

Compaction Management For Mirror Bend Achromats

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Abstract

We describe a means of setting the momentum compaction in a mirror bend achromat design.

Introduction

The mirror bend achromat (MBA) [1] is an inherently linear, large acceptance beam deflection system of potential use in compact ERLs. Its utility is, however, limited by the rather restricted range of momentum compactions available in a conventional mirror bend design. Though modest in magnitude, these compactions are completely constrained by the gross MBA geometry (bend radius and angle) and are inherently positive and linear. They therefore require use of additional bending modules (likely of less linear design, such as chicanes) when, for example, negative compaction and/or correction of aberrations (such as RF waveform curvature) are/is required.

In the following, we describe a design methodology freeing the MBA from both the close coupling of compaction to bend geometry and the inherently positive compaction. Using this process, it is readily possible to design an MBA with a specific momentum compaction spectrum (M_{56} , T_{566} , W_{5666}, \dots) while retaining the very large momentum acceptance that is characteristic of such systems.

Concept

Figure 1 illustrates the geometry of a conventional MBA. It is clear that lower energy beam components (such as path B) invariably travel a shorter distance than ones at higher energy (such as path A) and that therefore the compaction is inherently positive. Modification of the compactional properties of mirror bends can however be achieved by developing a mechanism to lengthen these lower energy orbits in a controlled fashion, so as to “match” them to the length of the higher energy component while holding the beamline footprint fixed.

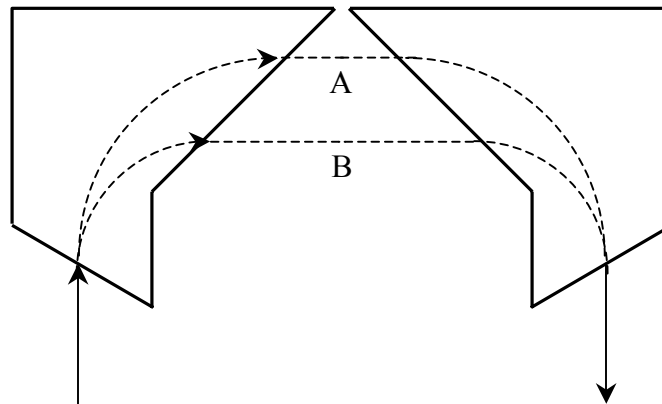


Figure 1: MBA geometry

This mechanism is illustrated in a particular case (that of a 180° mirror bend) in Figure 2, which shows the first half of a modified or compaction-managed MBA (CMMBA). We select a high momentum reference orbit (rather than one at a central energy) A, which sets the overall geometry of the CMMBA by defining the maximum radius of interest ρ_{ref} and the drift length d_{ref} from bend magnet to beamline centerline. The trajectory B of a lower momentum component will then lie on a smaller radius $\rho_B(\delta) = \rho_{\text{ref}} \delta$, where δ is the *fractional* momentum of the beam on orbit B relative to that on A ($\delta = p_B/p_A$, rather than the usual perturbative momentum offset $(p_B - p_A)/p_A$). We impose a chicane on the selected low momentum component B by *extending* the active magnetic region of the exterior dipole, and by introducing a reverse bending region at the center of the CMMBA. The additional bend angle $\theta(\delta)$ so introduced lengthens orbit B; proper selection of this angle and of the length $d_B(\delta)$ of the adjacent drift allows the length of the low momentum orbit to be matched to that of the high momentum reference while holding the footprint (beamline width or radius) fixed to that defined by the reference orbit as well. The use of the central reverse bending region ensures the orbit resolves to the correct angle (in the illustrated case, 90°) and that the complete system (when mirror imaged about the centerline) is dispersion-suppressed to all orders.

