JLAB-TN-00-020
13 September 2000

Longitudinal Phase Space Management in the IR Upgrade FEL Driver

D. Douglas

Abstract
We discuss issues pertaining to longitudinal phase space management in the IR Upgrade FEL driver. Particular isses of interest are the generation of ultrashort (<100 fsec) bunches at the wiggler and energy compression of the large (~10% relative energy spread) exhaust phase space, which may require the use of high (octupole) order corrections in the recirculator.

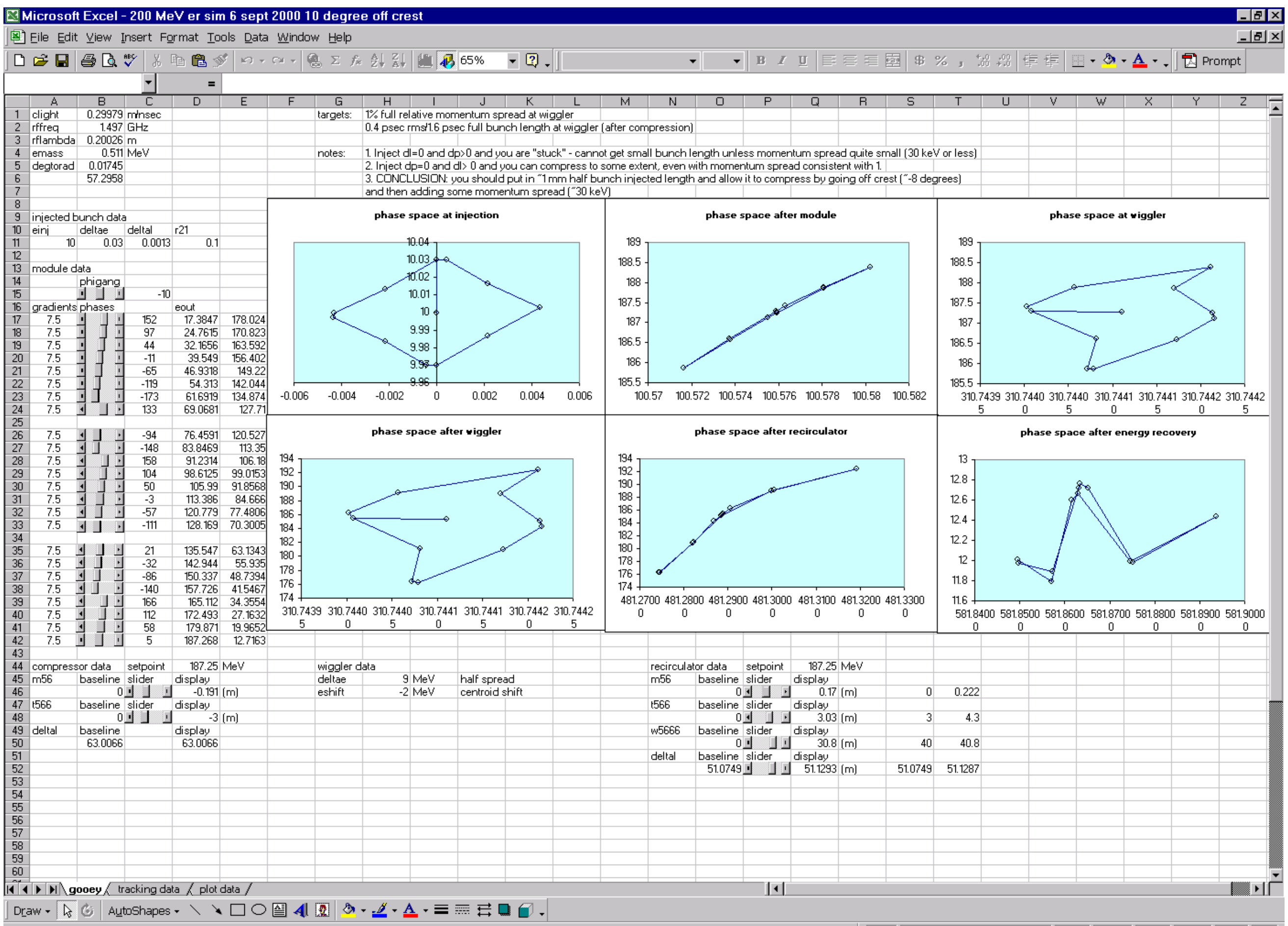
Overview
The issue of longitudinal phase space management in FEL drivers was investigated for both the design [1] and the commissioning [2] of the IR Demo. The scheme implemented in the Demo [3] will be employed in the Upgrade driver design [4]. Certain issues must therefore be addressed:
1. production of the appropriate longitudinal phase space at the wiggler (in the presence of CSR and other collective effects), and
2. energy recovery and energy compression of a large energy spread (~10% relative) exhaust beam over a larger dynamic range (20:1) than that employed in the Demo (4:1).

