Preliminary plan

1. Introduction
2. Inclusive and semi-inclusive DIS (structure functions)

   Basics of collinear PDFs at tree level (definition, gauge link)
3. Basics of collinear PDFs (interpretation)

   Basics of TMDs at tree level (definition, gauge link, interpretation)

- Basics of factorization
- Basics of TMD evolution

- Phenomenology of unpolarized SIDIS
- Phenomenology of polarized SIDIS

Next lectures

- Apr 29, 3:00 PM in L104

Results of last lecture

18 structure functions

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+ S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

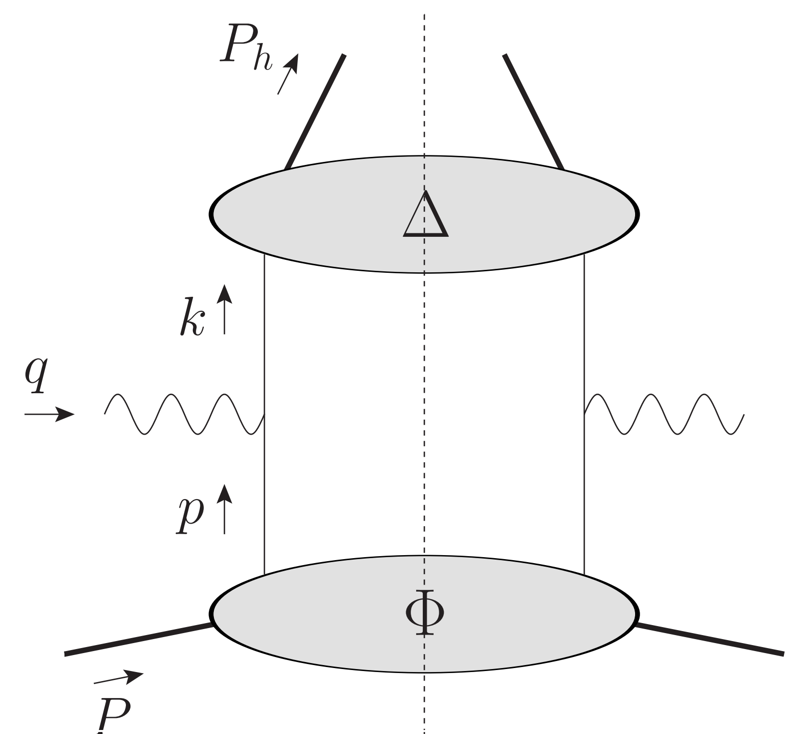
$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Starting formula (tree level)

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{I}\Big[\mathrm{Tr}(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu)\Big]$$



$$\mathcal{I}\Big[\cdots\Big] \equiv \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T-\frac{\boldsymbol{P}_{h\perp}}{z}-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big]$$

TMDs and their probabilistic interpretation

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

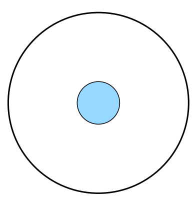
Twist-2 TMDs

TMDs in black survive transverse-momentum integration

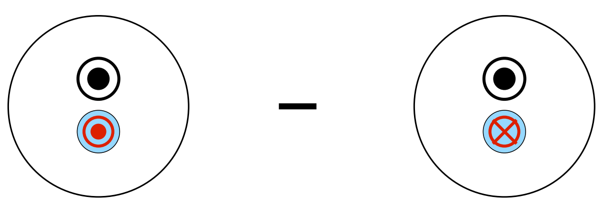
TMDs in red are T-odd

TMDs and their probabilistic interpretation

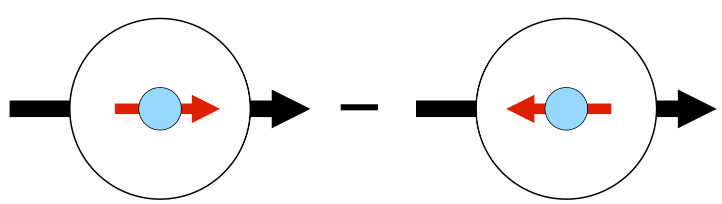
$f_1 =$



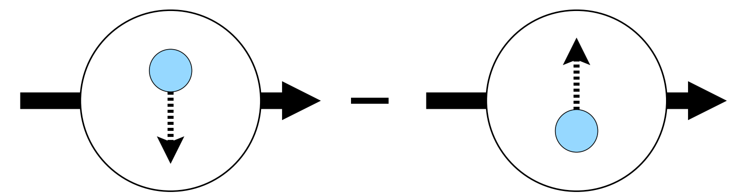
$g_1 =$



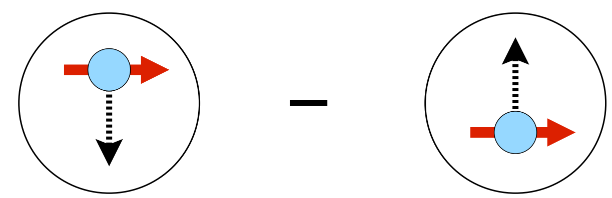
$h_1 =$



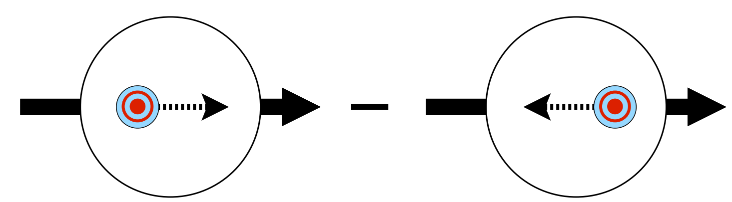
$f_{1T}^{\perp} =$



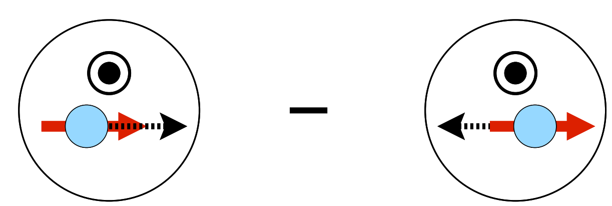
$h_1^{\perp} =$



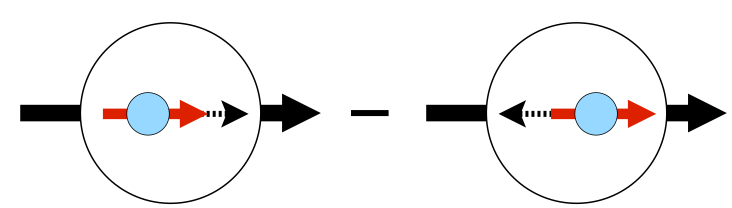
$g_{1T} =$



$h_{1L}^{\perp} =$



$h_{1T}^{\perp} =$



Unpolarized sector

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\right)\right],$$

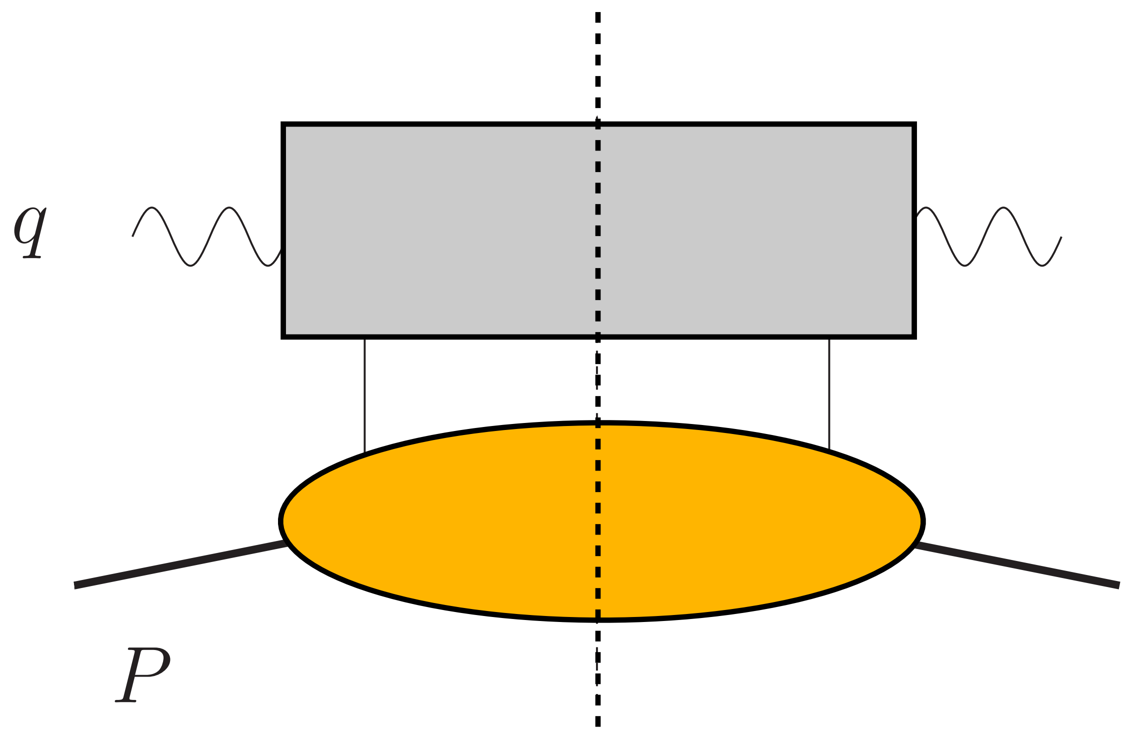
$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\big)\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\big) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right],$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,\delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\,w(\boldsymbol{p}_T, \boldsymbol{k}_T)\,f^a(x, p_T^2)\,D^a(z, k_T^2),$$

Essential ideas on factorization

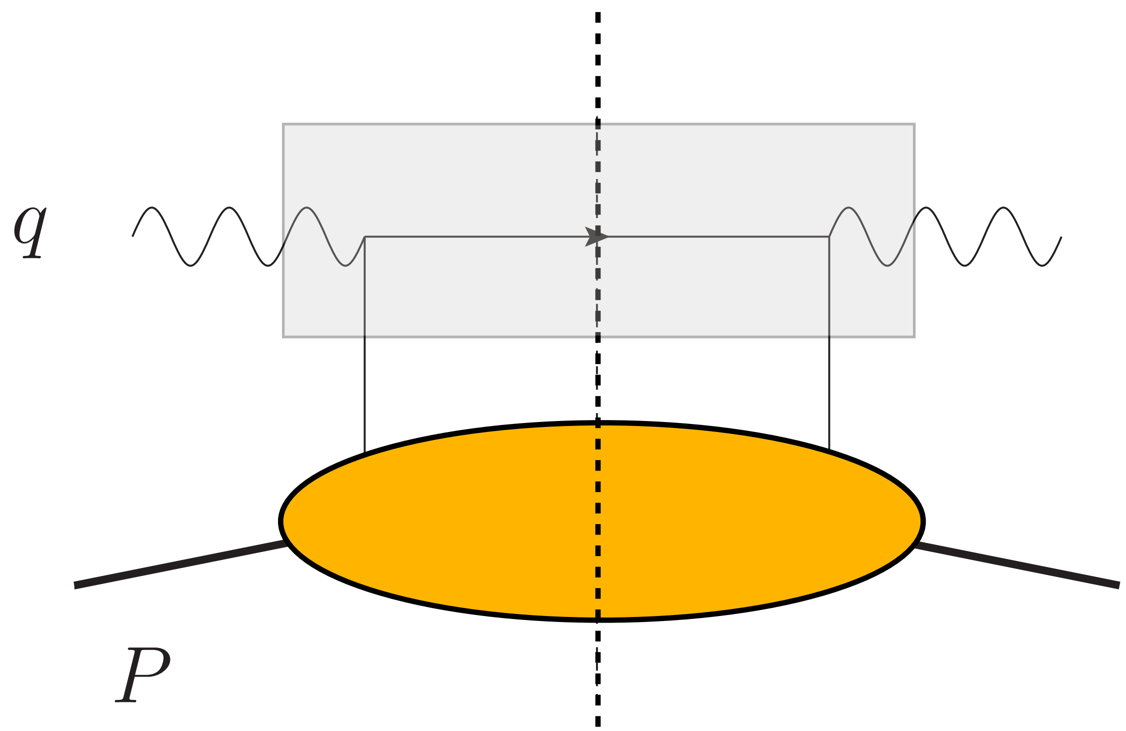
Inclusive DIS

$$F(x,Q^2) = x\sum_a \int_x^1 \frac{d\hat{x}}{\hat{x}}\, f^a\Big(\frac{x}{\hat{x}},\mu_F^2\Big)\; H_a\Big(\hat{x},\ln\frac{\mu_F^2}{Q^2}\Big)$$



