# A Skew-Quad Eigenmode Exchange Module (SQEEM) for the FEL Upgrade Driver Backleg Transport 

D. Douglas


#### Abstract

We describe a five-skew-quad FODO module that can be non-invasively embedded in the 3 F region of the FEL Upgrade Driver. It orthogonally interchanges decoupled incoming horizontal and vertical phase sub-spaces. With proper upstream and downstream matching, it will transparently out-couple incident steering errors (such as HOM-driven steering that could induce the BBU instability) while leaving betatron matching unperturbed.


## Motivation

Installation of the FEL Zone 3 cryomodule, with its documented high-Q HOMs [1], has led to concern that the Driver may become susceptible to the BBU instability. Todd Smith has provided considerable advice on management of this instability. In particular, he suggests use of $\mathrm{H} / \mathrm{V}$ phase space interchange and/or point-to-point transverse focusing as means of control of instability thresholds [2]. In this note, we describe a noninvasive modification to the IR Upgrade Driver transport system that will allow implementation of the former method. In a subsequent note we describe a "phase trombone [3]" that can be used to implement the latter [4].

## Description

The 3F "backleg" region of the Upgrade Driver is a FODO quadrupole array nominally comprising six periods of $90^{\circ}$ transport. It is sketched in Figure 1. We propose installing five skew and two additional normal quads as shown in Figure 2. The skew quads will be excited to form a so-called five-quad rotator, which will exchange the horizontal and vertical phase spaces. The central four normal quads will be de-excited during rotator operation, and the upstream and downstream normal quads will be used as betatron matching telescopes. These will allow transverse matching of the phase spaces across the rotator, so that the 3 F region, in its entirety, remains transparent to the rest of the machine (save for the exchange of transverse emittances and incoming steering).


Figure 1: Nominal 3F FODO quad array.

Figure 2: 3F FODO quad array during rotator operation. Center normal quads (green, ghosted) are de-excited. Additional normal quads (blue) are made available in the upstream and downstream array to support matching. Five skew quads (violet) in a symmetric FODO array are used to form the rotator.

The additional quads are centered at the mid-point of the drifts between existing quads. The "upstream normal quad telescope" thus has a single FODO cell and a triplet, while the "downstream normal quad telescope" has a triplet followed by a single FODO cell. The rotator comprises a symmetric pair of FODO cells.

Given this author's criminally weak understanding of coupled betatron motion, much of what follows is probably nonsense, but the codes give the answer we're groping for, so here goes! Coupled transverse motion is parameterized by 10 terms: the 6 "normal" mode betatron functions (beta, alpha, and phase for each transverse plane), two coupling angles and two coupling amplitudes. In principle, one might then fantasize that the coupling angles could be made $90^{\circ}$ (thereby mapping x to y and y to x ) using a two parameter system [5]. Were this done using, say, only two skew quads, the behavior might however be untoward because of strong focusing. We therefore adopt a more conservative viewpoint, and use a pair of skewed FODO cells. By properly fitting the quad strengths, we can make the $6 \times 6$ transport matrix for the rotator take the following form, wherein each entry is a $2 \times 2$ sub-block; 0 and $I$ have the obvious meaning.

$$
\left(\begin{array}{ccc}
0 & M & 0 \\
M & 0 & 0 \\
0 & 0 & I
\end{array}\right)
$$

The $2 \times 2$ sub-block transport matrix $M$ is the same for both exchanges ( x to $\mathrm{y}, \mathrm{y}$ to x ). For the parameters at hand (3F FODO half-pitch (quad center-to-quad center distance) of $\sim 1.58 \mathrm{~m}$ and QX quadrupoles of length 0.15 m ) $M$ turns out to be a betatron stable matrix with "tune" around $42^{\circ}$ and matched beta of about 4.9 m . Given the reflective symmetry of the system, the matched alpha is zero.

Thus, the rotator cleanly exchanges the horizontal and vertical phase spaces. If the incoming beam is matched (using the upstream telescope) to the acceptance beta/alpha of $4.9 \mathrm{~m} / 0$ radians in both planes, it will image to the same values downstream. These outputs can then be matched back to the accelerator acceptance (specifically, the envelopes at the start of the 4 F region), so that the entire 3 F region is transparent to the rest of the machine - save for the exchange of horizontal and vertical emittance, the interchange of incident steering, and possible differences in phase advance, nothing is changed.

