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# AN RF SEPARATOR SYSTEM FOR BUNCH LENGTH MEASUREMENT

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## Introduction

The performance requirement for the energy spread of the extracted CEBAF beam is $\frac{\sigma_E}{E} \leq 10^{-4}$. To achieve such a stringent specification, the phase length of the microbunches produced in the injector region (E ≈5 MeV) must be quite short (≈1° @ 1.5 GHz). Hence, a direct measurement of the microbunch length at the ≈5 MeV point in the injector region would provide an extremely valuable diagnostic tool. There are also strong reasons for the desirability of microbunch length measurement at energies of from one to six GeV. A method for bunch length measurements at several GeV has been developed and will be more formally documented in the near future. However, the bunch length measurement system discussed in this Technical Note is, given CEBAF design parameters, appropriate only for energies of several MeV.

## Theory

An rf separator followed by a drift may be used to determine the length of the electron microbunches. The measurement is simply based upon the fact that the deflection received by the electrons is a function of their position along the microbunch[1–4]. This may be most easily seen by considering the case of an rf separator using a traveling wave (TW) structure. For a TW rf separator of length $\ell$, an electron traveling at the speed of light and with a phase $\phi_0$ with respect to the deflecting wave will have a transverse angle ($\theta_{RF}$) and transverse displacement ($X_{RF}$) at the output of the rf structure given by:

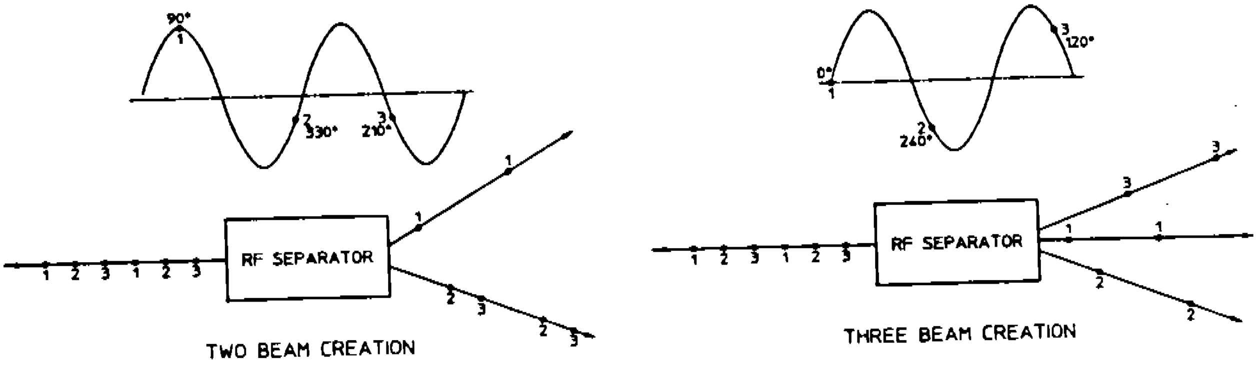
$$\theta_{RF} = \frac{p_\perp}{p_\parallel} \cdot \sin\ \phi_0 \tag{1}$$

$$X_{RF} = \frac{p_\perp}{2p_\parallel} \cdot \ell \cdot \sin\ \phi_0 \tag{2}$$

Where $p_\perp$ is the total transverse momentum gained by an electron riding on the crest ($\phi_0 = 90°$) of the deflecting wave in the rf separator, and $p_\parallel$ is the longitudinal momentum of the electron. After the electron has drifted to an observation point a distance $L$ downstream of the rf separator, the transverse displacement of the electron is given by:

$$X_{obs} = X_{RF} + L \cdot \theta_{RF} \tag{3}$$

The CEBAF rf separator system, which will be used to extract beam current modulo 499 MHz, will be a standing wave (SW) structure operating at a frequency of 998 MHz, 2/3 of the primary accelerating frequency of 1497 MHz. By appropriate phasing of the rf separator system the beam may be split into either two or three separate beams. See Figure 1. Differential loading of the charge per microbunch in the injector region at a frequency of 499 MHz will allow independent control of the current in each of the three individual beams. By using an rf separator system operating at a frequency of 998 MHz, the length of every third microbunch may be independently measured. Although the bunch length measurement method discussed in this note would benefit from the use of a higher frequency rf system, the beam extraction application benefits from a lower frequency, and therefore, the prudent use of available resources dictates the choice of 998 MHz for the frequency of the rf system to be used for microbunch length measurement.



TWO BEAM CREATION

THREE BEAM CREATION

Figure 1: Two and three beam creation for the CEBAF extraction system.