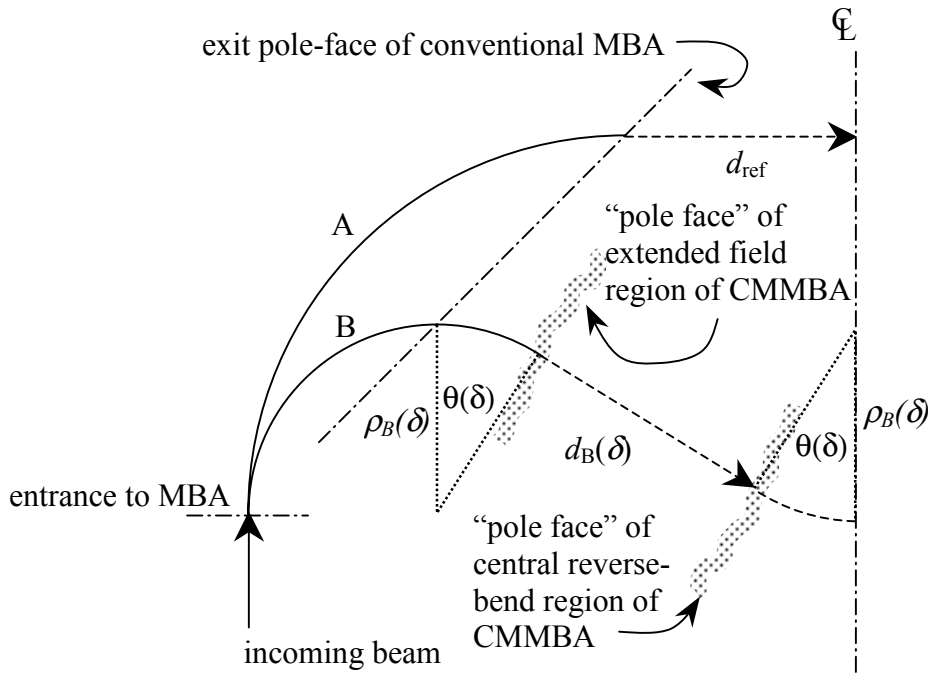


Figure 2: CMMBA geometry: first half of mirror symmetric layout.

The beamline geometry and compaction properties are then described as follows:

$$\text{Path length:} \quad L = \frac{\pi}{2} \rho_{\text{ref}} + d_{\text{ref}} = \frac{\pi}{2} \rho_B(\delta) + 2\theta(\delta)\rho_B(\delta) + d_B(\delta) + F(\delta)$$

$$\text{Beamline "radius":} \quad R = \rho_{\text{ref}} + d_{\text{ref}} = \rho_B(\delta) + 2\rho_B(\delta)\sin\theta(\delta) + d_B(\delta)\cos\theta(\delta)$$

$F(\delta)$ is a compaction function characterizing the desired dependence of orbit length on momentum and can be related to the usual compaction spectrum M_{56} , T_{566} , W_{5666} , By solving these two equations for the two unknowns $\theta(\delta)$ and $d_B(\delta)$ at variety of momenta δ , one can easily specify the location of the pole faces of the bending regions. Assuming an origin at the entry point of the first dipole, the location of trajectory B (at momentum δ) at the exit pole of the first magnet and the entrance to the reverse bend are as follows:

$$\begin{aligned} \text{Exit pole of first dipole:} \quad x(\delta) &= \rho_B(\delta) (1 + \sin\theta(\delta)) \\ y(\delta) &= \rho_B(\delta) \cos\theta(\delta) \end{aligned}$$

$$\begin{aligned} \text{Entrance to reverse bend:} \quad x(\delta) &= \rho_B(\delta) (1 + \sin\theta(\delta)) + d_B \cos\theta(\delta) \\ y(\delta) &= \rho_B(\delta) \cos\theta(\delta) - d_B \sin\theta(\delta) \end{aligned}$$

Examples

Figures 3 through 5 show pole face contours and orbit geometries for three example CMMBAs. These were evaluated in each case by solving the above system for $\theta(\delta)$ and $d(\delta)$ at ten momenta ranging from $\delta=1$ (where by definition $\theta(\delta=1) = 0$ and $d(\delta=1) = d_{\text{ref}}$) to $\delta=0.1$ in steps of $\Delta\delta=0.1$. A compaction function $F(\delta)=M_{56}\times(1-\delta)$ was used to illustrate how a specific compaction spectrum may be imposed on the CMMBA. Here the particular choice of M_{56} of selected value and all nonlinear compactions zero is appropriate to an FEL driver ERL with 750 MHz RF, accelerating -20° off-crest and using third harmonic RF linearization for bunch length and energy compression.

