Lattice QCD Calculations of Hadron Structure

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[Lattice Hadron Physics Collaboration]

- GPD's and generalized form factors (GFF's).
- Summary of LHPC hadron structure program.
- Some preliminary lattice results.

LHPC Hadron Structure project on USQCD resources

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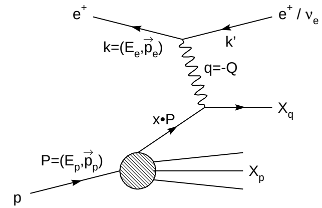
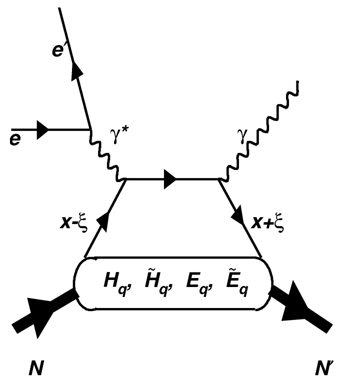
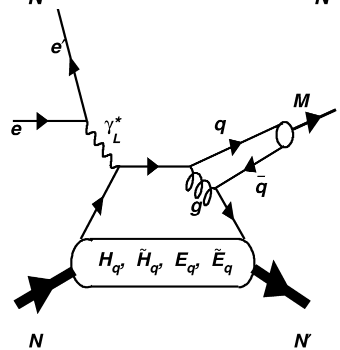
PDF's and Generalized PDF's (GPD's)



- Thanks to QCD factorization, very little about the structure of hadrons is required to understand inclusive DIS reactions.
  - Good: Can treat Tevatron and LHC as *"quark colliders."*
  - Bad: One dimensional view of hadrons, *i.e.* $u(x, Q^2)$, ...



- A lot of information about hadron structure is lost in the inclusive sum over final states of remnant.
- Deeply virtual semi-inclusive and exclusive processes used to build two-dimensional pictures of hadrons, *e.g.* DVCS, DVMP, ...
- Generalize the notion of PDF's to include giving the parton a kick and then putting it back into the hadron: GPD's.
  - More kinematic variables: $x = \frac{1}{2}(x_f + x_i)$, $\xi = \frac{1}{2}(x_f - x_i)$, t.
  - Think of GPD's as **form factors** of the collection of quarks in hadron with same **fixed** x.
  - As $\xi \to 0$ and $t \to 0$, GPD's must reduce to ordinary PDF's.
  - t dependence at **fixed** x should contain information about **spatial distribution** of all quarks with same x.



GPD's and Generalized Form Factors (GFF's)

- Experimentalists measure matrix elements of light cone operators

$$\langle P'S'|\mathcal{O}^q_\Gamma|PS\rangle = \left\langle P'S'\left|\overline{q}\left(-\frac{x^-}{2}\right)\ \Gamma\,\mathcal{P}\exp\left[-ig\int_{x^-/2}^{-x^-/2}A^+(y)dy\right]q\left(\frac{x}{2}\right)\right|PS\right\rangle$$

- They can be written in terms of generalized parton distributions (GPD's) [1]

$$\int\frac{dx^-}{2\pi}e^{ix^-\overline{p}^+x}\langle P'S'|\mathcal{O}^q_\Gamma|PS\rangle$$
$$= \overline{U}(P',S')\left[H^q(x,\xi,\Delta^2)\Gamma + E^q(x,\xi,\Delta^2)\frac{i\sigma\cdot\Delta}{2M}\right]U(P,S)$$

- Eight GPD's in all: H, E, $\widetilde{H}$, $\widetilde{E}$, H_T, E_T, $\widetilde{H}_T$, $\widetilde{E}_T$
- Using OPE, light cone operators replaced by tower of local twist two operators

$$\langle P'S'|\mathcal{O}^{\mu_1\cdots\mu_n}_\Gamma|PS\rangle = \left\langle P'S'\left|\overline{q}(x)iD^{(\mu_1}\cdots iD^{\mu_{n-1}}\Gamma^{\mu_n)}q(x)\right|PS\right\rangle$$