To develop the equations of motion for this case, the dipole fields in a SW structure may be assumed to be the sum of forward ($E_F$) and backward ($E_B$) waves. The total deflecting field is then given by:

$$E_T = E_F + E_B = -2 \cdot A \cdot \cos(\omega t) \cdot \sin(kz) \tag{4}$$

where:

$A = \dfrac{c \cdot p_\perp}{\ell}$

$\omega = 2 \cdot \pi \cdot$ frequency

$\lambda =$ wavelength

$t =$ time

$z =$ longitudinal position along rf separator structure

$\omega t = 2\pi \dfrac{z}{\lambda} = kz$

Because the electrons at several MeV are not fully relativistic, the electrons will slip in phase as they pass through the structure by an amount $\Delta\phi$:

$$\Delta\phi = kz\left(\frac{1}{\beta_e} - \frac{1}{\beta_w}\right) = kz\left(\frac{1}{\beta_e} - 1\right) \tag{5}$$

where:

$c =$ velocity of light
$\beta_e = v_e/c$, relative electron velocity
$\beta_w = 1$, relative wave phase velocity

Therefore, $E_T$ may be written as:

$$E_T = -\frac{2 \cdot c \cdot p_\perp}{\ell} \cdot \cos\left(\frac{kz}{\beta_e} + \phi_0\right) \cdot \sin(kz) \tag{6}$$

where: $\phi_0 =$ reference phase of the electron with respect to the deflecting wave.

Noting that:

$$d\theta = \frac{E_T}{c \cdot p_{||}} dz$$

$$\theta = \int_0^{z'} d\theta = \int_0^{z'} -\frac{2p_\perp}{\ell\, p_{||}} \cos\left(kz''/\beta_e + \phi_0\right) \cdot \sin\, kz''\, dz'' \tag{7}$$

$$X = \int_0^{z} \theta\, dz' \tag{8}$$

Evaluation of equations 7 and 8 for the case where $\beta_e = 1$ leads to the the result that the beam, at the output of the rf separator, will have a transverse position $(X_{RF})$ and angle$(\theta_{RF})$ with respect to the undeflected orbit of:

$$\theta_{RF}(\beta_e = 1) = \frac{p_\perp}{p_{||}} \sin\phi_0 \tag{9}$$

$$X_{RF}(\beta_e = 1) = \frac{p_\perp}{2p_{||}}\{\ell \cdot \sin\phi_0 - \frac{1}{k}\cos\phi_0\} \tag{10}$$

The value of $\theta_{RF}$ is equivalent for both the TW and SW case. However, the value of $X_{RF}$ for the SW case has the additional term $(p_\perp \cdot \cos\phi_0)/(2p_{||} \cdot k)$. The quantitative effect of this additional term on the bunch length measurement may be determined by evaluating $dX_{obs}/d\phi_0$ for the SW and TW case. The difference between $(dX_{obs}/d\phi_0)_{SW}$ and $(dX_{obs}/d\phi_0)_{TW}$, assuming the values of $\ell = 1.2$ m, L = 7 m, $p_\perp/p_\parallel = 7.24 \times 10^{-2}$, and $\phi_0 = 0$ to 15°, ranges from zero to $< 10^{-3}$, and therefore, is a negligible effect on the performance of the bunch length measurement.

For the more general case of $\beta_e \neq 1$, the beam will, at the output of the rf separator, have a transverse position ($X_{RF}$) and angle ($\theta_{RF}$) with respect to the undeflected orbit of:

$$\theta_{RF} = \frac{p_\perp}{\ell \cdot p_\parallel \cdot k}\left\{\frac{\cos\left(k \cdot \ell \cdot [1 - 1/\beta_e] - \phi_0\right) - \cos(\phi_0)}{(1 - 1/\beta_e)} + \frac{\cos\left(k \cdot \ell \cdot [1 + 1/\beta_e] + \phi_0\right) - \cos(\phi_0)}{(1 + 1/\beta_e)}\right\} \tag{11}$$

$$X_{RF} = \frac{p_\perp}{\ell \cdot p_\parallel \cdot k^2}\left\{\frac{\sin\left(k \cdot \ell \cdot [1 - 1/\beta_e] - \phi_0\right) + \sin(\phi_0) - \left(1 - 1/\beta_e\right) \cdot k \cdot \ell \cdot \cos(\phi_0)}{\left(1 - 1/\beta_e\right)^2} + \frac{\sin\left(k \cdot \ell \cdot [1 + 1/\beta_e] + \phi_0\right) - \sin(\phi_0) - \left(1 + 1/\beta_e\right) \cdot k \cdot \ell \cdot \cos(\phi_0)}{\left(1 + 1/\beta_e\right)^2}\right\} \tag{12}$$

When the rf separator is off, the beam at $X_{obs}$ will have a finite size ($\pm 2\sigma_x$) given by:

$$2\sigma_x = \sqrt{\epsilon \cdot \beta}$$

where:

$\epsilon =$ beam transverse emittance
$\beta =$ machine beta function

When the rf separator is on, the beam size at $X_{obs}$ will be due to the beam emittance ($\pm 2\sigma_x$), and the phase dependent deflection of the rf separator due to both the energy spread of the beam $\left(\frac{\Delta E}{E}\right)$ and the microbunch length. The last effect is used to determine the bunch length. The other effects plus those of the rf amplitude and phase variation determine the resolution of the measurement.

