyperonMomentum() - select the hyperon momentum in lab

(6) Displacement() - determine lab decay point for hyperon

(7) Store the lab values of the hyperon, decay point($x_{Lab}, y_{Lab}, z_{Lab}$), momenta (p_x, p_y, p_z) and weight for the event in a file.

(8) Analysis of this file can be done by creating histograms using the weights determined in step 7. The yields of hyperons per electron are absolute because we employ equation 26, which sums over target length, kaon transverse momentum and absolute momentum.

0.3 Electroproduction using Virtual Photons

The code has been written and tested and is in the Appendix. This section to be filled in yet.

a_1 $\mu b/GeV^2$	a_2 $\mu b/GeV$	a_3 GeV	a_4	a_5 $(GeV/c^2)^{-1}$	a_6 $(GeV/c^2)^{-1}$
368 ± 21	1.91 ± 2.74	1.91 ± 0.05	1.15 ± 0.06	-5.91 ± 0.07	-1.74 ± 0.06

Table 4: Parameters for equation 40 for kaon production $\gamma + p \rightarrow K + X$.

0.4 Appendix

0.4.1 Wiser's invariant cross section

Wiser's invariant cross section formula (IV-A-2).

$$E\frac{d^3\sigma}{dp^3} = (a_1 + \frac{a_2}{\sqrt{s}})(1 - x_R + \frac{a_3^2}{s})^{a_4} exp(a_5 M_L) exp(a_6 P_T^2/E) \tag{40}$$

The parameters for kaon production in equation 40 are shown in table 4.

0.4.2 example of Kaon production code from Wiser's data

```
// calculate the invariant cross section from Wiser thesis, siginv2.cpp
#include <iostream>
#include <cstdlib>
#include <ctime>
#include <cmath>
#include <fstream>
#include <cctype>
#include <cstring>
//Appendix A - Wiser kinematics
//k0, kmin = photon end point energy, minimum photon energy
//kaon cross sections for gamma+p->kaon + X
//pcmax = cm momentum for gamma+p->kaon+lambda, for endpoint energy
//s = cm energy squared
//pLab = kaon lab momentum
//pT = transverse kaon momentum in lab and cm
//pLcm = longitudinal momentum of kaon in cm for endpoint energy
//pcm = kaon cm momentum given by pT and pLcm
//dEQ = Equivalent Photons per electron(see Wiser thesis for definition)
main()
{
double k0, mp=0.93827, mkplus=0.494, mlambda=1.1157, pcmax, x, s;
double a1=368.,a2=1.91,a3=1.91,a4=1.15,a5=-5.91,a6=-1.74;
double pLab,pT, ML, xR,E,pLcm,pcm,pLabL,kmin=0.,theta,ekplus,abst;
double t1, t2a, t2, t3, t4, siginv;
double k,e0,t=0.025,r,kIapprox, z1, z2;
double Fapprox, dk, sum=0., norm, dEQ;
double beta, gamma, pzlab;
```

```
int nk=50, i;
char bl = ' ', eol = '\n';
std::cout<<"enter photon energy k0 in GeV and radiation length t as a fraction"<<eol;
std::cin>>k0>>t;
beta=k0/(k0+mp);
gamma=1/sqrt(1.-beta*beta);
s = (mp+k0)*(mp+k0)-k0*k0;
x = s*s+(mkplus*mkplus-mlambda*mlambda)*(mkplus*mkplus-mlambda*mlambda);
x = (x -2.*(mkplus*mkplus + mlambda*mlambda)*s)/4./s;
pcmax = sqrt(x);
std::cout<<"pcmax = "<<pcmax<<eol;
std::cout<<"enter pLab pT ";
std::cin>>pLab>>pT;
theta = asin(pT/pLab);
std::cout<<"theta = "<<57.295*theta<<eol;
ekplus = sqrt(pLab*pLab+mkplus*mkplus);
E = ekplus;
kmin=(mlambda*mlambda-mkplus*mkplus-mp*mp+2.*mp*ekplus)/2./(mp - ekplus + pLab*cos(theta));
pzlab = pLab*cos(theta);
pLcm = gamma*(pzlab -beta*ekplus);

pcm = sqrt(pT*pT+pLcm*pLcm);
xR = pcm/pcmax;
ML = sqrt(pT*pT + mkplus*mkplus);
std::cout<<"E ML xR "<<E<<bl<<ML<<bl<<xR<<eol;

std::cout<<"kmin "<<kmin<<eol;
t1 = a1 + a2/sqrt(s);
t2a = 1. - xR + a3*a3/s;
t2 = pow(t2a,a4);
t3 = exp(a5*ML);
t4 = exp(a6*pT*pT/E);
abst = 2.*k0*pLab*(1.-cos(theta));
std::cout<<"pT*pT/E "<<pT*pT/E<<" 2*mp*|t|/s "<<2.*mp*abst/s<<eol;
siginv = t1*t2*t3*t4;
std::cout<<"siginv = "<<siginv<<" ub/GeV^2 for end point energy =  "<<k0<<" GeV"<<eol;

// integral cross section from kmin to k0
// Use first order approximation for photon spectrum by Tsai and Van Whitis
// SLAC-PUB-184, March 1966
dk = k0/nk;
k = 0.;
e0 = k0;
z1 = 4.*t/3.;
z2 = 7.*t/9.;
// first get the normalization for the photon spectrum shape
```

```
// This is the normalization done by Wiser to calculate siginv
// see Wiser thesis Appendix B eqn <B-1-3> p 195
for(i=0;i<nk-1;i++){
r = 1. - k/e0;
kIapprox = ( pow(r,z1) - exp(-z2))/(7./9. +4./3.*log(r));
sum = sum + kIapprox;
k = k + dk;
}
sum = sum*dk;
std::cout<<"sum = "<<sum<<eol;
norm = k0/sum;
dEQ = sum/k0;
std::cout<<"normalization for integral 0 to k0 = "<<norm<<eol;
sum = 0.;
dk = (k0-kmin)/nk;
k = kmin;
// integrate over photon spectrum to get average siginv
for(i=0;i<nk-1;i++){
r = 1. - k/e0;
kIapprox = ( pow(r,z1) - exp(-z2))/(7./9. +4./3.*log(r));
sum = sum + kIapprox/k;
k = k + dk;
}
siginv = siginv*norm*sum*dk;
std::cout<<"average cross section from k0 to kmin "<<siginv<<" ub/GeV^2"<<eol;
std::cout<<"pLab pT pLcm kmin siginv "<<pLab<<bl<<pT<<bl<<pLcm<<bl<<kmin<<bl<<siginv<<eol;
std::cout<<"number of photons between kmin and k0  per electron = "<<sum*dk<<eol;
std::cout<<"dEQ per electron = "<<dEQ<<eol;
}
```

