



Quark mass extrapolation of hadronic observables in lattice QCD

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Jefferson Lab

JLab Users Workshop & Annual Meeting

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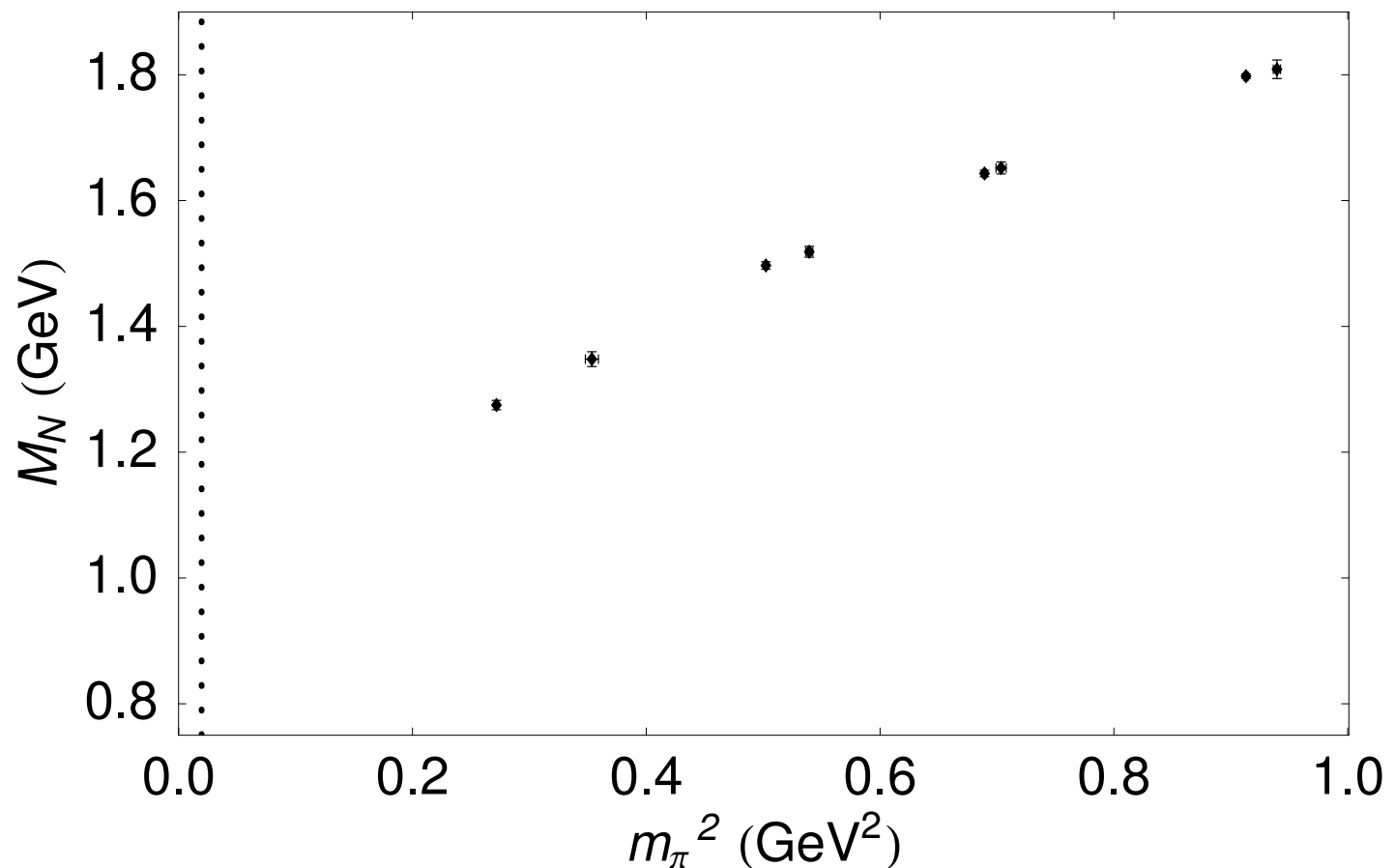


Introduction

- Chiral extrapolation of Lattice QCD results

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 - ▶ CP-PACS *PRD65:054505(2002)*



Introduction

- Chiral extrapolation of Lattice QCD results
- Low-energy QCD constraints
- Finite-Range Regularisation (FRR)
- Partial-quenching and chiral extrapolation
- Comment on G_M^s

