

A Refinement of the IR/UV Upgrade Quadrupole Systematic Multipole Specification

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Abstract

We review the tolerance for systematic multipoles in IR/UV upgrade quadrupoles and suggest a revision of the specification.

Introduction

Initial measurements [1] of a QX quadrupole prototype indicate the magnet (to be used in the IR/UV upgrade) does not meet the “0.1% @ pole radius” systematic multipole specification developed for the IR Demo [2]. These measurements (Figure 1) indicate that even after chamfering the dodecapole and icosapole moments each impose a 0.3% relative field deviation at the pole. We have therefore revisited this specification from the perspective of accumulated IR Demo operational experience; in the following, an analysis incorporating this information is used to evaluate the implications of a revision of the specification to match the larger observed relative field deviation.

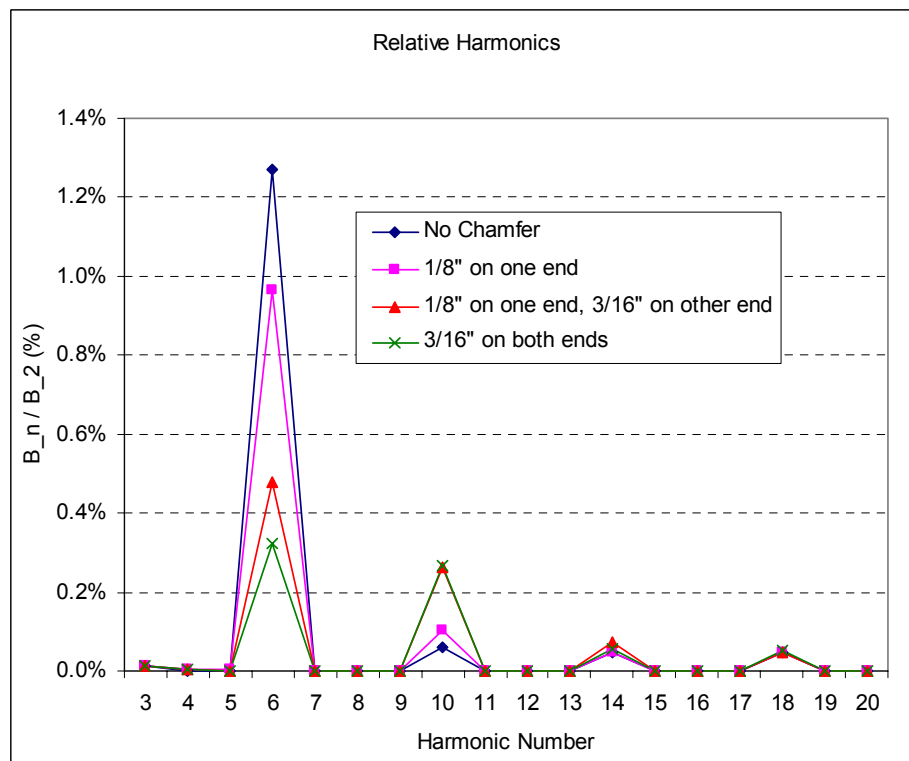


Figure 1: Results of QX measurements with various chamfers [3]. In this presentation, “harmonic number” n refers to the $2n$ -pole and characterizes a field variation of x^{n-1} .

Description of Field

Consider a quadrupole with a $2n+2$ -pole inhomogeneity. The field in such a magnet may, in the midplane, be described as follows.

$$B(x) = B_0'x + B_n x^n$$

Here B_0' is the ideal gradient; the $2n+2$ multipole coefficient B_n can be specified in terms of the relative field error at the pole (here at $x=a$) by observing that $B(a)=B_0'a+B_n a^n$, so that the $2n+2$ -pole tolerance $T_n=\Delta B(a)/B(a)=(B(a)-B_0'a)/B(a)$ is for modest inhomogeneities reasonably given by $T_n=B_n a^n/B_0'a$. Thus, the multipole coefficient is given in terms of T_n by

$$B_n = B_0' a T_n / a^n ,$$

with the field being given by

$$B(x) = B_0'x + B_0' a T_n \left(\frac{x}{a} \right)^n .$$

Note that $B_0'a$ is the ideal poletip field and the field error at position x in the midplane is $\Delta B(x)=B_0'a T_n (x/a)^n$.