## Simulation

Simulations have been done using equations (11) and (12) to determine the efficacy of such a system. The following design parameters which are compatible with CEBAF design values have been assumed:

$$\text{Beam Energy } (E_0) = 5 \text{ MeV}$$

$$\text{Total Energy Spread } \left(\frac{\Delta E}{E_0}\right) = 5 \times 10^{-3}$$

$$\text{Structure length } (\ell) = 1.2 \text{ m}$$

$$\text{rf frequency } (f) = 0.998 \text{ GHz}$$

$$\frac{p_\perp}{\ell} = 3 \times 10^{-4} \, \frac{\text{GeV/c}}{\text{m}}$$

$$\text{Distance to observation point } (L) = 7 \text{ m}$$

$$\text{emittance } (\epsilon) = 4 \times 10^{-7} \text{ m}\cdot\text{rad}$$

$$\text{Machine beta function at observation point } (\beta) = 2 \text{ m}$$

There will be a contribution to the beam size for a zero length microbunch due to the finite energy spread of the beam. This effect is reduced by the appropriate choice of $\phi_0$ as demonstrated in Figures 2 through 4. *Note that Figures 3 and 4 were calculated assuming a zero emittance to more clearly demonstrate the reduction in beam size due to the energy spread.* While this effect is of intellectual interest, it would, if not accounted for, result in only ≈ 5% over estimate of the microbunch length.

