Field Quality Specification for IR FEL Driver Sextupoles

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Abstract

The impact of variations in field quality of driver sextupoles is discussed. A specification for sextupole field quality is given.

Introduction

In this note we use “DIMAD” nomenclature [1], in which the field of a mid-plane symmetric magnet has the following multipole expansion.

$$B(x) = (B\rho)\sum_n k_n x^n$$

The sum is from n=0 upward, with 0 denoting dipole, 1 quadrupole, 2 sextupole, and so forth. In a sextupole, the higher order relative field variation (of primary concern in this note) is then given by the following expression.

$$\frac{\Delta B}{B} = \sum_{n=1}^{\infty} \frac{k_{n+2}}{k_2} x^n$$

An octupole error thus induces a linear variation from the intended sextupole field; a decapole error similarly causes a quadratic variation from design. We will ignore lower-order (dipole and quadrupole) effects, as they can individually be eliminated by appropriate alignment of the measurement apparatus within the sextupole bore (*i.e.*, by “centering the probe in the magnet”) [2] and can operationally be locally compensated using adjacent steering coils and trim quads.

Field Variations in the Prototype Sextupole

Karn [3] reports the prototype SC sextupole has observed multipole amplitudes as presented in Table 1. He notes that these fall into two classes, with n=8, 14, … “allowed” harmonics and all others “error” harmonics. Allowed harmonics are due symmetries inherent in the magnet design; error harmonics are induced by construction and excitation error driven asymmetries. The allowed harmonics will take on more or less systematic values from magnet to magnet; the error harmonics can in principle vary significantly from magnet to magnet.

In each case, the given amplitude is the relative integrated field error at a 0.1 m radius due to the indicated order of multipole. We note that there is no phase information provided; the field error can be due to either normal or skew fields or to a superposition thereof.

Table 1: SC Multipole Amplitudes

harmonic	3	4	5	6	7	8	9	10	11	12	13	14
|ΔBL/BL| (%)	0.41	0.17	0.13	0.04	0.30	3.3	0.14	0.09	0.1	0.1	0.05	6

Effect of Multipoles

Error multipoles in sextupoles can lead to unexpected focussing effects with attendant beam growth and transport system performance degradation. We have analytically estimated and numerically modeled the impact of these errors. As with other magnets in the FEL driver, we require the induced degradation in beam quality be small in a relative sense and conform to an error budget [4] over a "6σ + beam handling" aperture allowance. This, as elsewhere, is numerically $6(\sqrt{\beta\varepsilon}+|\eta\sigma_{\delta p/p}|)\pm 20\text{ mm}$ which, for FEL driver parameters is about 14 cm full aperture or 7 cm radial aperture.

Analytic estimates were based on the angular error induced by a local deviation ΔB/B of the sextupole field from design. Electrons at the location of a field error suffer an imposed angular error, which is simply the relative discrepancy of the local bend angle from the ideal:

$$\delta\theta = \frac{\Delta B}{B}\frac{Bl}{B\rho} = (\frac{\Delta B}{B}) l k_2 x^2$$

At the limit of the working aperture (0.07 m), a driver sextupole (k_2~10/m^3, l=0.15 m) will thus have an error response as follows.

$$\delta\theta = 7.5\times 10^{-3}(\frac{\Delta B}{B})$$

A 1% error relative at the edge of the working aperture thus leads to a 75 μrad angular error. To set the scale of this effect, we list a few relevant angular beam and lattice parameters.

- Beam betatron angular divergence:

$$\sqrt{\varepsilon/\beta} \sim \sqrt{0.1\text{ mm - mrad}/10\text{ m}} \sim 100\ \mu\text{rad}$$

- Beam dispersive angular divergence:

$$\eta'(\delta p/p) \sim 0.5\times 0.001 \sim 0.5\text{ mrad (tune - up beam)}$$
$$\sim 0.5\times 0.01 \sim 5\text{ mrad (enegy recovered beam)}$$

- Beam centroid angular offset, beam steered to edge of working aperture:

$$x' \sim x/\beta \sim 0.07\text{ m}/10\text{ m} \sim 7\text{ mrad}$$

The induced angular error is thus consistent with the beam betatron size and the momentum spread-induced size of the tune-up beam. It is therefore not an issue for tune-up, inasmuch as the tune-up beam does not fill the working aperture. Moreover, the angular error induced at the edge of the aperture is only a small fraction of the angular offset (7 mrad) required to reach the edge of the aperture; this will therefore not significantly affect difference orbit measurements. Finally, the "smear" imposed on a large momentum spread beam is only a small fraction of the full angular beam size (a multiple of 5 mrad). Error multipoles at this level will therefore not significantly degrade the quality of the energy-recovered beam.

The performance impact of a ~1% field error at the working aperture is thus modest. Analytic considerations therefore suggest that the field quality of the SC sextupole should be consistent with 1% total relative field deviation at a 7 cm radius.

Numerical Simulations were performed using DIMAD. Results were consistent with the above analytic discussion. Observations are as follows:

- General comments –

  Systematic errors (either allowed harmonics, or systematic error harmonics) are somewhat suppressed by betatron phasing. The lattice is therefore less sensitive to systematic errors than to random.

  The lattice is more sensitive (in both betatron focussing parameters and orbit effects) to errors in the sextupoles at the high dispersion points. This is unsurprising, inasmuch as the beam can be at large amplitude in these magnets.

- Allowed harmonics –

  Some cancellation of effects occurs due to betatron phasing between the sites of these systematic errors.