Inclusive DIS: tree level

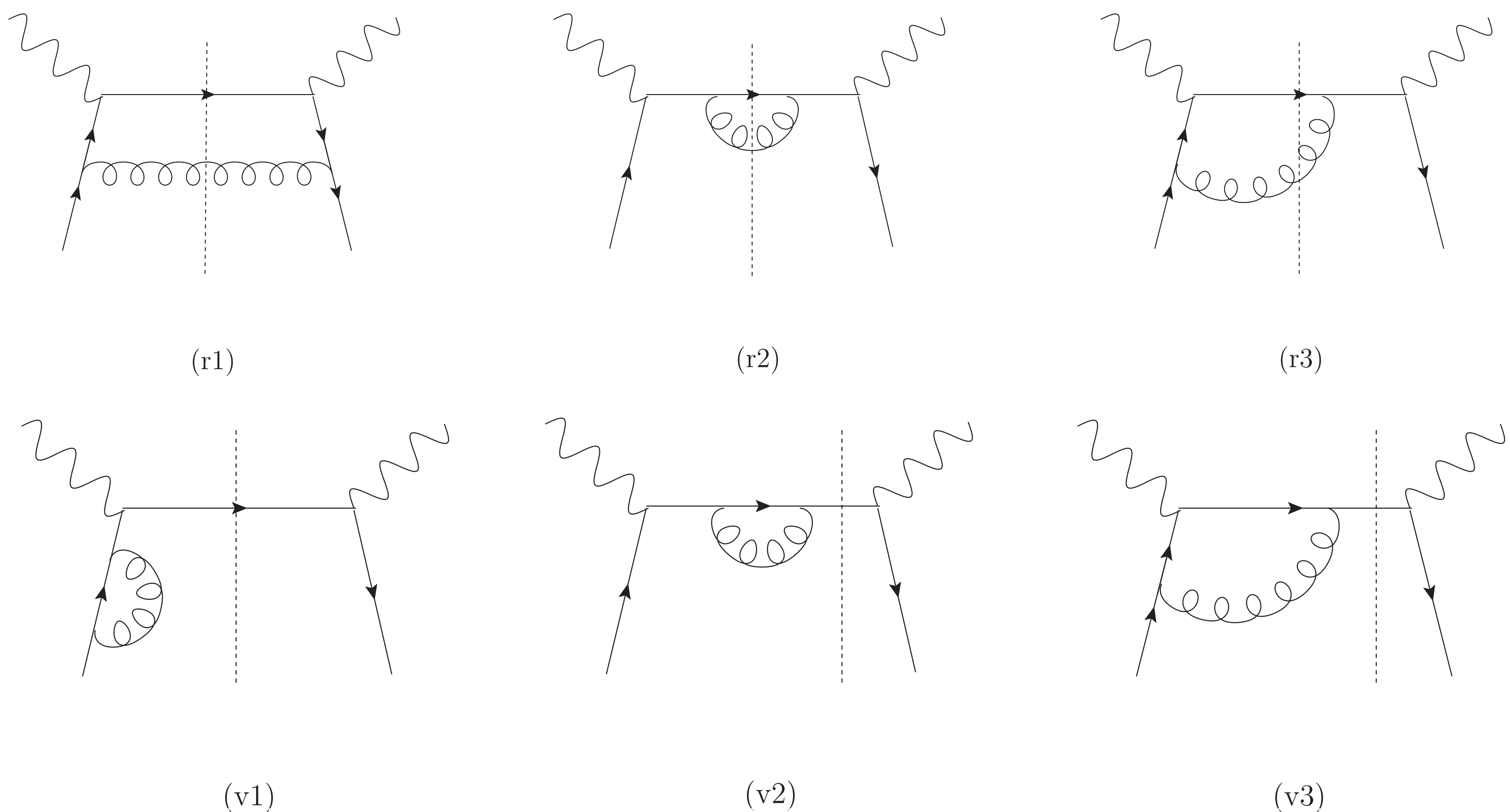
$$F(x,Q^2) = x \sum_a \int_x^1 \frac{d\hat{x}}{\hat{x}}\, f^a\!\left(\frac{x}{\hat{x}}, \mu_F^2\right)\, H_a\!\left(\hat{x}, \ln\frac{\mu_F^2}{Q^2}\right)$$



$$H_{UU,T}^{a(0)}(\hat{x}) = \sum_b e_b^2\, \delta^{ab}\, \delta(1-\hat{x})$$

$$H_{UU,L}^{a(0)}(\hat{x}) = 0$$

One-loop level



These diagrams have all sorts of divergences:

- ★ultraviolet
- ★collinear (if gluon and quark mass → 0)
- ★soft (if gluon mass → 0)

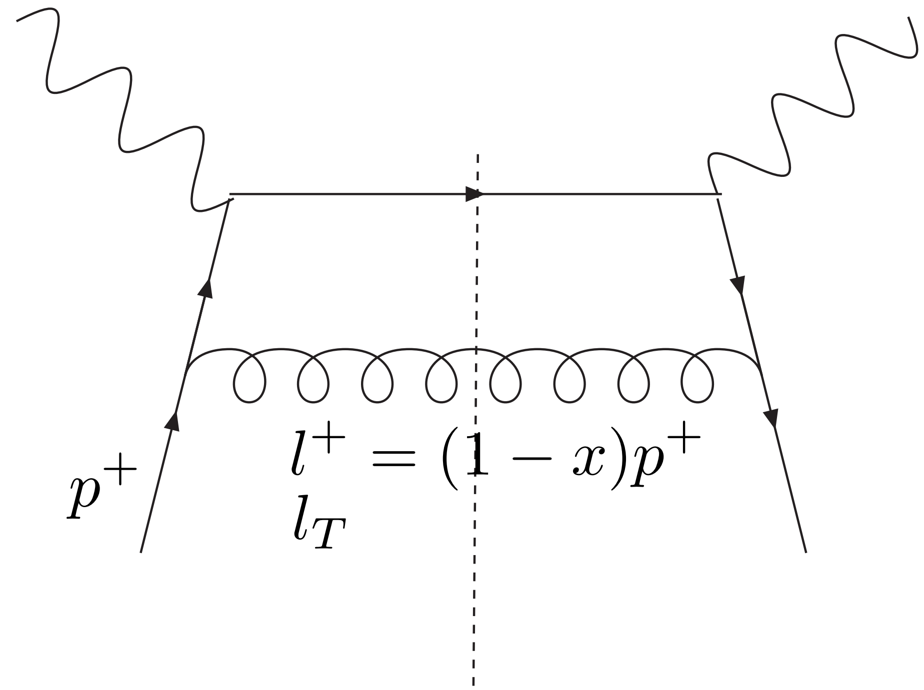
A general rule

The more you integrate, the more you cancel divergences.
For instance, the total cross section is free of any divergence (infrared safe)

Cancellations in inclusive DIS

All soft divergences disappear in inclusive DIS, thanks to cancellations between real and virtual diagrams

Collinear divergences



$$\int d^2 l_T \, dl^+ \, \frac{1}{l_T^2}$$

➡*regularize and include into PDFs*

$$\int d^{(2+\epsilon)} l_T \, dl^+ \, \frac{1}{l_T^2}$$

dimensional

$$\int d^2 l_T \, dl^+ \, \frac{1}{l_T^2 + \lambda^2}$$

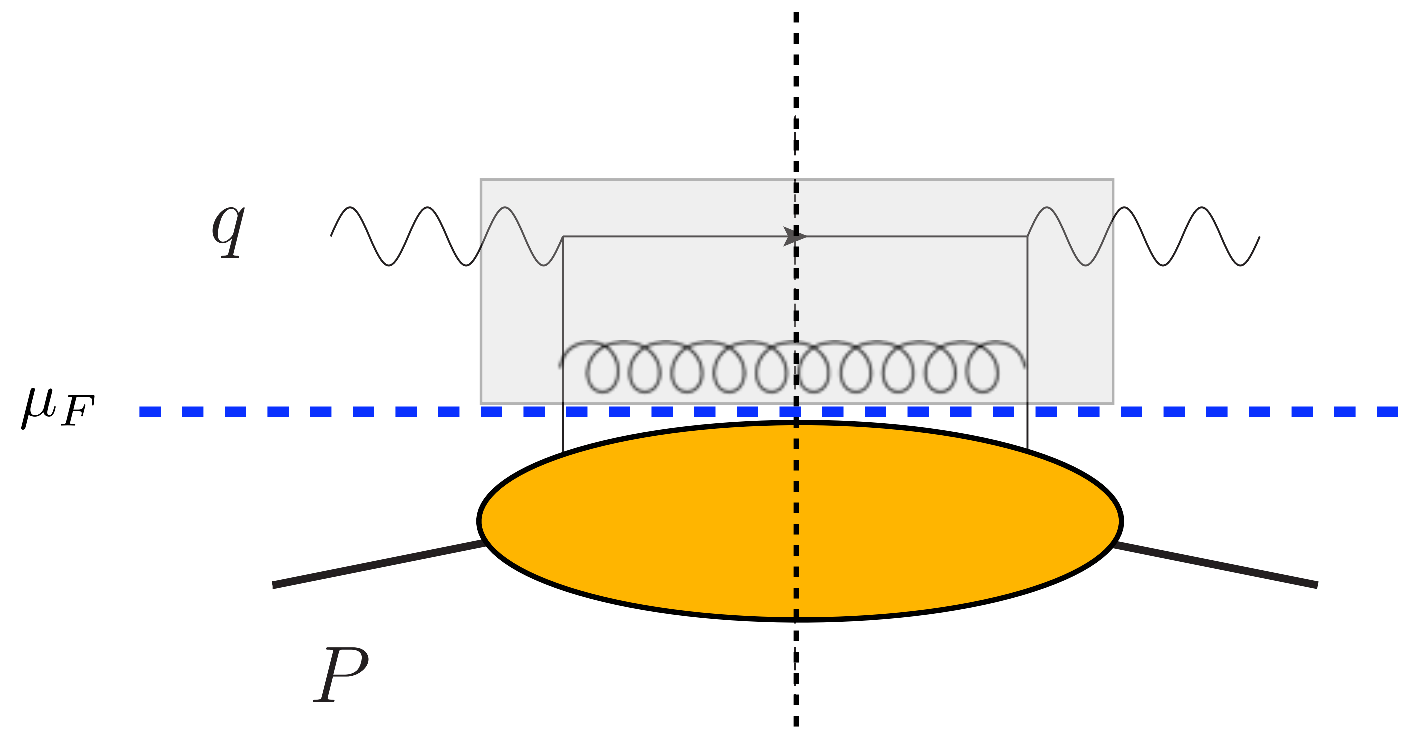
mass

$$\int_\lambda d^2 l_T \, dl^+ \, \frac{1}{l_T^2}$$

cutoff

Factorization scale

$$F(x,Q^2) = x \sum_a \int_x^1 \frac{d\hat{x}}{\hat{x}} f^a\left(\frac{x}{\hat{x}}, \mu_F^2\right) H_a\left(\hat{x}, \ln \frac{\mu_F^2}{Q^2}\right)$$

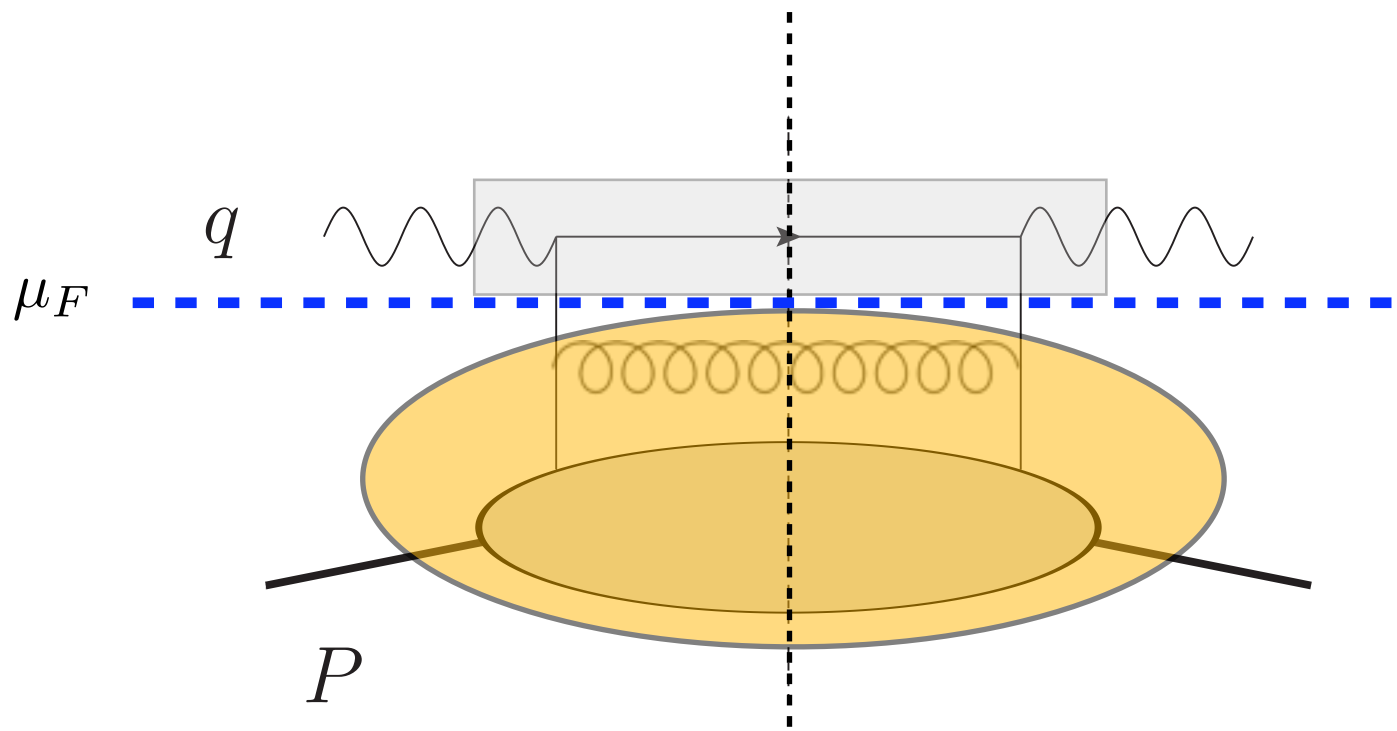


The factorization scale determines how much we put in the PDF and how much in the hard scattering

$$\int_\lambda^{\mu_F} d^2 l_T \, dl^+ \, \frac{1}{l_T^2}$$

Factorization scale

$$F(x,Q^2) = x \sum_a \int_x^1 \frac{d\hat{x}}{\hat{x}}\, f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big)\; H_a\Big(\hat{x}, \ln\frac{\mu_F^2}{Q^2}\Big)$$