## Performance

A DIMAD model of this system behaves as expected from the above discussion. Use of the $1^{\text {st }}$ four quads of the region provides a match of typical operationally observed beam envelopes from the start of the backleg to the rotator acceptance; use of the final four gives a compensatory match from the rotator to the end of the backleg. As expected, emittances and orbits are interchanged, but envelopes are unchanged. Table 1 gives the matrices for the rotator and the full transport. Figure 3 gives the beam spot sizes (assuming incident horizontal and vertical normalized emittances of, respectively, 30 and 15 mm -mrad). Figure 4 shows principle rays through the rotator and the entire backleg; cosine-like and sine-like launches for each of the two transverse planes map completely from the launch plane into the other transverse plane. Table 2 gives the layout coordinates for each component of the system.

All quad strengths were well within the excitation range of QX magnets. Though the above solution did not use the additional normal quads, we nonetheless recommend installation of these magnets. First, they will provide greater dynamic range for the match - our solution is simply an "existence proof" using a single set of incident and target envelopes from a particular machine setup. Secondly, one of the matching quads is getting a tad strong; an additional quad would help alleviate this. Finally, though only four magnets are needed in principle to provide an envelope match, additional magnets are needed to provide said match while simultaneously managing phase advance. By installing the extra two magnets, we can (at least in principle) control betatron phase advance around the ring as well. This opens another avenue for BBU control - point-topoint imaging - in the context of transverse plane interchange.

We have done a simple momentum scan (Figure 5) using the DIMAD "rmatrix" and "line geometric aberration" analysis tools. The first tool indicates the momentum dependence of the transverse matrix is modest across the momentum range appropriate to this region (Figure 5a). In particular, decoupling (magnitude of the $x x$ and yy submatrix elements) remains small. This is confirmed by examining the submatrix determinants (Figure 5b), which show the system remains essentially fully coupled across the momentum range of interest (on-diagonal submatrices have (near) zero determinant). It is therefore reasonable to propagate the initial phase space using decoupled betatron envelope expressions and the off-diagonal matrix elements. Results of this calculation are shown in Figure 5c, and suggest the momentum sensitivity of the system is acceptable. This conclusion is further confirmed by the "line geometric" simulation, which gives very similar results (Figure 5d). As it is based on ray-tracing the coupled lattice and fitting to the ray-traced data, it includes any potential momentum-offset/coupling crossterms

A final caveat - this analysis was done across the backleg only. It may be prudent to repeat this for the entire linac-to-wiggler transport to ascertain if there are constructive or destructive interferences amongst the arc, FODO transport, and matching telescope elements. And we'll do it, in the copious spare time (as Steve Benson discussed in his review presentation [6]) we have.

It will be entertaining to see what impact operation of this module may have on potential collective effects other than BBU. Given, for example, potential collusions between arc and chicane, will it suppress or enhance CSR?

## Acknowledgments

Thanks to several people for very useful discussions on this topic. Thanks to even more people for listening patiently while I blathered on about this with a coherence level deemed, at best, marginal. In particular, Jim Boyce, Michelle Shinn, George Neil, Steve Benson, Rich Evans, Eduard Pozdeyev, Dick Walker, Carlos Hernandez-Garcia, Kevin Jordan, Joe Gubeli, Fred Dylla,... OKAY, THE WHOLE FEL TEAM!!! were (was) victimized. Thanks to Jim Boyce for help with the graphs. A special thanks to George Biallas for his pathological obsession with the notion of putting skew quads into the driver. It made "Fatal Attraction" look like "The Sound of Music", and were it not for his nearly delusional enthusiasm, the project would not have been done before it ever started.

## References

[1] Merminga, Pozdeyev, Simrock Tennant, and probably et al.. At least, they were in the gallery and made some FLOG entries about it, and so at least share in the guilt.
[2] Rand, R.E. and T.I. Smith, "Beam Optical Control of Beam Breakup In A Recirculating Electron Accelerator", Particle Accelerators, Vol. 11, pp.1-13, 1980.
[3] According to A. Bogacz (private communication) such a system has been installed and tested in CEBAF.
[4] Application of appropriate tuning algorithms to the Driver 3F region FODO transport allows independent point-to-point focusing of either transverse plane from any cavity in zone 3 back to itself. Douglas, D., "A Phase Trombone Based on the IR Upgrade Driver Backleg FODO Transport", to appear as a JLab Technical Note.
[5] Eduard Pozdeyev has reminded me that an axially symmetric incident beam can in fact be managed with a single parameter system - a solenoid. See reference 2.
[6] S. Benson, presentation at the IR Upgrade RF Zone 3 Preinstallation Review, May 6, 2004.