Figure 6 shows a conceptual implementation of the Figure 3 and Figure 5 arcs in a compact FEL driver of the type discussed elsewhere [2]

Comments

This method can be extended to arbitrary overall bend angle (it is not constrained to MBAs with 180° total angle) and choice of compaction function $F(\delta)$. It therefore provides a basis for a variety of applications requiring large acceptance and longitudinal phase space management. In particular, the ability to set the entire compaction spectrum at design time can be used in the design of compact FEL driver ERLs using only a single RF frequency – harmonic linearization is not needed; proper selection of T_{566} and higher order compaction components will allow magnetostatically based management of the system energy compression [3].

Transverse focusing properties are yet to be evaluated, but given that a) the pole-face rotation angles local to various beam momentum components are smaller than in a

conventional MBA, b) conventional MBAs admit stable solutions, and c) the central bend provides additional vertical focusing, it is likely that acceptable betatron envelope behavior can be achieved.

In practical designs, the reference momentum will likely be “virtual” – well above the actual intended maximum. This will avoid the “cusp” in the central reverse bend while still providing extremely large momentum acceptance.

Notes and References

- [1] D. Douglas, “A Nearly Isochronous Arc With Unlimited Momentum Acceptance”, JLAB-TN-02-020, 30 May 2002; D. Douglas, “A Compact Mirror-Bend-Achromat-Based Energy Recovery Transport System for an FEL Driver”, JLAB-TN-02-026, 24 July 2002.
- [2] D. Douglas, “A Compact Mirror-Bend-Achromat-Based Energy Recovery Transport System for an FEL Driver”, *op. cit.*
- [3] This is in contrast to the use of harmonic RF linearization in designs based on conventional MBAs; *ibid.*

Figure 3: $M_{56} = -0.2$ m (appropriate for acceleration at 750 MHz -20° off-crest)

Half-achromat; beams in steps of momentum equal to 10% of full momentum: 0.1 x full, 0.2 x full, 0.3 x full, ..., full

Path length vs. radius (\sim momentum)

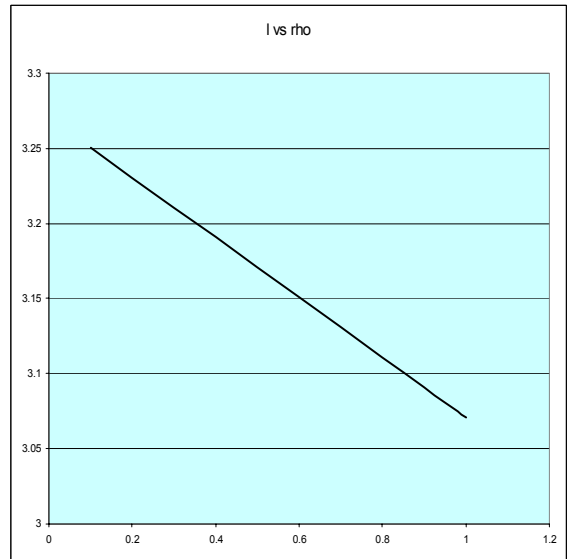
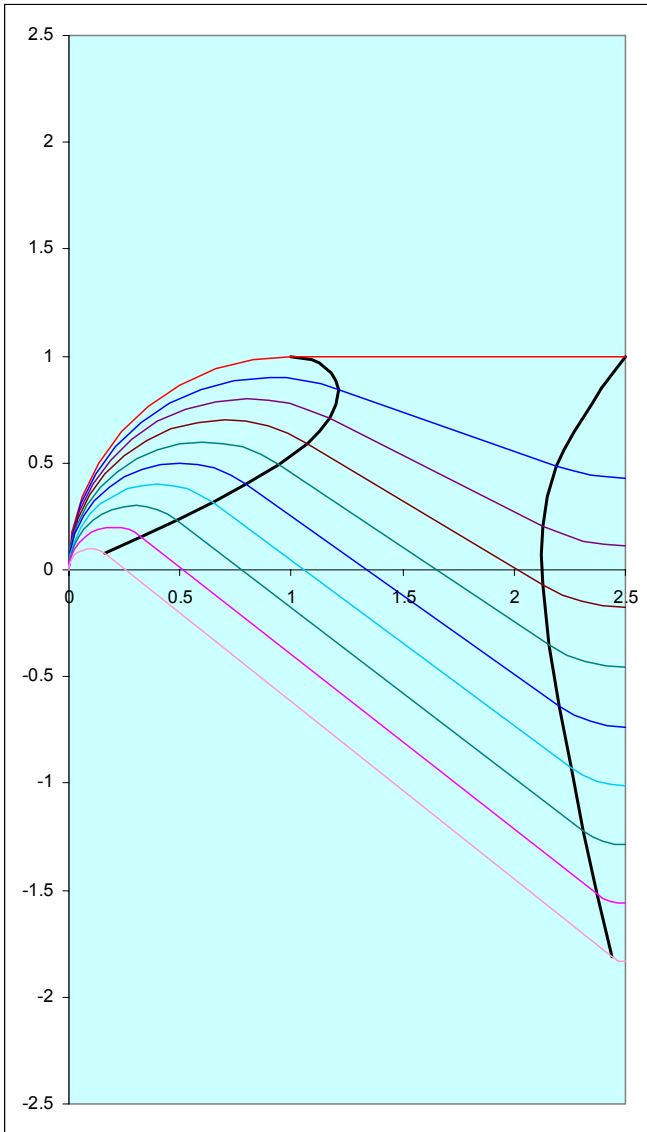