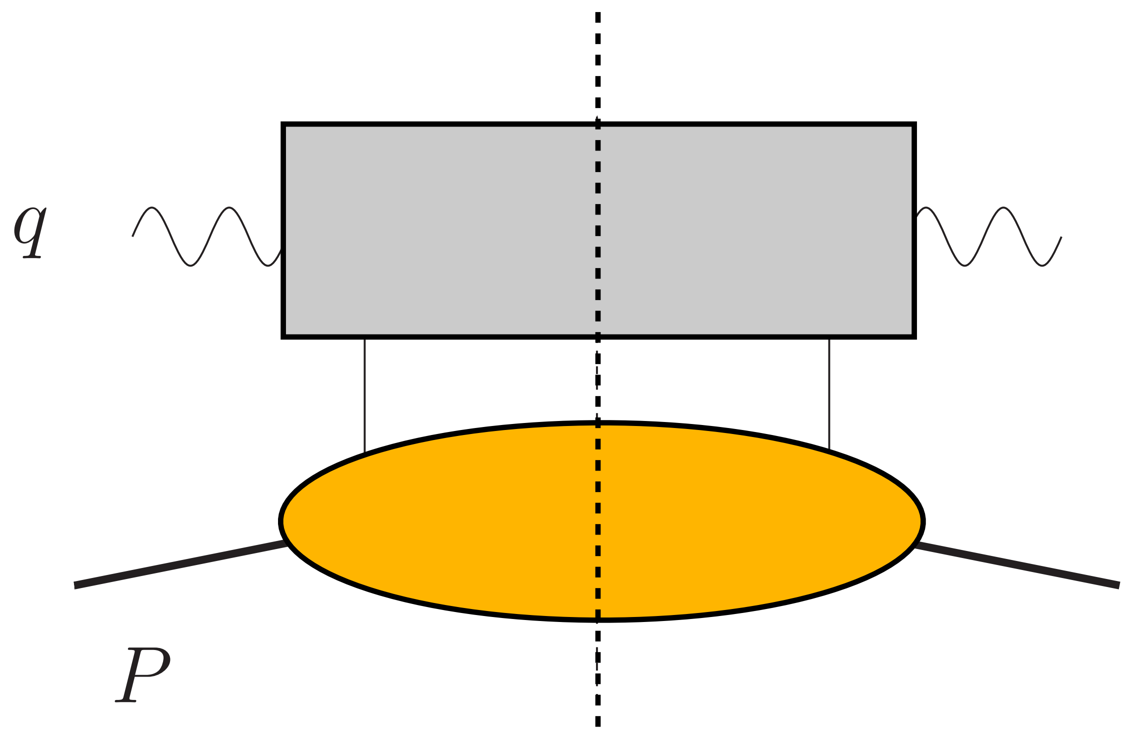


The factorization scale determines how much we put in the PDF and how much in the hard scattering

$$\int_\lambda^{\mu_F} d^2 l_T\, dl^+ \frac{1}{l_T^2}$$

Factorization theorem

$$F(x,Q^2) = x\sum_a \int_x^1 \frac{d\hat{x}}{\hat{x}}\, f^a\!\left(\frac{x}{\hat{x}},\mu_F^2\right)\, H_a\!\left(\hat{x},\ln\frac{\mu_F^2}{Q^2}\right)$$



Collins, Soper, Sterman (1988), hep-ph/0409313

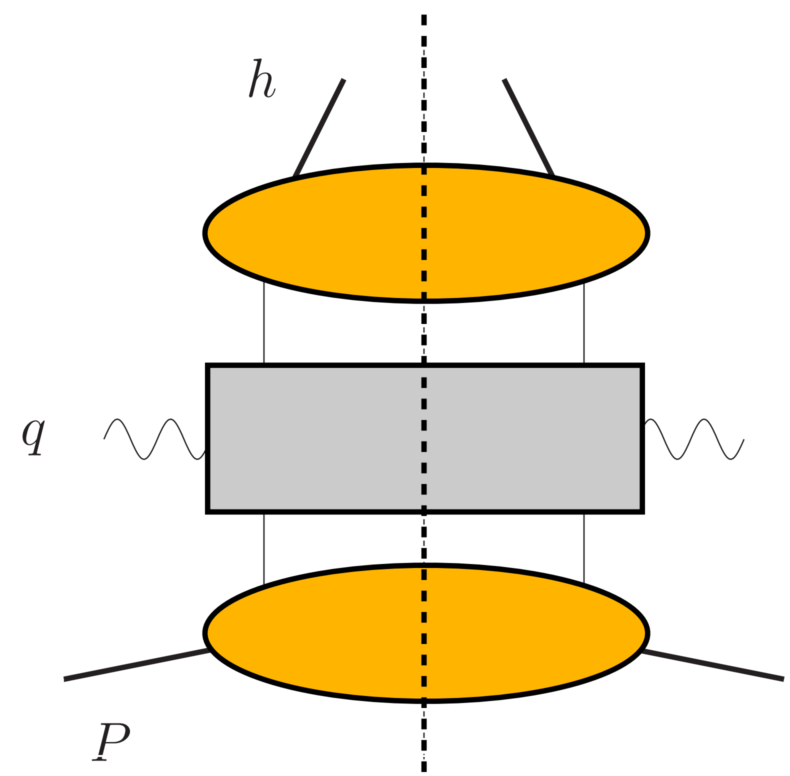
See: Handbook of Perturbative QCD, CTEQ, http://www.phys.psu.edu/~cteq/

Evolution equations

- The factorization scale μ_F is put in “by hand” to separate perturbative from nonperturbative
- The final result for the structure function cannot depend on μ_F
- The dependence of the PDFs on μ_F can be computed (DGLAP evolution equations) if $\mu_F >> \Lambda_{\mathrm{QCD}}$
- The PDFs at a low scale are nonperturbative and have to be extracted from the experiments

SIDIS integrated over transverse momentum

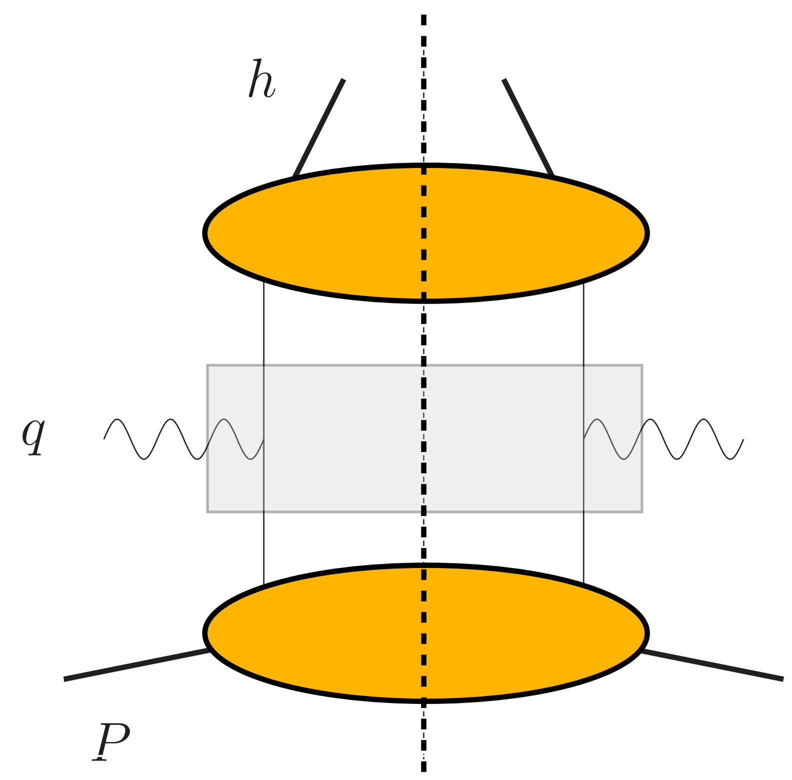
$$F(x,z,Q^2) = x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \, f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big) \, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big) \; H_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$



analogous to theorems for Drell-Yan or e^+e^- annihilation, see previous references

Integrated SIDIS: tree level

$$F(x,z,Q^2) = x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \, f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big) \, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big) \; H_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$

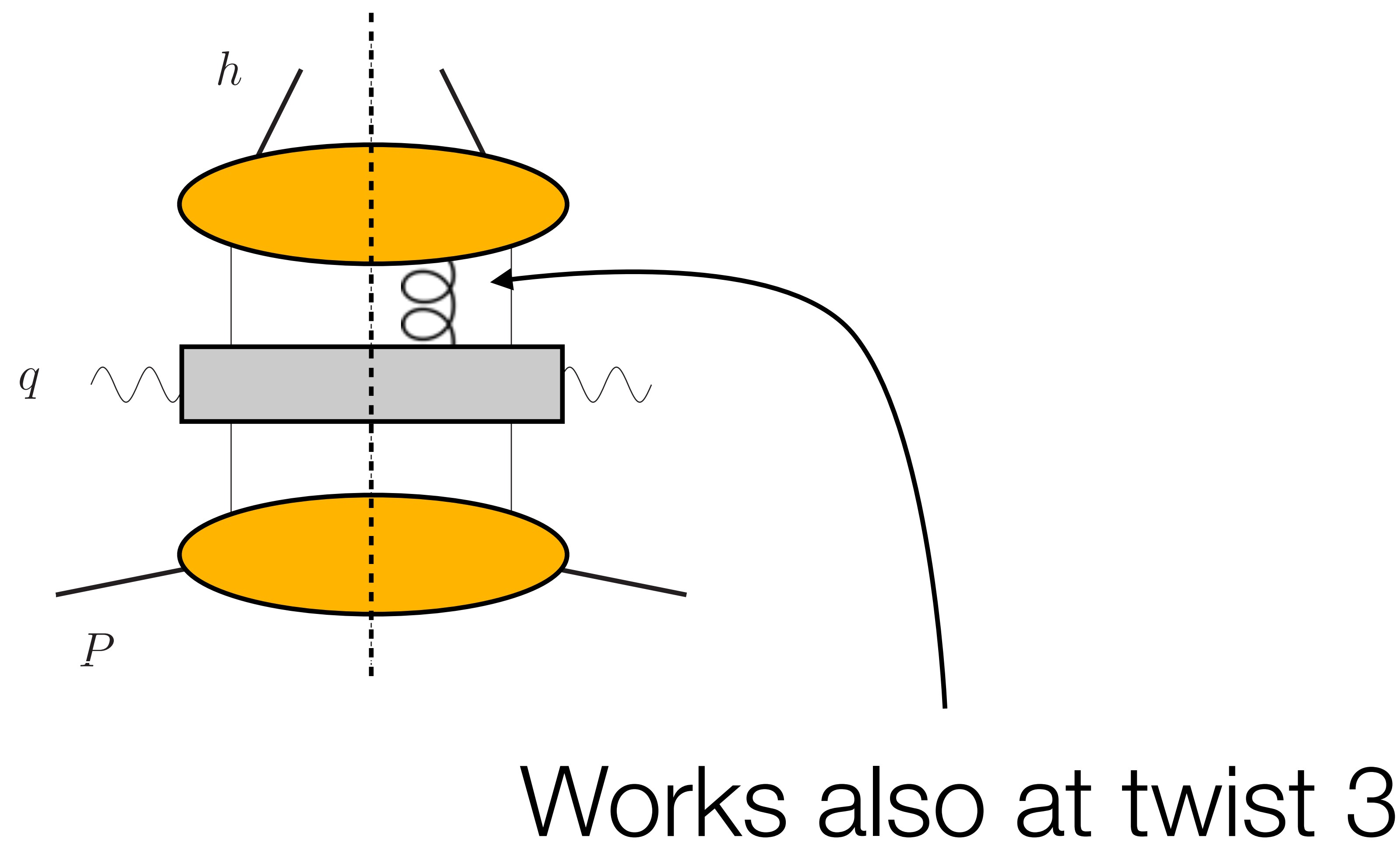


$$H_{UU,T}^{ab(0)}(\hat{x}) = e_b^2 \, \delta^{ab} \, \delta(1-\hat{x})\delta(1-\hat{z})$$