- They can be parameterized by generalized form factors (GFF's), *i. e.*

$$\left\langle P'S'\left|\mathcal{O}^{\mu_1\mu_2}_q\right|PS\right\rangle =$$
$$\overline{U}(P',S')\left[A^q_{20}(Q^2)\gamma^{(\mu_1}\Delta^{\mu_2)} + B^q_{20}(Q^2)\frac{i\sigma^{(\mu_1\alpha}\Delta^\alpha}{2M}\Delta^{\mu_2)} + C^q_2(Q^2)\frac{\Delta^{(\mu_1}\Delta^{\mu_2)}}{2M}\right]U(P,S)$$

- Nine GFF's in all: A_{ni}, B_{ni}, C_n, $\widetilde{A}_{ni}$, $\widetilde{B}_{ni}$, A_{Tni}, B_{Tni}, $\widetilde{A}_{Tni}$, $\widetilde{B}_{Tni}$

[1] X. D. Ji, hep-ph/9807358. Recent review: M. Diehl, Phys. Rept. **388**, 41-277 (2003).

Equivalence of GPD's and GFF's

- GPD's and GFF's are formally equivalent by Mellin transformation *e. g.*

$$\int_{-1}^{1} dx\; x^{n-1} H^q(x,\xi,Q^2) = \sum_{i=0,\ \text{even}}^{n-1} A^q_{ni}(Q^2)(-2\xi)^i + \delta_{n,\ \text{even}} C^q_n(Q^2)(-2\xi)^n$$

$$\int_{-1}^{1} dx\; x^{n-1} E^q(x,\xi,Q^2) = \sum_{i=0,\ \text{even}}^{n-1} B^q_{ni}(Q^2)(-2\xi)^i - \delta_{n,\ \text{even}} C^q_n(Q^2)(-2\xi)^n$$

- Choice of GPD's *vs.* GFF's depends on physics.

GPD: PDF's and transverse PDF's

GFF: elastic form factors and nucleon spin

- In Euclidean lattice QCD, only GFF's can be computed directly.
- Many GFF's are familiar experimental quantities:
  - $A^q_{10}(Q^2) = F^q_1(Q^2)$, $B^q_{10}(Q^2) = F^2_2(Q^2)$
  - $\widetilde{A}^q_{10}(Q^2) = G^q_A(Q^2)$, $\widetilde{B}^q_{10}(Q^2) = G^q_P(Q^2)$,
  - $J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$, $\frac{1}{2}\Delta\Sigma^q = \widetilde{A}^q_{10}(0)$
  - $L^q = J^q - \frac{1}{2}\Delta\Sigma^q$
  - $\left\langle x^{n-1}\right\rangle_q = A^q_{n0}(0)$, $\left\langle x^{n-1}\right\rangle_{\Delta q} = \widetilde{A}^q_{n0}(0)$, $\left\langle x^{n-1}\right\rangle_{\delta q} = A^q_{Tn0}(0)$

Summary of LHPC hadron structure program

- Long term program to compute all $n \leq 4$ GFF's in dynamical lattice QCD.
- Current pion masses $m_\pi \approx 350 - 750$ MeV and lattice spacing $a \approx \frac{1}{8}$ fm.
- Status of the calculation

Operators	Matrix elements	Operator renorm.	GFF extraction	Analysis
$\overline{q}\Gamma_\mu q$	Done!	Done!	Done!	Starting
$\overline{q}\Gamma_{(\mu}D_{\nu)}q$	Done!	Done!	Done!	Starting
$\overline{q}\Gamma_{(\mu}D_\nu D_{\rho)}q$	Done!	Done!	Done!	Starting
$\overline{q}\Gamma_{(\mu}D_\nu D_\rho D_{\sigma)}q$	Not yet	Done!	Not yet	Not yet

- Only isovector flavor combinations for GFF's in this round.
- Finite perturbative renormalization needed to quote results in $\overline{\text{MS}}$ scheme.

$$\langle P'S'\,|\mathcal{O}_\Gamma^{\mu_1\cdots\mu_n}|\,PS\rangle_{\overline{\text{MS}}} = Z\,\langle P'S'\,|\mathcal{O}_\Gamma^{\mu_1\cdots\mu_n}|\,PS\rangle_{\text{latt}}$$

- Lighter pion masses $m_\pi \approx 250 - 350$ MeV finished by next year.