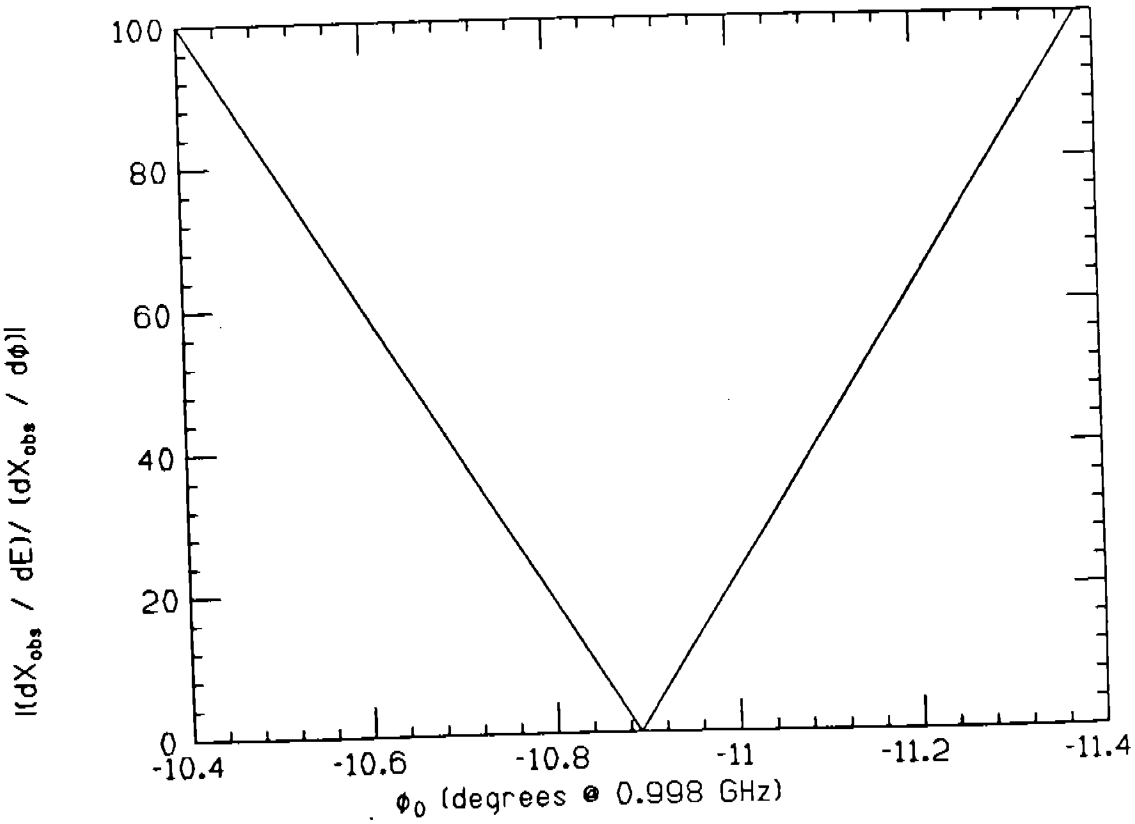


**Figure 2:** Minimization of the energy spread contribution to beam size at $X_{obs}$ by variance of $\phi_o$.

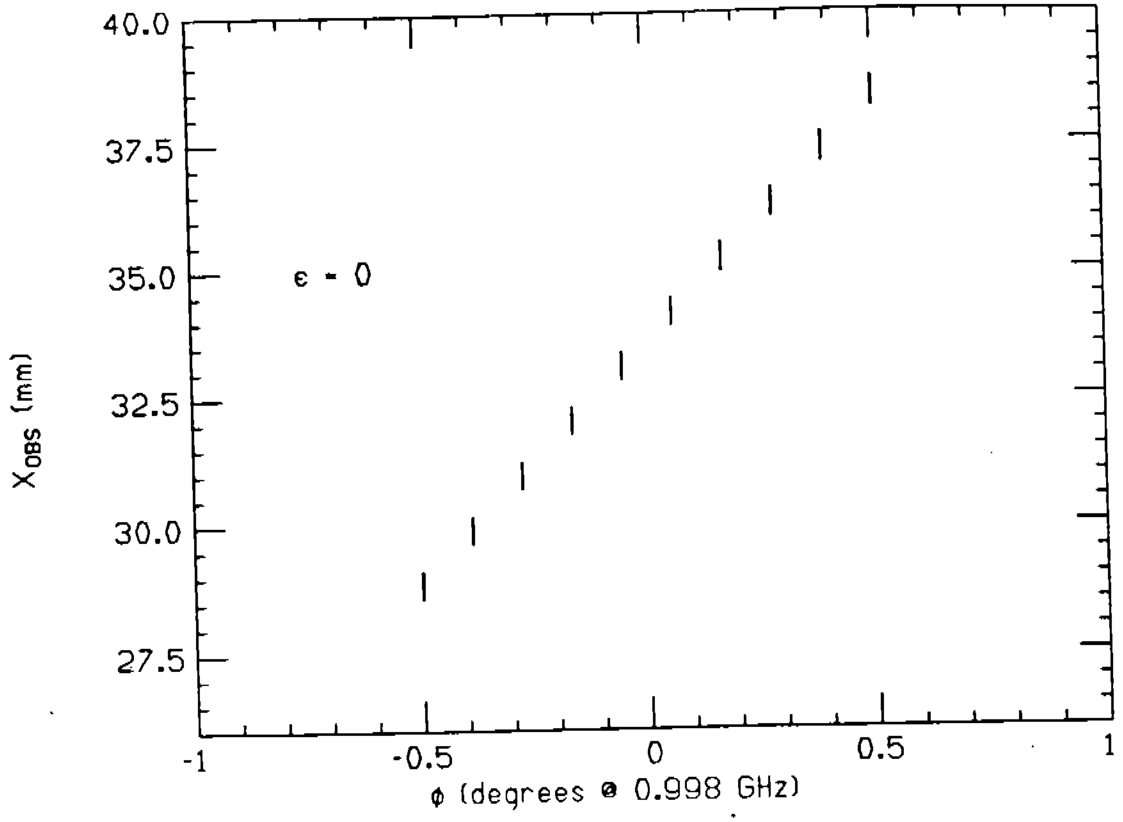


Figure 3: Beam distribution as $X_{obs}$ for $\phi_o = 0°$ @ 0.998 GHz.