Requirements
Essential to the operation of the IR Upgrade FEL are the following:
1. Generation of a (nominally longitudinally upright) 135 pC/bunch drive electron beam of 200 fsec rms bunch length with rms relative momentum spread under ½% (this to be done in the presence of CSR and other collective effects), and
2. energy compression to a final full width of order 1% during energy recovery from full energy to a final energy of ~10 MeV. Larger final energy spreads may be tolerable, depending on the dump design, but "less is more" inasmuch as smaller momentum spreads translate to simpler, more robust, lower cost dump implementations.

Production of Short Bunch at Wiggler
The initial design concept [5] mimics the IR Demo longitudinal phase space management scheme. An upright long, low momentum spread bunch of the type generated in the Demo injector is accelerated 10^{o} off crest induce a phase energy correlation. The –0.3 m momentum compaction (M_{56}) of the first endloop and the chicane around the upstream end of the optical cavity is used to rotate this bunch upright at the wiggler, resulting in a short bunch (0.2

psec rms) with resulting high peak current. Initial DIMAD simulations indicate this is readily possible and that CSR is not an issue [6]. Simulation of the full acceleration cycle using an Excel-based model [7] and Demo injection parameters also confirms that the required 200 fsec/½% phase space is accessible. Figure 1 presents a screen-shot from this simulation illustrating the longitudinal phase space through the acceleration/energy recovery cycle. The phase space at the wiggler has been manipulated to conform to the above requirements through appropriate adjustment of M_{56} and T_{566} in the first recirculation arc.

Figure 1: Demo longitudinal phase space management scheme applied to Upgrade.



We note that the phase space at the wiggler is ~200 fsec full length and has a full energy spread of ~3 MeV, or 1½%. These are well within specification. The phase space is "smaller" in the upgrade than at the corresponding point of the Demo by virtue of greater adiabatic damping provided by acceleration to higher energy.

Energy Compression During Energy Recovery

Initial simulation with DIMAD suggested energy recovery is straightforward provided octupoles are available [8]. This simulation did not however include effect of RF waveform curvature, which, when incorporated, produce a large, decapole-order induced energy spread at dump. This is clearly visible in the final frame of Figure 1. Moreover, the endloop W_{5666} required to produce even this poorly optimized result could require significant octupole strength. Futher improvement would, moreover, require the use decapole correction in the recirculator, which is not desirable philosophically or operationally.

To ascertain the source of this longitudinal aberration, we now review some features of an analysis performed originally as an IR Demo design exercise [9]. Consider the energy recovery of a bunch of momentum spread $\pm\Delta E$ and length zero at the wiggler. Denote by E_{linac} the linac maximum energy gain (or decrement, in energy recovery mode), and by E_0 and ϕ_0 the energy and phase of the bunch centroid. Note that ϕ_0 is simply the off-crest acceleration phase, give or take 180° and the odd minus sign or two. Let E and ϕ be the corresponding quantities for either the maximum or minimum energy component of the bunch. The energy $\overline{E}_0$ of the centroid and the energy $\overline{E}$ of the maximal initial energy component at the dump are then as follows.

$$\overline{E}_0 = E_0 - E_{linac}\cos\phi_0$$
$$\overline{E} = E - E_{linac}\cos\phi$$

We now note that $E = E_0 + \Delta E$, $\overline{E} = \overline{E}_0 + \Delta\overline{E}$, and $\phi = \phi_0 + \Delta\phi$; because of the initial zero bunch length at the wiggler, the entire phase difference $\Delta\phi$ is due to arc momentum compactions and energy offsets. Denoting $\Delta E/E_0$ by δ, $\Delta\phi$ is in fact specified as follows.

$$\Delta\phi = \left(\frac{2\pi}{\lambda_{RF}}\right)\left\{M_{56}\delta + T_{566}\delta^2 + W_{5666}\delta^3 + U_{56666}\delta^4 + \cdots\right\}$$

After differencing the first pair of equations, expanding in powers of $\Delta\phi$, inserting the expansion for the phase difference and re-expanding, the following expression (through $O(\delta^4)$ only) for the final energy offset $\Delta\overline{E}$ at the dump of a particle with initial energy offset δ.