Nonanalytic Behaviour

- Chiral extrapolation is nontrivial
- $m_\pi^2 \propto m_q$ (GOR relation)
- Chiral expansion:

$$M_N = c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + \dots$$

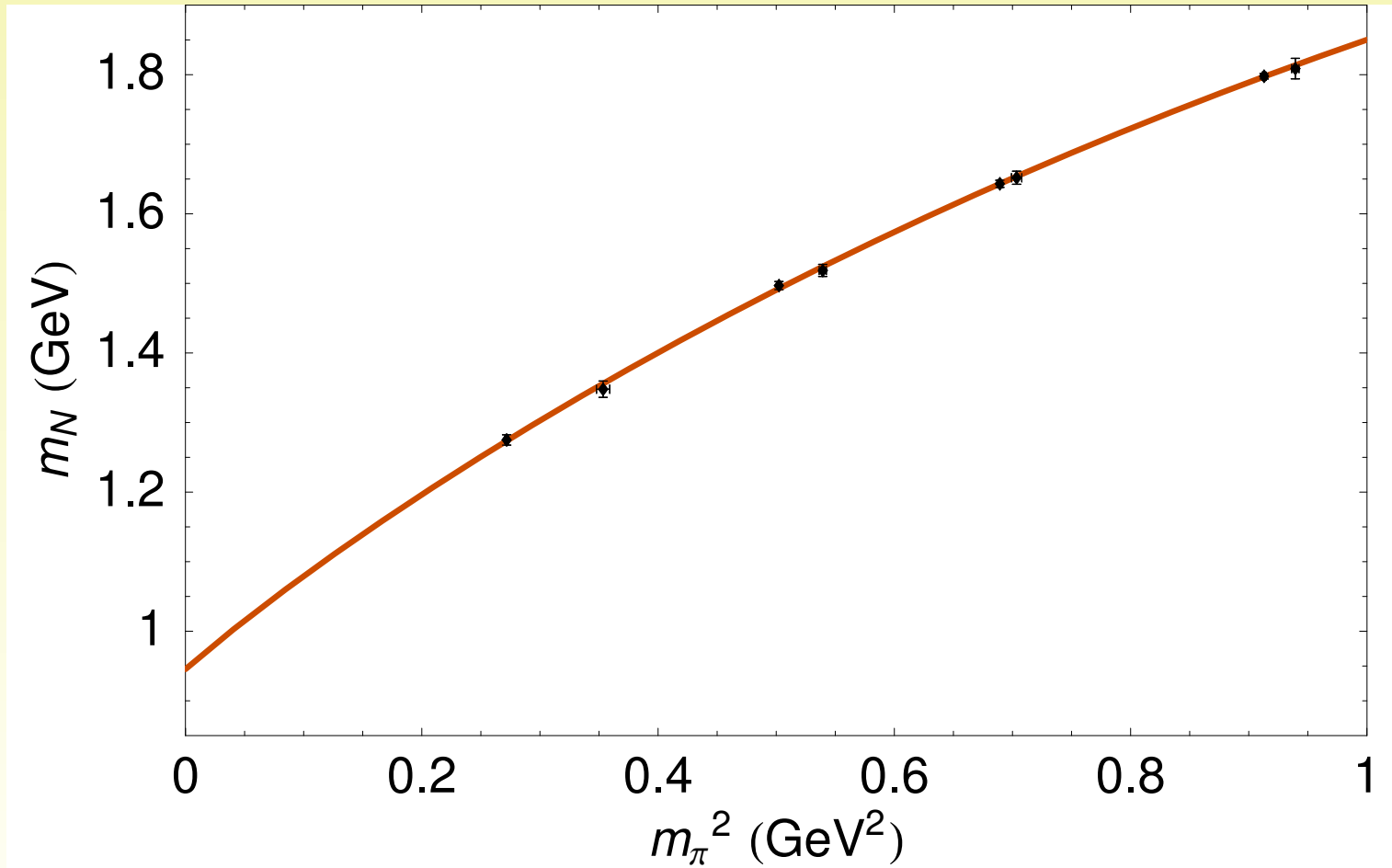
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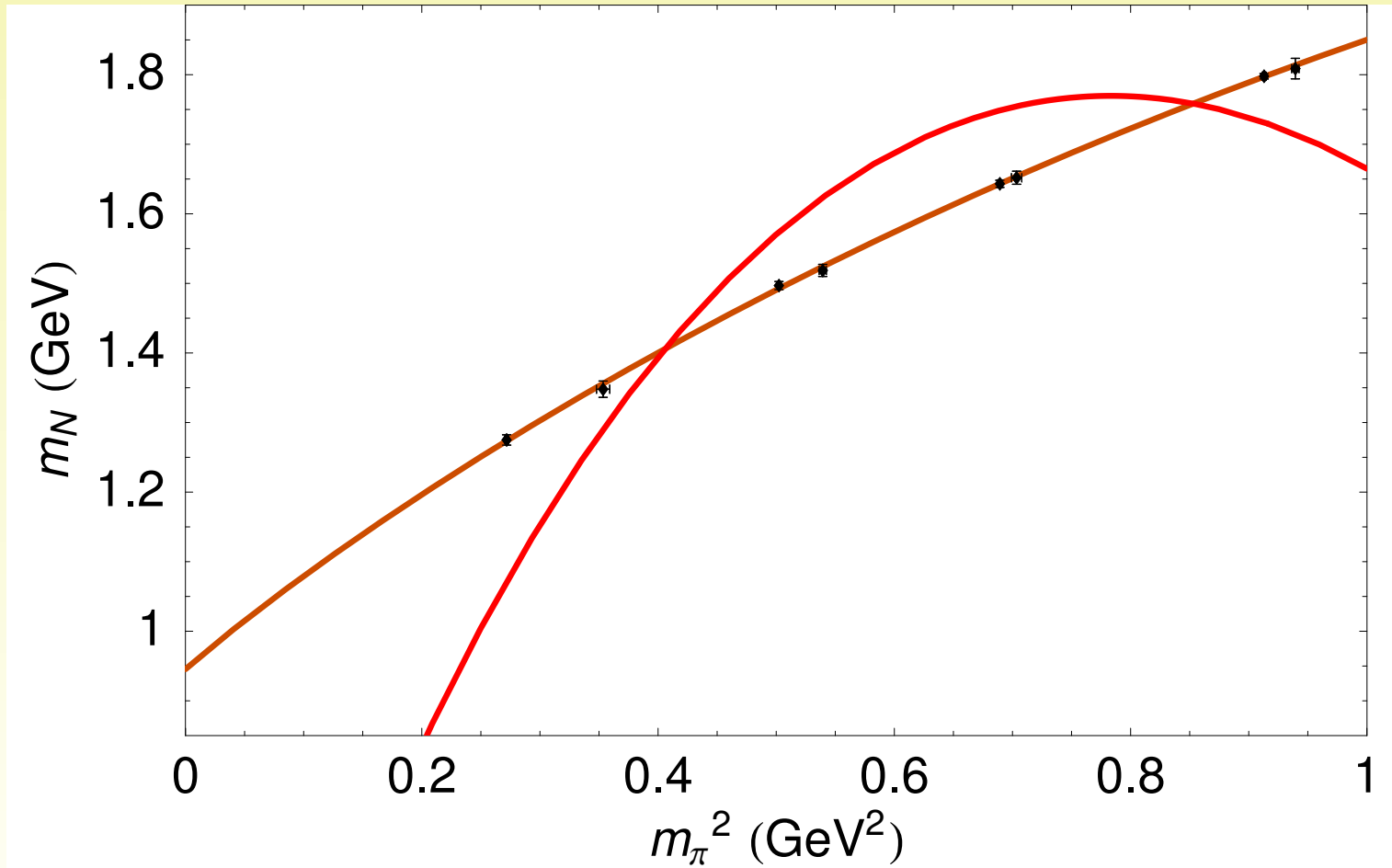
$$\chi_\pi = -\frac{3g_A^2}{32\pi f_\pi^2} \simeq -5.5 \text{ GeV}^{-2}$$

