Effect of Misaligning Driver Trim Quads and Sextupoles

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Abstract

We investigate the impact of misaligning IR FEL driver trim quadrupoles and sextupoles.

Introduction

Analytic methods were used to set alignment tolerances for primary IR FEL driver components (major dipoles and quadrupoles) [1]. The tolerances so derived were applied to trim components (quadrupoles and sextupoles) as well, inasmuch as they would clearly be sufficient (though tighter than necessary) to meet the performance specifications required of the weaker class of elements. These tolerances were then certified (and those for the sextupoles relaxed by a factor of two) by simulation [2].

Some difficulty was encountered in preliminary attempts at simultaneous alignment of the trim quads and sextupoles [3]. We therefore have reviewed the issue of trim quad and sextupole alignment and analytically evaluated the sensitivity of various beam performance parameters (steering, focussing, and coupling) to misalignment using trim-specific parameters.

Introduction

The magnetic field of a beamline element can be described using a multipole expansion.

$$Bl = l\left(Br\right)\sum_n k_n x^n$$

A beam traversing the element (which is assumed to be short) will experience

$$q = \frac{Bl}{Br} = l\sum_n k_n x^n$$

an angular kick of the following magnitude.

The beam will also be focussed by any field gradient; this focussing effect is quantified by the following focal length.

$$\frac{1}{f} = \frac{B'l}{Br} = l\sum_n n k_n x^{n-1}$$

We may estimate the effect of errors in trim quad and sextupole alignment by inserting in these expressions the element-specific multipole expansions and evaluating the results at offsets consistent with the expected misalignment values.

Trim Quad Misalignments

A trim quadrupole is characterized by a multipole expansion with nonzero k_1 and all other multipoles zero.

Angular deflections: If the quadrupole is displaced by an offset dx, the angle imposed on a beam at position x shifts by the following amount.

$$\Delta q = q\left(x + dx\right) - q\left(x\right) = l\, k_1 dx$$

The beam angular deflection thus suffers a position-independent error proportional to the misalignment of the quadrupole. The maximum trim quad k_1 encountered during normal recirculator operation is 1.5/m^2; the quadrupole length is 0.15 m. Thus, the maximum anticipated error in angular deflection will be as follows.

$$\Delta q = 0.225 dx / \mathrm{m}$$

A 1 mm misalignment thus leads to a 0.225 mrad angular error. This is normally manifested as, and characterized by, a DC downstream orbit error. Using an rms response matrix (M_{12}) value of $5/\sqrt{2}$ m, which is characteristic of the IR FEL driver [4], this will generate an rms orbit error value of ~0.7 mm. This is a relatively small effect, and is not of particular concern. This effect has been simulated using the Excel-based machine model [5]; results of the numerical study are consistent with the analytic estimate.

Focussing Errors: The multipole expansion for a quadrupole has only nonzero k_1. Consequently, the focal length associated with the quadrupole is (for good field quality), position independent (in the above expression for focal length, n=1, so x^{n-1}=1). Focussing due to trim quadrupoles is therefore not expected to depend on misalignment in any significant manner.

Summary: Trim quadrupole alignment errors at the 1 mm level are not of major concern. The only aspect of this issue to keep in mind is the fact that the recirculator central orbit position/dipole buss excitation is operationally established by use of centering in the trim quadrupoles. The resolution of this procedure is limited by the alignment precision of the quadrupole; a 1 mm alignment tolerance allows order 10^{-3} resolution for absolute calibration of the dipole buss excitation, and hence a similar absolute energy resolution.

Sextupole Misalignments

A sextupole is characterized by a multipole expansion with nonzero k_2 and all other multipoles zero.