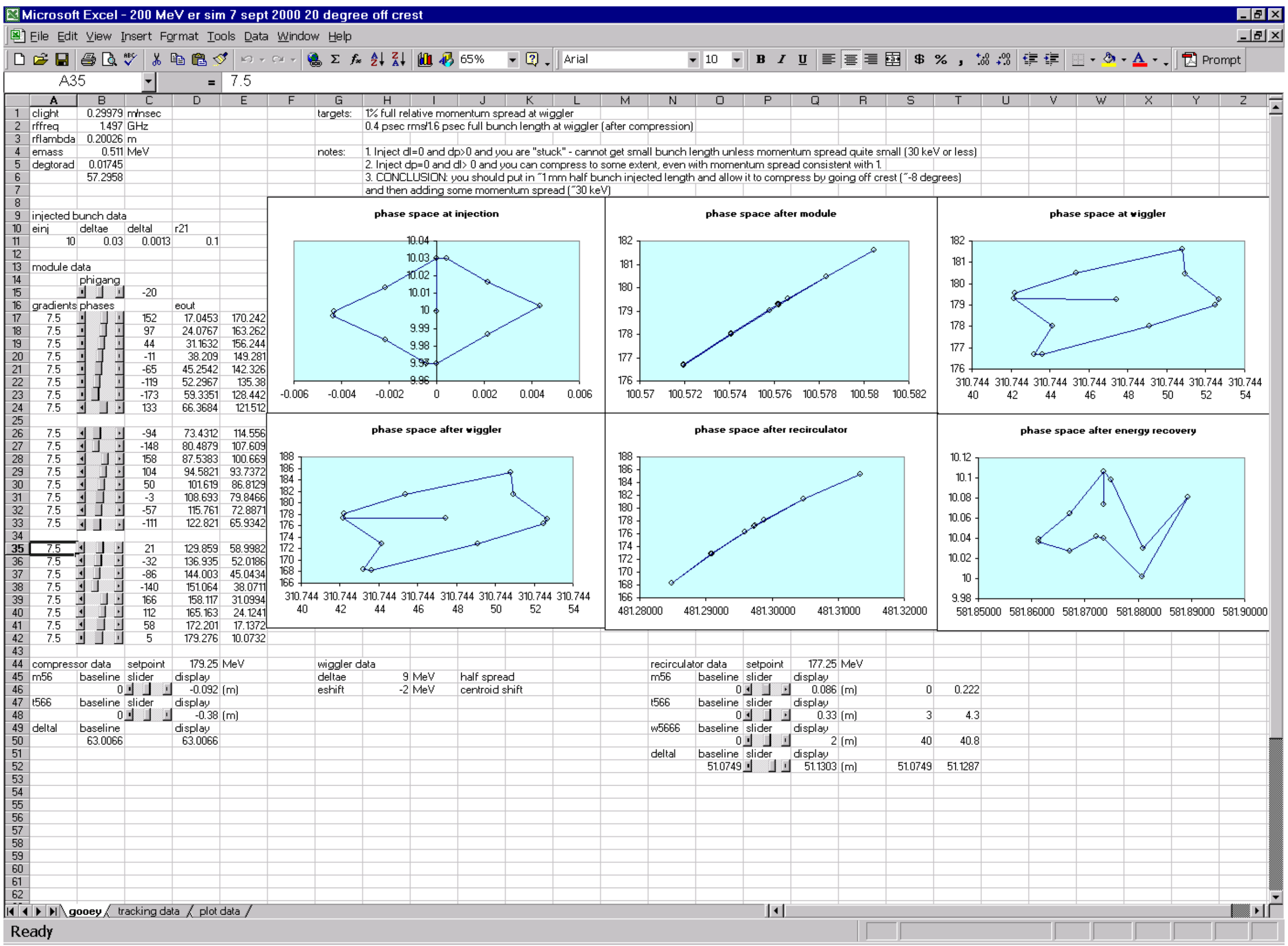
$$\Delta\overline{E} = E_0\delta + E_{linac}\Bigg\{\delta\left[\left(\frac{2\pi}{\lambda_{RF}}\right)M_{56}\sin\phi_0\right]$$
$$+\delta^2\left[\left(\frac{2\pi}{\lambda_{RF}}\right)T_{566}\sin\phi_0 + \frac{1}{2}\left(\frac{2\pi}{\lambda_{RF}}\right)^2(M_{56})^2\cos\phi_0\right]$$
$$+\delta^3\Bigg[\left(\frac{2\pi}{\lambda_{RF}}\right)W_{5666}\sin\phi_0 + \left(\frac{2\pi}{\lambda_{RF}}\right)^2 M_{56}T_{566}\cos\phi_0$$
$$-\frac{1}{6}\left(\frac{2\pi}{\lambda_{RF}}\right)^3(M_{56})^3\sin\phi_0\Bigg]$$
$$+\delta^4\Bigg[\left(\frac{2\pi}{\lambda_{RF}}\right)U_{56666}\sin\phi_0 + \left(\frac{2\pi}{\lambda_{RF}}\right)^2 M_{56}W_{5666}\cos\phi_0$$
$$+\frac{1}{2}\left(\frac{2\pi}{\lambda_{RF}}\right)^2(T_{566})^2\cos\phi_0 - \frac{1}{2}\left(\frac{2\pi}{\lambda_{RF}}\right)^3(M_{56})^2T_{566}\sin\phi_0$$
$$-\frac{1}{24}\left(\frac{2\pi}{\lambda_{RF}}\right)^4(M_{56})^4\cos\phi_0\Bigg]\Bigg\}$$