$$H_{UU,L}^{a(0)}(\hat{x}) = 0$$

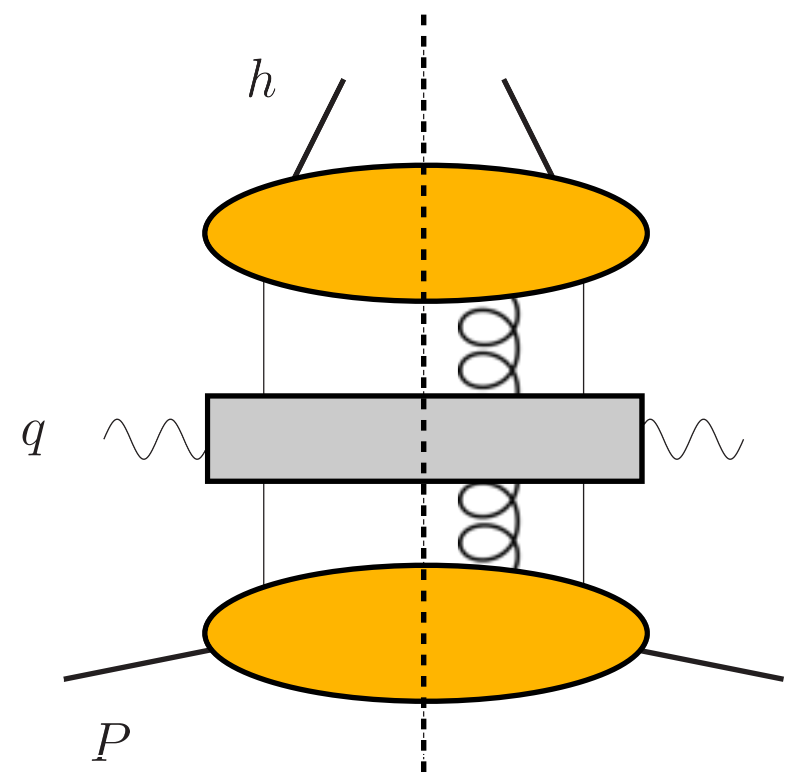
Integrated SIDIS: twist 3

$$F(x,z,Q^2) = x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big)\, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big)\; H_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$



Integrated SIDIS: problems at twist 4

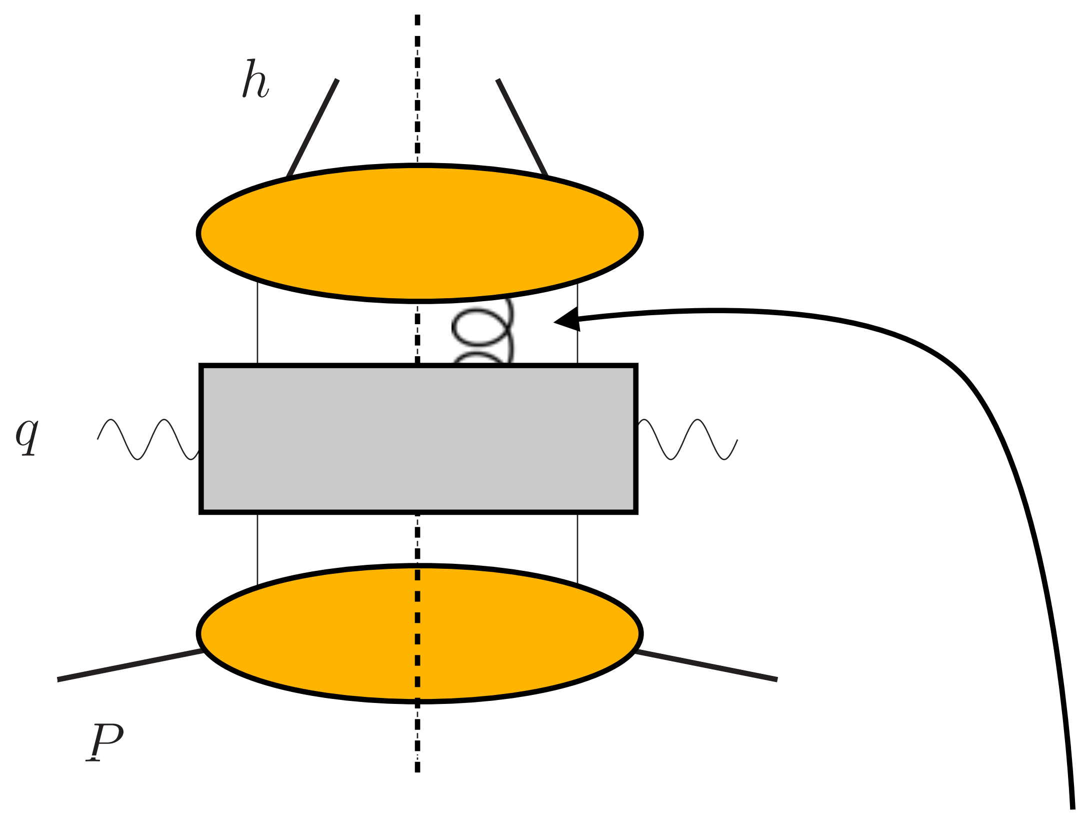
$$F(x,z,Q^2) = x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \, f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big) \, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big) \; H_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$



SIDIS at high transverse momentum

$$F(x,z,Q^2) = \frac{1}{Q^2 z^2}\, x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\Big(\frac{P_{h\perp}^2}{Q^2 z^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

$$\times\ f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big)\, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big)\ H'_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$

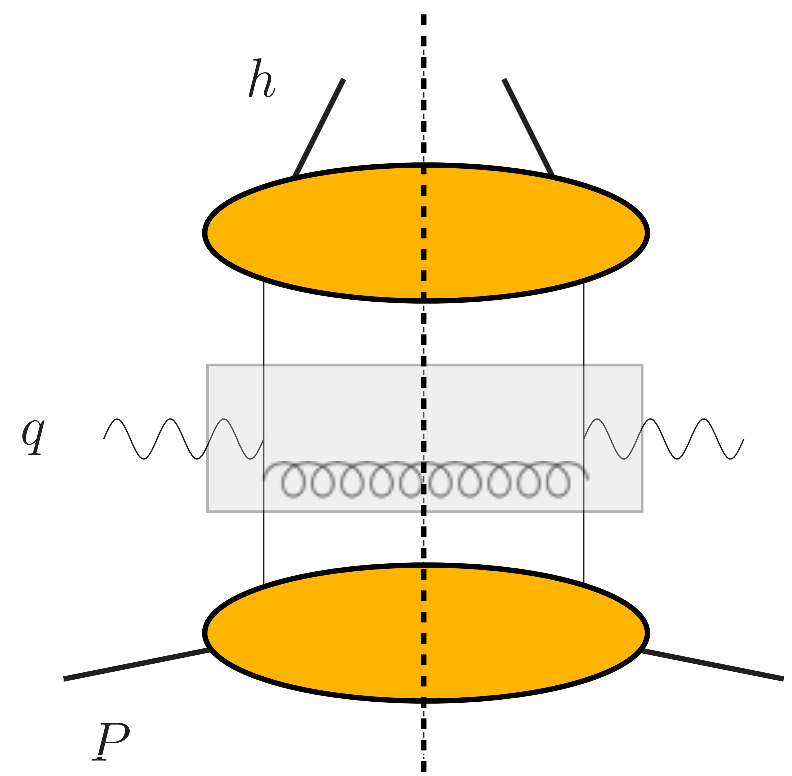


Works also at twist 3

SIDIS at high transverse momentum

$$F(x,z,Q^2) = \frac{1}{Q^2 z^2}\, x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\Big(\frac{P_{h\perp}^2}{Q^2 z^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

$$\times\ f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big)\, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big)\, H'_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$



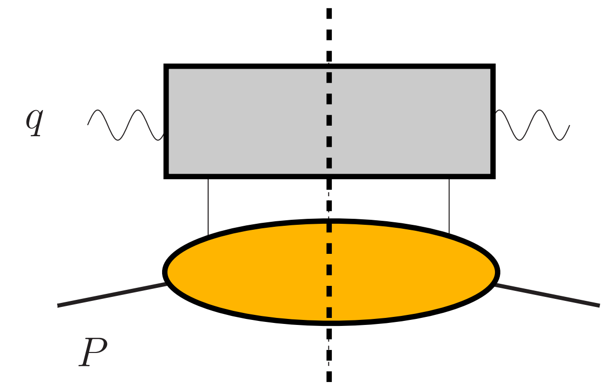
Starts at order α_s

Important messages

- Factorization theorems are the only rigorous way to define what are the objects we call “parton distribution functions”
- The intuitive idea, based on parton model and handbag diagram, of PDFs being probability densities is slightly modified by the factorization theorems.
- What is important is that the PDFs are nonperturbative objects, they describe the partonic structure of the nucleon, they can be extracted from experiments

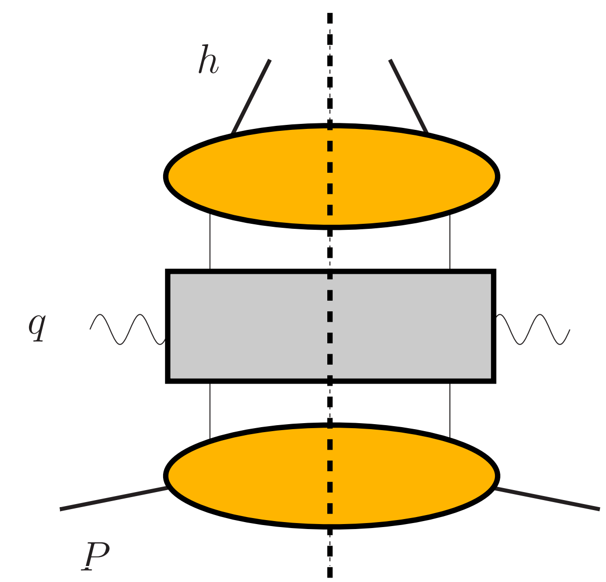
Factorization theorems

- Inclusive DIS
  up to twist 4 ✔



- Integrated SIDIS
  up to twist 3 ✔

- SIDIS at high transverse mom.
  up to twist 3 ✔



- SIDIS at low transverse mom. ?

TMD factorization: relevant literature

- Collins, Soper, NPB 193 (81)
- Collins, Soper, Sterman, NPB 250 (85)
- Collins, Acta Phys. Polon. B34 (03)
- Ji, Ma, Yuan, PRD 71 (05)
- Collins, Rogers, Stasto, PRD 77 (08)
- Collins, arXiv:0808.2665 [hep-ph]
- Coming up at JLab: talks by F. Yuan (Apr 27) and A. Stasto (May 20)

No cancellations

The problem is that soft divergences do not cancel anymore and a new class of divergences (light-cone or rapidity divergences) appear

Regulate divergences

To regulate divergences we can use:

μ ultraviolet cutoff

m quark mass

λ gluon mass

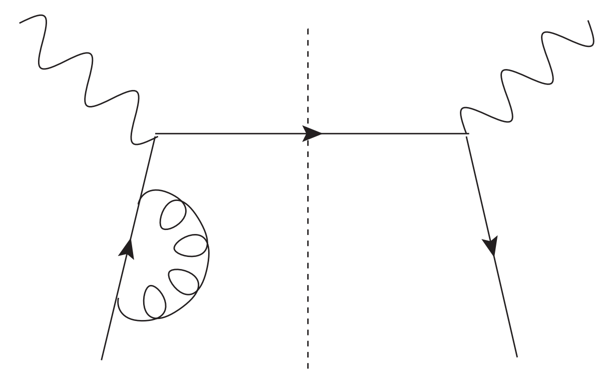
Light-cone divergences

$$d^{\mu\nu}(l;v) = -g^{\mu\nu} + \frac{l^\mu v^\nu + l^\nu v^\mu}{l\cdot v} - \frac{l^\mu l^\nu}{(l\cdot v)^2}\, v^2 \qquad \int d^2 l_T\, dl^+ \,\frac{1}{l\cdot v} = \int d^2 l_T\, dl^+ \,\frac{1}{l^+}$$

To regulate these divergences,
we give a + component to v so that

$$v = v^- n_- + \frac{2P^{+2}}{\zeta^2}\, n_+$$

Factorizing soft divergences



(v1)

Soft divergence

$$\delta(1-x_B)\delta(1-z_h)\delta^2(P_{h\perp})\Big[1+2\frac{\alpha_s C_F}{4\pi}\Big(-\ln\frac{\mu^2}{\lambda^2}+3\ln\frac{m^2}{\lambda^2}-4\Big)\Big]$$