Figure 4: $M_{56} = 0$ (example of strictly isochronous transport)

Half-achromat; beams again in 10% in momentum

Path length vs. radius (\sim momentum)

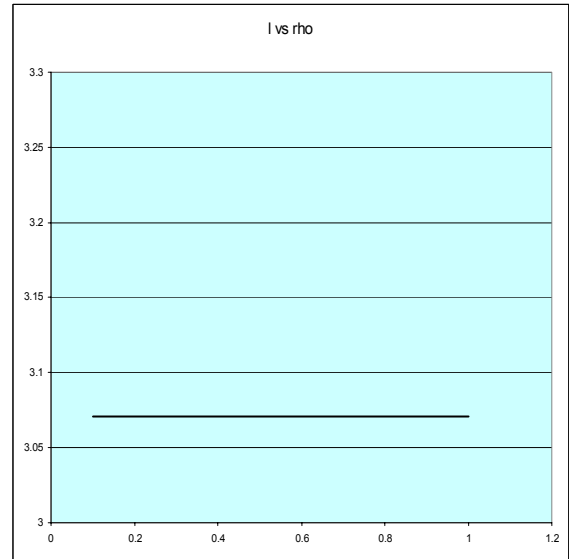
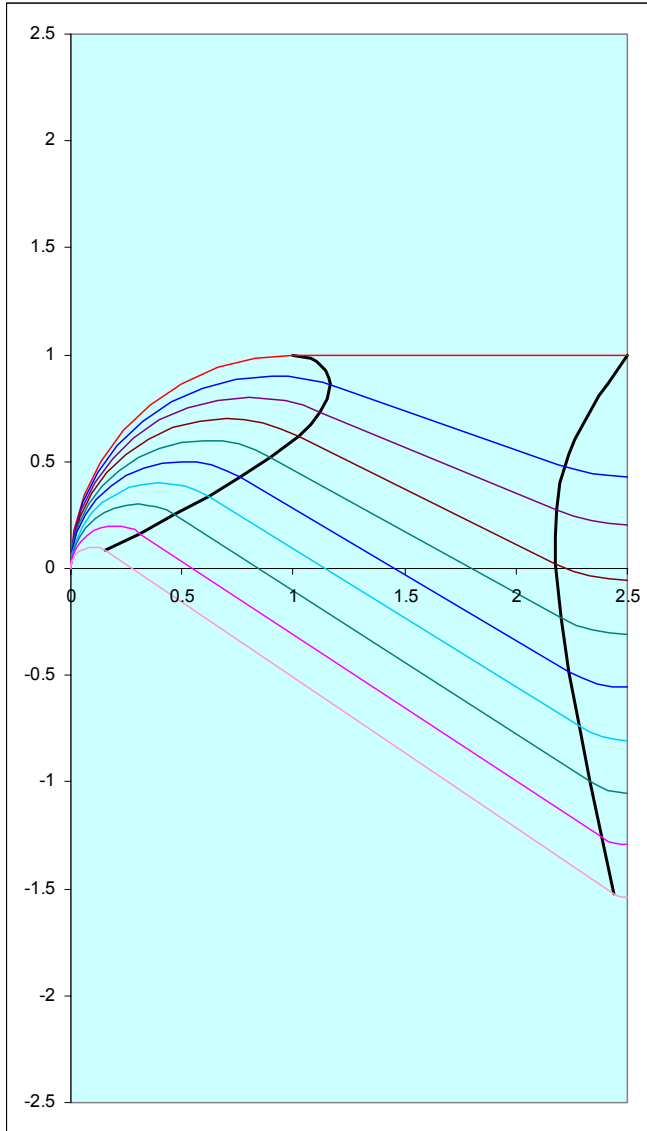