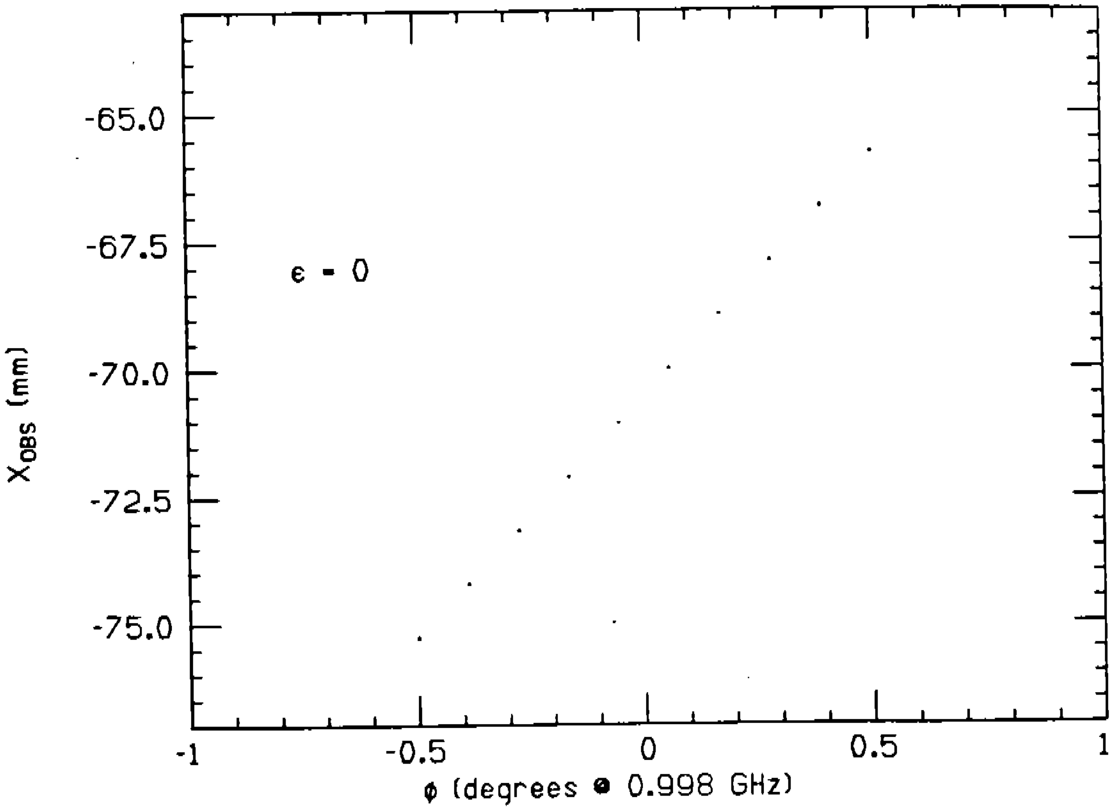


Figure 4: Beam distribution as $X_{obs}$ for $\phi_o = -10.8913°$ @ 0.998 GHz.

Figure 5 displays the distribution at the observation point ($X_{obs}$) given the parameter values specified above and including the effect of emittance. Clearly, given the paramters specified, the major limitation to the measurement resolution is the emittance. However, this effect may be measured and subtracted simply by turning off the rf separator. In addition, by appropriate collimation, the beam emittance may easily be reduced during bunch length measurements by approximately a factor of three ($\epsilon \simeq 1.3 \times 10^{-7}$ m · rad).

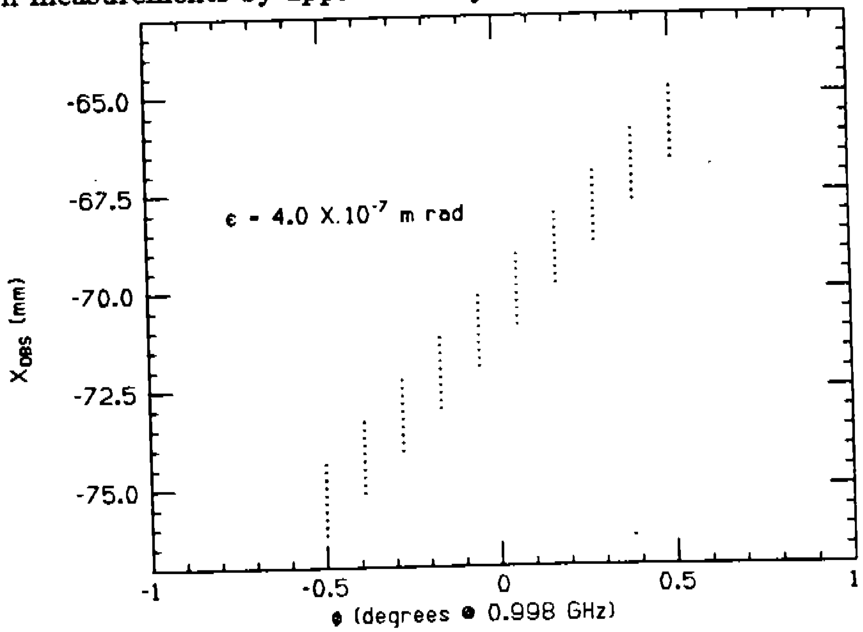


**Figure 5:** Beam distribution at $X_{obs}$ for $\phi_o$ = -10.8913° @ 0.998 GHz and finite emittance.

Figure 6 shows the effect of the reduced emittance. The contribution to the beam size at $X_{obs}$, given the paramters specified above, are ≈9% and ≈5% for emittance values of $4 \times 10^{-7}$ m · rad and $1.3 \times 10^{-7}$ m · rad respectively.

