# Some Features of the FEL Upgrade $\pi$-Bends 

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#### Abstract

Two features of the upgrade $\pi$-bends are discussed. These are 1) the field integral deviation required for generation of a specified path length change, and 2) the effect of a field integral shortfall induced by the presence of a path-length correction coil.


## Introduction

The FEL Upgrade driver accelerator [1] will utilize a pair of large dipoles in the recirculation end loops. These dipoles, colloquially referred to as the " $\pi$-bends", will move the central orbit through a $180^{\circ}$ circular arc on a 1 m radius over a beam momentum range of $80 \mathrm{MeV} / \mathrm{c}$ to $210 \mathrm{MeV} / \mathrm{c}$. The dipoles must therefore provide a core field of 2.6685 kG to 7.0048 kG with field quality specifications similar to those required in the IR Demo [2].

In order to conserve space in the relatively congested region adjacent to the $\pi$ bends, path length correction in the FEL Upgrade will be provided not by ganged horizontal correctors external to the dipoles (as in the IR Demo driver) but instead by correction coils embedded in the ends of the dipoles themselves [2]. A schematic plan view of poles and coils is provided in Figure 1. The first purpose of this note is to document an estimate of the field integral deviation that must be provided by these poleend correctors to produce a specified change in path length.


Figure 1: Configuration of path length correctors in Upgrade $\pi$-bends.

The coil used to drive the path length correction trim is to be placed in a notch machined into the dipole pole. This notch is parallel to the pole face, approximately 3 cm wide, and displaced inward approximately 10 cm from the pole [3]. Given the aforementioned magnet parameters (particularly the 1 m bend radius), we note that the choice of a notch parallel to the pole face is a fortuitous one, in that it makes the dipole essentially a $167^{\circ}$ dipole embedded between two $6^{1} 2^{\circ}$ parallel-faced dipoles. The radial focusing provided by the dipole is thus unaffected by changes in the path-length correctors - meaning that dispersion matching in the lattice will be decoupled from path length adjustment, though some variation will be induced in the vertical betatron matching.

Of further interest is the effect of the notch on the field in the magnet. Modeling indicates that this feature will locally depress the magnetic field, resulting in a field integral shortfall [5]. The second purpose of this note is to estimate the impact of this shortfall on the central orbit.

## Path Length Correction

The mechanism for path length correction is shown in Figure 2 [6], wherein quantities are shown accurately to first order in small perturbations. An impulse $x$ ' is applied to the beam at the entrance to the magnet, resulting in a displacement of the orbit. The resulting shifted orbit, still of radius $\rho$, now subtends an angle $\pi+2 x$, with a resulting orbit length $\left(\pi+2 x\right.$ ) $\times \rho$. This represents a path length differential of $2 x^{\prime} \rho$ from the nominal orbit length of $\pi \rho$.


Figure 2: Path length adjustment by steering in $\pi$-bend.

To phase the driver accelerator for energy recovery, we require this differential cover a range of $\pm \lambda_{\mathrm{RF}} / 2$. In the IR Demo, this correction was to be provided over the full energy range of the lattice (by design, 35-79 MeV). In the Upgrade, this has been limited to $\pm \lambda_{R F} / 2$ at only the turn-on momentum of $80 \mathrm{MeV} / \mathrm{c}$, so as to confine the scope of the correction to field integrals that can be manageably generated in the available space. Should such large corrections be required at higher energy, they will be generated by moves in the position of the $\pi$-bends.

Given the energy specification, the path length range, and noting that there are two $\pi$-bends available to generate the path length correction, we arrive at the following expression specifying the required field integral.

$$
\delta l=2 \text { dipoles } \times 2 x^{\prime} \rho(\text { from each dipole })=4\left(\frac{B l}{B \rho}\right) \rho= \pm \frac{\lambda_{R F}}{2}
$$

As the beam rigidity $B \rho$ is $33.3654 \mathrm{kG}-\mathrm{m} /(\mathrm{GeV} / \mathrm{c}) \times p$ we have the following result for the required field integral range.

$$
B l= \pm \frac{\lambda_{R F}}{8} \frac{33.3564 p}{\rho} \mathrm{kG}-\mathrm{m}
$$

Lengths are in meters, momentum is in $\mathrm{GeV} / \mathrm{c}$; inserting the 0.2 m RF wavelength, the bend radius of 1 m , and the $0.08 \mathrm{GeV} / \mathrm{c}$ specification momentum indicates a field integral range of order $\pm 6700 \mathrm{~g}$-cm is required. The present $\pi$-bend design provides twice this range, thereby allowing for a full wavelength of path length adjustment either short or longer at $80 \mathrm{MeV} / \mathrm{c}$, or half a wavelength at the design operating point of $145 \mathrm{MeV} / \mathrm{c}$ [7].

