End-Field Specifications for IR FEL Driver Dipoles

D. Douglas

Abstract

A specification has been set for the IR driver dipole end-field roll-off integral K_1 [1]. This specification defines the range in which the average K_1 value of any dipole family must fall. J. Karn [2] has noted that this specification *does not* define the allowed variation in K_1 within a family, nor does it address the effect of the "second integral," K_2. This note examines the impact of variations in K_1 within a family and effect of the second integral K_2. Specifications for each parameter are developed.

Definitions

The vertical focussing effect of a dipole end is given by M_{34}. TRANSPORT [3] family codes use the following relation for this matrix element.

$$M_{34} = -\frac{1}{\rho_0}\tan(\beta - \psi)$$

Here, ρ_0 is the bend radius, β the entrance or exit angle and ψ the following function of various dipole edge parameters.

$$\psi = K_1\left(\frac{g}{\rho_0}\right)\left(\frac{1+\sin^2\beta}{\cos\beta}\right)\left[1 - K_1 K_2\left(\frac{g}{\rho_0}\right)\tan\beta\right]$$

In the preceding,

$$K_1 = \int_{-\infty}^{\infty}\frac{B_y(z)[B_0 - B_y(z)]}{gB_0^2}dz$$

is the "first" end-field roll-off integral, which leads to a depression of the vertical focussing from the dipole edge due to bowing of the field lines across the finite gap, g. In this expression B_0 is the "core" field in the center of the dipole and the integral is evaluated along a path perpendicular to the pole-face tangent plane. K_1 typically assumes values on the range of [0,1]. K_2 is a "second integral" related to the finite extent of the fringing fields that is typically numerically ~4. The impact of K_2 is suppressed below that of K_1 by a factor of K_1^2 (g/ρ_0) tan β; this, for small gaps, moderate bend radii, modest K_1 and moderate entry/exit angles, is ordinarily quite small.

Impact of Variations in K_1

A specification has been set for the end-field roll-off integral K_1. It constrains the average value of this parameter for any particular driver dipole family. It is as follows [4]:

$$K_1 = 0.27 \pm 0.05$$

This specification was set by estimating (and simulating numerically) the depression of vertical focussing introduced by K_1 and thereby establishing the range over which driver electron beam transport performance will likely be tolerant of variations in this parameter. The calculation assumed that K_1 is a systematic value within a family. The specification thus simply constrains the average value for each family to lie within the stated range.

J. Karn has noted that K_1 will in fact vary from magnet to magnet within a family and has inquired as to the tolerance for this variation [5]. This tolerance is readily established by noting that such intra-family variations are random effects that are appropriately considered a source of random focussing errors, and that limits have been set on such errors [6]. A random deviation δK_1 in K_1 will lead to a variation $\delta\psi$ in ψ of the following magnitude (to first order in the small parameter (g/ρ_0)).

$$\delta\psi = \delta K_1 \left(\frac{g}{\rho_0}\right)\left(\frac{1+\sin^2\beta}{\cos\beta}\right)$$

Appending this deviation to ψ in the expression for M_{34}, expanding the tangent using tan (A+B)=(tan A+tan B)/(1+tan A tan B), and retaining only terms of low order in the small parameter $\delta\psi$, we obtain the following expression for the deviation δM_{34} in M_{34} due to a variation δK_1 in K_1.

$$\delta M_{34} \sim \frac{\delta K_1}{\rho_0}\left(\frac{g}{\rho_0}\right)\left(\frac{1+\sin^2\beta}{\cos\beta}\right)(1-\tan^2\beta)$$

Thus, under assumptions valid for the IR FEL driver and using values consistent with driver dipoles (g~0.05 m, ρ_0~1 m, β~1/8 radian), we find M_{34} exhibits the following sensitivity to variations in K_1.

$$\delta M_{34} \sim 0.05\,\delta K_1/\mathrm{m}$$

Note that typical values of M_{34} for driver dipoles are of order 1/(4 m) (all have ~1 m bend radii and ~1/8 radian entry/exit angles). Note also that we have generally specified that focussing terms in the driver will be controlled to the $0.5\text{x}10^{-3}$ rms relative error level [7]. Thus, we may bound δK_1 by requiring it not impose an rms relative error in M_{34} with magnitude in excess of 1/2 part per thousand.

$$0.5\times10^{-3} > \frac{\delta M_{34}}{M_{34}} \sim \frac{0.05\,\delta K_1/\mathrm{m}}{1/(4\,\mathrm{m})} \sim 0.2\,\delta K_1$$

Consequently, δK_1 is to be constrained as follows.

$$\delta K_1 < \frac{0.5\times10^{-3}}{0.2} = 0.0025$$

We observe this is approximately 1% of the specified K_1 dipole-family average value of 0.27±0.05, and therefore conclude that the rms intra-family relative variation $\delta K_1/K_1$ about the family average will be appropriately constrained at the 10^{-2} level.