Focusing effects are characterized by the quadrupole gradient, which, from the above expression is just the following.

$$B'(x) = B_0' + n B_0' T_n \left(\frac{x}{a} \right)^{n-1}$$

B_0' is the ideal gradient; the gradient error at position x in the midplane is $\Delta B'(x)=n B_0' T_n (x/a)^{n-1}$.

Effect of Field Errors

Errors in field and gradient affect beam orbit and envelopes. Field variations will result in nonlinear response of the orbit to steering; gradient errors will generate beam envelope mismatch and nonlinear phase space distortion. The magnitude of these effects may be controlled by appropriate specification of the tolerance T_n . To make this specification we will first determine the orbit and envelope sensitivity to such a multipole error. This will then be evaluated for both IR Demo and IR/UV Upgrade parameters. Based on a comparison of these results (and with the perspective of IR Demo operational experience), the application of the IR Demo specification – limiting individual systematic multipoles to a relative field error contribution of 0.1% at the pole – can be reviewed in the context of the Upgrade.

In the discussion below, we closely follow the treatment of Reference [3].

Orbit Effects – The response of the beam to a single kick $x' = Bl/B\rho$ is given by the transfer matrix element M_{12} from the kick to the end of the beam line.

$$x = M_{12}^{kick \rightarrow end} x'$$

The (linear) response of the beam to N kicks at different indexed locations “ n ” is given by the sum of the individual kicks.

$$x = \sum_{n=1}^N M_{12}^{n \rightarrow end} x'_n$$

The matrix element may be represented in terms of local beam envelope functions as

$$M_{12}^{n \rightarrow end} = \sqrt{\beta_n \beta_{end}} \sin \psi^{n \rightarrow end} \quad (1)$$

and, were the various kick errors under consideration uncorrelated, the above sum could be averaged over an ensemble of errors and across all betatron oscillations through the system to yield the following scaling law for the average beam response $\langle x \rangle$ at the end of a beam line to N kicks of average magnitude $\langle x' \rangle$ distributed along the line in which the average beam envelope is $\bar{\beta}$.

$$\langle x \rangle = \sqrt{\frac{N}{2}} \bar{\beta} \langle x' \rangle \quad (2)$$

In the case of the quadrupole field inhomogeneity-induced errors discussed above, the “average kick” is nominally

$$\langle x' \rangle = \left\langle \frac{\Delta Bl}{B\rho} \right\rangle = \frac{\langle B'_0 \rangle l}{B\rho} a T_n \left(\frac{x_{average}}{a} \right)^n = \frac{a}{\langle f \rangle} T_n \left(\frac{x_{average}}{a} \right)^n, \quad (3)$$

where $\langle f \rangle$ is the average quadrupole focal length in the beam line, a the quadrupole radial bore, and $x_{average}$ the average beam position in the quadrupoles. This displacement could, for example, be the result of remnant orbit errors, beam extent, or deliberate steering during difference orbit measurements. In such cases, equation (2) represents the (nonlinear) error in beam response to the stimulating effect – the beam offset in the quads in the presence of the errors described by equation (3). If due to remnant orbit, $\langle x \rangle$ characterizes the failure of the orbit to reproduce; in the case of beam extent, $\langle x \rangle$ characterizes the nonlinear growth in spot size, and in the case of diagnostic steering, $\langle x \rangle$ is the deviation of the orbit response from linearity. The resulting discrepancy can be summarized as follows.

