ten times the nominal emittance) by introducing wiggler errors at ten times the nominal level, with misalignments of ten times the nominal beam size or angular divergence at the wiggler. Even in this case, the phase spaces are not wildly irregular, merely distorted.

In summary, chromatic behavior is generally good and the system remains fairly linear even for alignment and field errors an order of magnitude larger than specification. Geometric aberration behavior over a phase space of ten times the nominal beam emittance remains good for wiggler errors as specified and misalignments consistent with the nominal beam size, and becomes somewhat irregular (phase space distortions giving relative emittance errors of 100%) over this large region only when wiggler errors are pushed to ten times the nominal value and misalignments pushed to ten times the nominal beam size or angular divergence at the wiggler.

The insensitivity to errors appears to be due in part to the small beam envelope function values at the wiggler.

Beam behavior becomes quite erratic (particularly the geometric aberrations, which cause the phase space to become quite irregular) when wiggler errors are pushed to 100 times specification.

Conclusion

The specified wiggler field quality appears adequate to insure appropriate beam transport system performance in the IR Demo Driver Accelerator.

References

[1] S. Benson, "Technical Specifications for the IR Demo FEL Wiggler", CEBAF Spec. 09510-S-002, 1 April 1996.
[2] DIMAD manual and source code; see D.R. Douglas, J.Y. Tang, and R.C. York, "The Effects of Inhomogenieties in Arc Transport System Quadrupoles", CEBAF-TN-0175, 10 October, 1989, for a discussion of multipoles as implemented in the code.
[3] D. Douglas, IR Driver Baseline Design of 3 April, 1996; CEBAF-TN in progress.
[4] D. Douglas, "Chromatic Correction in the CEBAF Beam Transport System", Proc. 1991 I.E.E.E. Part. Accel. Conf., San Francisco.
[5] D. Douglas, *op. cit.*

- Momentum scan (“detailed chromatic analysis” operation):

  This is good (beam envelope function variations of only a factor of a couple) for the ideal beamline and wiggler and the wiggler with errors over a momentum range of ±3% around nominal, even for wiggler misalignments as large as ten times the nominal beam size or angular divergence at the wiggler.

  It is not untoward (beam envelope function variations of only a factor of a few) for ten times the nominal wiggler multipole error, over the full 6% momentum range and with wiggler misalignments of up to ten times the nominal beam size or angular divergence at the wiggler.

- Linear matrix from tracking (“rmatrix” operation):

  This is good (within a few to several percent of ideal) for the beamline with either ideal wiggler or wiggler with errors, even for wiggler misalignments as large as ten times the nominal beam size or angular divergence at the wiggler. H/V coupling is observed with wiggler errors activated, but is very weak (The submatrix determinants $|A|$ and $|D|$ are 1.000, $|B|$ and $|C|$ are 0.000, to DIMAD printout precision; off-diagonal submatrix elements are numerically of order 0.03 rad or 0.01 m, in agreement with analytic estimates).

  The linear matrix from tracking is good (to within a few tens of percent of ideal) even for wiggler multipoles of ten times nominal, with wiggler misalignments of up to ten times nominal beam size or angular divergence at the wiggler, with only weak x/y coupling observed (The submatrix determinants $|A|$ and $|D|$ are of order ~0.99, $|B|$ and $|C|$ are of order ~0.01).

- Geometric aberrations (“line geometric aberrations” operation):

  Geometric aberrations were examined over a phase space region of ten times the nominal beam emittance of ~0.16 mm-mrad at 42 MeV, through a momentum range of ±3% around nominal. Cases with equal and unequal (by a factor of ~4) emittances were examined.

  Very limited phase space distortion (relative emittance errors of ~20-30%) is observed in the ideal beam line, even with wiggler misalignments of up to ten times the beam size or angular divergence at the wiggler for either the ideal wiggler or the wiggler with nominal multipole errors, for equal or unequal emittances, through the full momentum range.

  Beam envelope functions and emittance distortion differ by only a few percent between the ideal case and the case when the wiggler has nominal errors.

  Significant phase space distortion (relative emittance errors of order unity) can be produced over the phase space under consideration (which is, recall, at

The fed-down octupole is, at 0.1 cm offset, roughly a fifth of the quad contribution, and will therefore have smaller impact. Moreover, we expect better steering in the wiggler (0.01 cm), so this error source will be quite negligible. The skew multipole contribution can be treated similarly

Numerical Simulations

Numerical simulations have been performed using the above multipole information. Simulation results for linear focussing effects are consistent with the results of analytic estimates given above, thus verifying the numerical model against an analytic benchmark. These simulations give, in addition, information about multipole-driven nonlinear phase space distortion.

Simulation Technique . The integrated multipoles of Table 1 were subdivided into 39 identical multipoles and distributed (amongst 40 wiggler periods) in a DIMAD model of the wiggler. This was used in a simulation of the performance of the IR driver baseline design of 3 April 1996 [5].

