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Transparent Reflections: How to Turn a SQUIB Off and On Without Influencing Betatron Matching

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Abstract

In prior notes [1, 2, 3], we have discussed the use of a "Skew Quad Eigenmode Exchange Module", or SQEEM, for the control of the beam breakup instability in the IR FEL Upgrade driver. We should have really called it a "Skew Quad Uncoupled Eigenmode Exchange Module", because that would have the acronym SQUEEM - more indicative of both the desired pronunciation "sqweem", as in "squeamish" (describing my feelings about fully coupled systems) and the fact that as a *fully* coupled system, a SQUEEM isn't really "coupled" – it *completely* maps each plane into the other. Of late, being in a Harry Potter kind of mood, we instead have gravitated to the name "SQUIB", for "Skew Quad Uncoupled Interchange Beamline", which retained the notion of the complete separation – with exchange – of the eigenmodes by the end of the system, and invokes the notion, as in a Potteresque "squib", of a completely unmagical creature (a bunch of rotated quads) living in a land of magic (coupled beamlines).

Anyway….

Here we hope to describe some simple constraints on SQUIB behavior that will immediately provoke an awareness of how to simply and locally activate and deactivate the interchange of horizontal and vertical degrees of freedom without having any impact on betatron matching of the decoupled beam envelopes external to the skew quad system. It will, hopefully, become almost immediately apparent how to bring the machine global transfer matrix from the "problem child" zone 3 back to itself into not only the desirable form of a reflection, but moreover to the very desirable form of an exactly imaging reflection wherein the off-diagonal sub-matrices are I and –I.

Behavior Foisted Upon SQUIBs by Nature

Consider a beamline consisting of a collection of normal quads and drifts. It will be described by a decoupled 4×4 matrix $\mathbf{M}$ built of 2×2 sub-blocks defined as follows.

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} \\ 0 & 0 & M_{43} & M_{44} \end{pmatrix} \equiv \begin{pmatrix} \mathbf{M_x} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_y} \end{pmatrix}$$

If this beamline is rotated 45°, its transfer matrix will be as follows.

$$\widetilde{\mathbf{M}} = \mathbf{RMR^t},$$

R is the usual rotation matrix, which we will conveniently write as below.

$$\mathbf{R} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I} & \mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{pmatrix}$$

The transfer matrix for the skewed system will then "mix" the sub-matrices for the system in "normal" coordinates.

$$\widetilde{\mathbf{M}} = \mathbf{RMR^t} = \frac{1}{2}\begin{pmatrix} \mathbf{I} & \mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{pmatrix}\begin{pmatrix} \mathbf{M_x} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_y} \end{pmatrix}\begin{pmatrix} \mathbf{I} & -\mathbf{I} \\ \mathbf{I} & \mathbf{I} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \mathbf{M_x} + \mathbf{M_y} & -\mathbf{M_x} + \mathbf{M_y} \\ -\mathbf{M_x} + \mathbf{M_y} & \mathbf{M_x} + \mathbf{M_y} \end{pmatrix}$$

So, there you have it. If $\mathbf{M_x} = \mathbf{M_y} = \mathbf{M}$, the skewed matrix is decoupled and block diagonal in form with on-diagonal blocks equaling the 2×2 sub-block.

$$\widetilde{\mathbf{M}} = \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix}$$

If $-\mathbf{M_x} = \mathbf{M_y} = \mathbf{M}$, the matrix is fully coupled – with the diagonal sub-blocks zero and the off-diagonal sub-blocks equaling the 2×2 matrix (which is how the IR Upgrade 3F rotator [1, 2, 3] looks…).

$$\widetilde{\mathbf{M}} = \begin{pmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{pmatrix}$$

This immediately suggests a way to think about the skew quad system that even I can understand: configure a system of normal quads to have a desirable 2×2 matrix in each transverse plane, and then rotate the whole thing. If you make the unrotated system have identical sub-matrices, it will stay decoupled when rotated. If the matrices are equal in magnitude but opposite in sign – for example, if the two "normal modes" have the same matched beam envelopes but betatron phase advances differing by 180^o – the resulting skewed system will be fully coupled, interchanging the horizontal and vertical planes, but will maintain betatron matching from plane to plane.