$$\langle x \rangle = \sqrt{\frac{N}{2}} \bar{\beta} \frac{a}{\langle f \rangle} T_n \left(\frac{x_{average}}{a} \right)^n \quad (4a)$$

We note however that the kick errors in this situation are generated because the beam is off axis in nonideal quadrupoles; the kicks thus depend on the displacement and therefore are correlated with the betatron phase. Furthermore, inasmuch as the kicks have as their source systematic multipoles in the quadrupoles, the errors are systematic not only in betatron phase, but also in error amplitude (two quadrupoles with the same field and the same beam displacement will impose the same error). Consequently, the above expressions (which exhibit the “root- N ” growth rate characteristic of uncorrelated errors) will give at best an order of magnitude estimate of the effect and should in fact be considered a lower bound for the magnitude of the error effect.

A more meticulous treatment of the problem would average equation (1) over an ensemble of betatron orbits through the beam line while maintaining the betatron phase and magnetic field amplitude relationship. Such a treatment involves a sum over beamline-specific parameters and is thus beyond the scope of this note, in which we utilize only scaling relations and need only determine order-of-magnitude estimates of sensitivities. Here, we will simply replace each term in (1) by a value average to or characteristic of the beam line, and sum over the N error sources. Thus we set as averages

$$\begin{aligned}\langle \sqrt{\beta_n \beta_{end}} \rangle &= \bar{\beta} \\ \langle \sin \psi^{n \rightarrow end} \rangle &= 1/\sqrt{2}\end{aligned}$$

and take $\langle x' \rangle$ as in (3) to give the following result for the orbit deviation.

$$\langle x \rangle = \frac{N}{\sqrt{2}} \bar{\beta} \frac{a}{\langle f \rangle} T_n \left(\frac{x_{average}}{a} \right)^n \quad (4b)$$

This result for the orbit deviation may be regarded as something of an upper bound on the effect, inasmuch as any phase- or symmetry-induced cancellations are suppressed, being represented only through the average value taken for the sinusoid. It exhibits the “ N ” growth rate expect from the response to a systematic error.

Equations (4a) and (4b) provide a range within which we may expect the orbit response to a particular systematic multipole to lie. This response depends sensitively on the value of $x_{average}$. As noted above, this may be taken to be a remnant orbit error, the extent of the beam or the orbit displacement induced during a difference orbit measurement. In the IR Demo project, we viewed this displacement as the either the quad half aperture (50% of the quadrupole bore; $x_{average}=0.5a$) or, alternately, the full working aperture (80% of the quadrupole bore; $x_{average}=0.8a$) – both of which drive the relatively stringent constraint that T_n be less than 0.001 at the pole radius. The resulting machine performance was commensurately good, with little or no obvious nonlinear amplitude dependence in the orbit. We note however that the IR/UV upgrade quadrupole does not at present meet this specification on T_n . Moreover, the parameter set for the upgrade suggests (through both (4a) and (4b')) that the upgrade will be somewhat more sensitive to such errors than was the Demo. Table 1 presents the relevant parameters; at this time we simply need note that the Upgrade has 1.5 times as many quadrupoles and 1.5 times

larger bore than the Demo, while the “ β/f ratio” is about the same in both systems. For a fixed *relative* radius (such as 80% of the bore) we would therefore expect the Upgrade to be ~ 3 times more sensitive to multipole errors than the Demo was; coupled to the measurement suggesting the Upgrade QA quad has thrice the relative field error of the Demo QB quad, we would nominally expect an order of magnitude worse performance from the Upgrade.