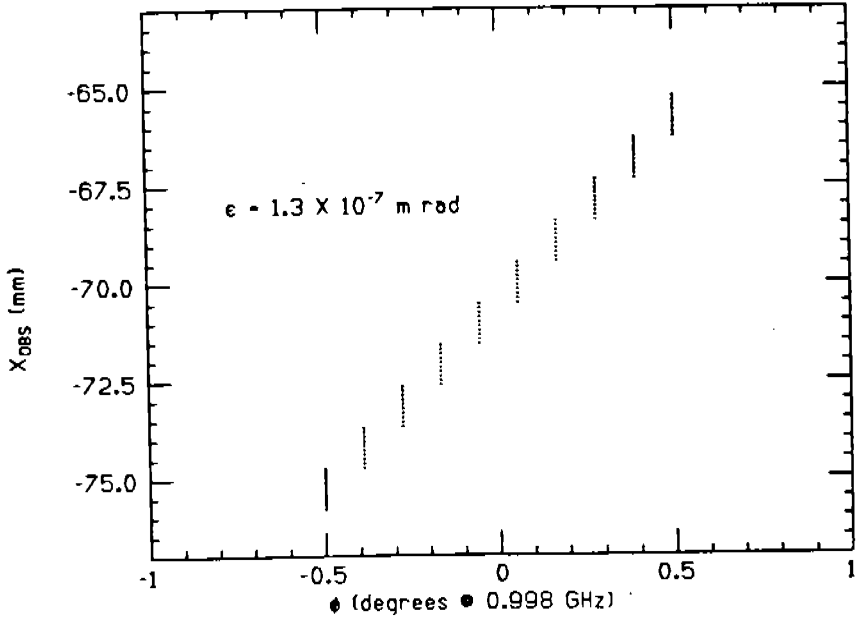
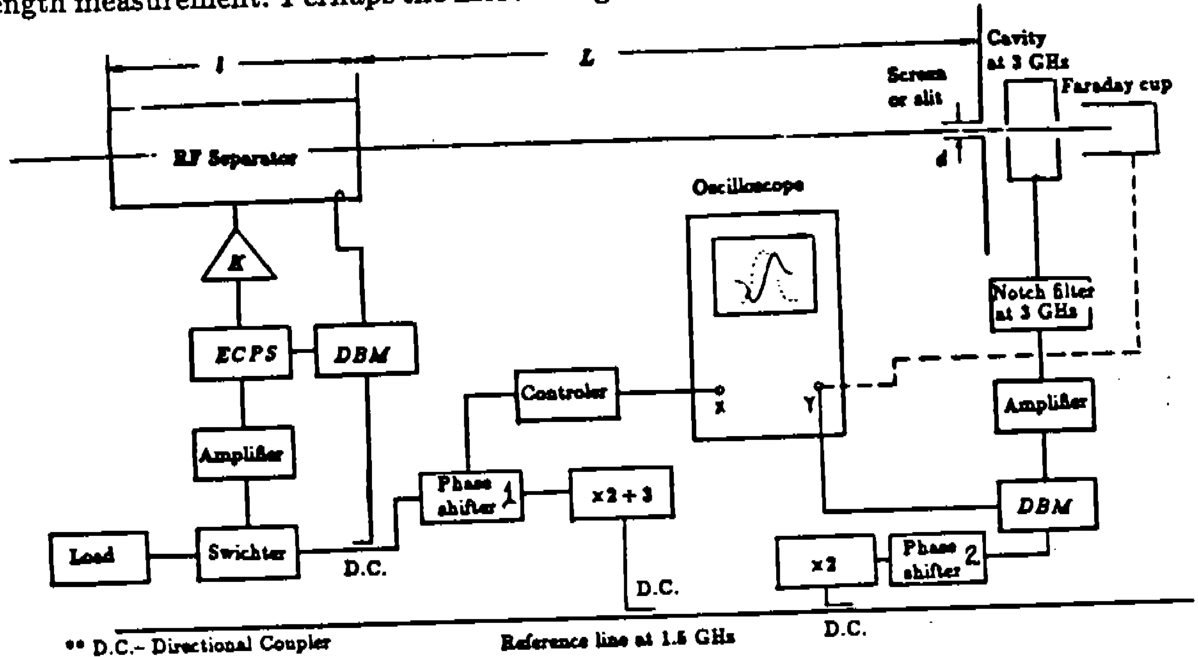