A particularly simple configuration can then be obtained by laying out a number of FODO cells and tuning them to a rational phase advance (shades of the second order achromat!) For example, if you use four cells, you could set the tunes of both planes to 90^o, and you would get an uncoupled overall transfer matrix that is the identity.

$$\widetilde{\mathbf{M}}_u = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

If, however, you tuned the normal mode horizontal plane to 45^o/cell, and the vertical to 90^o, you would have -$\mathbf{M_x}$ = $\mathbf{M_y}$ = $\mathbf{I}$, with a net skewed system matrix consisting of off-diagonal identities.

$$\widetilde{\mathbf{M}}_c = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}$$

In either case, the system consists of excited skew quads, and betatron matching across the system is maintained by the identity matrix transport. The exchange of planes (or absence thereof) is simply controlled by the normal mode phase advances.

A practical system might consist of fewer quads – say, 3 periods at 60^o and/or 120^o tune in either plane. A total of only six quads would then be required. Sadly, the 3F rotator in the IR Upgrade driver has only five quads – too few to take advantage of this method (in the available configuration, the system to betatron unstable: five quads is too few to get a wavelength in one plane and a half in the other). If the 3F03a and 3F09a normal quads were also rotated, this method could be used. The usefulness of this will hopefully be apparent after the following section.

Matching The Rest of the Machine

Typical (I *wish*) linac optics for the IR Upgrade driver are shown in Figure 1. The beam is axially symmetric at the center of Zone 3 with equal envelopes on both passes. To leverage a rotator of the type described above, one would set up the transport system from Zone 3 to the rotator (denoted conceptually by the black box in the backleg of Figure 1) to provide an axially symmetric image; this could be done, for example, using a matrix of the following block form.

$$\mathbf{T}_1 = \begin{pmatrix} \mathbf{t} & \mathbf{0} \\ \mathbf{0} & \pm\mathbf{t} \end{pmatrix}$$

Assuming the rotator is set up in either of the aforementioned forms $\widetilde{\mathbf{M}}_u$ or $\widetilde{\mathbf{M}}_c$, it is then useful to configure the transport system from rotator back to Zone 3 to produce a matrix as follows.

$$\mathbf{T}_2 = \begin{pmatrix} \mathbf{t}^{-1} & \mathbf{0} \\ \mathbf{0} & \mu\,\mathbf{t}^{-1} \end{pmatrix}$$

The difference in sign can be imposed either before or after the rotator. It would be implemented in practice by choice of betatron phase advance: the transport plane with negative sign would have the same matched beam envelopes as, but the betatron phase would be retarded or advanced by half a wavelength relative to, the other plane.

