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Reflections on Rotators,
Or,
How to Turn the FEL Upgrade 3F Skew Quad "Rotator" Into a Skew Quad Rotator

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Introduction

A previous note [1] describes a simple skew quad system that can be used to exchange horizontal and vertical motion in the IR Upgrade recirculator and thereby influence BBU thresholds. This system was devised and installed with the assumption that the relevant HOMs driving the instability would stimulate either pure horizontal or pure vertical deflections, and could thus be counteracted simply by out-coupling the kick to the other transverse plane.

After successful operation of this system – with an increase of the observed BBU threshold from around 2 mA to over 8 mA – it has been noted that it is in fact not a true "skew quad rotator" but rather is a "reflection". It has further been noted that suppression of BBU driven by HOMs of arbitrary polarization is desirable [2]. A thorough quantitative investigation of the relationship between "reflections" and "rotations" is underway [3]; here, we give only a simple method for converting the specific reflection installed in the IR Upgrade into a true rotation.

Reflections on Rotators

The 3F "rotator" described in Reference [1] simply interchanges horizontal and vertical phase spaces, imaging each identically into the other. This is characterized by the following transfer matrix, wherein the 2 x 2 off-diagonal sub-blocks M are identical and in this case betatron stable, imaging an upright ellipse (α=0) characterized by beam envelope β back to itself in the other transverse plane.

$$\begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}$$

$$\text{with } M = \begin{pmatrix} \cos\psi & \beta\sin\psi \\ -\frac{1}{\beta}\sin\psi & \cos\psi \end{pmatrix}$$

This will map a horizontal kick into the vertical plane, and a vertical kick to the horizontal, and thus should suppress BBU driven by modes with these specific polarizations. However, it will equally clearly map a *diagonal* displacement and kick *(x,x'y,y')* back into a similar diagonal displacement and kick, thus potentially failing to suppress BBU driven *via* an HOM of arbitrary polarization. We were however gently reminded by Todd Smith [4] that earlier work [5] shows a true rotation – with a matrix interchanging both planes with equal amplitudes but differing rotational *phasing* – will provide the desired suppression. The task at hand is thus to convert the above matrix to one with the following form using components installed in the 3F region of the Upgrade Driver

$$\begin{pmatrix} 0 & \mp M \\ \pm M & 0 \end{pmatrix}$$

Introducing the desired sign differences immediately suggests use of the existing "rotator" led (and followed) by upstream (and downstream) decoupled transport of rational phase advance – quarter wave, half wave, or full wave, to produce an overall system with the desired rotational phase properties. We note that that half- or full-wave transport is described by $-I$ or I, and quarter-wave can be described by the following matrix, where the leading notation is deliberately chosen to suggest a potential approach.

$$i = \sqrt{-I} = \begin{pmatrix} 0 & \beta \\ -\dfrac{1}{\beta} & 0 \end{pmatrix}$$

Inspection of the situation then suggests a solution: Lead the "rotator" with a matrix that is negative identity in one plane (half-wavelength) and a quarter wavelength in the other, trail it with a similar module with the planes exchanged. The net transfer matrix is then either

$$\begin{pmatrix} I & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} 0 & M \\ iM & 0 \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} 0 & M \\ iMi & 0 \end{pmatrix}$$

or

$$\begin{pmatrix} i & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & iM \\ M & 0 \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & iMi \\ M & 0 \end{pmatrix}$$

The product of the 2 x 2 sub-matrices is evaluated under the assumption that the β in i is the betatron stable "matched envelope" characteristic of the existing eigenmode exchange module.

$$iMi = \begin{pmatrix} 0 & \beta \\ -\dfrac{1}{\beta} & 0 \end{pmatrix}\begin{pmatrix} \cos\psi & \beta\sin\psi \\ -\dfrac{1}{\beta}\sin\psi & \cos\psi \end{pmatrix}\begin{pmatrix} 0 & \beta \\ -\dfrac{1}{\beta} & 0 \end{pmatrix} = \begin{pmatrix} -\cos\psi & -\beta\sin\psi \\ \dfrac{1}{\beta}\sin\psi & -\cos\psi \end{pmatrix} = -M$$

The resulting matrices thus have the desired form of a true rotation:

$$\begin{pmatrix} 0 & \mu M \\ \pm M & 0 \end{pmatrix}$$

We note that full wave (I) and three-quarter wave (-*i*) will work equally well, should that be otherwise called for.

Implementation

This section is called “Implementation” rather than “Practical Implementation” for specific reasons. See the “Comments” below.

To convert the 3F region to a pure rotation, we first recall that the preferred polarity is as shown in Figure 1, and the installed system is configured (during “reflector” operation) as in Figure 2. We can lead with –I in either plane and a quarter-wave in the other, for the purposes of this example we chose to start with the half wave in the horizontal. After a bit of fiddling, we found that an additional pair of quads was useful in making the upstream and downstream telescopes symmetrical and to get a robust match. Given this configuration (Figure 3) it was trivial to obtain the desired characteristics and get an overall transfer matrix that was a pure rotation (Table 1).