The following items were examined from the front of the wiggler to the reinjection point in front of the cryomodule:

- Second order transfer matrix:

    This is good for the ideal beamline and wiggler, (though T_{336}, T_{346} are a tad larger than desired).

    It is also good for nominal wiggler errors - the chromatics are essentially the same, and some weak x/y linear coupling through wiggler skew quad and nonlinear coupling through geometrics such as T_{114}, T_{124},..., T_{334}, T_{344},... starts occurring. Linear coupling strengths are consistent with analytic estimates and nonlinear coupling is via second order matrix elements with numerical values of a few to several 10's of rad^{-1} or meter-radian^{-1}.

    The matrix is still not bad for ten times the nominal wiggler errors, but the chromatics and geometrics get larger (numerical values of a few 100s of rad^{-1} or meter-radian^{-1}).

- Beam envelope functions are small (peaks ~10 m) and only weakly dependent (at the ~1% level in beta and 0.01 rad in slope) on the presence or absence of multipoles in the wiggler.

For various misalignments of wiggler (in x, x’, y, and y’ individually, and x and y together), the following items were examined from the front of the wiggler to the reinjection point in front of the cryomodule:

$$m_{12},\ m_{34} \sim \sqrt{\beta_{\mathrm{wiggler}}\beta_{\mathrm{average}}}\sin\psi \sim \sqrt{0.5\ \mathrm{m} \times 5\ \mathrm{m}} \times \frac{1}{2} \sim 0.8\ \mathrm{m}$$

$$m_{22},\ m_{44} \sim \sqrt{\beta_{\mathrm{wiggler}}/\beta_{\mathrm{average}}}\cos\psi \sim \sqrt{0.5\ \mathrm{m}/5.0\ \mathrm{m}} \times \frac{1}{2} \sim 0.2\ \mathrm{rad}$$

Thus, the off-diagonal submatrix elements are estimated as follows:

$$m_{31},\ m_{13} = \frac{m_{34},\ m_{12}}{f} \sim \frac{0.8\ \mathrm{m}}{30\ \mathrm{m}} \sim 0.03\ \mathrm{rad}$$

$$m_{41},\quad m_{23} = \frac{m_{44},\ m_{22}}{f} \sim \frac{0.2\ \mathrm{rad}}{30\ \mathrm{m}} \sim 0.007\ \mathrm{m}^{-1}$$

Off-diagonal matrix elements are therefore expected to be of order 0.03 m or 0.01 radian when nominal wiggler skew quad errors are introduced. These represent very weak H/V coupling, and are consistent with simulation results discussed below.

Fed-Down Focussing From Higher-Order Multipoles. Higher order multipoles provide both linear and nonlinear focussing. Nonlinear focussing has been addressed through use of DIMAD; results are discussed below. Linear focussing errors are due to multipole feed-down. The field integral at a midplane displacement x in the wiggler is $BL = b_1 x + b_2 x^2 + b_3 x^3$, so the gradient integral is as follows:

$$B'L = b_1 + 2b_2 x + 3b_3 x^2$$

The first term was discussed above and is just the 50 g quadrupole contribution. The second term is the fed-down sextupole contribution to field gradient at offset x. For a (rather large) orbit offset of 0.1 cm, this contribution is

$$2b_2 x \sim 2 \times 100\ \mathrm{g/cm} \times 0.1\ \mathrm{cm} \sim 50\ \mathrm{g}$$

The fed-down sextupole is, at 0.1 cm offset, the same as the quad contribution, and will therefore have similar impact. However, we expect better steering in the wiggler (0.01 cm), so this error source will be negligible. The skew multipole contribution can be treated similarly.

The second term is the fed-down octupole contribution to the field gradient at offset x. For a (rather large) orbit offset of 0.1 cm, this contribution is

$$3b_3 x^2 \sim 3 \times 300\ \mathrm{g/cm}^2 \times (0.1\ \mathrm{cm})^2 \sim 9\ \mathrm{g}$$

The beam envelope function variation at a point ψ in phase advance away from an error focal length f_{error} at a point of beta function β_{error} is as follows [4].

$$\left(\frac{\Delta\beta}{\beta},\Delta\alpha\right) \sim \frac{\beta_{error}}{f_{error}}(\sin 2\psi, \cos 2\psi)$$

(This ignores the beam envelope function slope α at the observation point; we assume this is small, which is in fact the case at typical points of interest such as the reinjection point). For the case of interest, β_{error} < 1 m, f_{error} ~30 m, and the sinusoids are of no more than order unity. Thus,

$$\left(\frac{\Delta\beta}{\beta},\Delta\alpha\right) \leq (0.04, 0.04) \ ,$$

that is, the induced beam envelope function error is less than 4%.