We note, as an aside, that the central orbit position midway through the dipole varies by $\pm x ’ \rho$ around the design trajectory as the path length is adjusted over the full dynamic range. For the full range of correction $x^{\prime}=B l / B \rho= \pm \lambda_{R F} / 8 \rho$, so with the above parameters we see that the beam will be steered by $\pm 25 \mathrm{mrad}$ and can thus occupy a positional span of $\sim 5 \mathrm{~cm}$ around the central orbit. With the nominal $7+\mathrm{cm}$ dispersive + betatron beam size and required 4 cm working aperture at this point, this suggests the horizontal stay-clear and good field must be 16 cm or greater. The present engineering design allows 20 cm [8], which would allow additional aperture to generate greater path length adjustment through use of the presently over-specified field integral. It would not, however, allow use of the full available range at $80 \mathrm{MeV} / \mathrm{c}$.

## Effect of Field Integral Shortfall

Additional path length adjustment capability (either broader range at low energy or full range at higher energy) is a virtuous feature for the design. It does, however, come at a cost. T. Schultheiss of AES has conducted a computational study of the $\pi$-bend design and has thereby determined that the notch accommodating the correction coil leads to a local depression of the magnetic field. A plot of his results - magnetic field as a function of distance along the design orbit, from 1 m upstream of the magnet to the $90^{\circ}$ point - is provided in Figure 3 [9].


Figure 3: B vs. position from 1 m upstream of magnet through magnet center on nominal orbit [9].

The field depression due to the notch is clearly visible with the minimum $\sim 17 \mathrm{~cm}$ into the magnet and a FWHM (full-width-half-minimum) of $\sim 8 \mathrm{~cm}$. The full depth of the notch is about 1.4 kG , or $20 \%$ of the 7 kG core field. The integral decrement is therefore of order of $0.1 \times 7 \mathrm{kG} \times 8 \mathrm{~cm}$ (FWHM field decrement $\times$ FWHM) or $5600 \mathrm{~g}-\mathrm{cm}$. Observe that this is roughly the full range excitation of the path length correction integral this is intended to allow, though it will be this large only when the magnet is excited to accommodate the full $210 \mathrm{MeV} / \mathrm{c}$ momentum.

At full momentum, this integral error corresponds to a point deflection of $\sim 8$ mrad. Propagating this forward to the pole face (nominally at $1 \mathrm{~m}, 17 \mathrm{~cm}$ away), the central orbit would emerge displaced approximately 1.4 mm from the central orbit. A more detailed picture is given in Figure 4. Here, the notch-induced field integral decrement $\delta B l$ is viewed as the source of an angular impulse $x{ }^{\prime}=\delta B l / B \rho$ imposed a horizontal distance $L$ from the exit face of the magnet. This kick displaces the circular arc of the orbit from the nominal center (the origin, $(0,0)$ ) to an offset location $\left(x_{n}, y_{n}\right)$. Given the geometry of Figure 4, this can cast in terms of magnet parameters and the impulse angle as follows.

$$
\begin{aligned}
& x_{n}=-L+\rho \sin \left(\theta+x^{\prime}\right) \\
& y_{n}=\sqrt{\rho^{2}-L^{2}}-\rho \cos \left(\theta+x^{\prime}\right)
\end{aligned}
$$



Figure 4: Orbit geometry with localized field error due to correction coil notch.
After the kick, the beam moves on the circle $\left(x-x_{n}\right)^{2}+\left(y-y_{n}\right)^{2}=\rho^{2}$. The intersection of this circle with the exit of the magnet (the line $x=0$ ) specifies the exit point $\left(x_{0}, y_{o}\right)$ of the perturbed orbit; it is as follows.

$$
\begin{aligned}
& x_{o}=0 \\
& y_{o}=y_{n}+\sqrt{\rho^{2}-x_{n}^{2}}
\end{aligned}
$$

The residual angular error in the exit orbit is then $\phi=\operatorname{atan}\left(\left(x_{n}-x_{o}\right) /\left(y_{n}-y_{o}\right)\right)$. From Figure 4, it is evident that the perturbed orbit offset from design is simply $y_{n}$. Expansion \& linearization of the above expressions in the small variable $x$ ' provides the following approximate results for the offset and angle.

$$
\begin{aligned}
& y_{n}=L x^{\prime} \\
& \phi=x^{\prime} \cos \theta
\end{aligned}
$$

As $\theta=\operatorname{asin}(L / \rho) \sim 0.17, \cos \theta \sim 1$; the results then return to the values estimated above a 1.4 mm orbit offset at a 8 mrad angle.

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## References

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