**Figure 6:** Beam distribution at $X_{obs}$ for $\phi_o$ = -10.8913° @ 0.998 GHz and finite emittance.

## Experimental arrangement

Figure 7 is a block diagram of several possible experimental configurations for a bunch length measurement. Perhaps the most straight forward method is to use a screen or other



** D.C.- Directional Coupler

** DBM- Double Balance Mixer

** ECPS- Electronic Controled Phase Shifter

Figure 7: Block diagram of bunch length measurement using an rf separator.

beam profile monitoring device at $X_{obs}$ a distance L beyond the end of the rf separator. The beam size would be measured with the rf separator off to determine the effect of the transverse emittance. The transverse optics and any emittance reducing collimation would be adjusted at this time to produce the minimum beam size. The value of $p_\perp/p_\parallel$ would be determined experimentaly by adjusting the phase of the rf separator to achieve the maximum deflection ($\phi_0 = 90°$). Finally, the rf separator phase would be set at the value appropriate to mimimize the contribution to the beam size due to the finite energy spread, and then the beam size would be measured. Note that, as discussed in the section on simulation, the setting of the rf separator phase so as to minimize the contribution due to the energy spread is not critical ($\approx$5% over estimate in bunch length). Comparisons between equations 1 and 2 and 11 and 12 show that, for the parameter values specified in the section on simulation, the data can be evaluated using equations 1 and 2 without introducing significant (<1%) error. Assuming a beam profile measurement resolution of 100$\mu$m and the parameters specified in the proceeding section, a bunch length resolution of $< 0.2°$ @ 1.5 GHz appears to be achievable. This resolution will be more than adequate for the measurement of the bunch length at CEBAF at the $\approx$5 MeV point.

Beyond the direct measurement of the beam size, there are several alternative methods to experimentally determine the bunch length. See Figure 7. A possible experimental procedure would utilize, in lieu of a beam profile monitor, a slit of adjustable width $d$ followed by a current monitoring device such as a Faraday cup. As before, the beam size with the rf separator off would be determined and the slit width would be set equal to this size. Then with the rf separator on, the phase of the rf separator with respect to the microbunch train would be varied over a small range (several degrees) by the application of a sweep voltage to phase shifter 1. At the same time the current passing through the slit would be measured providing not only the information necessary for the bunch length measurement, but also the charge distribution along the microbunch. As shown schematically in Figure 7, the current with respect to the phase of the rf separator would be displayed directly on the screen of an oscilloscope. The bunch length would then be determined based upon the calibration of phase shifter 1. The resolution of this method would be approximately the same as that given for the previous method.

Two additional methods are possible. These have not been as fully developed, and therefore, are discussed at the conceptual level. First, a 3 GHz cavity, instead of a Faraday cup (see Figure 7), could be used to measure the sixth harmonic of the 500 MHz bunch train. When segments of the microbunch from different longitudinal positions, as defined by the slit and rf separator parameters, pass through the 3 GHz cavity a phase shift three times as large as the phase interval between the slices of the bunch at 1 GHz will be detected at the output of the 3 GHz cavity. The major advantage of this method is a higher sensitivity in the phase measurement.

Assuming the charge in the bunch has a Gaussian distribution $I(\phi)$, the amplitude response of a double balanced mixer may be modeled as:

$$V_{out} = \alpha I(\phi_b) \cdot \{\cos(\phi_b - \phi_r) + V_{offset}\} \tag{13}$$

where

$\alpha =$ a linear response coefficient
$\phi_b =$ the phase of the beam signal
$\phi_r =$ the phase of the CW reference
$V_{offset} =$ a dc offset term

Given these assumptions, the waveform which would be displayed on the oscilloscope is given in Figure 8. The bunch length ($6\sigma$) would be determined from this measurement.

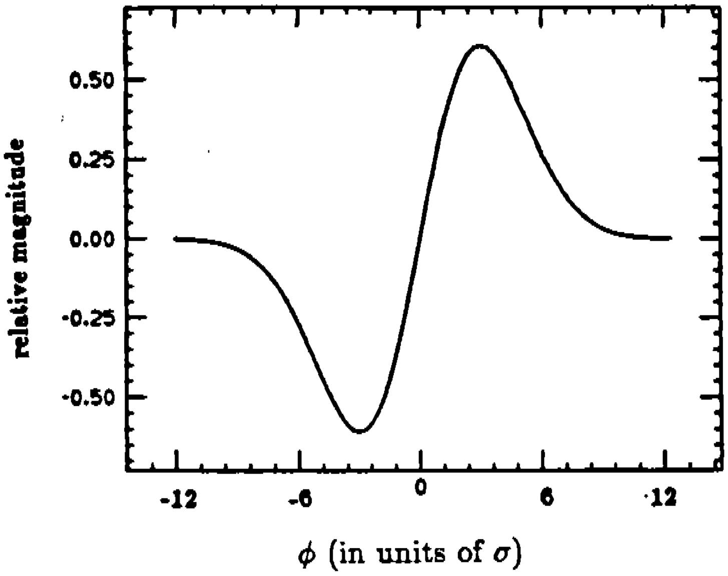


Figure 8: $V_{out}$ versus $\phi$ in units of $\sigma$.