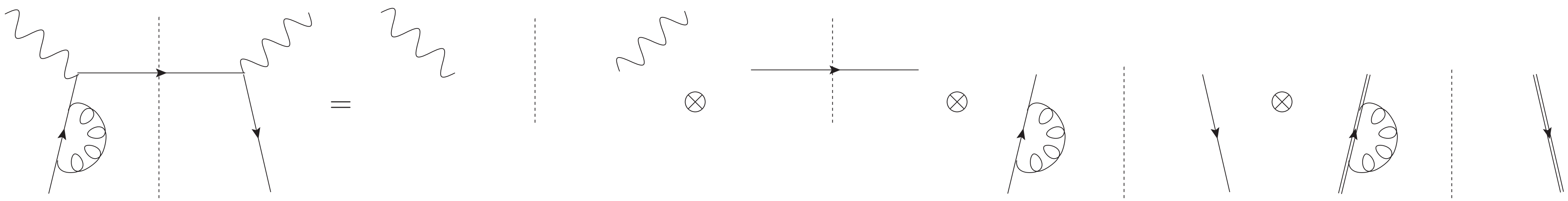
Collinear divergence

μ ultraviolet cutoff

m quark mass

λ gluon mass

Factorizing soft divergences



Soft divergence

$$\delta(1-x_B)\delta(1-z_h)\delta^2(P_{h\perp})\left[1+2\frac{\alpha_s C_F}{4\pi}\left(-\ln\frac{\mu^2}{\lambda^2}+3\ln\frac{m^2}{\lambda^2}-4\right)\right]$$

Collinear divergence

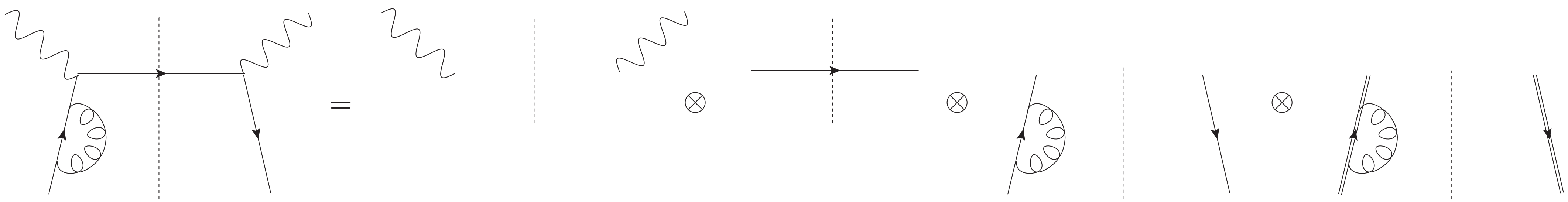
μ ultraviolet cutoff

m quark mass

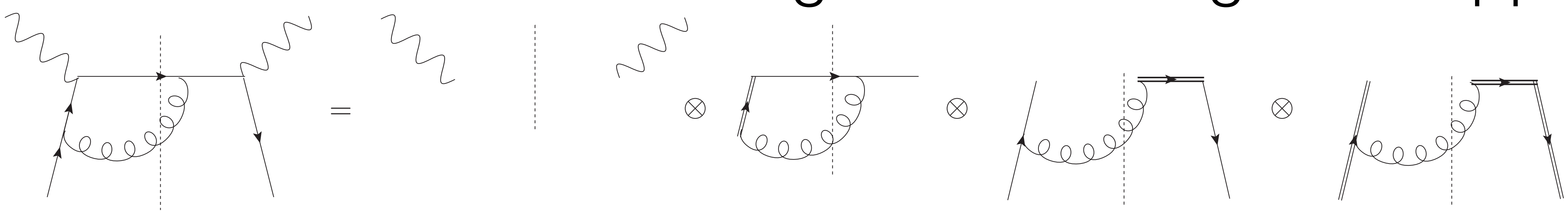
λ gluon mass

Factorizing soft divergences

Hard FF PDF Soft factor



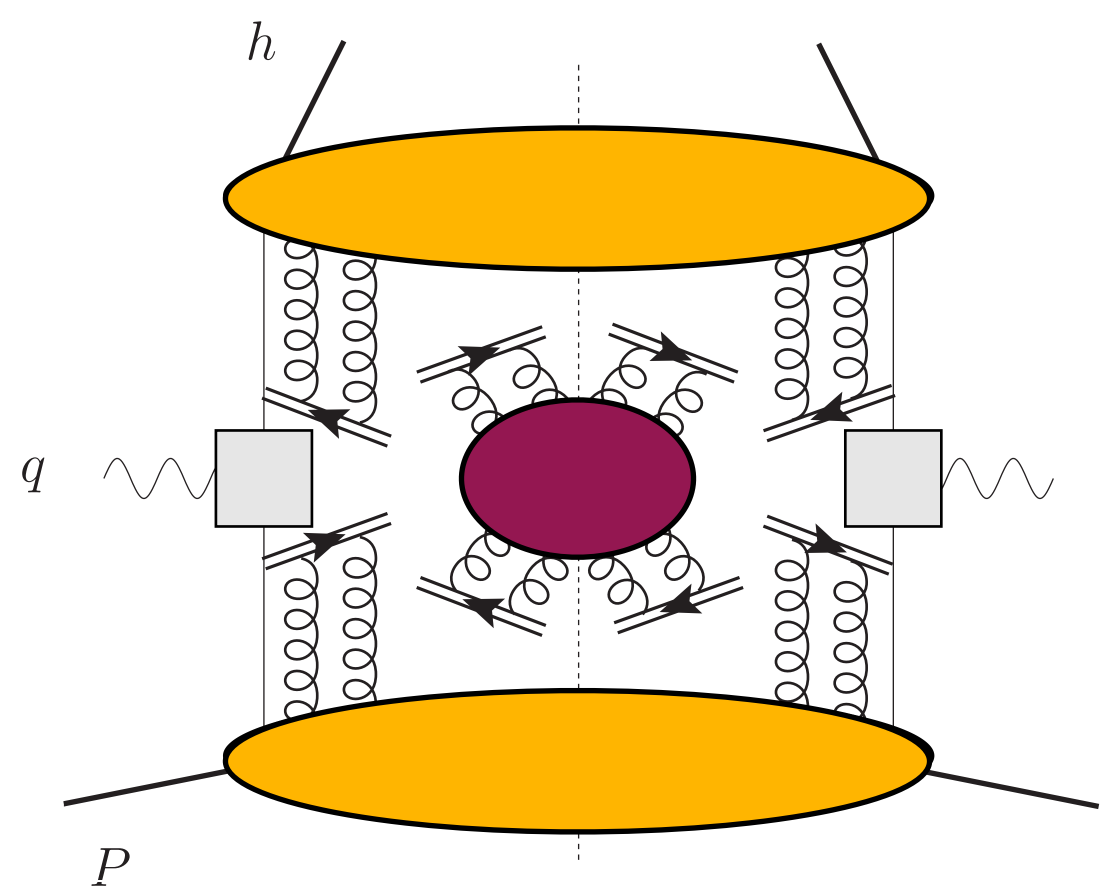
Light-cone divergences appear



the light-cone regulators determine what goes in the FF, PDFs, and SF

TMD factorization

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)



$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}'\left[f_1 D_1\right]$$

$$= H(Q^2,\mu^2,\zeta,\zeta_h)\int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\right)$$

$$x\sum_a e_a^2\, f_1^a(x,p_T^2,\mu^2,\zeta)\, D_1^a(z,k_T^2,\mu^2,\zeta_h)\, U(l_T^2,\mu^2,\zeta\zeta_h)$$

Hard part

TMD PDF

TMD FF

Soft factor

TMD factorization

- TMD factorization at the one-loop level has been proven in the work of Ji, Yuan, and Ma, extending the earlier work of Collins, Soper, Sterman, etc.
- Factorization should work for SIDIS, Drell-Yan, and e^+e^- annihilation
- The extension to all order is probably just a conjecture
- Some subtleties have been pointed out by Collins, but I am not aware of any statement that says that the work of Ji, Yuan, and Ma is wrong

TMD evolution

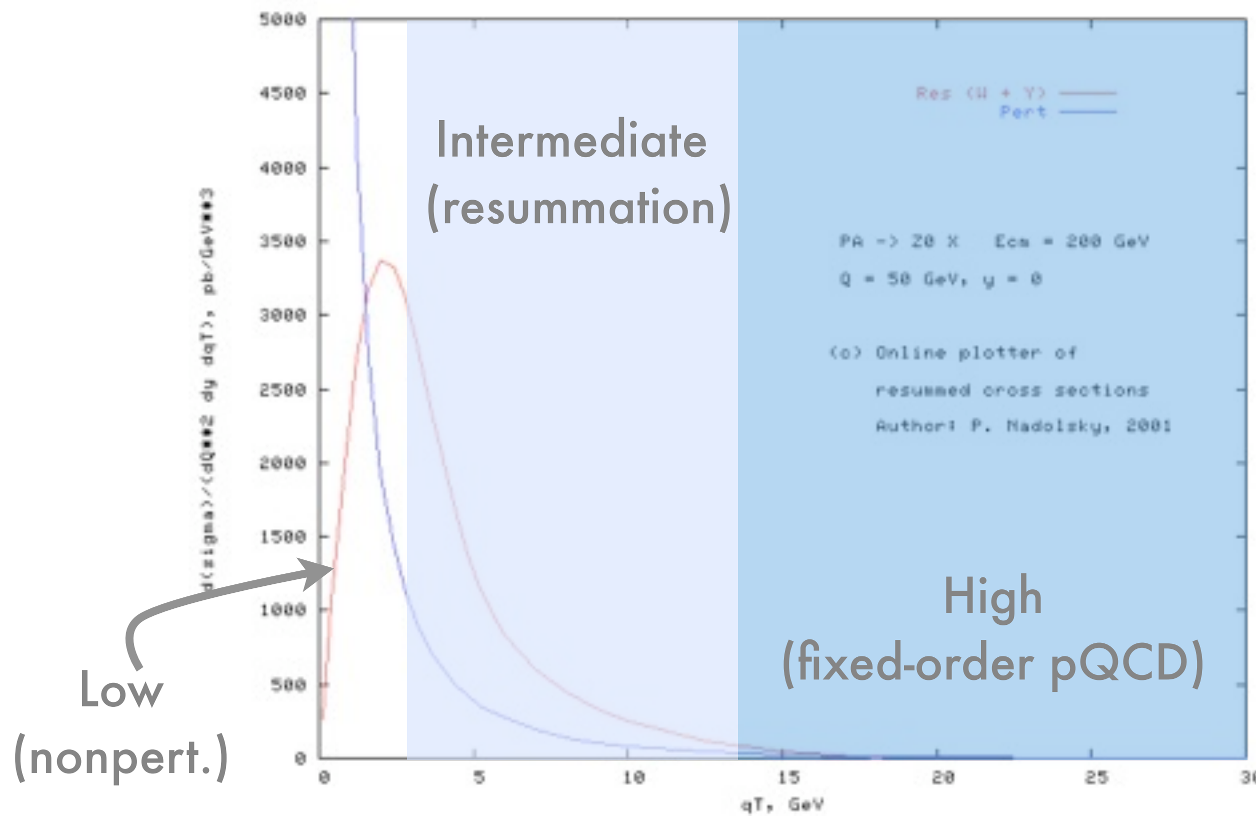
- The light-cone regulators are put in “by hand” to separate what belongs to PDFs, FFs, SF.
- The final result for the structure function cannot depend on the regulators
- The dependence on the light-cone regulators can be computed (Collins-Soper evolution equations) in the region where the transverse momentum is >> $\Lambda_{\rm QCD}$
- The component of the TMDs at small transverse momentum is nonperturbative and has to be extracted from the experiments
- Everything is done in *b* space