Table 1: Machine/Magnet Parameters for IR Demo and IR/UV Upgrade

Parameter	IR Demo	IR/UV Upgrade
Type/Number of Quads, N	QB/28	QX/42
Average inverse focal length, $1/f$	0.6/m	0.5/m
Radial bore, a	2.125"/2 = 2.7cm	3.125"/2 = 3.97cm
Average beam envelope, $\bar{\beta}$	8 m	10 m
Maximum beam envelope, $\hat{\beta}$	40 m	50 m
Relative field error from dodecapole, T_5	0.001	0.003
Relative field error from icosapole, T_9	0.001	0.003

This view is however probably too pessimistic. The increase in bore of the Upgrade quad was not provided to increase the nominal working aperture (which remains at the ~ 2 cm radial value used in the Demo) but rather to provide additional physical aperture for halo management. The larger bore of the QX will then provide *better* field quality over the 2 cm radial working aperture than was available in the QB, even though the relative errors at the QX pole (Figure 1) are larger than those in the QB. Hence, working-aperture related issues such as remnant orbit reproducibility, diagnostic orbit linearity, and (as to be seen below) envelope mismatch and phase space distortion will actually be improved in the Upgrade. The only potential degradation from the ideal will be in phase space distortion in the halo. At present, we have set no detailed specification on this effect, other than to allow physical aperture for the propagation of the halo. Heuristically, this is still a better situation than that provided by the Demo QB quad – a distorted halo in clear aperture is much less likely to be a problem than an undistorted halo hitting the quad pole! We will in a subsequent section address the level of envelope mismatch to be expected both from the “in spec” QB and the “out of spec” QX.

At a more detailed level, we note that the circumstance used to set the IR Demo specification – an average displacement of 80% of the quad bore – is not a physically realizable situation. Because of modulation of the betatron amplitude (typically $\bar{\beta}$ differs significantly from $\hat{\beta}$), the average displacement of a betatron oscillation that reaches the pole radius in the quad at the point of maximum beam envelope function will be far less than “most” of the quad bore in most of the other quadrupoles. The betatron displacement through the beam line is given as follows.

$$x(s) = \sqrt{\beta(s)\varepsilon_0} \cos(\psi(s) + \delta)$$

By an appropriate choice of the initial betatron phase, the beam can be steered to x_{max} , the maximum excursion allowed in the beam line. This is limited by the quad bore a , occurs at the point of maximum beam envelope function $\hat{\beta}$, and defines the orbit emittance ϵ_0 .

$$a = \sqrt{\hat{\beta}\epsilon_0}$$

Thus

$$x(s) = a\sqrt{\beta(s)/\hat{\beta}} \cos(\psi(s) + \delta)$$

and substituting “average” values for $\beta(s)$ and the sinusoid provides the following estimate for the largest average displacement that allows loss-less beam transmission through the system.

$$x_{average} = \frac{a}{\sqrt{2}} \sqrt{\frac{\bar{\beta}}{\hat{\beta}}} \quad (5)$$

Noting (Table 1) that the average beam envelope is ~ 5 times smaller than the maximum in both the Demo and Upgrade, we see $\sim 1/3$ of the quad bore is a more physically reasonable average displacement for use in estimation of the effects of a particular multipole content.

Use of (5) in (4a & b) gives the following expressions bounding the anticipated error in orbit response to steering to the aperture limit.

$$\langle x \rangle = \sqrt{\frac{N}{2}} \bar{\beta} \frac{a}{\langle f \rangle} T_n \left(\sqrt{\frac{\bar{\beta}}{2\hat{\beta}}} \right)^n \quad (4a')$$

$$\langle x \rangle = \frac{N}{\sqrt{2}} \bar{\beta} \frac{a}{\langle f \rangle} T_n \left(\sqrt{\frac{\bar{\beta}}{2\hat{\beta}}} \right)^n \quad (4b')$$

We note that (4a' & b') assume steering upstream of the point of maximum beam envelope so as to generate marginal beam loss at that point only. It is of course operationally possible to induce loss at points of smaller beam envelope and to generate an average orbit excursion larger than that defined by (5). This is, for example, why the working aperture is defined to be of order 2 cm ($\sim 80\%$ of the QB bore and $\sim 50\%$ of the QX bore). With the enlarged working aperture, it is possible to steer locally to large amplitudes for diagnostic and operational purposes. In such cases, the quad count N will of course decrease – the beam will not see the inhomogeneous region of all quadrupoles when the large amplitude motion is locally constrained.