Operationally, the lattice parameters are selected (through proper excitation of trim quads, sextupoles, *etc.*) to eliminate each term in this expression. M_{56} nominally cancels E_0, T_{566} nominally cancels the $(M_{56})^2$ term, and so forth. This choice yields the following expressions for the required lattice properties.

$$M_{56} = -\frac{\lambda_{RF}}{2\pi}\left(\frac{E_0}{E_{linac}}\right)\frac{1}{\sin\phi_0}$$
$$T_{566} = -\frac{1}{2}\left(\frac{2\pi}{\lambda_{RF}}\right)(M_{56})^2\frac{\cos\phi_0}{\sin\phi_0}$$
$$W_{5666} = -\left[\frac{1}{6}+\frac{1}{2}\frac{\cos^2\phi_0}{\sin^2\phi_0}\right]\left(\frac{2\pi}{\lambda_{RF}}\right)^2(M_{56})^3$$
$$U_{56666} \propto \left(\frac{2\pi}{\lambda_{RF}}\right)^3(M_{56})^4, \text{etc.}$$

The source of the decapole order effect is thus made clear – it is due to a feed-up of the linear momentum compaction through the curvature of the RF waveform and the momentum-spread driven bunch length. A solution is also clear – accelerate and energy recover farther off crest. If ϕ_0 is moved from 10^o to $20\ ^o$, M_{56} is halved, and the required higher order terms fall off more quickly. U_{56666}, for example, goes down by a factor of 16. The resulting energy spread at the dump will then fall commensurately. This is in fact the case as illustrated by Figure 2, wherein the acceleration/energy recovery cycle of Figure 1 is repeated at an acceleration (energy recovery) phase shift of 20°. The resulting energy spread at the dump is smaller – on the order of 100 keV rather than 1 MeV, as are the required momentum compactions.

Figure 2: Demo longitudinal phase space management scheme applied to Upgrade with acceleration/energy recovery phase offset of 20°.