Perturbative renormalization of twist two matrix elements

Tree level: $Z = 1$, One loop HYP corrections: $< 10\%$.

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.68	0.971	1.07
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.765	0.964	0.99
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.821	0.987	0.989
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.986	0.968	0.929
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.972	0.962	0.925
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.206	0.982	0.898
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	8.78×10^{-3}	2.88×10^{-3}	1.26×10^{-3}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.191	0.98	0.898
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.375	0.989	0.876
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.018	0.991	0.945
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.967	0.973	0.983
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.931	0.937	0.947

Table 11.17: Full $\overline{MS}$ to lattice renormalization coefficients for $M = 1.7$ and 1-loop expression for g. By chiral symmetry matrix elements are the same (except for parity) with and without γ_5, and this is indicated by the $[\gamma_5]$ notation where the upper parity arises in the absence of γ_5.

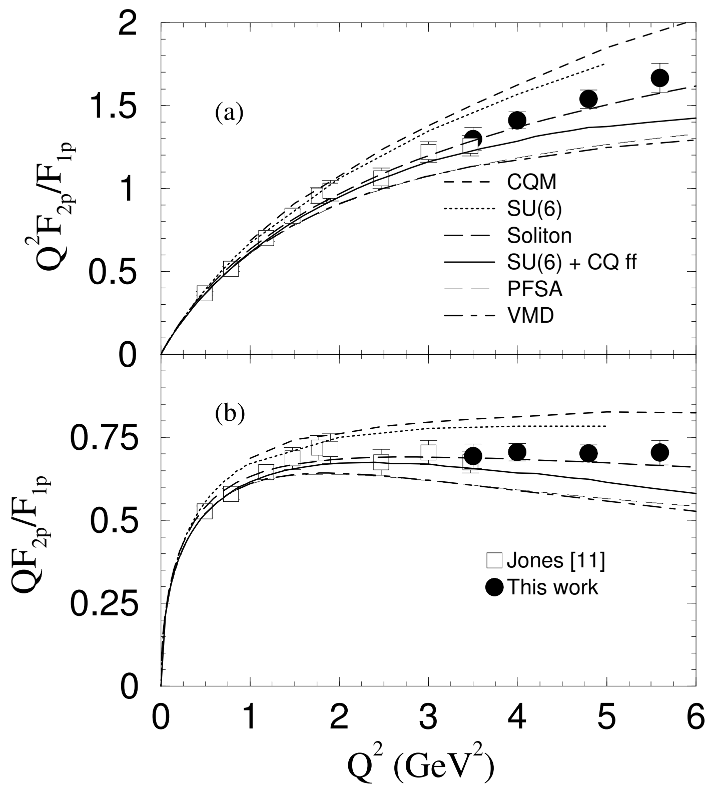
B. Bistrovic, Ph. D. Thesis, MIT, 2005

The Nucleon Electromagnetic Form Factors

- Brodsky-Lepage predicted $F_1 \sim Q^{-4}$ and $F_2 \sim Q^{-6}$ as $Q^2 \to \infty$.
- Older L/T separation experiments observed predicted behavior.
- New polarization transfer experiments do not see expected scaling.
- *One Photon Exchange* approximation may not be justified.
- Softer scaling also possible (hep-ph/0212351)

$$\frac{Q^2}{\log^2(Q^2/\Lambda^2)}\frac{F_2(Q^2)}{F_1(Q^2)} \sim \text{const}$$

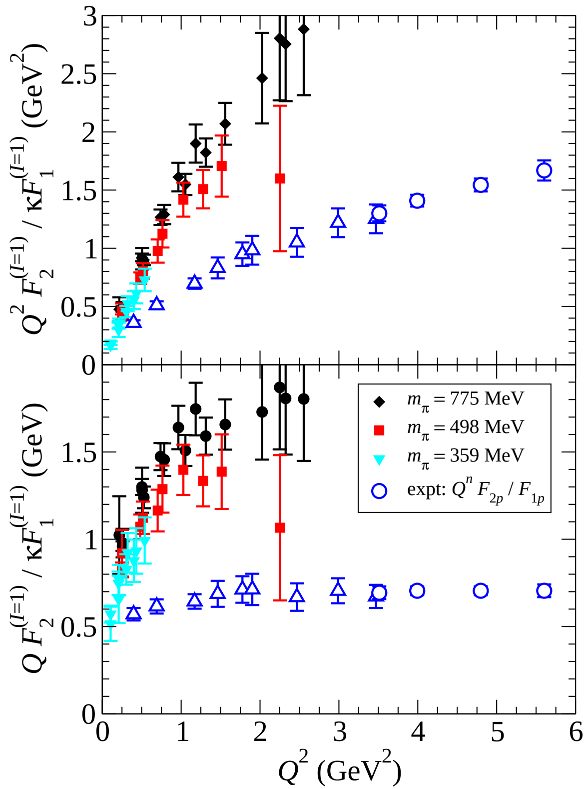
Phys. Rev. Lett. **88**, 092301 (2002)