TMD factorization: *b* space

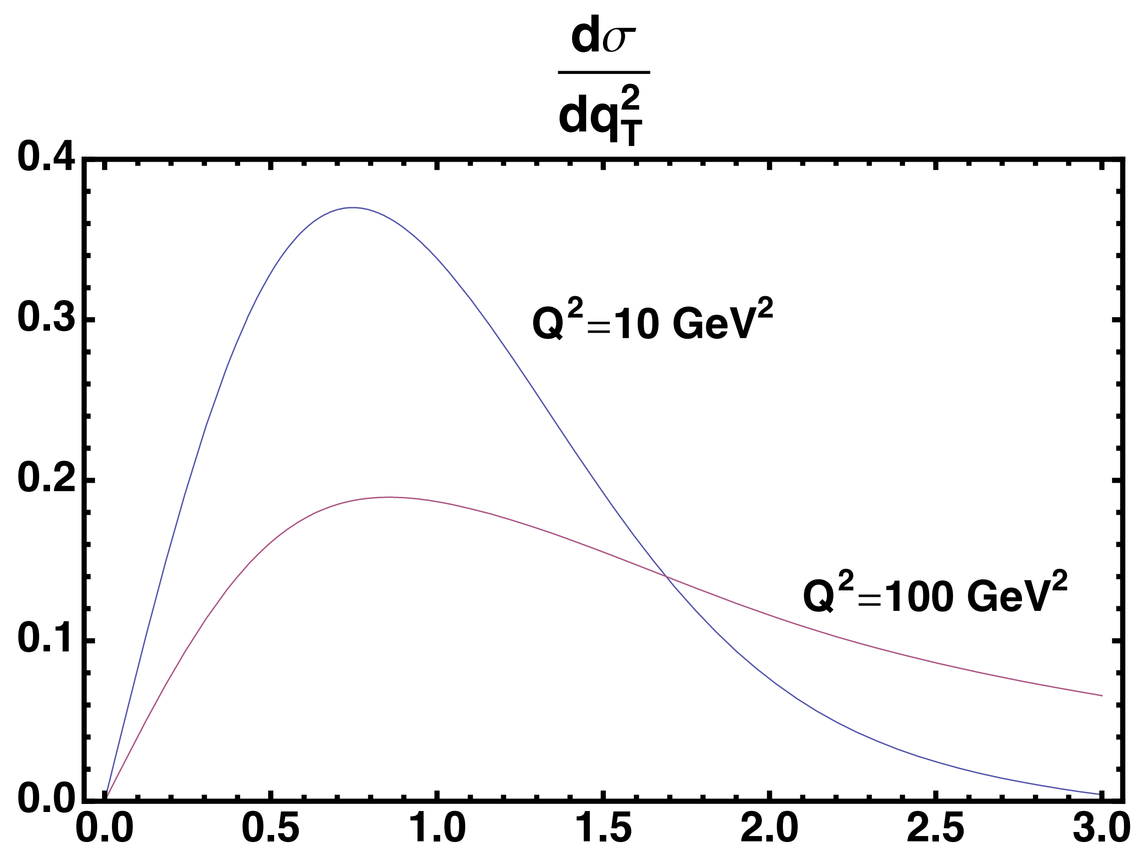
Sudakov form factor

$$F_{UU,T}(x,z,b,Q^2) = x\sum_a e_a^2 \left[(f_1^i \otimes \mathcal{C}_{ia})\,(\mathcal{C}_{aj} \otimes D_1^j)\, e^{-S} e^{-S_{NP}} \right]$$

collinear PDF and FF   calculable with pQCD   nonperturbative part of TMDs

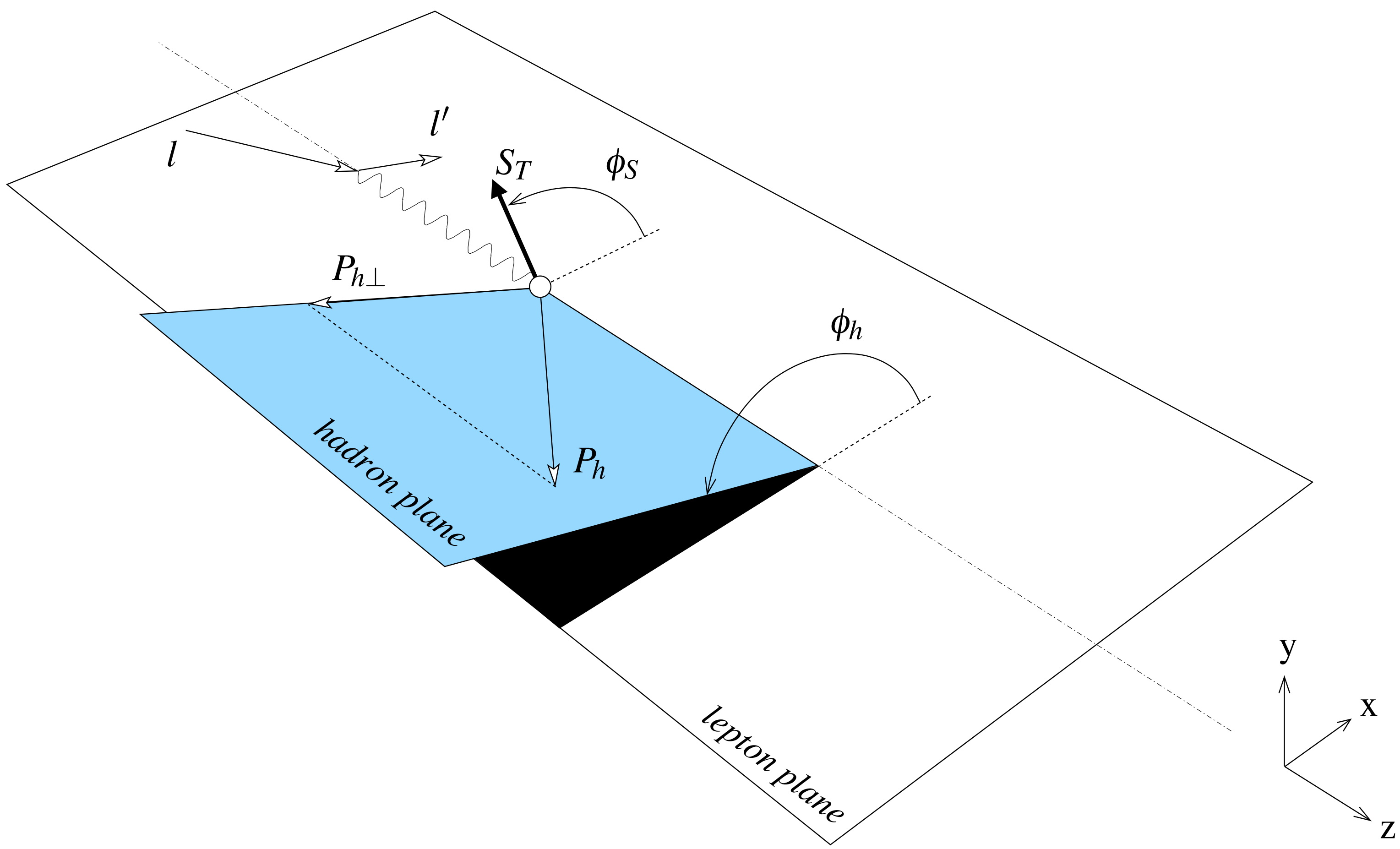


Evolution equations for TMDs



High and low transverse momentum

SIDIS once again



$$Q = \text{photon virtuality}$$

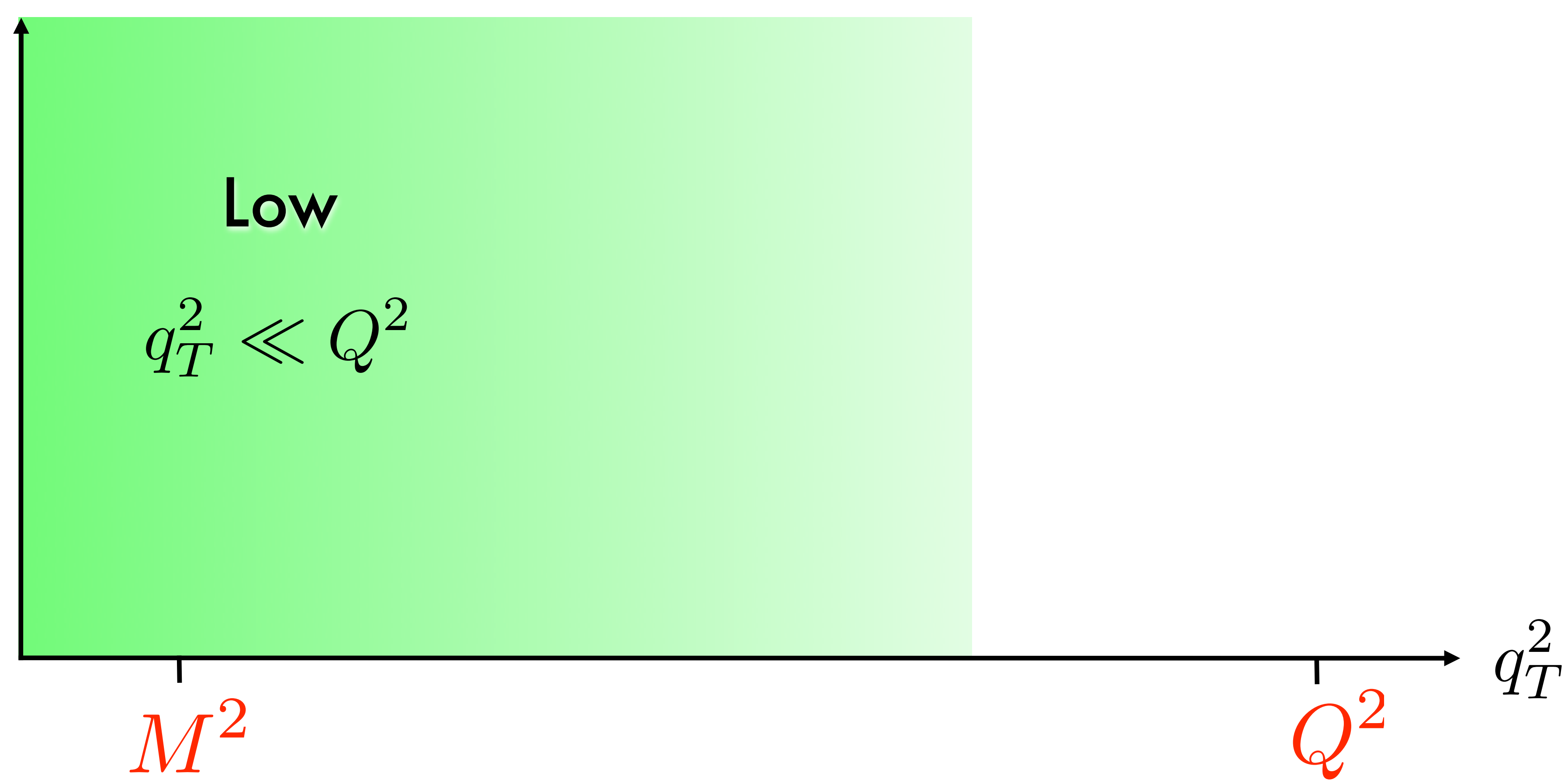
$$M = \text{hadron mass}$$

$$P_{h\perp} = \text{hadron transverse momentum}$$

$$q_T^2 \approx P_{h\perp}^2 / z^2$$

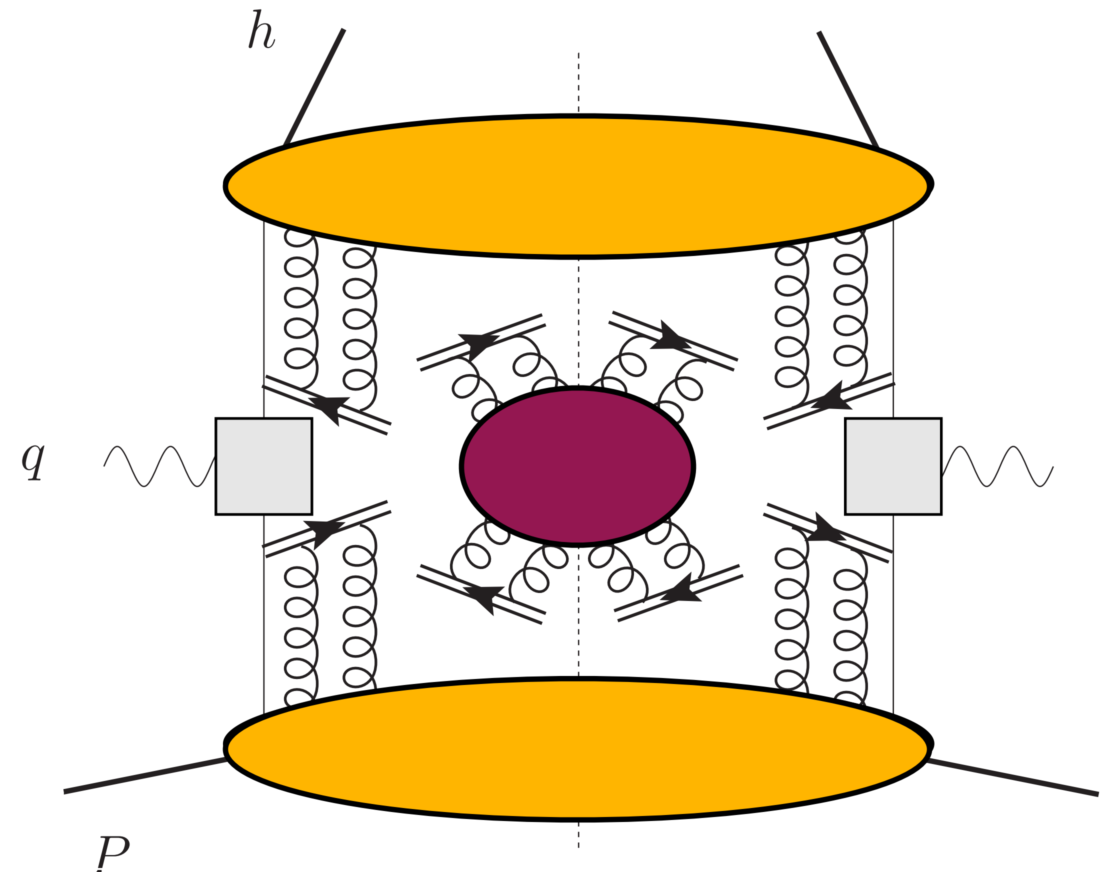
Low and high transverse momentum

AB, D. Boer, M. Diehl, P.J. Mulders, JHEP 08 (08)



