The first electron beam polarization measurement with a diamond micro-strip detector

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A diamond multi-strip detector was used for the first time, to track Compton scattered electrons in a new electron beam polarimeter in the experimental Hall C at Jefferson Lab. We report the first high precision beam polarization measurement with electrons detected in diamond multi-strip detectors. The analysis technique leveraged the high resolution of the detectors and their proximity to the electron beam $(\gtrsim 0.5 \text{ cm})$. The polarization was measured with a statistical precision of < 1%/hr, and a systematic uncertainty of 0.59%, for a 1.16 GeV electron beam with currents up to 180 μ A. This constitutes the highest precision achieved for polarization measurement of few-GeV electron beams.

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INTRODUCTION

High precision nuclear physics experiments using po-2 larized electron beams rely on accurate knowledge of 27 3 beam polarization to achieve their ever improving pre-28 4 cision. A parity violating electron scattering (PVES) ex- 29 5 periment in the experimental Hall C at Jefferson Lab $^{\rm 30}$ 6 (JLab), known as the $Q_{\rm weak}$ experiment, is the most re- $^{\rm 31}$ cent example [1, 2]. The goal of the Q_{weak} experiment ³² is to measure the Standard Model parameter known as ³³ 9 the weak mixing angle, at a low energy (relative to the ³⁴ 10 Z^0 mass) with unprecedented precision. With a goal of ${}^{_{35}}$ 11 < 1% uncertainty, determination of electron beam polar- ³⁶ 12 ization is one of the greatest technical challenges of the ³⁷ 13 Q_{weak} experiment. The experiment utilized an existing ³⁸ 14 Møller polarimeter [2, 3] and a new Compton polarime- 39 15 ter [2, 4] to monitor the electron beam polarization. The 16 Compton polarimeter was the only polarimeter at JLab 41 17 Hall C that could non-destructively monitor the beam $_{\scriptscriptstyle 42}$ 18 polarization at very high beam currents. A novel aspect $_{\scriptscriptstyle 43}$ 19 of this polarimeter was the first use of diamond detector $_{\scriptscriptstyle 44}$ 20 technology for this purpose. 21 45

The use of *natural* diamond in the detection of charged 46 22 particles and radiation has a long history; but the use of 47 23

synthetic diamond grown through a process known as "chemical vapor deposition" (CVD) is a relatively recent development. Detailed reviews of diamond as charged particle detectors can be found in [12–14]. Thin sheets of centimeter-sized diamond are grown using the CVD process and the plates of diamond are then turned into charged particle detectors by depositing suitable electrodes on them [15].

Compared to the more commonly used silicon detector, the signal size in a diamond detector is smaller, but the higher electron and hole mobility of diamond leads to a faster and shorter duration signal. However, the well-established radiation hardness of diamond [16, 17] is by far the most important consideration for the use of diamond detectors in nuclear and particle physics experiments.

The use of Compton scattered electrons and/or backscattered photons to measure the Compton asymmetry and thereby the electron beam polarization, is a well established polarimetry technique [5-10]. Most previous Compton polarimeters, other than the one used in the SLD experiment [7], relied primarily on detection of the scattered photons to measure the beam polarization. The SLD Compton polarimeter, which detected scat-

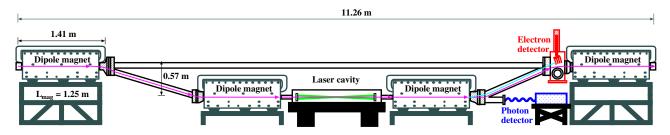


FIG. 1: Schematic diagram of the JLab Hall C Compton polarimeter.

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tered electrons (and used detection of photons as a cross- 90 48 check), was operated at a beam energy of 50 GeV and 91 49 reported a precision of 0.5%. The relatively low energy 92 50 of the electron beam at JLab leads to a smaller Compton 93 51 analyzing power, and makes it significantly more chal-94 52 lenging to achieve the same level of precision. Nonethe-95 53 less, the Compton polarimeter in Hall A at Jefferson Lab 96 54 has reported a relative precision of $\sim 1\%$ by detecting $_{97}$ 55 the Compton scattered electrons at a beam energy of $_{98}$ 56 3 GeV [11].57