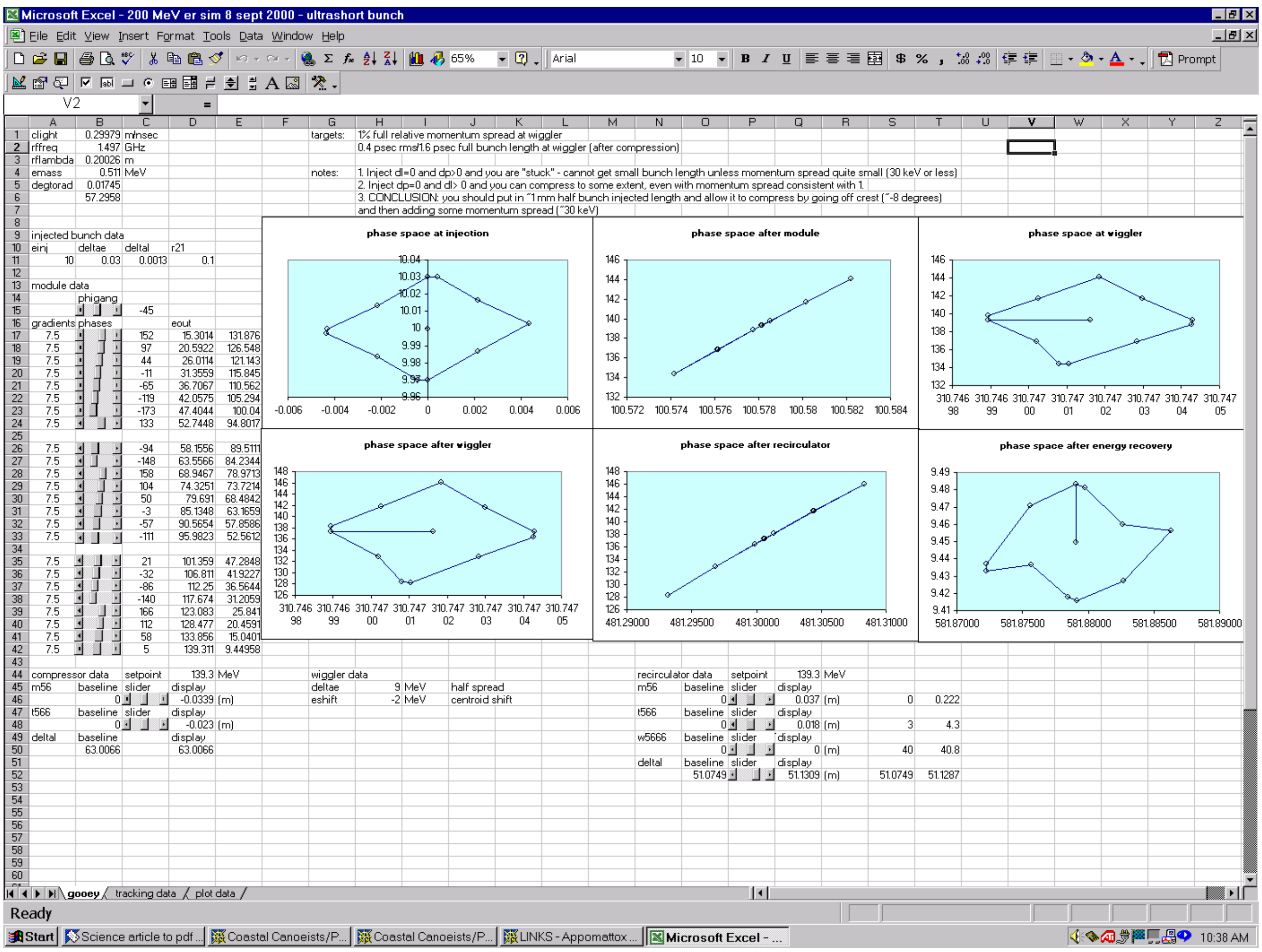
A secondary benefit of the larger (20°) phase offset is immediately obvious – the bunch length at the wiggler is of order 100 fsec full length!

Production/Energy Recovery of "Ultrashort" Bunches

In the previous section, acceleration and energy recovery were performed farther from the crest and trough of the waveform so as to increase the phase/energy correlation imposed by the RF, thereby reducing the required linear momentum compaction. The acceleration phase is, however, linked to the bunch length at the wiggler. By accelerating farther off-crest, the induced correlated momentum spread increases and the phase space becomes "longer and skinnier". When it is rotated upright at the wiggler, the bunch length can in principle be extremely short.