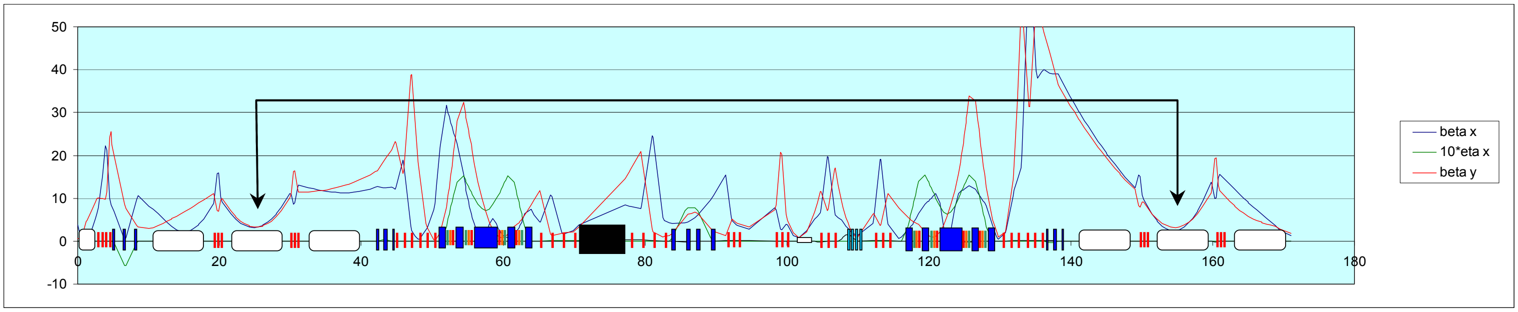

Figure 1: "Typical" beam envelope solution for IR Upgrade driver; with linac observation points of interest indicated

With this arrangement, the beam envelopes are automatically matched regardless of rotator coupling status; a **t** of some polarity always maps across the rotator *via* an **I**, and is then followed by a $\mathbf{t}^{-1}$ of some polarity. The envelopes therefore image. When uncoupled, the overall transfer matrix is the identity in one plane, –**I** in the other.

$$\mathbf{T}_2\tilde{\mathbf{M}}_u\mathbf{T}_1 = \begin{pmatrix}\mathbf{t}^{-1} & \mathbf{0} \\ \mathbf{0} & \mu\,\mathbf{t}^{-1}\end{pmatrix}\begin{pmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{t} & \mathbf{0} \\ \mathbf{0} & \pm\mathbf{t}\end{pmatrix}$$
$$= \begin{pmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}\end{pmatrix}$$

Of more interest is the fully coupled case, which produces a true four dimensional rotation of the type prescribed by Pozdeyev and Tennant [4] – with null angular response – from the first pass to the second.

$$\mathbf{T}_2\tilde{\mathbf{M}}_c\mathbf{T}_1 = \begin{pmatrix}\mathbf{t}^{-1} & \mathbf{0} \\ \mathbf{0} & \mu\,\mathbf{t}^{-1}\end{pmatrix}\begin{pmatrix}\mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mathbf{t} & \mathbf{0} \\ \mathbf{0} & \pm\mathbf{t}\end{pmatrix}$$
$$= \begin{pmatrix}\mathbf{0} & \mathbf{t}^{-1} \\ \mu\,\mathbf{t}^{-1} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mathbf{t} & \mathbf{0} \\ \mathbf{0} & \pm\mathbf{t}\end{pmatrix}$$
$$= \begin{pmatrix}\mathbf{0} & \pm\mathbf{I} \\ \mu\,\mathbf{I} & \mathbf{0}\end{pmatrix}$$

By varying the FODO-rotator tunes as described above, you can – without influencing the external betatron matching – toggle between these states.

Conclusions

1. A six- (or more) quad rotator based on multiple FODO periods at rational tunes can be used to make a skew system with nice imaging properties and selectable coupling.
2. Proper matching to and from the rotator can readily yield an overall de- and fully-coupled betatron solution that can be switched to and from a pure reflection state.
3. We should consider skewing QS3F03A and QS3F09A: if we can cook up a nice match to/from the resulting rotator, it'd make SQUIB operation a SNAP (Systematic Naturally Applicable Procedure).

References

[1] D. Douglas, "A Skew-Quad Eigenmode Exchange Module (SQEEM) for the FEL Upgrade Driver Backleg Transport", JLAB-TN-04-016, 12 May 2004.

[2] D. Douglas, " Reflections on Rotators, Or, How to Turn the FEL Upgrade 3F Skew Quad "Rotator" Into a Skew Quad Rotator", JLAB-TN-04-023, 4 August 2004.

[3] D. Douglas, "Operation of the FEL Upgrade with Skew Quad Reflection and Rotation", JLAB-TN-04-024, 17 August 2004.

[4] E. Pozdeyev & C. Tennant, "Equation for the Multi-Pass Beam Breakup Threshold Current for a Single Mode and a 4x4 Recirculation Matrix", JLAB-TN-04-019, 8 July 2004.