The JLab Hall C Compton polarimeter detects the₁₀₀ 58 scattered electrons in a set of tracking detectors. The low₁₀₁ 59 energy of the electron beam (1.16 GeV) and other oper-102 60 ating parameters of the Q_{weak} experiment, presented the₁₀₃ 61 most challenging set of conditions to achieve the goal of_{104} 62 < 1% uncertainty in measurement of the beam polariza-105 63 tion. For example, it constrained the tracking detector to₁₀₆ 64 be placed as close as 0.5 cm from the electron beam. Fur-107 65 ther, the polarimeter was operated at the highest beam₁₀₈ 66 current (180 μ A) ever used by any experiment at JLab₁₀₉ 67 and ran for over 5000 hrs, thereby subjecting the elec-110 68 tron detectors to a rather large cumulative radiation dose₁₁₁ 69 (> 100 kGy, just from electrons). In order to withstand₁₁₂ 70 the large radiation dose, a novel set of diamond micro-113 71 strip detectors were used to track the scattered electrons.₁₁₄ 72 In this letter we report the first high precision measure-115 73 ment of electron beam polarization with this device. 74 116

75 THE HALL C COMPTON POLARIMETER

A schematic of the Compton polarimeter in Hall C121 76 at JLab is shown in Fig. 1. It consists of four identi-122 77 cal dipole magnets forming a magnetic chicane that dis-123 78 places a 1.16 GeV electron beam vertically downward by₁₂₄ 79 57 cm ($\sim 10.13^{\circ}$). A high intensity ($\sim 1 - 2 \text{ kW}$) beam₁₂₅ 80 of $\sim 100\%$ circularly polarized photons is provided by an₁₂₆ 81 external low-gain Fabry-Pérot laser cavity which consists¹²⁷ 82 of an 85 cm long optical cavity with a gain between 100₁₂₈ 83 and 200, coupled to a green (532 nm), continuous wave, 129 84 10 W laser (Coherent VERDI). The laser light is focused₁₃₀ 85 at the interaction region ($\sigma_{\text{waist}} \sim 180 \ \mu\text{m}$), and it is₁₃₁ 86 larger than the electron beam envelope ($\sigma_{\rm x/v} \sim 40 \ \mu m_{^{132}}$ 87 when optimally tuned). The degree of circular polariza-133 88 tion was determined by two methods; first by monitoring₁₃₄ 89

the polarization state of the transmitted laser light and using a transfer function to translate it to the Compton interaction point, and second, a more precise method of measuring the leakage of the back-reflected power from the laser cavity.

The laser was operated in ~ 90 second cycles, where it is active for ~ 60 s (laser on period) and blocked off (laser off period) for the rest of the cycle. The laser off data were used to measure the background. The helicity of the laser beam was reversed very infrequently (6 times during the entire experiment).

The maximum scattered photon energy was approximately 46 MeV. A calorimeter consisting of a 2×2 matrix of 3 cm \times 3 cm PbWO₄ scintillating crystals attached to a single photo-multiplier tube was used to measure the scattered photon energy. The signal from the photon detector was digitally integrated with zero threshold over a full helicity state ($\sim 1 \text{ ms}$) using a 200 MHz flash analog to digital converter.

The Compton scattered electrons were momentum analyzed by the third dipole magnet of the chicane. The maximum separation between the primary electron beam and the Compton scattered electrons, just in front of the fourth dipole, was ~ 17 mm. The deflection of the scattered electron with respect to the primary electron beam, from the maximum down to distances as small as \sim 5 mm, was tracked by a set of four diamond microstrip detectors. This range allowed the detection of a large fraction of the Compton electron spectrum, from beyond the kinematic maximum (strip 55 in Fig. 3) down past the zero-crossing point (~ 8.5 mm from the primary beam) of the Compton asymmetry. The electron detectors are made from 21 mm \times 21 mm \times 0.5 mm plates of CVD diamond [18]. Each diamond plate has 96 horizontal metalized electrode strips with a pitch of 200 μ m (180 μ m of metal and 20 μ m of gap) on one side (front) and a single metalized electrode covering the entire diamond surface on the opposite (back) side. Details can be found in Ref. [2]. A photograph of a single detector plane is shown in Fig 2.