Nucleon Electromagnetic Form Factors on the Lattice

PRELIMINARY

- Only $I = 1$ form factors computed so far to avoid disconnected diagrams. $F_1^{I=1} = F_{1p} - F_{1n}$ but F_{1n} not known accurately for $Q^2 \gtrsim 1$ GeV^2.
- Our normalization is $F_2(Q^2) \rightarrow \kappa$ as $Q^2 \rightarrow 0$.



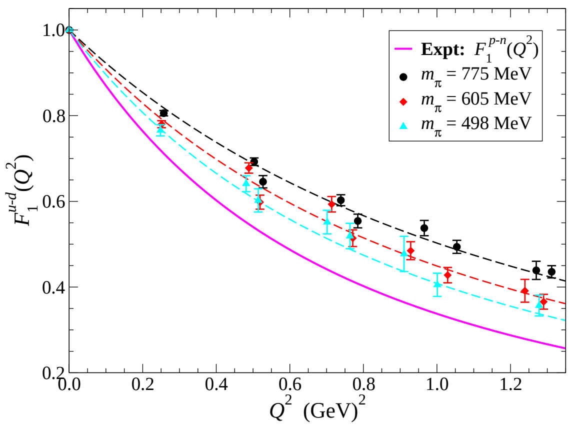
Nucleon Isovector F_1 on the Lattice

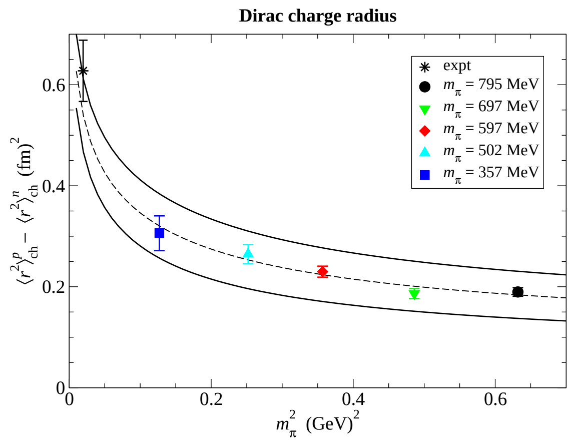
- We know the right answer in the forward limit: $F_1^{I=1} \to 1$ as $Q^2 \to 0$.
- Experimentally, nucleon charge radii $\langle r^2 \rangle_{\rm ch}^{p}$ and $\langle r^2 \rangle_{\rm ch}^{n}$ are accurately determined.
- For $Q^2 \lesssim 1$ GeV^2, fitting lattice data to dipole *ansatz* gives the Dirac charge radius $\langle r^2 \rangle_{\rm ch}^{u-d}$.
- Chiral extrapolation using leading analytic and non-analytic terms and finite range regulator (PRL **86** 5011).

$$\langle r^2 \rangle_{\rm ch}^{u-d} = a_0 - 2\frac{(1+5g_A^2)}{(4\pi f_\pi)^2}\frac{1}{2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

- Best fit: $\Lambda \approx 740$ MeV.