Figure 3 illustrates a potential configuration providing ~60 fsec full bunch lengths at the wiggler. This was achieved by accelerating ~45° off-crest and making an appropriate selection of M_{56} and T_{566} for the first recirculation endloop.

Figure 3: Upgrade longitudinal phase space management configured to provide ~60 fsec full bunch length at the wiggler. Acceleration/energy recovery phase offset is 45°.



In practice, a number of issues can impede the production and utilization of short bunches. Phase slippage between the bunch and the optical mode may (and for the IR Upgrade probably will [9]) preclude good longitudinal overlap over the complete length of a long wiggler. Energy variations and dipole buss ripple can cause temporal jitter at the wiggler. Bunch lengthening due to intrinsic momentum spread can degrade the bunch length. Beam transport system lattice aberrations can generate bunch length dilution. Finally, wakefield and CSR effects are aggravated by short bunch length and may result in intolerable momentum spread and emittance dilution. In the following section, we look at timing and time of flight issues; subsequently, we examine lattice and CSR effects at short bunch lengths in the Upgrade.

Tolerances – To set a scale, note that 1 fsec corresponds to 0.3 μm travel at the speed of light. If we allow variations of ~20% of 1 rms bunch length (0.2 x 15 fsec = 3 fsec in the preceding "best case"), we can tolerate ~1 μm path length error. Arc M_{56} values are typically of order 0.1 m or smaller.

Energy stability – couples to path length through the arc M_{56} as follows.

$$\delta l = \frac{\Delta E}{E} M_{56}$$

Setting a 1 μm limit on the path length error implies the following constraint on the energy stability.

$$\frac{\Delta E}{E} < \frac{\delta l}{M_{56}} \sim \frac{10^{-6}\text{ m}}{10^{-1}\text{ m}} = 10^{-5}$$

This has been achieved in CEBAF.

Dipole Field Stability – couples to path length through the arc M_{56} as follows.

$$\delta l = \frac{\Delta B}{B} M_{56}$$

Setting a 1 μm limit on the path length error implies the following constraint on the dipole field stability.

$$\frac{\Delta B}{B} < \frac{\delta l}{M_{56}} \sim \frac{10^{-6}\text{ m}}{10^{-1}\text{ m}} = 10^{-5}$$

This has been achieved in CEBAF and in the IR Demo, both of which use dipole power supplies with 10 ppm stability.

Velocity-Spread-Induced Bunch Lengthing – The beam momentum spread will induce bunch lengthening *via* velocity-spread induced time of flight variations. This effect is included in the simulations on which Figures 1-3 are based; the minimum-bunch-length observation point in these figures is at the center of the wiggler. The bunch length elsewhere in the wiggler can be estimated as follows.

$$\begin{aligned} \delta l &= \delta v\, t \\ &= \frac{\delta v}{v} L \\ &= -\frac{(\delta\gamma/\gamma)}{\gamma^2 - 1} L \end{aligned}$$

Here, L is the wiggler half length (~2.5 m); evaluating this at 200 MeV (γ~400) suggests the following limit on $\delta\gamma/\gamma \sim \sim \Delta E/E$.

$$\frac{\delta\gamma}{\gamma} < (\gamma^2 - 1)\frac{\delta l}{L} \sim 160000\,\frac{10^{-6}\ \mathrm{m}}{2.5\ \mathrm{m}} = 0.064$$

Thus, the full energy spread must be less than ~6.4%. Figure 3 suggests the full "best case" energy spread is of order (144 MeV – 134 MeV)/140 MeV = 7%, consistent with this constraint. Alternately, one can view such effects as a restriction on the wiggler length; an excessively long wiggler will result in dilution of the bunch length over some portion of the wiggler length. This is somewhat similar in nature to the optical mode/electron beam phase slip issue noted above, which will in fact probably preclude use of such short bunches in IR Upgrade applications [10].

Effect of lattice aberrations and CSR with ultrashort bunches – The bunch formation process detailed above has been simulated in DIMAD to ascertain the impact of beam transport system aberrations and to study, using a simple model [11], the effect of CSR on the beam.