  10% errors at full aperture are very bad, leading to particle loss during simulation.

  3% errors at full aperture pole are not bad, or are at least survivable, when the beam is within the acceptance used by a 5% momentum spread, well steered beam. This is essentially half the sextupole aperture. In this case, there is little change in the focussing (beam envelopes) or orbit as functions momentum when errors are included in the model. Significant changes occur when errors are activate and the beam is outside the "5% acceptance" – that is, if the momentum spread is larger than the design 5% or if the beam is poorly steered.

The allowed harmonics are very high order, and the beam does not respond to them until it is sampling the outlying region of the sextupole. When the beam is near the edge of the magnet, the response is quite nonlinear and sensitively depends on the magnitude of the error. At a relative error of ~3%, the response seems to "take off"; 1% errors seem to be away from the edge where the beam response is taking becoming pathological, and are thus to be preferred as a specification.

Conclusion – for ½% extraction efficiency, a few percent systematic error at the pole is okay, if the beam is well steered. If higher output power is desired, the sextupoles must provide ~1% field quality (relative error due to allowed harmonics) at the pole and beam must be well steered.

- Error harmonics – may be random from magnet to magnet, and, in this case, will experience no cancellation from betatron phasing.

  The lattice is, again, apparently less sensitive to systematic errors than it is to random.

  High order error multipoles behave like the allowed harmonics, giving little effect at small aperture but inducing large response at large aperture. The effect falls off fast as the beam moves in from the pole. The beam may be more sensitive to random high order multipoles than it is to systematic high order multipoles inasmuch as betatron phasing does not lead to cancellation.

  A 1% field deviation at the pole due to a random error harmonic is not "nice", 0.1% is okay. For example, a "large" relative error, such as the 0.4% octupole at the pole, is noticeable over the nominal 5% momentum aperture due to changes from design of the momentum dependence of the orbit, but does not lead to significant focussing errors.

  Low order effects are more significant than high order effects. 1% octupole or decapole content at the pole leads to little focussing effect (small change in the beam envelope dependence on momentum) but can lead to potentially observable variations in central orbit over the "5% acceptance", with changes of perhaps a few hundred microns. As these can occur from skew as well as normal multipoles, this is not desirable inasmuch as it potentially generates x/y coupling. Effects of order 1% errors are significant for the "10% acceptance".

At 0.1% relative error at the pole, low order error multipoles lead to tens of microns variations in central orbit with momentum. This is tolerable within the 5% acceptance, and is probably survivable over the 10% acceptance.

Conclusion – we should develop a specification that is operationally viable for a 5% momentum spread beam and that does not preclude recirculating a 10% momentum spread beam. Thus, we will need ~1% allowed, and 0.1% error harmonics at the pole, and will need ~0.1% relative field error at half aperture (which corresponds to roughly the 5% acceptance if well steered). The total relative error at the pole, summed over several harmonics, will then be a few percent; as this is due to nonlinear field variations, the field error may be expected to fall to order 1% at the 7 cm working aperture limit ($(7/10)^n$ being a small number for moderate to large n). Moving further inward to half aperture should reduce the total error to well below the 1% level, consistent with the desired 0.1% deviation.

This aggressive specification is driven by to momentum acceptance issues. The beam betatron size is modest throughout the system; it does not couple to the field inhomogeneities under consideration. The dispersion is however very large in the sextupoles, so the off-momentum orbit is very sensitive and nonlinear in its response to such errors. This nonlinearity can lead to operational difficulties; it cannot be compensated for example, through use of the installed with trim quadrupoles.

SC Field Quality Specification

SC sextupoles should conform to the following field quality specification.

- Allowed harmonics should not contribute a relative integrated field error at the pole radius (0.1 m) of order in excess of 1% of the base sextupole field integral at the pole (*but see comment below*).
- Error harmonics should not contribute a relative integrated field error at the pole radius (0.1 m) of order in excess of 0.1% of the base sextupole field integral at the pole.
- The total of all relative integrated field errors at half aperture (0.05 m) should not be of order in excess of 0.1% of the base sextupole field integral at half aperture.

Comment: The above specification conforms to the comprehensive driver transport system error budget [5] and, in addition, allows for ready extension of machine operations to higher powers through generation of momentum spreads (of order 10%) that will fill the available physical aperture. If "aperture filling" exercises are disallowed and/or approached only as an upgrade, the behavior of the allowed harmonics in the existing prototype sextupole is adequate for 1 kW operation *provided the beam is well steered*. We note that improvements should be made in the error harmonic performance to ensure conformance with the error budget.

Notes and References

[1] R. V. Servranckx *et al.*, "A Users Guide to the Program DIMAD", SLAC Report SLAC-285, UC-28, May, 1985.

[2] Thanks to L. Harwood for pointing out that observed dipole and quadrupole terms can be attributed to misalignment of the probe within the sextupole.

[3] J. Karn, "Overview of the SC Sextupole Harmonic Measurement Probe", *unpublished*; values for n=10 and above are taken (eyeballed) from a chart by J. Karn entitled "P8A Probe Commissioning with SC Prototype Sextupole", *unpublished*.

[4] D. Douglas, "Error Estimates for the IR FEL Transport System", CEBAF-TN-96-035, 15 July 1996; D. Douglas, "Lattice Design for a High-Power Infrared FEL", to appear in the Proceedings of the 1997 IEEE Particle Accelerator Conference (Vancouver, May 1997).

[5] *ibid*.