0.4.3 Virtual Photon calculation from Tiator and Wright

```
//virtual photon spectrum from L. Tiator and L.E. Wright, NPA379(1982)407
#include <iostream>
#include <cstdlib>
#include <ctime>
#include <cmath>
#include <fstream>
#include <cctype>
#include <cstring>
// gamma + Mi -> pi + residual
// q0 = kpi + pf : momentum conservation in photoproduction, photon momentum
//(kpi,theta_pi,phi_pi) pion 3 momentum desired
//Mi: target mass, e.g. proton
```

```
//Mf: residual mass, e.g. neutron
//R recoil factor
//D, D0 denominators
//ki, kf : initial and final electron energy
main()
{
double mpi=0.13957, mp=0.93827,w0,kpi,Epi,theta_pi,ei,ef0,ki,kf0, pi=3.14159;
double D0,Ei, Mi, Mf, mn=0.93946,rad=57.2958, q0,R, me = 0.000511, Ne, sigp,sige;
double ar, br, arp, brp, a, b, ap, bp, delta, alpha=0.00729735, f1, f2, f3;
char bl = ' ', eol = '\n';

// determine the photon energy for a given pion momentum and theta_pi
std::cout<<"enter pion momentum(GeV/c) and theta_pi: ";
std::cin>>kpi>>theta_pi;
Epi = sqrt(mpi*mpi + kpi*kpi);
q0=(mn*mn+kpi*kpi-mp*mp-Epi*Epi+2.*Epi*mp)/2./(mp-Epi+kpi*cos(theta_pi/rad));
w0 = q0;
std::cout<<"photon energy = "<<q0<<eol;

// find the recoil factor, very sensitive R near the end point energy(q0)
std::cout<<"enter initial electron momentum, ki(GeV/c): ";
std::cin>>ki;
kf0 = ki - q0;
ei = sqrt(ki*ki + me*me);
ef0 = sqrt(kf0*kf0 + me*me);
Mi = mp;
D0 = Mi + ei - Epi +ef0*(kpi*cos(theta_pi/rad)-ki)/kf0;
R = (Mi + w0 -(Epi/kpi)*w0*cos(theta_pi/rad))/D0;
std::cout<<"recoil factor = "<<R<<eol;

//determine the virtual photon spectrum, Ne
ar = Mi + ei - Epi; //eqn 16
br = ef0*(kpi*cos(theta_pi/rad)-ki)/kf0;
a = me*me -ei*ef0;
b = ki*kf0;
delta = -me*me*br*(ei/ef0 - 1.)/ar;
ap = a - delta;
bp = b + delta;
f1 = alpha/2./pi*(w0*w0/ki/ki)*(b/bp)*(ar+br)/(ar - (ap/bp)*br);
f2 = (ap-bp)*(ar+br)/(ap+bp)/(ar-br);
f3 = (1.-2.*a/w0/w0)*log(f2) -4.*b/w0/w0;
Ne = f1*f3;
std::cout<<"enter photoproduction cross section (ub/sr): ";
std::cin>>sigp;
sige = (Ne/w0)*R*sigp*1.e3;
std::cout<<"sige = "<<sige<<bl<<"nb/GeV/sr"<<eol;
```

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[2] *Thick target Bremsstrahlung and Target Consideration for Secondary Particle Production by Electrons*, Y. S. Tsai and Van Whitis, March 1966, Stanford Linear Accelerator Center, SLAC-PUB-184

[3] *A measurement of the Strange Quark Contributions to the Electromagnetic Form Factors of the Nucleon*, Jianglai Liu, 2006, University of Maryland, College Park, PhD Thesis

[4] *Virtual Photons in Pion Electroproduction*, L. Tiator and L.E. Wright, NPA379(1982) 407-414

[5] *Beam-recoil polarization transfer in the nucleon resonance region in the exclusive $ep \rightarrow e'K^+\Lambda$ and $ep \rightarrow e'K^+\Sigma^0$ reactions at the CLAS spectrometer*, D.S.Carman, *et al.*, PhysRev **C79**, 065205(2009)
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[6] *Relativistic Kinematics, A guide to the kinematic problems of high energy physics*, R.Hagedorn and J.D.Jackson, W.A.Benjamin, Inc, 1963
Classical Electrodynamics, 2nd Edition, J.D.Jackson, John Wiley & Sons, 1975