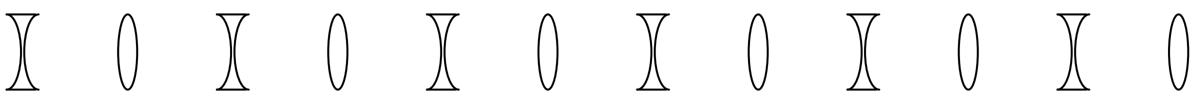

Figure 1: “Natural” 3F configuration

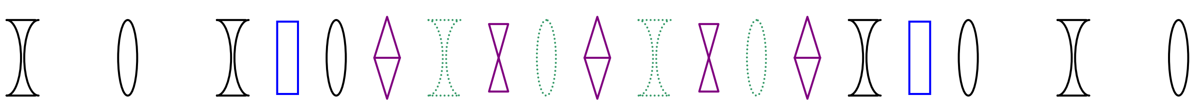

Figure 2: 3F with reflection activated

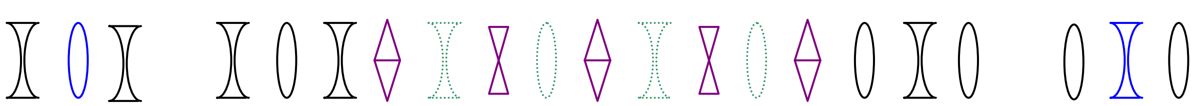

Figure 3: 3F with rotation activated; nonexistent elements in blue.

Table 1: DIMAD output for 3F transfer matrix in rotator (rather than reflector) mode. The elements M_{14}, M_{23}, M_{32}, M_{41} match to only the 5- or 6-digit level because that's the resolution used to set the beta value in fitting for the quarter-wave transform.

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                              ******************************
                              *   TRANSFORMATION  MATRIX   *
                              ******************************


          FIRST ORDER MATRIX

-  0.7443778E-15  0.1101679E-14 -0.7397106E+00 -0.3300114E+01  0.0000000E+00  0.0000000E+00
-  0.1075734E-15 -0.7345646E-15  0.1372159E+00 -0.7397106E+00  0.0000000E+00  0.0000000E+00
-  0.7397106E+00  0.3300006E+01 -0.6019490E-15 -0.6425416E-14  0.0000000E+00  0.0000000E+00
- -0.1372204E+00  0.7397106E+00 -0.8326673E-16 -0.2220446E-15  0.0000000E+00  0.0000000E+00
-  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.1000000E+01  0.0000000E+00
-  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.1000000E+01
```

Comments

The only skew elements are the embedded reflection

This is an exercise only; I wouldn't necessarily recommend running off and installing the two extra quads for four reasons. Firstly, they might not absolutely be necessary; the computation was done using telescopes with two triplets simply because we knew we could get the answer quickly. We haven't checked to be sure we can't do it with a doublet/triplet configuration. Secondly, this demonstration doesn't actually solve the right problem: we really want a rotation from an unstable cavity back to itself, not just across the backleg. The appropriate unstable cavity hasn't, however, yet been identified, and, even if it were, knowledge of the polarization of the offending mode would be needed and could in fact be such that a reflection, or even just a simple shift in phase advance, would provide adequate operational control of the instability. Thirdly, even if the *matrix* is right, the internal details of the transport could preclude successful machine operation. Though the instability might be suppressed by the appropriate choice of matrix, the envelope mismatch internal to the system could potentially be severe enough that performance could be loss-limited, not instability limited.

Finally, additional quads may well not be required to meet the desired conditions. With 30 other main quadrupoles available in the linac and recirculator, it is valid to question whether imposing the constraint that 3F be a pure rotation (with potential nasty internal mismatch) is the right operational approach. One may do better simply by running the rotator as we do now – using the upstream and downstream quads to match into and out of it, and using the 2F and 5F telescopes to massage the overall transfer matrix into the proper form.

We note that operational practice often ignored BBU-imposed constraints altogether. Were we instability limited (to, say 1 or 2 mA) we frequently just tuned up the transport system to be better matched and thereby adequately cured the instability limit as well (that is, we pushed it out to over 5 mA).

Acknowledgments
Thanks to Geoff Krafft for trying to get us to pay attention to this, and to Todd Smith for succeeding in getting us to pay attention to this. Be sure to study Eduard Pozdeyev's treatment of the reflection/rotation; the initial note looks great and it will be done rigorously, generally, and in detail. Thanks to Chris Tennant for causing the ruckus to begin with by documenting BBU suppression in the Upgrade.

References
[1] D. Douglas, "A Skew-Quad Eigenmode Exchange Module (SQEEM) for the FEL Upgrade Driver Backleg Transport", JLAB-TN-04-016, 12 May 2004.

[2] G. Krafft initially attempted to instruct us of this. As always, he was ignored. T. Smith, however, is NOT to be ignored, and wasn't, when *he* brought it up. T. Smith, private communication; E. Pozdeyev, private communication and unpublished note; C. Tennant *et al.*, to be presented at the 2004 FEL Conference (Trieste, Aug. 2004)

[3] E. Pozdeyev, work in progress.

[4] *Op. cit*.

[5] Rand, R.E. and T.I. Smith, "Beam Optical Control of Beam Breakup In A Recirculating Electron Accelerator", *Particle Accelerators*, Vol. 11, pp.1-13, 1980.