Baseline simulation parameters were as follows:

- Revision 1.0 lattice, linac-to-wiggler transport with M_{56} = -0.034 m, and T_{566} = 0, appropriate for "best case" short bunch generation (Figure 3) with acceleration to ~140 MeV at ~45^{o} off crest.
- Beam parameters: 140 MeV, 10 mm-mrad normalized emittance, 135 pC/bunch; bunch at wiggler to be upright, 15 fsec (4.5 μm) rms length with relative *full* momentum spread of 7%.
- Beam was modeled by a 6-d truncated (at 2σ radially, for a 4 σ full width) Gaussian distribution of 10000 particles.

Simulation suggests that octupole correction of the lattice W_{5666} is necessary (because of the large relative momentum spread of $\sigma_{\delta p/p} \sim$ 1.75%) to achieve the desired short bunch at the wiggler. A single family consisting of a pair of modestly strong octupoles (k_3=-7.04/m^4) at each end of the π-bend provided an rms bunch length of ~5 μm at the wiggler (for a full-width 4σ distribution). In addition, lattice aberrations lead to a perceived emittance degradation of ~7% horizontally and ~5% vertically, possibly due to momentum-spread induced variations in the orbit and beam envelopes at the wiggler. When CSR was activated, the rms bunch length increased to ~6 μm at the wiggler and the horizontal emittance an additional grew ~5% beyond the dilution observed from aberrations alone.

These results suggest that neither lattice nor CSR effects need preclude production of very short bunch lengths at IR Upgrade parameter values. However, caution is warranted as the model used here is very rudimentary and observed sensitivity to simulation parameters (such as octupole strengths, initial distribution, *etc.*) implies significant effort may be required to get down to rms bunch lengths of a few tens of femtoseconds.

Conclusions

Appropriate choice of accelerating/energy recovery phases and lattice momentum compactions will allow generation of the desired beam properties at both the wiggler and the dump in the Upgrade. Though not necessarily immediately useful to the IR Upgrade, it may be possible to produce bunches with rms lengths of order of 10 fsec. These may be useful for driving a short wiggler to generate x-rays via Thomson-scattering.

Acknowledgements

Thanks to Geoff Krafft, Byung Yunn, Jim Boyce and Steve Benson for useful discussions of this topic.

References

[1] D. Douglas *et al.*, “Analytic Modeling and Lattice Scaling Relations for FEL Driver Accelerators”, CEBAF-TN-95-015, 19 November 1996; D. Douglas, “Lattice Issues Affecting Longitudinal Phase Space Management During Energy Recovery, Or, ‘Why the FEL Needs Sextupoles’”, JLAB-TN-98-024, 9 July 1998.

[2] D. Douglas, “Modeling of Longitudinal Phase Space Dynamics In Energy-Recovering FEL Drivers”, JLAB-TN-99-002, 14 January 1999; D. Douglas and G. Biallas, “Beam Transport Issues in the Fall/Winter 1998 FEL Run”, JLAB-TN-99-007, 9 April 1999; D. Douglas, “Beam Transport

Issues in the Winter/Spring 1999 FEL Run", JLAB-TN-99-008, 9 April 1999.

[3] D.Douglas, "The Jefferson Lab 1 kW IR FEL", to appear in the Proceedings of Linac2000, 21-25 August 2000, Monterey, CA.

[4] D. Douglas, "IR FEL Upgrade Driver Accelerator Design, Revision 1.0", JLAB-TN-00-013, 8 June 2000; D. Douglas *et al.*, "Driver Accelerator Design for the 10 kW Upgrade of the Jefferson Lab IR FEL", to appear in the Proceedings of Linac2000, 21-25 August 2000, Monterey, CA.

[5] *ibid.*

[6] D. Douglas, "Preliminary Estimate of CSR Effects in the IR Upgrade FEL Driver", JLAB-TN-00-017, 29 August 2000.

[7] D. Douglas, "Modeling of Longitudinal Phase Space Dynamics In Energy-Recovering FEL Drivers", *op. cit.*

[8] D. Douglas, "IR FEL Upgrade Driver Accelerator Design, Revision 1.0", *op. cit.*

[9] S. Benson, private communication.

[10] *ibid.*

[11] D. Douglas, "Preliminary Estimate of CSR Effects in the IR Upgrade FEL Driver", JLAB-TN-00-017, 29 August 2000.