The second conceptual experimental arrangement is shown in Figure 9. A tunable generator, phase-locked to the reference signal of 1497 MHz (or a low frequecy generator and mixer),

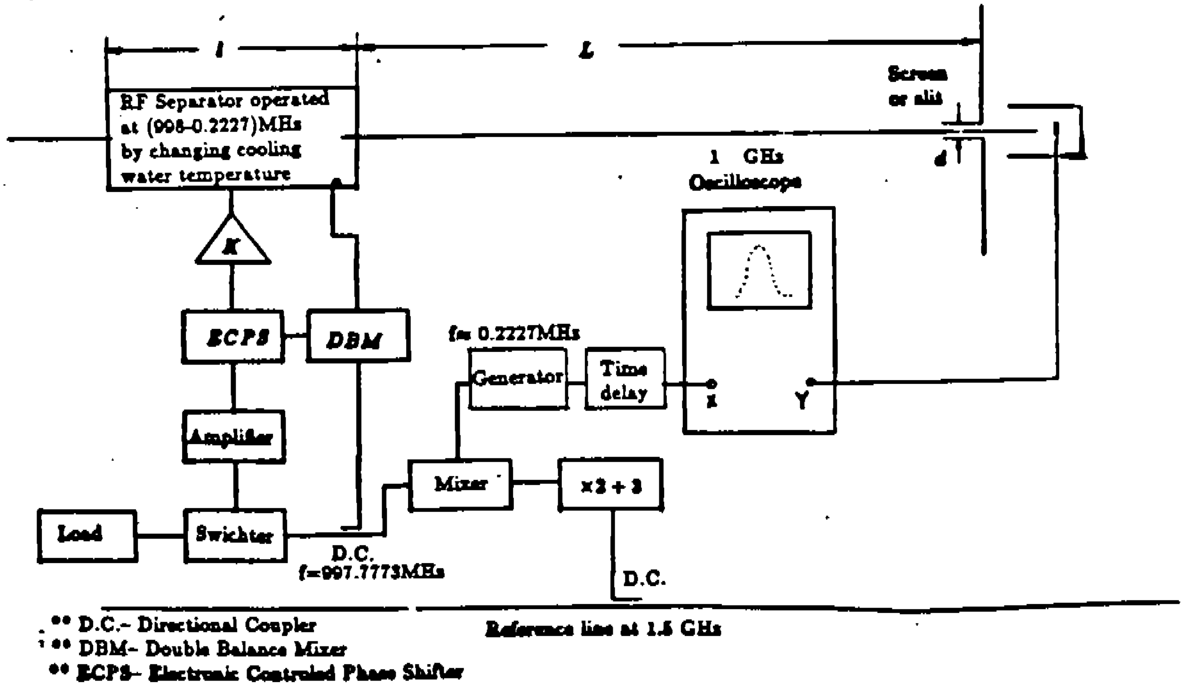


Figure 9: Block diagram of bunch length measurement using an rf separator.

with a frequency of 998 MHz + $df$ would be used as the input source to the rf separator klystron. The rf separator would be tuned to 998 MHz $+df$ by changing the temperature of the cooling water. The low frequency signal would also be used to trigger the oscilloscope through a delay line and a Faraday cup would be used to measure the beam current signal. In this case the bunch phase with respect to the deflecting wave will shift after each rf cycle by an amount, $d\phi$, that is related to the frequency difference $df$ by:

$$d\phi = 360^0 \cdot \frac{df}{f} \tag{14}$$

Where $f$ = 998 MHz. The charge-phase distribution in the bunchs displayed on the oscilloscope would then be composed of a series of points with phase interval of $2d\phi$ (not $d\phi$ because the frequency of bunchs detected is 499 MHz). A significant feature of this measurement method is that it is an absolute measurement of bunch length, and therefore, phase calibration is not needed.

## Conclusion

A SW rf separator may be used to measure the microbunch length of the CEBAF electron beam at energies of ≈5 MeV with a resolution of $< 0.2$ @ 1.5 GHz. It would, therefore, provide an invaluable diagnostic tool for the CEBAF injector commissioning and operating processes.

In addition, the implementation of the rf separator system as a diagnostic for the Front End Test will provide an early and important opportunity to thoroughly test the rf separator design concepts.

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