TMD factorization

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

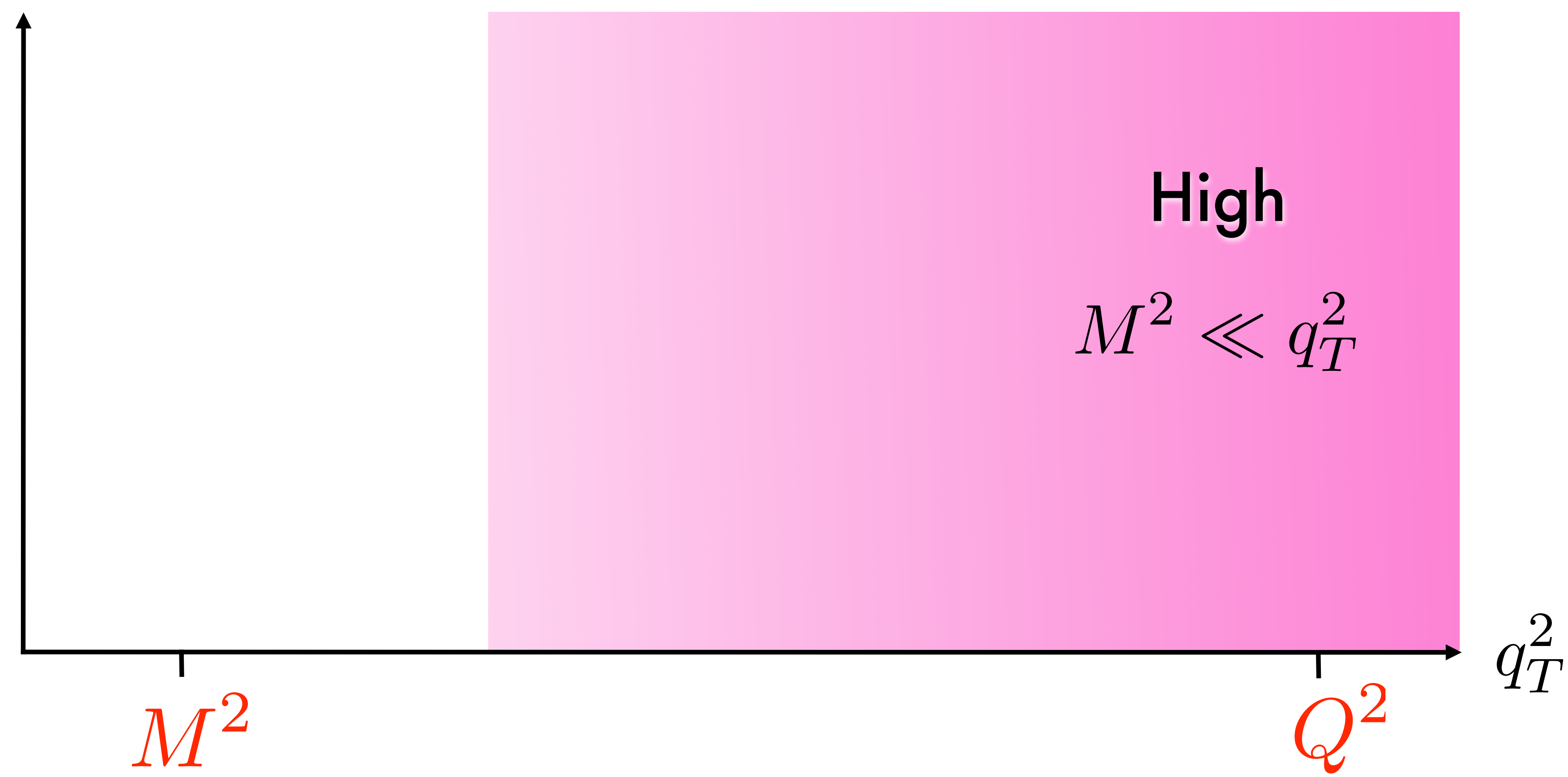


$$F_{UU,T}(x, z, P_{h\perp}^2, Q^2) = \mathcal{C}'\big[f_1 D_1\big]$$

$$= H(Q^2, \mu^2, \zeta, \zeta_h) \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x, p_T^2, \mu^2, \zeta)\, D_1^a(z, k_T^2, \mu^2, \zeta_h)\, U(l_T^2, \mu^2, \zeta\zeta_h)$$

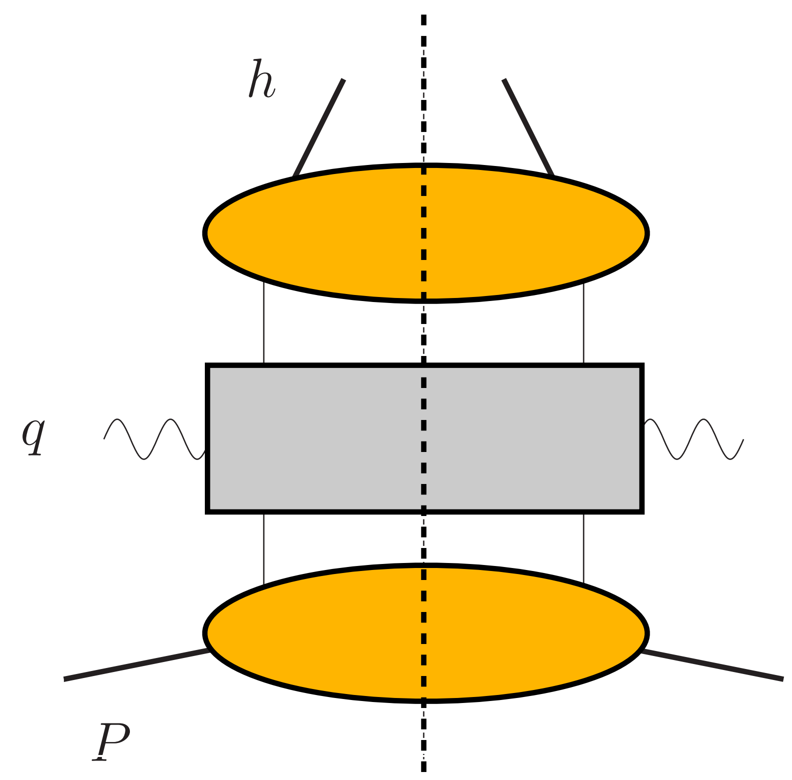
Low and high transverse momentum



SIDIS at high transverse momentum

$$F(x,z,Q^2) = \frac{1}{Q^2 z^2}\, x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\Big(\frac{P_{h\perp}^2}{Q^2 z^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

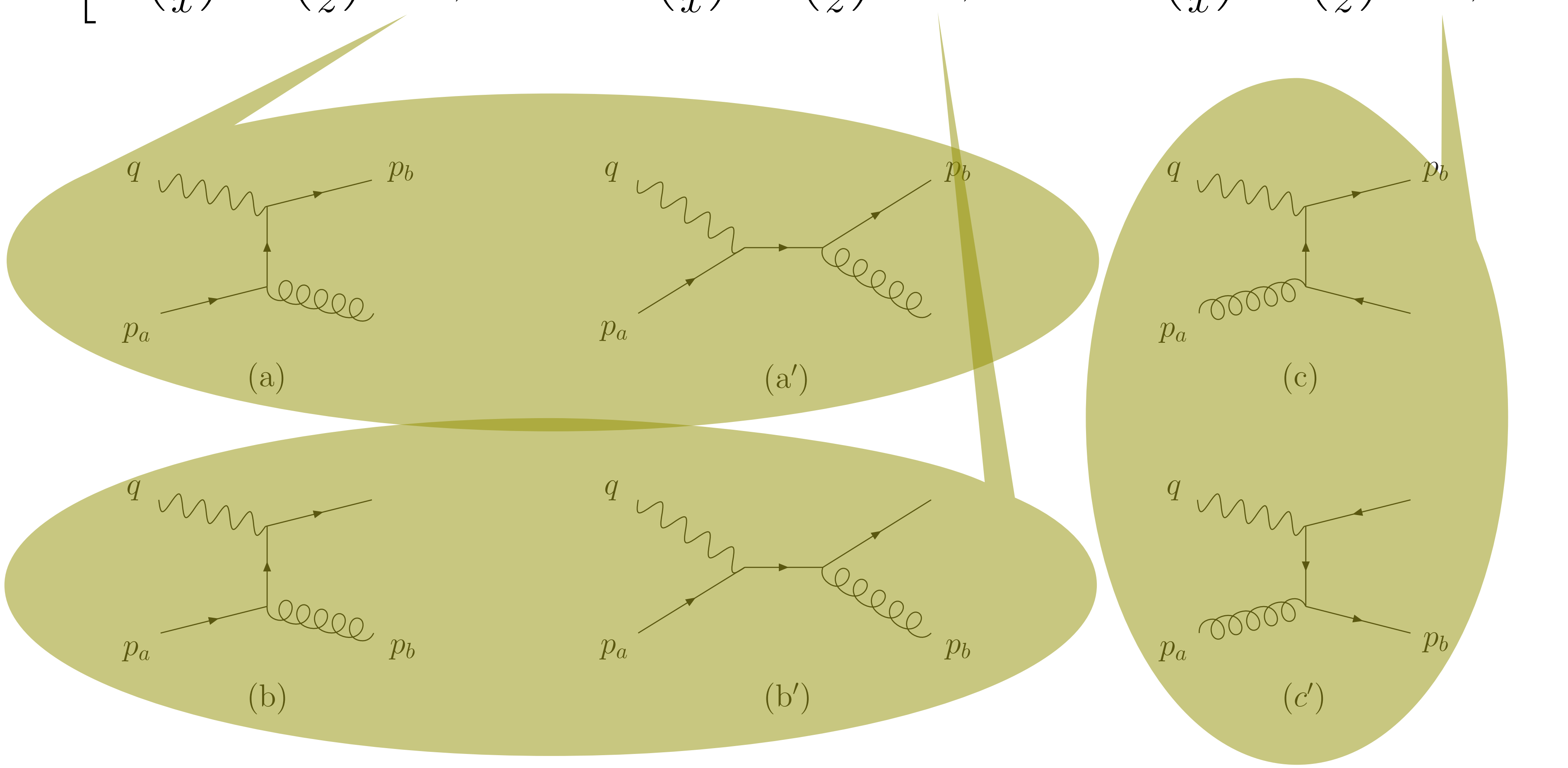
$$\times\; f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big)\, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big)\; H'_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$



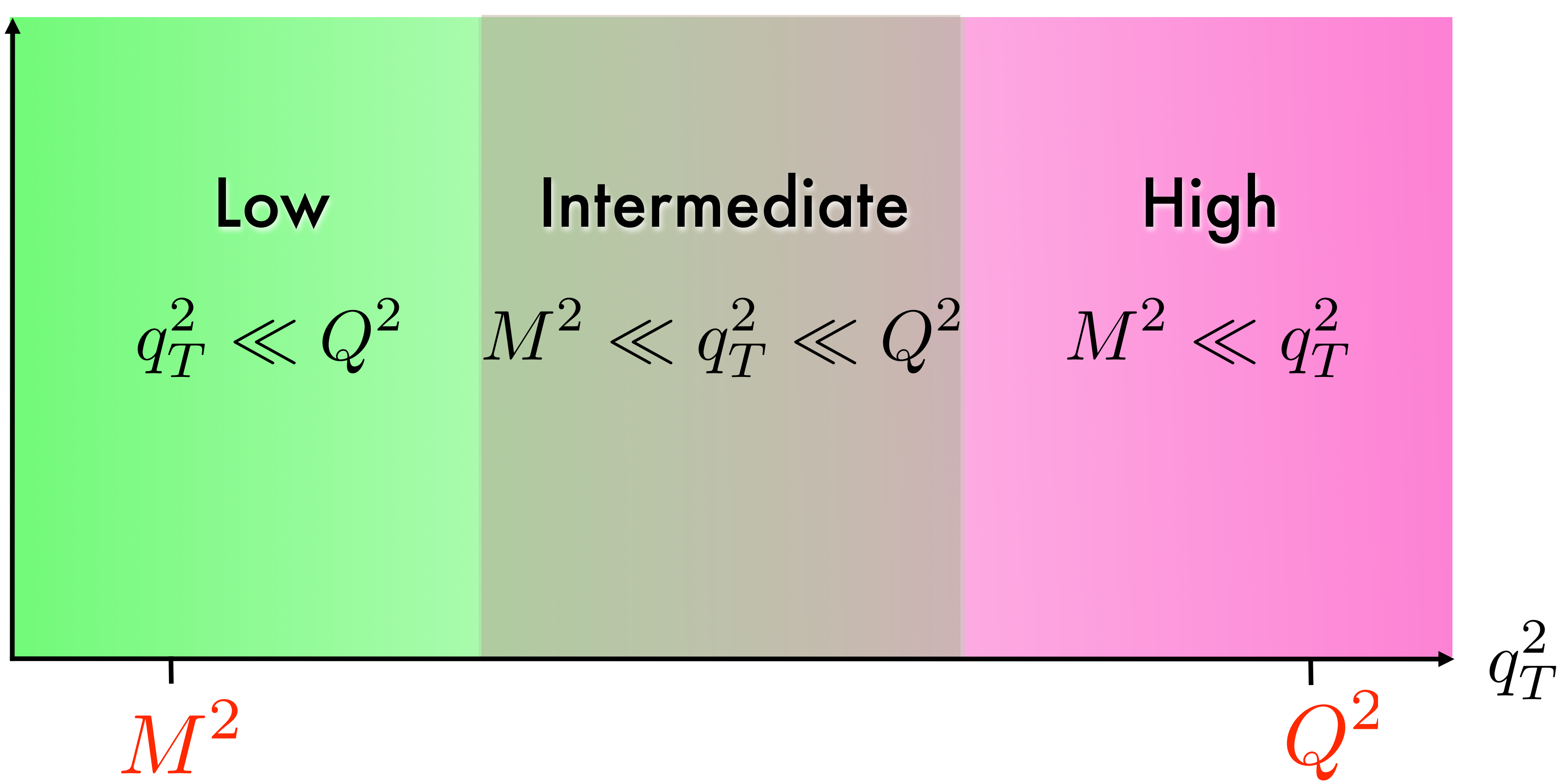
Example of high-transverse momentum result