Focusing Effects – The effect of multipole-induced gradient errors on beam envelopes can be treated in much the same manner as were the orbit perturbations. A localized gradient error $\Delta B'$ corresponding to an error focal length $\delta(1/f) = \Delta B' l / B \rho = n T_n (x/a)^{n-1} / f$ leads to deviations in downstream lattice parameters as follows, where terms have meanings equivalent to their corresponding use above.

$$\frac{\Delta\beta}{\beta} = -\delta(1/f) \beta^{error} \sin 2\psi^{error \rightarrow end}$$

$$\Delta\alpha = \delta(1/f) \beta^{error} \left(\cos 2\psi^{error \rightarrow end} - \alpha^{end} \sin 2\psi^{error \rightarrow end} \right)$$

Unlike the orbit errors discussed above, these effects propagate at twice the phase advance. Proceeding as in the case of orbit errors, with superposition over N errors along a beam line, averaging without and with correlation across elements, betatron oscillations, and/or initial phases, and with substitution of the above expressions for gradient we get the following relations.

uncorrelated:

$$\left\langle \frac{\Delta\beta}{\beta} \right\rangle = \langle \Delta\alpha \rangle = \sqrt{\frac{N}{2}} \frac{\bar{\beta}}{\langle f \rangle} n T_n \left(\frac{x_{average}}{a} \right)^{n-1} \quad (6a)$$

correlated:

$$\left\langle \frac{\Delta\beta}{\beta} \right\rangle = \langle \Delta\alpha \rangle = \frac{N}{\sqrt{2}} \frac{\bar{\beta}}{\langle f \rangle} n T_n \left(\frac{x_{average}}{a} \right)^{n-1} \quad (6b)$$

The preceding discussion regarding $x_{average}$ applies in this case as well. If we seek the effect on the envelopes when steering to the limits of the free aperture, we find the following.

uncorrelated:

$$\left\langle \frac{\Delta\beta}{\beta} \right\rangle = \langle \Delta\alpha \rangle = \sqrt{\frac{N}{2}} \frac{\bar{\beta}}{\langle f \rangle} n T_n \left(\sqrt{\frac{\bar{\beta}}{2\hat{\beta}}} \right)^{n-1} \quad (6a')$$

correlated:

$$\left\langle \frac{\Delta\beta}{\beta} \right\rangle = \langle \Delta\alpha \rangle = \frac{N}{\sqrt{2}} \frac{\bar{\beta}}{\langle f \rangle} n T_n \left(\sqrt{\frac{\bar{\beta}}{2\hat{\beta}}} \right)^{n-1} \quad (6b')$$

In analogy to the prior discussion, equations (6) may be regarded as providing bounding values on the expected magnitude of envelope discussion when the beam is missteered through, or has significant spatial extent in, a quadrupole with a multipole error.

Estimates of Effects in IR Demo and IR/UV Upgrade

The effect of the 0.003 multipole-driven relative error in the Upgrade QX quad design can be put into context by comparison to the impact of the 0.001 multipole-driven

error occurring in the Demo QB quadrupole. This comparison is provided in Tables 2 and 3, wherein Table 1 parameters are used to evaluate equations (4) and (6) at various significant apertures to determine average single quadrupole sensitivities as well as the sensitivity for summed uncorrelated and correlated errors along the entire beam line.

Table 2: Orbit and envelope response to $n=5$ error. Effects given for individual quads and for uncorrelated and correlated sums over the entire beam line using Table 1 values.