Fit Lattice Data



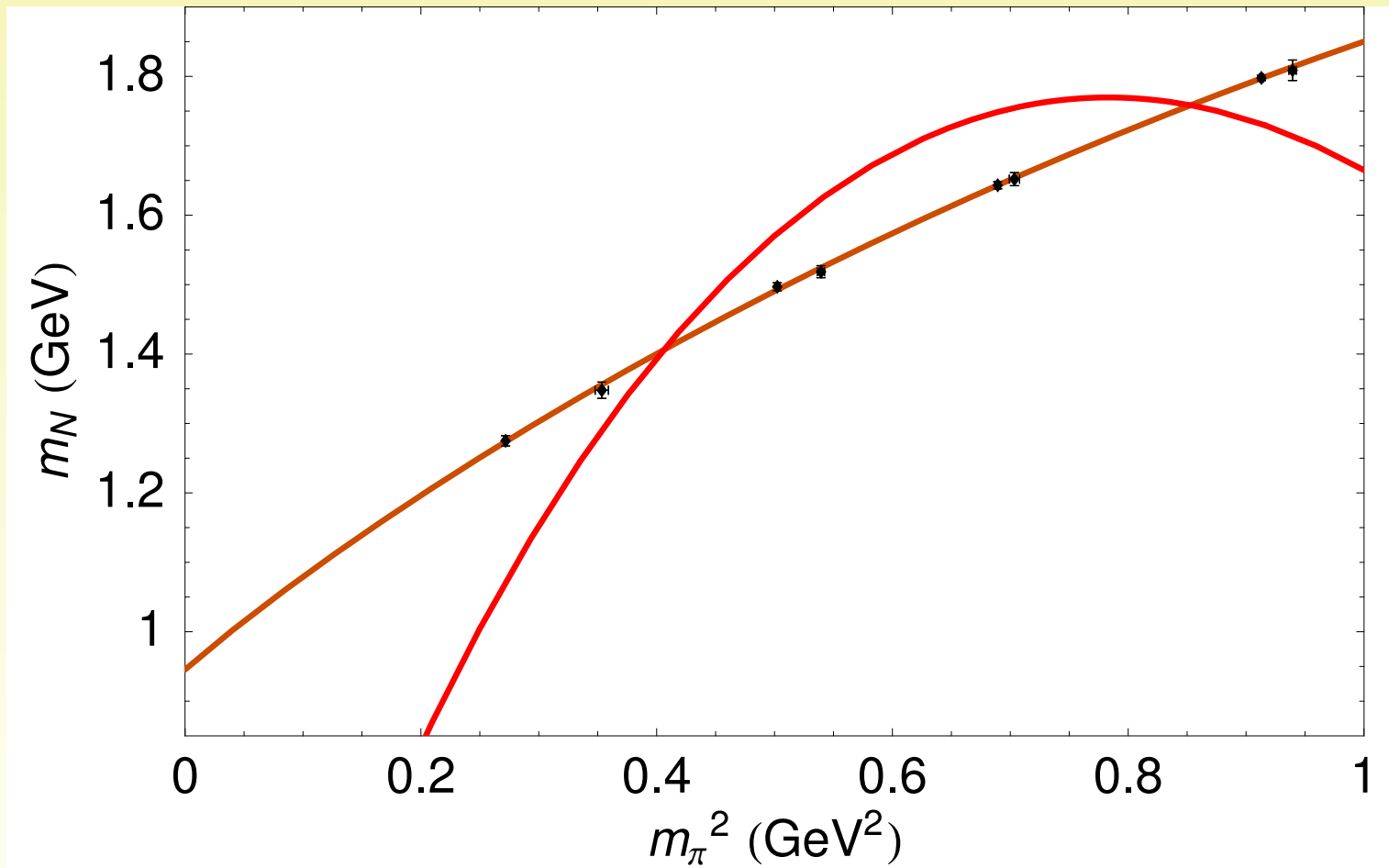
- $\chi_\pi = -0.63 \text{ GeV}^{-2}$! Order of magnitude too small!

Fit Lattice Data



- $\chi_\pi = -5.5 \text{ GeV}^{-2}$, fixed to phenomenological value

Fit Lattice Data



- Need to do more work

Expansion of Nucleon Mass

- Expansion about chiral limit ($m_q = 0$)

$$M_N = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + a_6 m_\pi^6 + \chi_\pi I_\pi(m_\pi) + \chi_{\pi\Delta} I_{\pi\Delta}(m_\pi) + \chi_{\text{tad}} I_{\text{tad}}$$

$$I_\pi = \text{---} \overset{\text{---}}{\text{---}} \text{---}$$

$$I_{\pi\Delta} = \text{---} \text{---} \text{---}$$

$$I_{\text{tad}} = \text{---} \text{---}$$

Finite-Range Regularisation (FRR)