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{\hat{x}}{\hat{x}}\int_z^1 \frac{\hat{z}}{\hat{z}}\,\delta\Big(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

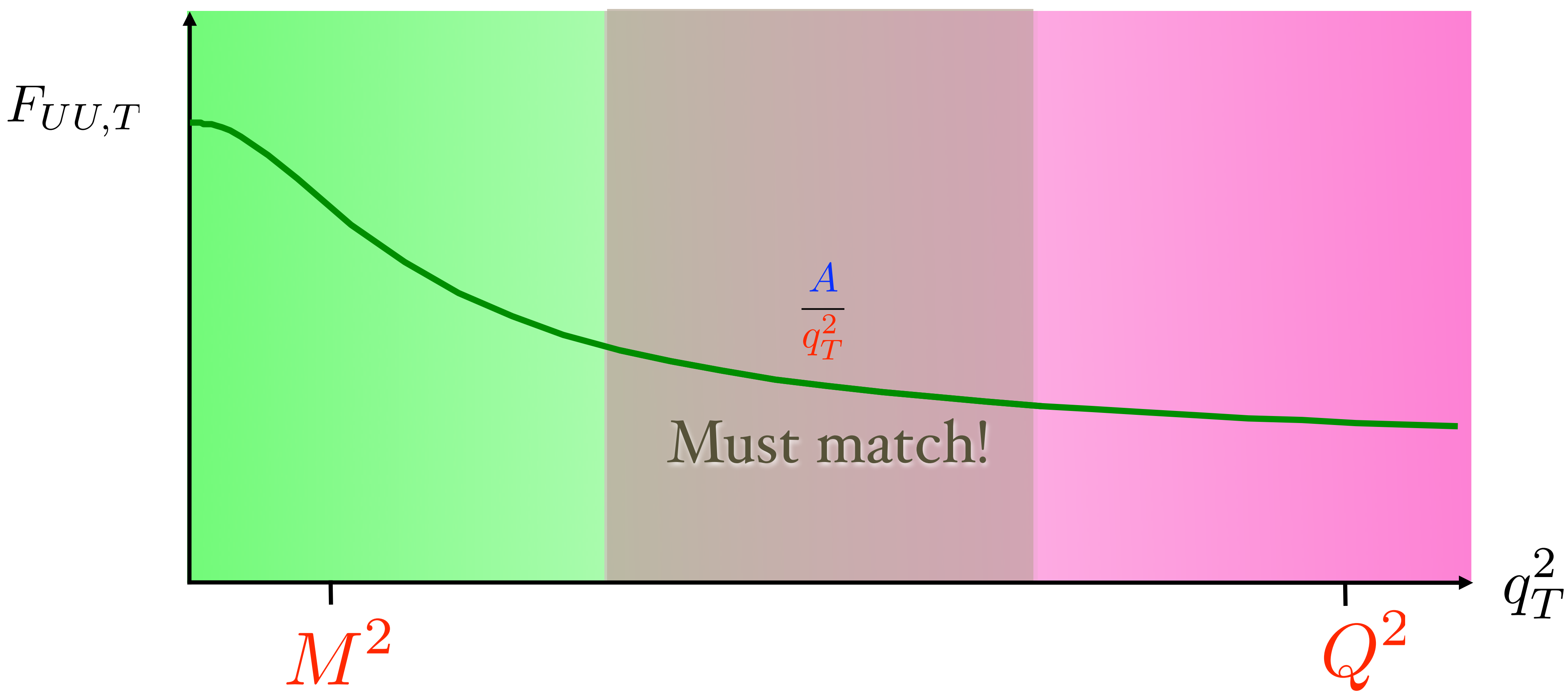
$$\times\Big[f_1^a\Big(\frac{x}{\hat{x}}\Big)D_1^a\Big(\frac{z}{\hat{z}}\Big)C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\Big(\frac{x}{\hat{x}}\Big)D_1^g\Big(\frac{z}{\hat{z}}\Big)C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\Big(\frac{x}{\hat{x}}\Big)D_1^a\Big(\frac{z}{\hat{z}}\Big)C_{UU,T}^{(\gamma^* g\to q\bar{q})}\Big]$$



Low and high transverse momentum



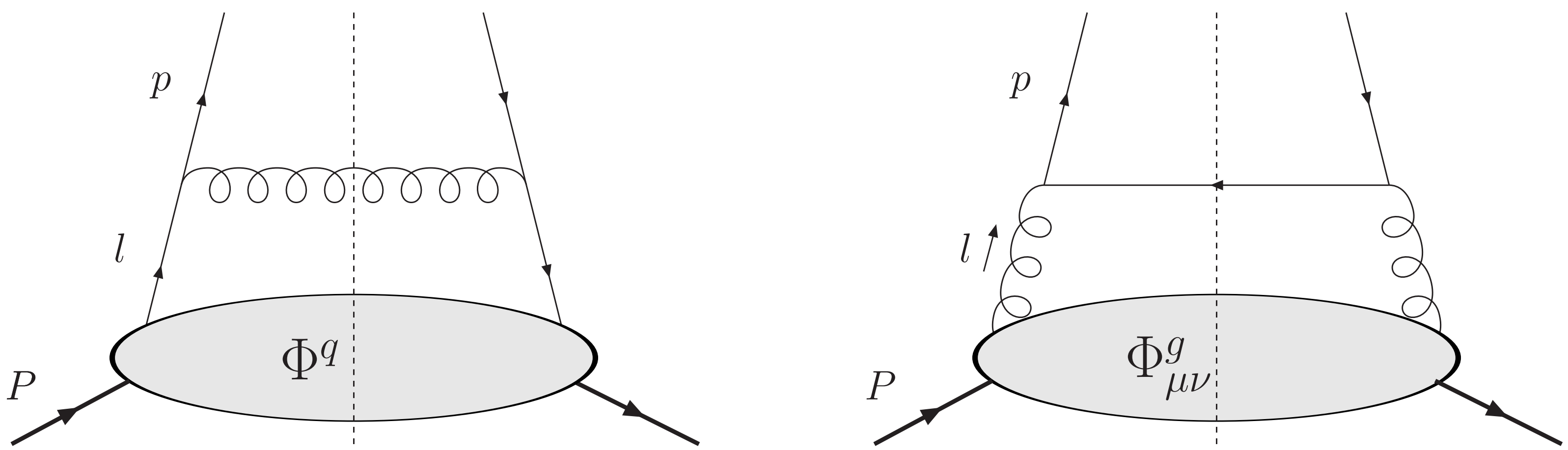
Matching



The leading high-q_T part is just the "tail" of the leading low-q_T part

Collins, Soper, Sterman, NPB250 (85)

Perturbative corrections to TMDs



(a) (b)

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2}\frac{1}{\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2} f_1^q(x) - C_F f_1^q(x) + \big(P_{qq}\otimes f_1^q + P_{qg}\otimes f_1^g\big)(x)\right],$$

$$F_{UU,T} = \frac{1}{q_T^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2 \Bigg[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z)$$

$$+ \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\, D_1^a(z)\Bigg]$$

Large log, needs resummation

DGLAP splitting functions

where $L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$

Other TMDs

$$x f^{\perp} \sim \frac{1}{\boldsymbol{p}_T^2}\, \alpha_s\, \mathcal{F}\big[f_1\big]\,,$$

$$\ldots$$

$$f_{1T}^{\perp} \sim \frac{M^2}{\boldsymbol{p}_T^4}\, \alpha_s\, \mathcal{F}\big[f_{1T}^{\perp(1)}, \ldots\big]\,,$$

$$\ldots$$

$$x f_L^{\perp} \sim \frac{1}{\boldsymbol{p}_T^2}\, \alpha_s^2\, \mathcal{F}\big[g_1\big]\,,$$

$$\ldots$$

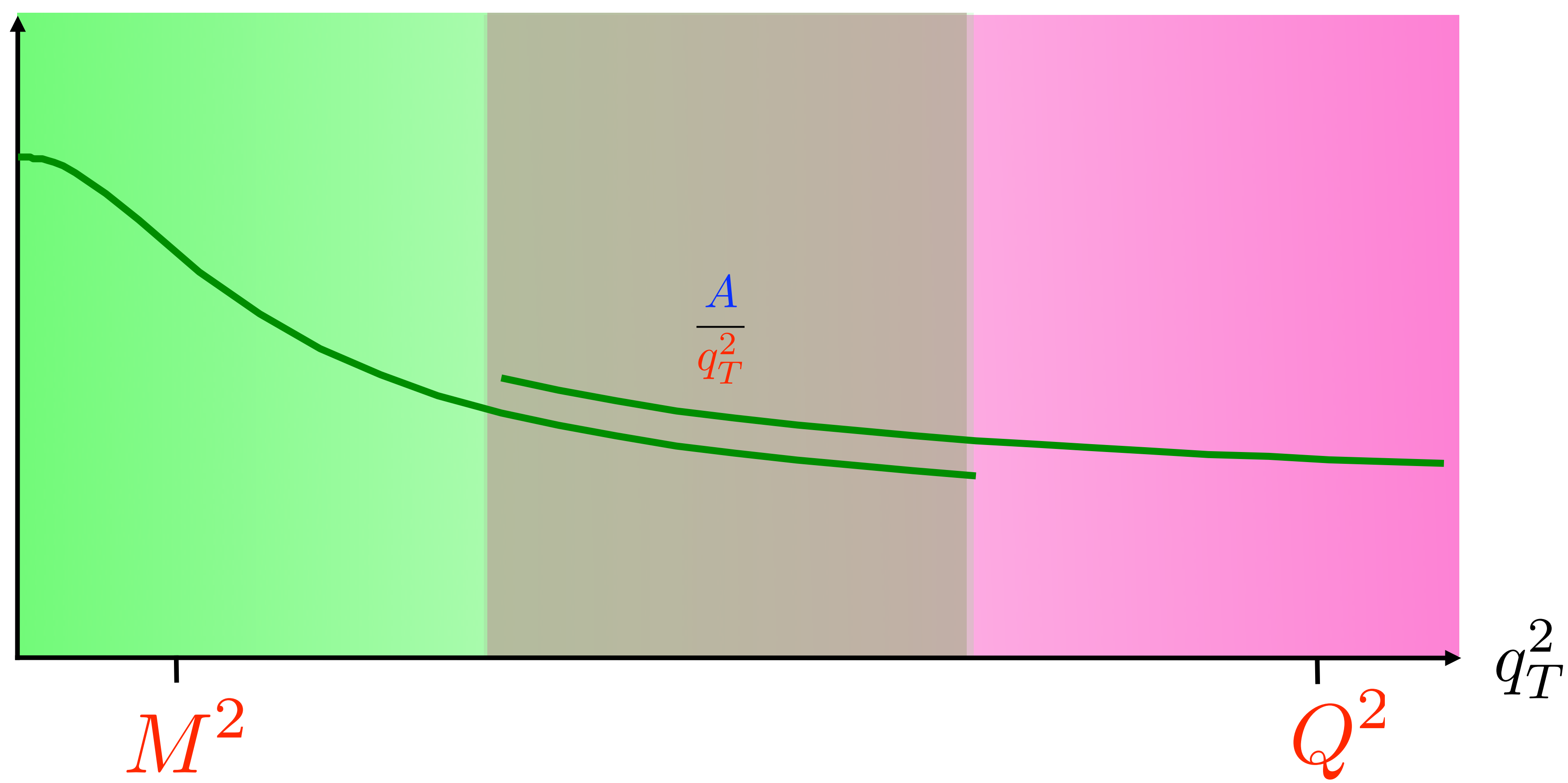
$$h_{1T}^{\perp} \sim \frac{M^2}{\boldsymbol{p}_T^4}\, \alpha_s^2\, \mathcal{F}\big[h_1\big]\,,$$

$$\ldots$$

AB, D. Boer, M. Diehl, P.J. Mulders, JHEP 08 (08)

Mismatches at twist 3

We are neglecting something that cannot be neglected...



Conclusions

- TMD factorization is proven, at least at the one-loop level
- There is a framework to study TMDs, including their evolution
- At the moment, this framework has been used since 1985, but only for unpolarized TMDs. Most recent work: Landry, Brock, Nadolsky, Yuan, PRD 67 (03)
- Everything else has been done at “tree-level” neglecting soft factor and evolution