# Effect of Betatron Mismatch on Projected Emittance Growth

- **If there is no coupling, there is no projected emittance growth.**
- **Local beam size or orbit blowup due to betatron mismatch, however, does play a role in exacerbating projected emittance growth in the presence of coupling**
- **In some sense this is only a necessary consequence of projected emittance growth over a distributed coupling source.**

**Projected Emittance Growth across a Skew Quad with Inverse Focal Length *k* (Assuming Initial Beam is Uncoupled)**

$$\varepsilon_{X\ Final}^{2} - \varepsilon_{X\ Initial}^{2} = \langle XX \rangle \langle YY \rangle k^{2} = \beta_X \beta_Y \varepsilon_X \varepsilon_Y k^{2}$$

**So, if beam or orbit has a large amplitude going through a coupling source, the growth in emittance (not amplitude) will be larger.**

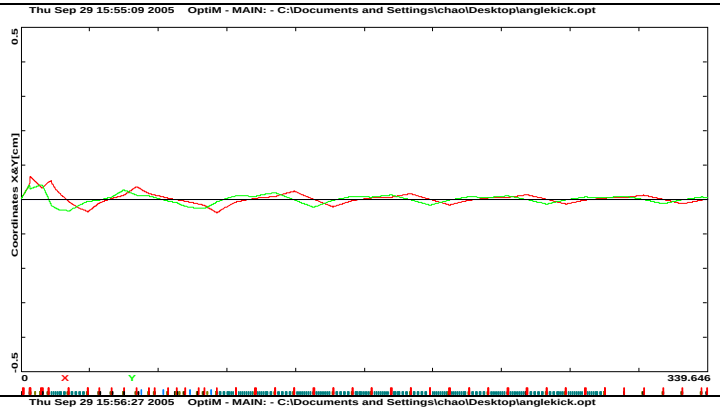
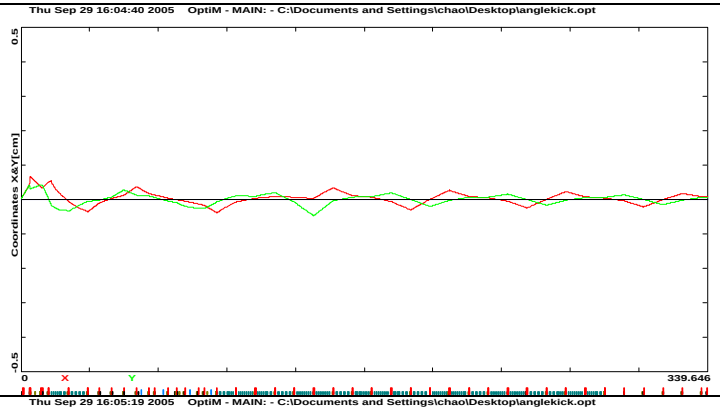
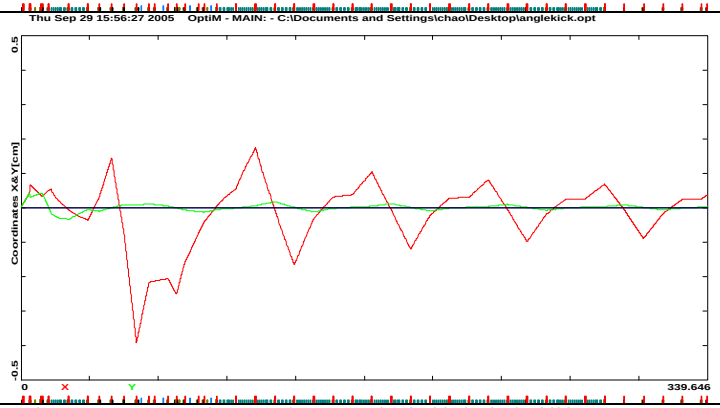
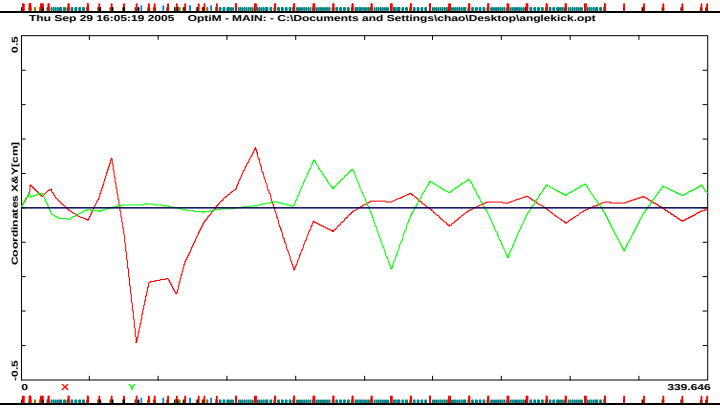
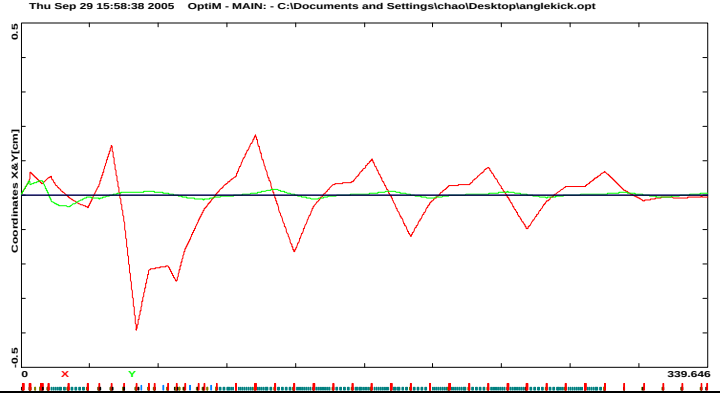
**If we don't carefully control the betatron matching locally, after a region of distributed coupling, the projected emittance growth will be worse than if the local betatron matching is under control.**

**If we had 4 skew quads at the end then this may not be a problem (theoretically). But we don't. So the emittance growth caused by betatron mismatch is unfixable at the end and looks just like real emittance growth.**

**→ Local betatron matching must be ensured to prevent projected emittance growth due to coupling buildup.**

**→ This is also true with PZT orbit propagation**

## Using North Linac Optim Deck as an Example.

| | No Intervening Skew Field | Slight Skew Field at 1L04-06 |
|---|---|---|
| **Matched Input Trajectory** |  |  |
| **Mismatched Input Trajectory** |  |  |
| **Re-Match at the End with Q1L24-26** |  | **???** **Cannot easily rematch since projected emittance is larger in absolute terms.** |