PRELIMINARY

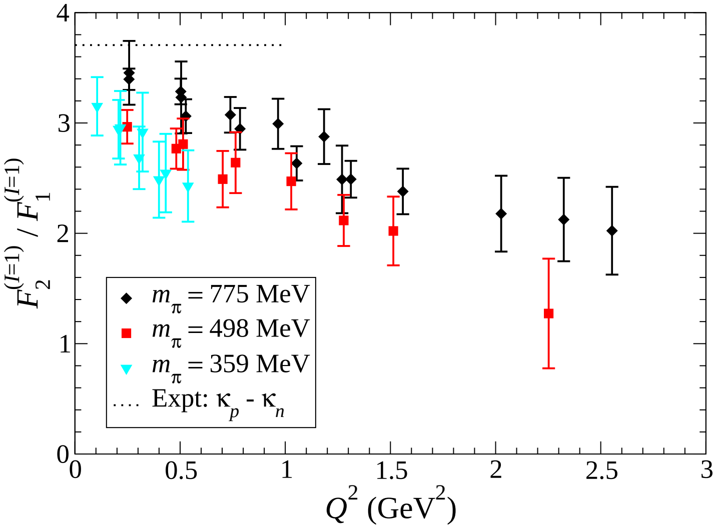




Nucleon Isovector F_2 on the Lattice

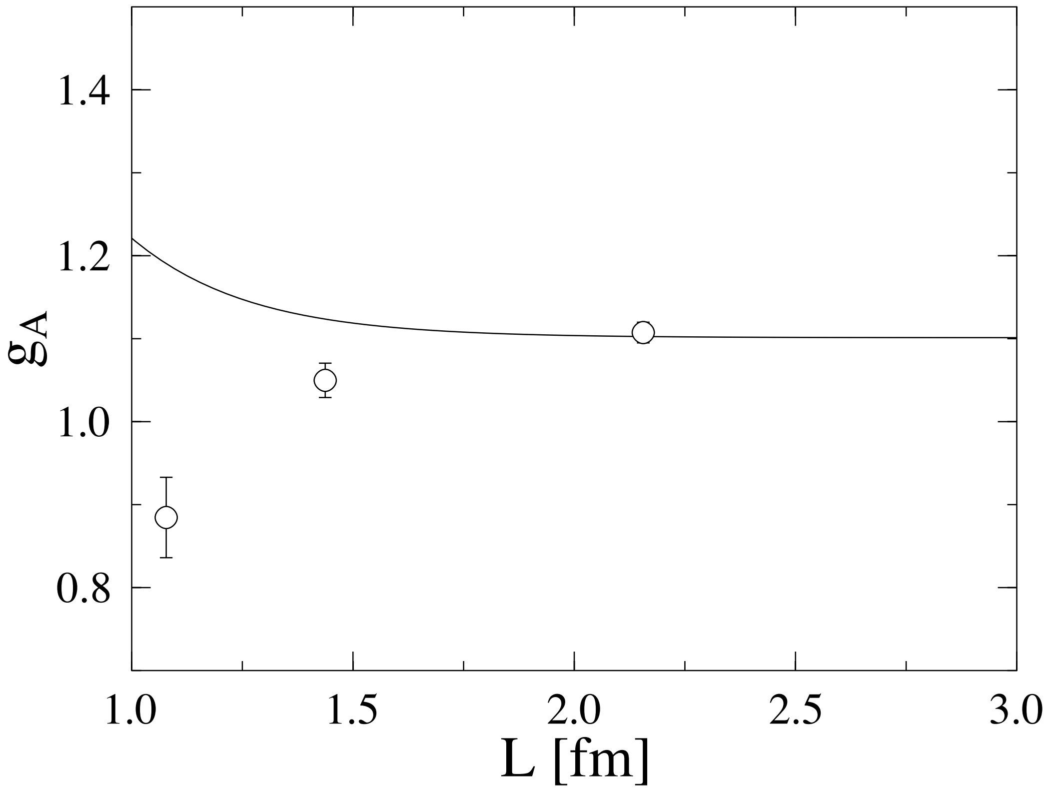
PRELIMINARY

- $F_2^{I=1}/F_1^{I=1} \to \kappa_p - \kappa_n$ as $Q^2 \to 0$.
- PDG: $\kappa_p = 1.792847351(28)$
- PDG: $\kappa_n = -1.91304273(45)$
- So, comparison of $I = 1$ with $p - n$ could be OK with proper chiral extrapolation.