Case		IR Demo		IR/UV Upgrade	
		Orbit: $\langle x \rangle$ (m)	Beam Envelopes $\langle \Delta\beta/\beta \rangle = \langle \Delta\alpha \rangle$	Orbit: $\langle x \rangle$ (m)	Beam Envelopes $\langle \Delta\beta/\beta \rangle = \langle \Delta\alpha \rangle$
$x_{average}=80\%$ of aperture	single quad	3.0E-05	7.0E-03	1.4E-04	2.2E-02
	uncorrelated	1.6E-04	3.7E-02	8.9E-04	1.4E-01
	correlated	8.4E-04	1.9E-01	5.8E-03	9.1E-01
$x_{average}=\text{working aperture}$ (2 cm radial)	single quad	2.0E-05	5.1E-03	1.4E-05	3.4E-03
	uncorrelated	1.1E-04	2.7E-02	8.9E-05	2.2E-02
	correlated	5.7E-04	1.4E-01	5.7E-04	1.4E-01
$x_{average}=\text{half aperture}$	single quad	2.9E-06	1.1E-03	1.3E-05	3.3E-03
	uncorrelated	1.5E-05	5.6E-03	8.5E-05	2.1E-02
	correlated	8.0E-05	3.0E-02	5.5E-04	1.4E-01
$x_{average}=\text{max accessible aperture}$	single quad	2.9E-07	1.7E-04	1.3E-06	5.3E-04
	uncorrelated	1.5E-06	9.0E-04	8.6E-06	3.4E-03
	correlated	8.1E-06	4.8E-03	5.6E-05	2.2E-02

Table 3: Orbit and envelope response to $n=9$ error.

Case		IR Demo		IR/UV Upgrade	
		Orbit: $\langle x \rangle$ (m)	Beam Envelopes $\langle \Delta\beta/\beta \rangle = \langle \Delta\alpha \rangle$	Orbit: $\langle x \rangle$ (m)	Beam Envelopes $\langle \Delta\beta/\beta \rangle = \langle \Delta\alpha \rangle$
$x_{average}=80\%$ of aperture	single quad	1.2E-05	5.1E-03	5.7E-05	1.6E-02
	uncorrelated	6.5E-05	2.7E-02	3.7E-04	1.0E-01
	correlated	3.4E-04	1.4E-01	2.4E-03	6.7E-01
$x_{average}=\text{working aperture}$ (2 cm radial)	single quad	6.2E-06	2.8E-03	8.8E-07	4.0E-04
	uncorrelated	3.3E-05	1.5E-02	5.7E-06	2.6E-03
	correlated	1.7E-04	7.8E-02	3.7E-05	1.7E-02
$x_{average}=\text{half aperture}$	single quad	1.8E-07	1.2E-04	8.2E-07	3.7E-04
	uncorrelated	9.5E-07	6.3E-04	5.3E-06	2.4E-03
	correlated	5.0E-06	3.3E-03	3.5E-05	1.6E-02
$x_{average}=\text{max accessible aperture}$	single quad	2.9E-09	3.1E-06	1.3E-08	9.5E-06
	uncorrelated	1.5E-08	1.6E-05	8.6E-08	6.2E-05
	correlated	8.1E-08	8.6E-05	5.6E-07	4.0E-04

Observations, Discussion & Conclusions

Tables 2 and 3 highlight regions of the multipole parameter space that should raise concern about machine performance. Orbit and envelope response that is beyond the “few percent effect” criterion of Reference [2] are highlighted – with stronger response called out in darker shades. We see that the Upgrade, with its higher multipole content and larger number of quads, typically manifests larger orbit and envelope distortion at a given relative aperture (by a factor of 4 or 5) than does the IR Demo. However, at a specific physical aperture – such as the 2 cm working aperture – the Upgrade actually performs better than the Demo. This is, of course, a consequence of the larger quad aperture to which the multipole field contribution is scaled. Furthermore, though the Upgrade does not maintain beam quality as well as the Demo at the “max accessible aperture” (a relative criterion), the Upgrade performance at “max accessible aperture” does surpass Demo performance at both “working aperture” and “half aperture”. These served as specification radii for the Demo, and we may expect the Upgrade performance to match or surpass that of the Demo over these apertures.