A statistical precision of < 1% per hour was routinely achieved with these detectors. The signal to background ratio was found range from 5–20 [2]. The observed Compton scattered electron rate, aggregated over all strips in each detector plane was $\sim 150-180$ kHz. By comparing

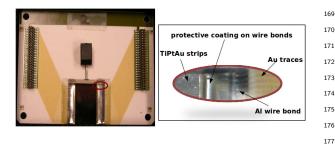


FIG. 2: A CVD diamond plate mounted on an alumina sub-¹⁷⁸ strate which forms a single detector plane (left). The red oval¹⁷⁹ indicates the area that has been shown in the enlarged view (right).

the expected to the observed rates, the detector efficiency was estimated to be $\sim 70\%$. The large separation between the detector and the readout electronics was the leading cause of the inefficiency.

The data acquisition (DAQ) system employed a set of 139 field programmable gate array (FPGA) based logic mod-140 ules [19] to find clusters of detector hits, and to imple-141 ment a track-finding algorithm, which generated a trigger 142 when the same cluster was identified in multiple active 143 planes. The size of the cluster was defined as 4 adjacent 144 strips. Only 3 detector planes were operational during 145 the experiment and the typical trigger condition was set 146 to 2 out of 3 planes. 147

Over the 2 year period of the Q_{weak} experiment, the
detectors were exposed to a radiation dose of ~ 100 kGy
(without including the dose from Synchrotron radiation).
No significant degradation of the signal size was observed
during this period, demonstrating the radiation hardness
of the diamond detectors.

154 DATA REDUCTION AND RESULTS

The electron beam helicity was reversed at a rate of 155 960 Hz in a pseudo-random sequence. In addition a180 156 half-wave plate in the polarized electron photo emission¹⁸¹ 157 source [20] was inserted or removed about every 8 hours₁₈₂ 158 to reverse the beam helicity relative to the polarization of 159 the source laser. The background yield measured during 160 the laser-off period was subtracted from the laser-on yield 161 for each electron helicity state, and a charge normalized 162 Compton yield for each detector strip was obtained for 185_{185}^{104} 163 the two electron helicities. The measured asymmetry was 164 built from these yields using, 165 187

$$A_{exp} = \frac{Y^+ - Y^-}{Y^+ + Y^-}, \qquad (1)_{\rm 189}^{\rm 188}$$

where $Y^{\pm} = \frac{N_{on}^{\pm}}{Q_{on}^{\pm}} - \frac{N_{off}^{\pm}}{Q_{off}^{\pm}}$ is the charge normalized Comp-¹⁹¹₁₉₂ ton yield for each detector strip, $N_{on/off}^{\pm}$ and $Q_{on/off}^{\pm}$ are₁₉₃ the detector counts and the beam charge accumulated₁₉₄

during the laser on/off period for the two electron helicity states (\pm) , respectively. The Compton yields were integrated over two different time intervals, ~250 thousand helicity cycles and 1 laser cycle. The asymmetries extracted over both time intervals, and averaged over an hour long run, were consistent with one another. A typical spectrum for an hour long run is shown in Fig. 3. The background asymmetry is consistent with zero within the statistical uncertainties, and given the large signal-tobackground ratio of the dilution to the measured asymmetry due to the background is negligible.

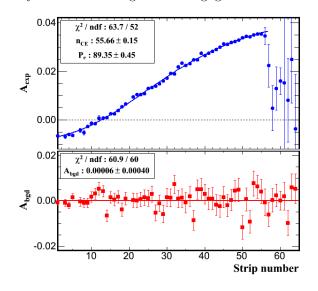


FIG. 3: The measured asymmetry as function of detector strip number for a single detector plane during the laser-on period (top) and the background asymmetry from the laser-off period (bottom). The strip number is linearly mapped to the displacement of the scattered electron from the primary beam. The dashed line in the top panel corresponds to $A_{exp} = 0$. The solid blue line (top) is a fit to Eq. 2 and the solid red line (bottom) is a fit to a constant value. Only statistical uncertainties are shown in this figure.

The electron beam polarization P_e was extracted by fitting the measured asymmetry to the theoretical Compton asymmetry using;

$$A_{exp}(y_n) = P_e P_\gamma A_{th}(y_n), \tag{2}$$

where P_{γ} is the polarization of the photon beam, y_n is the scattered electron displacement along the detector plane for the *n*-th strip, and A_{th} is the $\mathcal{O}(\alpha)$ theoretical Compton asymmetry for fully polarized electrons and photon beams. The radiative corrections to the Compton asymmetry were calculated to leading order within a low energy approximation applicable for few GeV electrons [21]. The relative change in the Compton asymmetry due to radiative corrections was <0.3%.