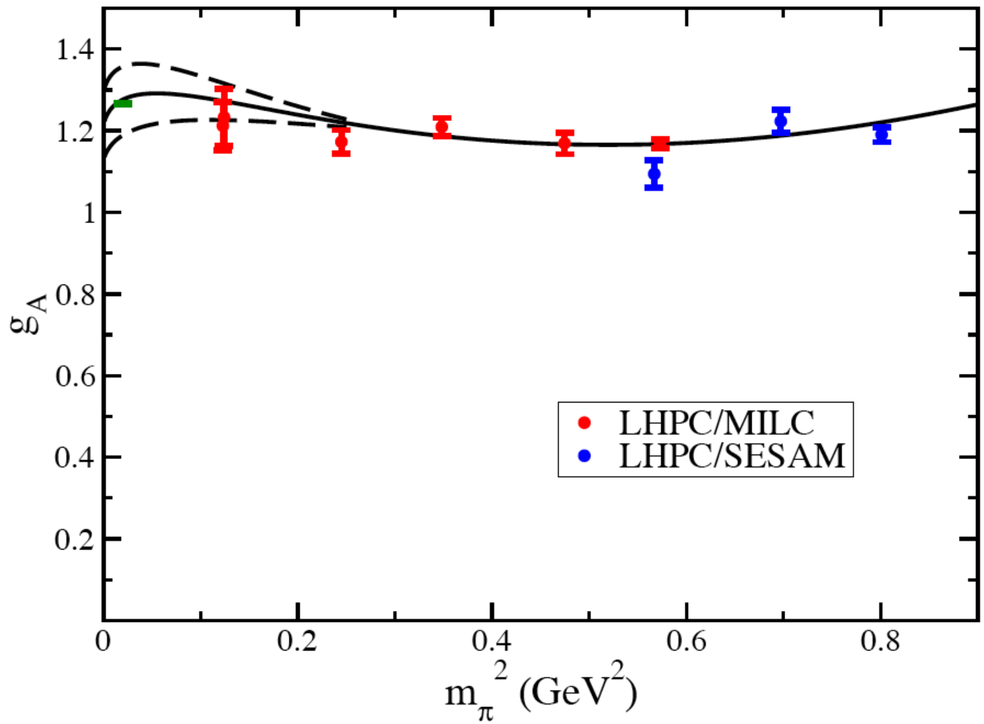
Nucleon axial charge g_A in a finite volume

- g_A is strongly suppressed by finite volume when $m_\pi L < 4$.
- Graph from hep-lat/0409161 (QCDSF). $m_\pi = 717$ MeV, curve is LO χPT.



NLO χPT (with Δ) extrapolation of g_A

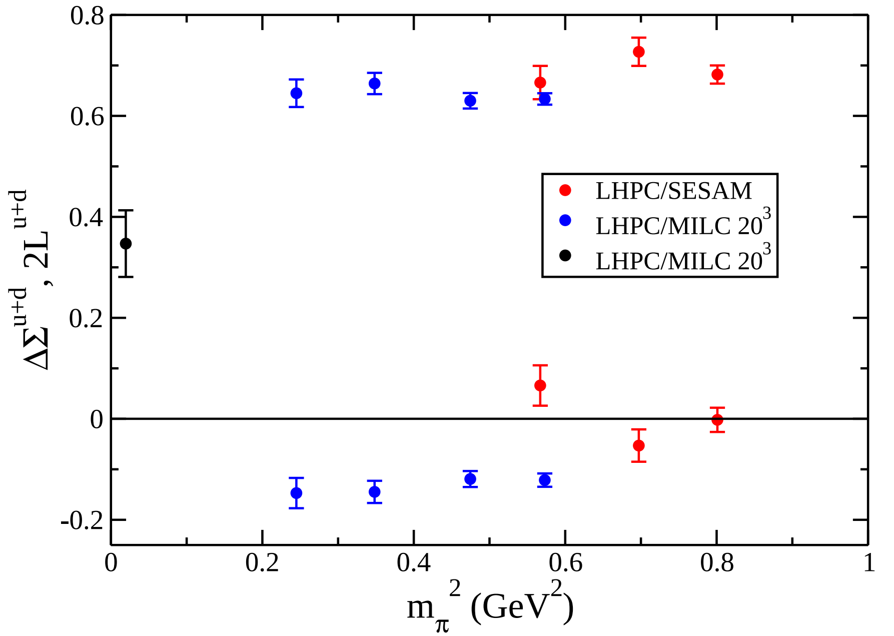
- Fitting formula proposed by Hemmert *et al.* hep-lat/0303002.
- Two free parameters: g^A_{NN} and $g^A_{\Delta\Delta}$.
- Other parameters fixed by phenomenology: f_π, $g^A_{N\Delta}$, $m_\Delta - m_N$, $B_9 - g_A B_{20}$.