Effect of K_2

The effect of K_2 can be determined by reference to the definition of ψ.

$$\psi = K_1\left(\frac{g}{\rho_0}\right)\left(\frac{1+\sin^2\beta}{\cos\beta}\right)\left[1-K_1K_2\left(\frac{g}{\rho_0}\right)\tan\beta\right]$$

Nominally g/ρ_0 is small and K_1 and β are fractional. Thus, K_2 can be considered a perturbation on ψ, leading to a deviation $\delta\psi$ (away from the K_2=0 value) of the following magnitude.

$$\delta\psi \sim K_1^2\left(\frac{g}{\rho_0}\right)^2\left(\frac{1+\sin^2\beta}{\cos\beta}\right)K_2\tan\beta$$

As with the above computation of the effect of δK_1, this deviation $\delta\psi$ leads to a variation δM_{34} in M_{34} of magnitude

$$\delta M_{34} \sim \frac{K_1^2}{\rho_0}\left(\frac{g}{\rho_0}\right)^2\left(\frac{1+\sin^2\beta}{\cos\beta}\right)K_2\tan\beta(1-\tan^2\beta);$$

now, however, the variation is systematic within a family. The effect of such errors can be addressed using relations given elsewhere. At the end of the recirculation beam line, these perturbations will induce the following deviations in the transported vertical beam envelopes [8].

$$\frac{\Delta\beta}{\beta} = -\sum_{k=1}^{N} \delta M_{34}^{k} \beta_k \sin 2\psi_{k\to end}$$
$$\Delta\alpha = \sum_{k=1}^{N} \delta M_{34}^{k} \beta_k (\cos 2\psi_{k\to end} - \sin 2\psi_{k\to end})$$
$$\Delta\psi = \frac{1}{2\pi} \sum_{k=1}^{N} \delta M_{34}^{k} \beta_k \sin^2 \psi_{k\to end}$$

Observe that the perturbation in beam envelope functions propagates at twice the betatron phase advance. Thus, systematic perturbations at homologous points a ¼ betatron wavelength apart cancel. In the driver, this means that any mismatch effect of K_2 in the DX, DQ and DY magnets of the energy recovery transport will self-suppress, due to the 5/4-integer tune of the end loops and 3/2-integer tune of the backleg. Only the chicanes will generate betatron mismatch, though all magnets will contribute to a vertical phase advance error.

To estimate the magnitude of chicane dipole K_2 induced mismatch, we simply attribute "average" values to all betatron parameters (β~5 m, α_{end}=0, all sinusoids are given the rms value $1/\sqrt{2}$, and $M_{34}^{k\to end}$~ $\beta/\sqrt{2}$ ~3.5 m), evaluate the sums, and get the following expressions. In this step, we consider only contributions from chicane (energy recovery, reinjection, and optical cavity) dipoles, so N=14.

$$\frac{\Delta\beta}{\beta} \sim \frac{N}{\sqrt{2}} \beta\, \delta M_{34} \sim (50\text{ m})\, \delta M_{34}$$
$$\Delta\alpha \sim \frac{N}{\sqrt{2}} \beta\, \delta M_{34} \sim (50\text{ m})\, \delta M_{34}$$
$$\Delta\psi \sim \frac{N}{4\pi} \beta\, \delta M_{34} \sim (5.6\text{ m})\, \delta M_{34}$$

K_2 enters δM_{34} though an expression given above. Using values defined earlier (g~0.05 m, ρ_0~1 m, β~1/8 radian, K_1=0.27) we find δM_{34}=(2.3x10^{-5}/m)K_2, implying the propagated beam envelopes have the following sensitivities.

$$\Delta\beta/\beta \sim 0.0012\, K_2$$
$$\Delta\alpha \sim 0.0012\, K_2$$
$$\Delta\psi \sim 0.00013\, K_2$$

Hence, a K_2 value of ~10 will lead to ~1% errors in beam envelopes and fraction of a degree phase advance error. These are an acceptable contribution to the total error budget.

Specification on δK_1 and K_2

Given the above results for driver sensitivity to variations in K_1 and the effect of K_2, we may state the following specifications on these parameters.

> ***The rms relative intra-family variation $\delta K_1/K_1$ of K_1 will be limited to the 1% level:***

$$\frac{\delta K_1}{K_1} < 0.01$$

> ***The "second integral" K_2 will be limited to a value below 10:***

$$K_2 < 10$$

Acknowledgments

I would like to thank Jeff Karn for his efforts to develop a complete and clearly defined specification set for IR-FEL driver magnets and the consequential motive he provided for this work.

References

[1] D. Douglas, "Error Estimates for the IR FEL Transport System," CEBAF-TN-96-035, 15 July 1996. This is available from the World Wide Web at http://www.jlab.org/~douglas/FEL/technote/CEBAFTN96035.ps.

[2] J. Karn, private communication.

[3] The following information is drawn directly from K. Brown, F. Rothacker, D. Carey, and Ch. Iselin, "TRANSPORT A Computer Program for Designing Charged Particle Beam Transport Systems," SLAC Report SLAC-91, Rev.2, UC-28 (I/A), May 1977.

[4] D. Douglas, *op. cit.*

[5] J. Karn, *op. cit.*

[6] See, for example, the quadrupole excitation tolerance specifications given in Table 3 of Reference 4.

[7] *Ibid*.

[8] See the Introduction of Reference 4.