## Table 1: Transport matrices

Table 1a: For rotator
FIRST ORDER MATRIX

| - | $0.4113260 \mathrm{E}-15-$ | -0.2335914E-14 | $0.7397106 \mathrm{E}+00$ | $0.3300006 \mathrm{E}+01$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | -0.1515715E-15 - | -0.4432218E-15 | -0.1372204E+00 | $0.7397106 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.7397106 \mathrm{E}+00$ | $0.3300006 \mathrm{E}+01$ | $0.5204170 \mathrm{E}-15$ | -0.1387779E-14 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| - | -0.1372204E+00 | $0.7397106 \mathrm{E}+00$ | -0.1942890E-15 | $0.3330669 \mathrm{E}-15$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1000000 \mathrm{E}+01$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1000000 \mathrm{E}+01$ |
| SECOND ORDER TERMS |  |  |  |  |  |  |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.4312257 \mathrm{E}-01$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1271229 \mathrm{E}+02$ |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.2991207 \mathrm{E}+01$ |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.6908279E+01 |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.5479328 \mathrm{E}+00$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.4312257 \mathrm{E}-01$ |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1053725 \mathrm{E}+01$ |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.2991207 \mathrm{E}+01$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.2991207 \mathrm{E}+01$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.6908279E+01 |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.4312257 \mathrm{E}-01$ |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1271229 \mathrm{E}+02$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1053725 \mathrm{E}+01$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.2991207 \mathrm{E}+01$ |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.5479328 \mathrm{E}+00$ |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.4312257 \mathrm{E}-01$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.5442012 \mathrm{E}+00$ |  | $0.8118421 \mathrm{E}+00$ | $0.4112290 \mathrm{E}+00$ | $0.1776283 \mathrm{E}+01$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  | $0.8711089 \mathrm{E}+01$ | $0.1776283 \mathrm{E}+01$ | -0.9261108E+01 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  | $0.5442012 \mathrm{E}+00$ | $0.8118421 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  | $0.8711089 \mathrm{E}+01$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |

Table 1b: For entire matched backleg
FIRST ORDER MATRIX

| - | -0.1359559E-16 - | -0.1861384E-14 | $0.6727265 \mathrm{E}+00$ | $0.1541859 \mathrm{E}+01$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | -0.3690335E-15 | $0.5261284 \mathrm{E}-15$ | -0.4889226E+00 | $0.3658993 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.1945898 \mathrm{E}+01$ | $0.4876622 \mathrm{E}+00$ | $0.1197901 \mathrm{E}-14$ | -0.1414935E-14 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.1506120 \mathrm{E}+00$ | $0.5516464 \mathrm{E}+00$ | -0.3688277E-15 | -0.2639629E-15 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1000000 \mathrm{E}+01$ | $0.0000000 \mathrm{E}+00$ |
| - | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1000000 \mathrm{E}+01$ |
| SECOND ORDER TERMS |  |  |  |  |  |  |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.5455416 \mathrm{E}+01$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.6903013 \mathrm{E}+01$ |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.4172146 \mathrm{E}+02$ |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $-0.7443658 \mathrm{E}+02$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.1325088E+00 |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.1444130E+01 |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.4460382E+01 |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.2118333 \mathrm{E}+02$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.7378630 \mathrm{E}+00$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.7845317E+01 |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.3419775E+01 |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.8174716 \mathrm{E}+01$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.0000000 \mathrm{E}+00$ |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.2019514 \mathrm{E}+02$ |
|  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.4244707 \mathrm{E}+01$ |
|  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | $0.1060217 \mathrm{E}+01$ |
|  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ | -0.4980900E+00 |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |
| $0.1973982 \mathrm{E}+02$ |  | $0.9514816 \mathrm{E}+01$ | $0.2578134 \mathrm{E}+01$ | -0.2200443E+01 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  | $0.3333421 \mathrm{E}+01$ | $0.2403534 \mathrm{E}+01$ | -0.4752453E+01 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  | $0.8534516 \mathrm{E}+01$ | -0.2289699E+02 | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  | $0.3023108 \mathrm{E}+02$ | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  | $0.0000000 \mathrm{E}+00$ | $0.0000000 \mathrm{E}+00$ |
|  |  |  |  |  |  | $0.0000000 \mathrm{E}+00$ |

Figure 3: Beam spots and projected emittances through system



Figure 4: Principle rays
Figure 4a: Through rotator


Figure 4b: Through entire backleg
prinicpal rays through entire backleg


Figure 5: Momentum scan
Figure 5a: Matrix elements as function of momentum. Decoupling remains small (diagonal subblock elements remain near zero).


Figure 5b: Submatrix determinants as a function of momentum. On momentum matrices have small determinant, confirming the system remains fully coupled ( $x$ to $y, y$ to $x$ ) across the full momentum range


Figure 5c: Variation in decoupled envelopes as a function of momentum, established by propagation using nominal initial envelopes and off-momentum, off-diagonal matrix elements of Figure 5a.


Figure 5d: Beam envelope variation with momentum, established using raytracing through fully coupled system and fitting of beam ellipses to imaged rays.