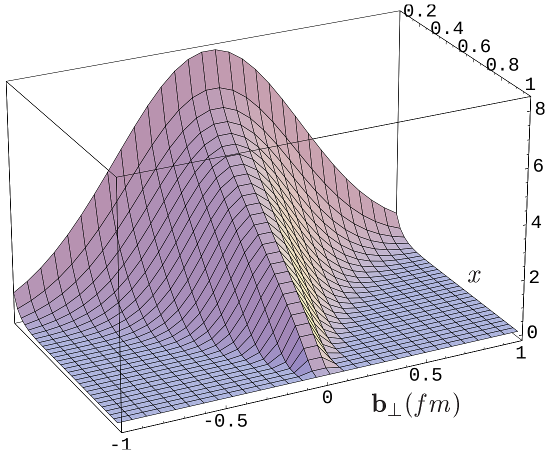
Orbital angular momentum

- *CAUTION:* missing disconnected contributions.
- Ji Sum Rule:

$$\Delta\Sigma^q = 2\widetilde{A}^q_{10}(0), \quad J^q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right], \quad L^q = J^q - \frac{1}{2}\Delta\Sigma^q.$$



Transverse quark distributions



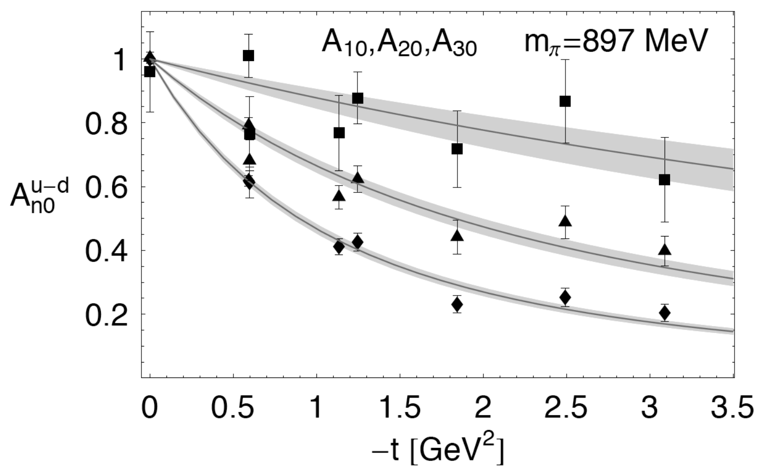
M. Burkardt hep-ph/0207047

$$A^q_{n0}(-\Delta^2_\perp) = \int d^2b_\perp e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \int_{-1}^{1} x^{n-1} q(x, \mathbf{b}_\perp)$$

$$\langle b^2_\perp \rangle^q_{(n)} = -4\frac{A^{q\prime}_{n0}(0)}{A^q_{n0}(0)}$$

$$\lim_{x\to 1} q(x, \mathbf{b}_\perp) \propto \delta(b^2_\perp)$$

- Higher moments A_{n0} weight $x \sim 1$.
- Slope of A^q_{n0} decreases as n increases.
- Slope of $A^{u-d}_{10}(0) =$ -0.93(4) (GeV)^2.
- Slope of $A^{u-d}_{30}(0) =$ -0.13(3) (GeV)^2.
- Will this continue at light pion masses?



D. Renner (LHPC/SESAM)

Summary and outlook

- Large scale computation of isovector matrix elements $(n \leq 3)$ is done. Data analysis is proceeding rapidly. Expect published results soon.
- Isoscalar and strange matrix elements are $\mathcal{O}(10) - \mathcal{O}(100)$ times harder to compute due to statistical noise. We'll make our first serious attempt this year.
- Perturbative renormalization complete. Bojan Bistrovic (MIT)
- Reaching higher Q^2 is high priority for nucleon form factors.