In fact, the actual error in a detailed computation will be even smaller, inasmuch as the average beam envelope function through the wiggler is more of order 0.5 m, and the phase terms will average to less than 1 due to the (nonzero) phase advance across the wiggler. Typical variations in beam envelope may be closer to 1 to 2% in beta and 0.01-0.02 radian in slope, which are the values observed in the numerical simulations discussed below.

Impact of Skew-Quad-Drive Coupling. As with the normal quadrupole error, the skew quad drives an error focal length f~30 m. This talks to the beam via the (decoupled) downstream transport matrix; the overall transport matrix from wiggler to a downstream point of interest (such as the energy-recovery reinjection point) is, in the thin-lens limit, as follows.

$$M_{\mathrm{TOTAL}} = \begin{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} m_{33} & m_{34} \\ m_{43} & m_{44} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1/f & 0 \\ 0 & 0 & 1 & 0 \\ 1/f & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & -m_{12}/f & 0 \\ m_{21} & m_{22} & -m_{22}/f & 0 \\ m_{34}/f & 0 & m_{33} & m_{34} \\ m_{44}/f & 0 & m_{43} & m_{44} \end{bmatrix}$$

Typical beam envelope function values in either plane are

$$\beta_{wiggler} \sim 0.5\ \mathrm{m}; \quad \beta_{average} \sim 5\ \mathrm{m}; \quad \beta_{maximum} \sim 10\ \mathrm{m}; \qquad \sin\psi\ , \cos\psi \sim \frac{1}{2}\ ;$$

so approximate matrix element values are as follows (ignoring α at the observation point):

Table 1 multipoles relate to DIMAD multipoles as follows.

$$k_n = \sqrt{a_n^2 + b_n^2} / (B\rho)$$

$$t_n = \frac{1}{n+1} \operatorname{atan}\left(\frac{a_n}{b_n}\right)$$

Using these gives Table 2, which provides values for DIMAD input and the following analytic estimates.

Table 2: IR Demo Wiggler Integrated Multipoles in DIMAD units

multiple order (n)	k_n	t_n
1	0.05047/m	22.5°
2	15.961/m^2	22.145°
3	3028.4/m^3	22.5°

Analytic Estimates

We now present analytic estimates of the impact of multipole-driven linear focussing errors. These give limits on the impact of multipole-driven linear focussing errors on transport system performance, and provide a benchmark for the numerical simulations discussed below. Three estimates have been performed; transport system parameters used in the estimates are for the IR Driver Baseline Design of 3 April 1996 [3].

Effect of Quadrupole-Driven Focussing Errors. The integrated normal quadrupole gradient is, from Table 1, ~50 g or 0.05kg. This corresponds to a focal length

$$f = \frac{B\rho}{B'l} = \frac{33.3564\ \text{kg-m/(GeV/c)} \times 0.042\ \text{GeV/c}}{0.05\ \text{kg}} \sim 30\ \text{m}$$

A typical IR recirculator quad has a focal length

$$f = \frac{1}{kl} = \frac{1}{4/\text{m}^2 \times 0.15\ \text{m}} \sim 1.7\ \text{m}$$

The effect of the wiggler integrated quad is therefore ~6% of that of a typical quad. This occurs, moreover, at a point of small beam envelope function, so the impact is correspondingly reduced.

Effect of Multipoles in the IR Wiggler

D. Douglas

Abstract

The impact of multipoles in the wiggler on IR driver beam transport has been analyzed analytically and numerically. Wiggler multipoles as specified by Benson [1] are not found to have significant impact on system performance.

Introduction

In this note, we examine the effect of multipoles in a wiggler on the performance of the IR Demo Driver beam transport system. These multipoles drive three performance issues. The first, the problem of orbit errors, is deemed negligible (as these can be operationally readily compensated) and will not be addressed here. The second issue, the problem of linear focussing and coupling errors, will be addressed analytically and numerically. The third issue, nonlinear phase space distortion, will be addressed numerically using DIMAD.

Multipole Information

The wiggler for the IR Demo FEL has integrated multipoles given in Table 1.

Table 1: Integrated multipoles for IR Demo FEL Wiggler

multiple order (n)	b_n	a_n
1	50 g	50 g
2	100 g/cm	200 g/cm
3	300 g/cm^2	300 g/cm^2

Nomenclature and values are from Reference [1]; I have interchanged the normal and skew multipoles, since [1] seems to have the primary wiggler field vertical (it is to be horizontal in the IR Demo FEL). In the following, we will employ DIMAD, which uses the following multipole representation [2]:

$$K_n = k_n e^{i\pi t_n / 180°}$$

Here, K_n is a complex integrated multipole, k_n is the integrated multipole amplitude in $1/m^n$ and t_n is the "tilt" of the multipole, in degrees. For a midplane symmetric magnetic field B_y, DIMAD uses a field expansion of $B_y L = B\rho \sum k_n x^n$, while Reference [1] uses a general field expansion of $(B_y + iB_x)L = \sum (b_n + ia_n)(x + iy)^n$. Thus,