As noted above, the beam cannot sample either 80% of the aperture or the entire 2 cm radial working aperture throughout the beam line. Performance over the “max accessible aperture (about 1/3rd of the physical aperture for both Demo and Upgrade parameters) is therefore the more meaningful figure of merit. Over this region, the Upgrade clearly meets the performance criteria established for the Demo. The orbit wanders on the order of (several) tens of microns – only a small fraction of the maximum offset (the 3 cm physical aperture) – and the beam envelopes vary by only a few percent. The resulting nonlinear variations in machine performance, though not zero, are at worst at the threshold of observation. This is consistent with Demo design and specification philosophy, and thus provides a reasonable specification.

There is an additional freedom in the specification of the Upgrade multipole content. The IR Demo is essentially all energy recovery recirculator. There is only a single quad telescope upstream of the FEL. The Upgrade, in contrast, comprises two functionally modular (though linked) systems – the transport to the wiggler, with $\sim 2/3^{\text{rd}}$ s of the quadrupoles, and the energy recovery transport, with about $1/3^{\text{rd}}$ of the quads. The effect quad number through the Upgrade transport can thus, for the purposes of these estimates, be reduced in principle by a factor of $1\frac{1}{2}$ to 3 – thereby improving the Table 2 and 3 estimates commensurately. This reinforces the above view that some relaxation of the specification is acceptable.

The effects of higher harmonics scale as $(x/a)^n$ (orbit) or $n(x/a)^{n-1}$ (envelopes). These are summarized for the first few harmonics at the “worst case” radius – 80% of the pole radius – in Table 4.

Table 4: Scaling of Higher Harmonics at $x=0.8a$

n	$(x/a)^n$	$n(x/a)^{n-1}$
5	0.33	2
9	0.13	1.5
13	0.05	0.9

Orbit effects thus converge fairly strongly, dropping by a factor of 2 to 3 over each harmonic; envelope effects fall off less rapidly, but nonetheless exhibit less sensitivity in the higher harmonics. We can therefore conclude the behavior shown in Figure 1 – where contributions from higher harmonics either roll off or at least do not exceed those from the lower – will not lead to pathological behavior.

These considerations suggest the relaxation of the systematic multipole specification by a factor of 3 – which will bring the existing quad into compliance – will not have significant adverse effect on Upgrade machine performance. Though the reduction in field quality will degrade large amplitude behavior to some extent, our primary concern is not with detailed control of such beam (that is, halo), rather only with transmission thereof. Over the 2 cm radial working aperture (which in the QX is half the aperture) and the “maximum accessible aperture”, the Upgrade performance should exceed that of the Demo at the Demo working aperture or the Demo half aperture. In analogy with Reference [2], we therefore set the following specification. Note that the random tolerance is unchanged from IR Demo levels.

Specification of QX Multipole Content

- systematic dodecapole should contribute relative field error of no more than 0.3% at the pole:

$$\frac{\Delta B_5}{B} < 10^{-3} \text{ at the pole}$$

- systematic icosapole should contribute relative field error of no more than 0.3% at the pole:

$$\frac{\Delta B_9}{B} < 10^{-3} \text{ at the pole}$$

- random multipoles should contribute a total rms relative field error of no more than 0.1% at half-aperture (with a 2σ cutoff assumed). Any individual random multipole should contribute an rms relative field error of no more than 0.1% at the pole (again with 2σ cutoff assumed):

$$\sum_n \left\langle \frac{\Delta B_n}{B} \right\rangle_{rms} < 10^{-3} \text{ at half aperture}$$

Acknowledgments

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References

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- [3] J. Karn, *op. cit.*