The Effect of Field Inhomogeneities In Upgrade Injection Line Dipoles

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Introduction

Detailed error estimates were performed for the IR Demo design [1]. In the following, we apply similar methods to ascertain the impact on machine and beam performance of nonideal dipole fields in the Upgrade injection line.

Field Error Effects

In the following, we utilize nomenclature illustrated by Figure 1, which presents "ideal" and "real" field profile within a dipole magnet. These differ by an absolute error $\delta B(x)$ and a relative error $\delta B(x)/B(x)$. Such discrepancies between the ideal and actual fields lead to at least two categories of readily observable effects – steering errors and focusing errors. Steering errors evolve from deviations of the field integral through the magnet from the ideal value; focusing errors result from deviations of the gradient integral from the ideal. Limits imposed on such effects by performance requirements allow us to set tolerances on the allowable magnitude of absolute and relative errors. In the following, we will denote by T the tolerance for the relative field error $\delta B/B$ across a working aperture x_0: $T=\delta B(x_0)/B(x_0)$.

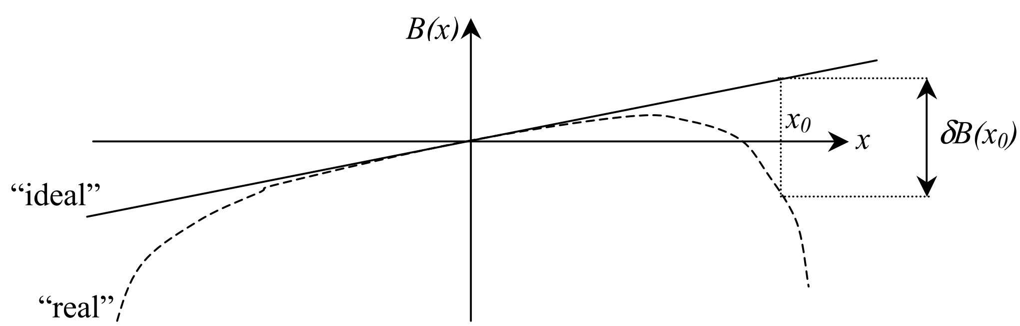

Figure 1: Ideal and nonideal field profiles within a dipole

<u>Steering Errors</u>: An error δB in field over a length L results in a field integral error $(\delta B)L$. This in turn steers the beam through an error angle $<x'>= (\delta B)L/B\rho$. Downstream of N such errors along a beam line, a betatron oscillation of the following rms amplitude will evolve (Ref. [1]).

$$\langle x\rangle = \sqrt{\frac{N}{2}}\,\beta\,\frac{(\delta B)L}{B\rho}$$

If $\delta B=\delta B(x)$ – if the field variation is position dependent (as in Figure 1), the beam amplitude response to steering upstream of the error source will exhibit a spurious response of magnitude $<x>$. This can, for example, lead to growth in spot size (different parts of the beam are steered differently), corrupt difference orbit measurements (beam response to steering is nonlinearly dependent on position) and impede convergence of orbit correction algorithms. The magnitude of the effect is readily related to the error tolerance $T=\delta B/B$ by noting $L/\rho=\theta$ is the bend angle.

$$\langle x\rangle = \sqrt{\frac{N}{2}}\,\beta\;T\theta$$

Evaluating this for IR Upgrade parameters ($\beta \sim 5$ m, $N = 3$, θ=1/3 radian) indicates $<x>\sim 2$ m × T. A 1% variation across the working aperture will thus lead to 2 cm spurious steering, a 0.1% variation leads to 2 mm spurious steering, and a 100 ppm variation leads to 0.2 mm spurious steering. Given the sub-millimeter resolution desired of difference orbit measurements and the millimeter level of orbit control desired through the accelerator, a 100 ppm field homogeneity specification is indicated.

<u>Focusing Errors</u>: An error $\delta B'$ in field gradient over a length L results in a gradient integral error $(\delta B')L$. This in turn leads to an error focal length $\delta(1/f)= (\delta B')L/B\rho$. Downstream of N such errors along a beam line, beam envelope and dispersion errors of the following rms amplitude will evolve (Ref. [1]).

$$\langle \Delta\beta/\beta\rangle \sim \langle \Delta\alpha\rangle = \sqrt{\frac{N}{2}}\,\beta\left\langle \delta\left(\frac{1}{f}\right)\right\rangle$$

$$\langle \Delta\eta\rangle = \sqrt{\frac{N}{2}}\,\beta\,\eta\left\langle \delta\left(\frac{1}{f}\right)\right\rangle$$

If the gradient error varies with position ($\delta B'=\delta B'(x)$), the beam phase space can experience distortions and or exhibit orbit dependence, with associated growth in spot size, machine/beam irreproducibility, and performance degradation [2]. The magnitude of this effect can be related to the error tolerance tolerance $T=\delta B/B$ by noting $L/\rho=\theta$ is the bend angle and taking $\delta B' \sim \delta B/x_0$ as an estimate of the gradient error. The associated error focal length is then $\delta(1/f)= T\theta/x_0$ and the induced errors are as follows.

$$\langle \Delta\beta/\beta\rangle \sim \langle \Delta\alpha\rangle = \sqrt{\frac{N}{2}}\,\beta\;T\theta/x_0$$

$$\langle \Delta\eta\rangle = \sqrt{\frac{N}{2}}\,\beta\,\eta\;T\theta/x_0$$

Using IR Upgrade parameters (N=3, β=5 m, θ=1/3 radian, and x_0 = 0.0375 m for the beam envelopes, N=1 [there's only 1 dipole at a dispersed point], β=5 m, η=0.4 m, θ=1/3 radian, and x_0 = 0.0375 m) gives $<\Delta\beta/\beta>\sim<\Delta\alpha>\sim 54\ T$ for the betatron sensitivity and $<\Delta\eta>\sim 12.5$ m × T for the dispersive sensitivity. A 1% dipole will thus lead to 50% errors in beam envelopes and 12 cm errors in dispersion at the end of the injection line; 1 ppt dipoles give 5% beam envelope errors, 12 mm dispersion error; 100 ppm dipoles will give 0.5% envelope errors and millimeter levels of dispersion error. The first is frightening, the second at roughly the level of diagnostic resolution and thus rather large for a single error source, and the last not unreasonable. A 100 ppm field homogeneity specification is, again, thus indicated.

References

[1] D. Douglas, " Error Estimates for the IR FEL Transport System", CEBAF-TN-96-035, 15 July 1996.

[2] For an object lesson on why dipole field inhomogeneity is a bad thing, see D. Douglas *et al.*, "In Search of Missing Dispersion: The Effect of Dipole End Fields in the FET Recirculation", CEBAF-TN-91-092, 10 December 1991.