Figure 5: $M_{56} = 0.2$ m (appropriate for energy recovery at 750 MHz when accelerating –
 20° off-crest)

Half-achromat; beams again in 10% in
momentum

Path length vs. radius (~momentum)

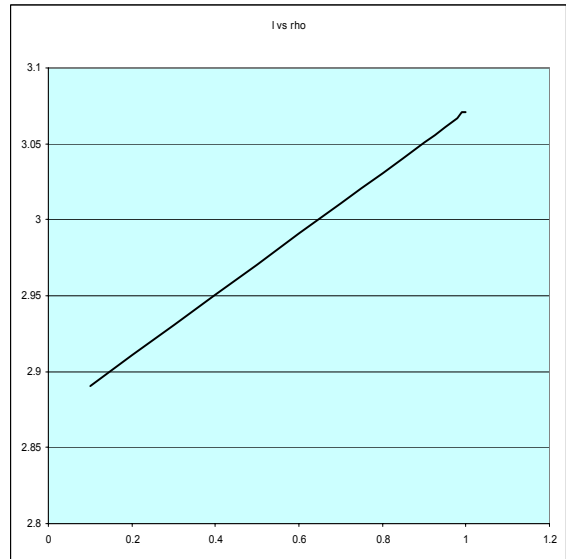
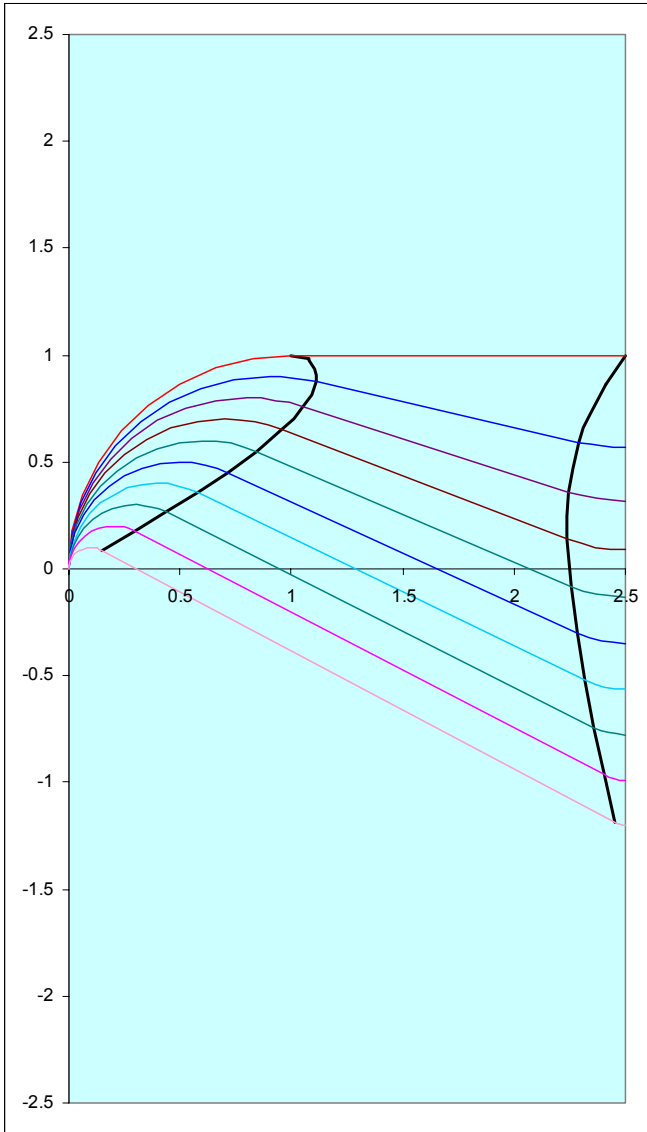


Figure 6: Conceptual use of CMMBA in an FEL driver ERL