The quantity A_{th} is typically calculated as a function of the dimensionless variable $\rho = E_{\gamma}/E_{\gamma}^{max}$, where E_{γ} and E_{γ}^{max} are the energy of the back-scattered photon and its

maximum value, respectively. In order to directly com-223 195 pare with the measured asymmetry, ρ was mapped, by a₂₂₄ 196 third order polynomial, to the displacement of the scat-225 197 tered electron along the detector plane y_n . Further, y_n is₂₂₆ 198 linearly related to detector strip number, and depends on₂₂₇ 199 several parameters, such as, dimensions and dispersion of₂₂₈ 200 the chicane magnets, and exact location of the detectors₂₂₉ 201 with respect to the third dipole. 202 230

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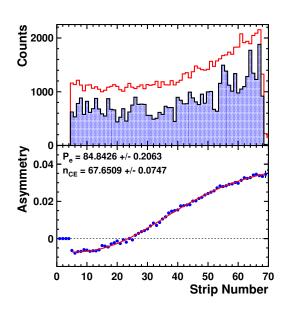


FIG. 4: (top) A typical Monte Carlo simulated Compton spec- $_{252}$ trum for a single detector plane, with (blue, shaded) and without (red) detector inefficiency. The counts have been scaled 253 by a factor of 10^{-3} . (bottom) The Compton asymmetry ex- 254 tracted from the simulated spectrum including detector ineffi- 255 ciency (blue circles), and a two parameter fit to the calculated 256 asymmetry (red line). The input asymmetry was 85%. $_{257}$

The measured asymmetry A_{exp} was fit to Eq. 2 for²⁵⁹ 203 each detector strip, with P_e and n_{CE} as the two free₂₆₀ 204 parameters. The number of degrees of freedom was typ-261 205 ically between 50 - 60, which was made possible by the²⁶² 206 high resolution of the detector, and the proximity of the²⁶³ 207 detector to the primary electron beam. The detection₂₆₄ 208 of a large fraction of the Compton electron spectrum,265 209 spanning both sides of the zero crossing of the Compton²⁶⁶ 210 asymmetry, significantly improved the robustness of the²⁶⁷ 211 fit and the analysis technique. A typical fit is shown in²⁶⁸ 212 Fig. 3. The χ^2 per degree-of-freedom of the fit ranges₂₆₉ 213 between 0.8 - 1.5 for all production runs reported here. 270 214 A Monte Carlo (MC) simulation of the Compton po-271 215 larimeter was coded in the Geant3 [22] detector simu-272 216 lation package. In addition to Compton scattering, the₂₇₃ 217 simulation included backgrounds from beam-gas inter-274 218 actions and beam halo interactions in the chicane ele-275 219 ments. The simulation also incorporated the effects of₂₇₆ 220 detector inefficiency, the track-finding trigger, and elec-277 221 tronic noise. A typical simulated strip-hit spectrum (with₂₇₈ 222

and without detector inefficiency), and the asymmetry extracted from simulated spectra are shown in Fig. 4. The simulation was used to validate the analysis procedure and to study a variety of sources of systematic uncertainties. For each source, the relevant parameter was varied within the expected range of uncertainty, and the change in the extracted polarization was listed as its contribution to the systematic uncertainty. The list of contributions is shown in Table I.

The MC simulation demonstrated that secondary par-232 ticles knocked out by the Compton scattered electron 233 passing through the first plane produced a 0.4% change 234 in polarization in the subsequent planes, consistent with 235 observation. A correction for the second and third planes 236 could be made but at the cost of a slightly higher sys-237 tematic uncertainty, and hence only the results from the 238 first detector plane are quoted here. 239

There were several sources of inefficiency associated with the DAQ system, such as the algorithm used to identify electron tracks and form the trigger, and the deadtime due to a busy (hold off) period in the DAQ. The entire DAQ system was simulated on a platform called Modelsim [23]. While in Monte Carlo simulations, events are generated based on the probability distribution for the relevant physics process, in contrast Modelsim is a simulation technique based on time steps. It employs the same firmware, written in the hardware description language for very high speed integrated circuits (VHDL), that operated the logic modules in the DAQ system. The DAQ simulation included signal generators that mimic the electron, the background and the noise signals, along with a detailed accounting of delays due to the internal signal pathways in the logic modules and the external electronic chain.