Angular Deflections: If the sextupole is displaced by an offset dx, the angle imposed on a beam at position x shifts by the following amount.

$$\Delta q = q(x+dx) - q(x) = 2l\,k_2\,x dx + l\,k_2 dx^2$$

The beam angular deflection thus suffers both position-dependent and position independent errors proportional to the misalignment of the sextupole. The position dependent term is appropriately considered a linear focussing effect and will be discussed immediately below. The position-independent term, which is quadratic in the offset, is a true central orbit deflection of the type considered above. The maximum trim sextupole k_2 encountered during normal recirculator operation is 10/m^3; the sextupole length is 0.15 m. Thus, the maximum anticipated position-independent error in angular deflection will be as follows.

$$\Delta q = 1.5 dx^2 / \mathrm{m}^2$$

A 1 mm misalignment thus leads to a 1.5 µrad angular error. As in the case of the trim quadrupole, this is manifested as, and characterized by, a DC downstream orbit error. Using an rms M_{12} value of $5/\sqrt{2}$ m, which is characteristic of the IR FEL driver [6], this will generate an rms orbit error value of order 5 µm. This is a very small effect, and is not of concern.

Focussing Errors: Misalignment by dx of a sextupole will lead to the following shift $d(1/f)$ in the local focussing.

$$d\left(\frac{1}{f}\right) = \frac{1}{f}(x+dx) - \frac{1}{f}(x) = 2k_2\,l\,dx$$

The variation in the focussing (essentially, the “gradient of the gradient”) is for a sextupole thus a constant depending on the positional offset. The maximum trim sextupole k_2 encountered during normal recirculator operation is 10/m^3; the sextupole length is 0.15 m. Thus, the maximum anticipated sensitivity of focussing to alignment error will be as follows.

$$d\left(\frac{1}{f}\right) = 3/\mathrm{m}^2\,dx$$

A 1 mm misalignment leads to an inverse focal length error of 0.003/m, corresponding to an error focal length of 333 m. This should be compared to a typical recirculator quadrupole, which has a focal length of 1/(5/m^2 x 0.15 m), or 1.33 m. The resulting relative focal error is on the order of ½% of the typical quadrupole strength. We note the powering tolerance on the quadrupoles is 0.1% rms [7]. The sextupoles, at 1 mm alignment error, therefore represent a relatively large, though almost certainly nonfatal, source of focussing error. The magnitude of the error imposed by a single sextupole is offset by the small number of sextupoles in the machine (8 sextupoles *vs.* ~33 main quadrupoles) and the fact that most of the sextupoles can be aligned to better than 1 mm rms [8]. The effect is, moreover, reproducible and not varying in time, inasmuch as the sextupoles are not moving around. Local compensation using adjacent trim quads could therefore be contemplated.

This error will most likely be manifested through the occurrence of spurious dispersion. We note the average dispersion at the sextupoles is of order 1 m; the above focussing error will thus lead to a 3 mrad error in dispersive slope h'. Using the M_{12} value cited above, the resulting downstream error dispersion value will be of the order of a centimeter. Sextupole misalignments thus may globally lead to a few cm of spurious dispersion, which can be locally compensated through adjustment of the trim quadrupoles. As noted above, such corrections will be reproducible inasmuch as the sextupole position is not varying.

<u>*Coupling Effects*</u>: The transfer map for a thin sextupole may be represented as follows.

$$\begin{aligned}
\bar{x} &= x \\
\bar{x}' &= x' + \frac{B_y l}{Br} = x' - k_2 l\left(x^2 - y^2\right) \\
\bar{y} &= y \\
\bar{y}' &= y' + \frac{B_x l}{Br} = y' + 2\,k_2 l\, xy
\end{aligned}$$

If the sextupole is misaligned by horizontal and vertical offsets *dx* and *dy*, the transfer map may be rewritten to include the effect of the misalignment.

$$
\begin{aligned}
&\bar{x} = x \\
&\bar{x}' = x' + \frac{B_y l}{Br} = x' - k_2 l\left((x+dx)^2 - (y+dy)^2\right) \\
&\bar{y} = y \\
&\bar{y}' = y' + \frac{B_x l}{Br} = y' + 2k_2 l(x+dx)(y+dy)
\end{aligned}
$$

Expansion of these relations shows that the misalignment imposes on the angular portion of the transfer map three terms in addition to the ideal dependences. The first terms are kicks, which lead to the following shifts in outgoing angle:

$$\Delta\bar{x}' = -k_2 l\left(dx^2 - dy^2\right)$$

$$\Delta\bar{y}' = 2k_2 l dx dy$$

As noted above, these are of order 1 µrad for 1 mm misalignments in the IR FEL driver. The second terms correspond to linear focussing, leading to the following shifts in outgoing angle:

$$
\begin{aligned}
\Delta\bar{x}' &= -\frac{1}{f_x} x \\
\Delta\bar{y}' &= \frac{1}{f_x} y
\end{aligned}
$$

In these expressions, $(1/f_x) = 2k_2 l dx$. The focal length, as noted above, is of order 333 m for 1 mm misalignments in the IR FEL driver. The final terms correspond to H/V coupling, and are of the same form as the linear focussing terms:

$$\Delta\bar{x}' = \frac{1}{f_y} y$$

$$\Delta\bar{y}' = \frac{1}{f_y} x$$

where $(1/f_y) = 2k_2 l dy$. The focal length, as noted above, is of order 333 m for 1 mm misalignments in the IR FEL driver. Note however the H/V coupling term is driven by *vertical* misalignment. Good control of vertical alignment has been achieved [9]; as a consequence we expect the misalignments driving this effect to be significantly smaller than the horizontal misalignments previously considered.

Even at the 1 mm level the effect is relatively small. The 333 m focal length, corresponds to a gradient integral of ~4 gauss at 40 MeV. This is the skew quad coupling associated with a single CEBAF cavity at 5 MV/m [10], so the effect is not dramatic. Operationally, we can estimate the effect of such coupling during machine diagnostic activities. For example, in the course of a difference orbit measurement with ±5 mm horizontal offset, the variation in outcoupled vertical angle will be (0.003/m) x (0.005 m), or 15 μrad. With the generic M_{12} value considered above, this will evolve to a downstream orbit offset of ~50 μm, which is at the limit of BPM resolution. Given that the vertical alignment has been constrained to ~1/4 mm [11], these coupling effects may well be unobservable.

Summary: Sextupole alignment errors at the 1 mm level will not lead to significant central orbit errors. They may lead to focussing errors with resultant spurious dispersion, which, in all likelihood, can be corrected using adjacent trim quadrupoles. Some H/V coupling effects may be generated by such sextupole misalignments. The expected out-of-plane coupling will be similar to that in a single CEBAF (not FEL) cavity, corresponding to a gradient integral of ~4 g; the resulting orbit effects will be at the threshold of observability. Such alignment errors are therefore not expected to create operational difficulties.

Current Status and Conclusions

The above estimates indicate trim quadrupole/sextupole alignment errors at a 1 mm rms level will not significantly affect IR FEL driver beam transport system performance.

Although initial attempts to align both trim quads and sextupoles were somewhat difficult, subsequent efforts resulted in excellent simultaneous alignment of both types of magnet [12]. The surveyed trim quad and sextupole positions are well inside any limit that could lead to observable effects in the IR FEL Driver beam transport system.

References

[1] D. Douglas, “Error Estimates for the IR FEL Transport System”, CEBAF-TN-96-035, 15 July 1996. This is available from the World Wide Web at http://www.jlab.org/~douglas/FEL/technote/CEBAFTN96035.ps.

[2] D. Douglas, “Simulation of Alignment and Powering Errors in the IR FEL Driver Beam Transport System”, CEBAF-TN-96-055, 18 October 1996. This is available from the World Wide Web at http://www.jlab.org/~douglas/FEL/technote/CEBAFTN96055.ps.

[3] J. Dahlberg and C. Curtis, private communication. Both trim quads and sextupoles were rigidly fixed to the same girder and were not independently adjustable in position. Documentation of the alignment of these objects is available in the Jefferson Lab Alignment Group Data Transmittal #461, 8 September 1998.

[4] D. Douglas, "Simulation of Alignment and Powering Errors in the IR FEL Driver Beam Transport System", *op. cit.*

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[6] D. Douglas, "Simulation of Alignment and Powering Errors in the IR FEL Driver Beam Transport System", *op. cit.*

[7] *ibid.*

[8} J. Dahlberg and C. Curtis, *op. cit.*

[9} *ibid.*

[10] D. Douglas, "FEL Driver Trim Magnet Field Specifications", JLAB-TN-97-015, 11 April 1997. This is available on the World Wide Web at http://www.jlab.org/~douglas/FEL/technote/JLABTN97015.ps.

[11] J. Dahlberg and C. Curtis, *op. cit.*

[12} *ibid.*