The difference between the triggered and the untriggered counts observed in the DAQ simulation was due to DAQ inefficiency. Also, the average DAQ inefficiency was found to be directly related to the aggregate detector rate. These results were used to determine the correction to the detector yield for each 1 hr run, based on the aggregate detector rate during the run. The DAQ inefficiency correction resulted in < 1% change in the extracted polarization. The validity of the corrections and the systematic uncertainty due to the corrections (listed in Table I) was determined by comparing the polarization extracted from triggered vs. un-triggered data over a wide range of beam currents (rates) and several different trigger conditions. Thus, the Modelsim simulation provided a robust method to determine the inefficiency of the DAQ.

Extensive simulation studies provided the comprehensive list of contributions to the systematic uncertainties, tabulated in Table I, with a net systematic uncertainty of 0.59% for the Compton Polarimeter. The extracted beam polarization for the entire second running period of the Q_{weak} experiment is shown in Fig. 5. Most of

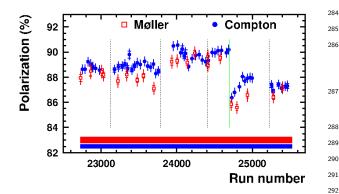


FIG. 5: The extracted beam polarization as a function of run-²⁹³ number averaged over 30 hour long periods, during the second²⁹⁴ run period of the Q_{weak} experiment (blue, circle). Also shown²⁹⁵ are the results from the intermittent measurements with the²⁹⁶ Møller polarimeter [2, 3] (red, open square). The inner error₂₉₇ bars show the statistical uncertainty while the outer error bar₂₉₈ is the quadrature sum of the statistical and point-to-point₂₉₉ systematic uncertainties. The solid bands show the additional normalization/scale type systematic uncertainty. The dashed³⁰⁰ and solid (green) vertical lines indicate changes at the electron³⁰¹ source.³⁰²

TABLE I: Systematic Uncertainties

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Source	Uncertainty	$\Delta P/P\%$]
Laser Polarization	0.18	0.18	307
Plane to Plane	secondaries	0.00	
magnetic field	0.0011 T	0.13	308
beam energy	$1 { m MeV}$	0.08	309
detector z position	$1 \mathrm{mm}$	0.03	310
inter plane trigger	1-3 plane	0.19	311
trigger clustering	1-8 strips	0.01	312
detector tilt(w.r.t x, y and z)	1 degree	0.06	313
detector efficiency	0.0 - 1.0	0.1	314
detector noise	up to 20% of rate	0.1	315
fringe field	100%	0.05	316
radiative corrections	20%	0.05	
DAQ inefficiency correction	40%	0.3	
DAQ inefficiency ptto-pt.		0.3	
Beam vert. pos. variation	$0.5 \mathrm{\ mrad}$	0.2	317
helicity correl. beam pos.	5 nm	< 0.05	318
helicity correl. beam angle	3 nrad	< 0.05	319
spin precession in chicane	20 mrad	< 0.03	320 321
Total		0.59	322
] 323

the variation in the polarization are due to changes at³²⁶
the electron source indicated by the dashed and solid³²⁷
(green) vertical lines. The Compton and Møller measure-³²⁹
ments [2, 3] were quantitatively compared by examining³³⁰
periods of stable polarization. The ratio of Compton to³³¹

³⁴ Møller measurements, when averaged over these stable ³⁵ periods using statistical and point-to-point uncertainties, ³⁶ was found to be 1.007 ± 0.003 .

CONCLUSIONS

The polarization of a 1.16 GeV electron beam was measured using a set of diamond micro-strip detectors for the first time. The high resolution of the detectors and their proximity to the primary beam helped record a large fraction of the Compton electron spectrum, spanning both sides of the zero crossing of the Compton asymmetry. These detectors, coupled with a robust analysis technique and rigorous simulations of the polarimeter and the DAQ system, produced a very reliable, high precision measurement of the polarization in a very high radiation environment. They demonstrate that diamond micro-strip detectors are indeed a viable option as tracking detectors, and they are the appropriate choice for tracking detectors that are exposed to very high radiation dose. We have also demonstrated that it is possible to achieve high precision with a Compton polarimeter operated at beam energies as low as ~ 1 GeV. This has very positive implications for the future PVES program at the upgraded JLab.

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