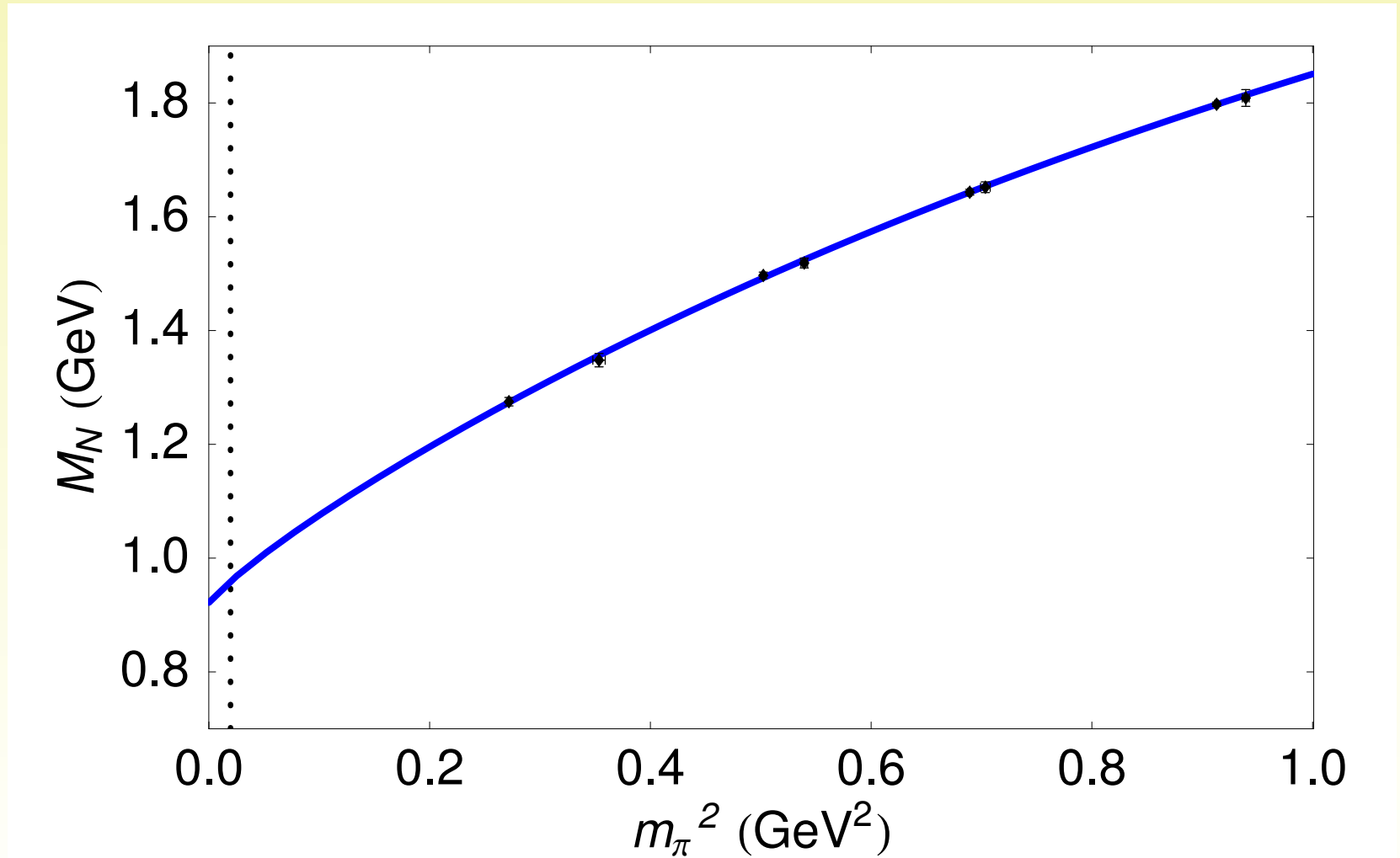
- Suppress loop integral above **finite** mass scale Λ
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Finite-Range Regularisation (FRR)

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- Functional cutoff in momentum space, eg. **DIPOLE**
- Alternatives: Sharp, Monopole, Gaussian...
- Mathematically equiv. to dimensional regularisation

M_N^{NLNA} Extrapolation

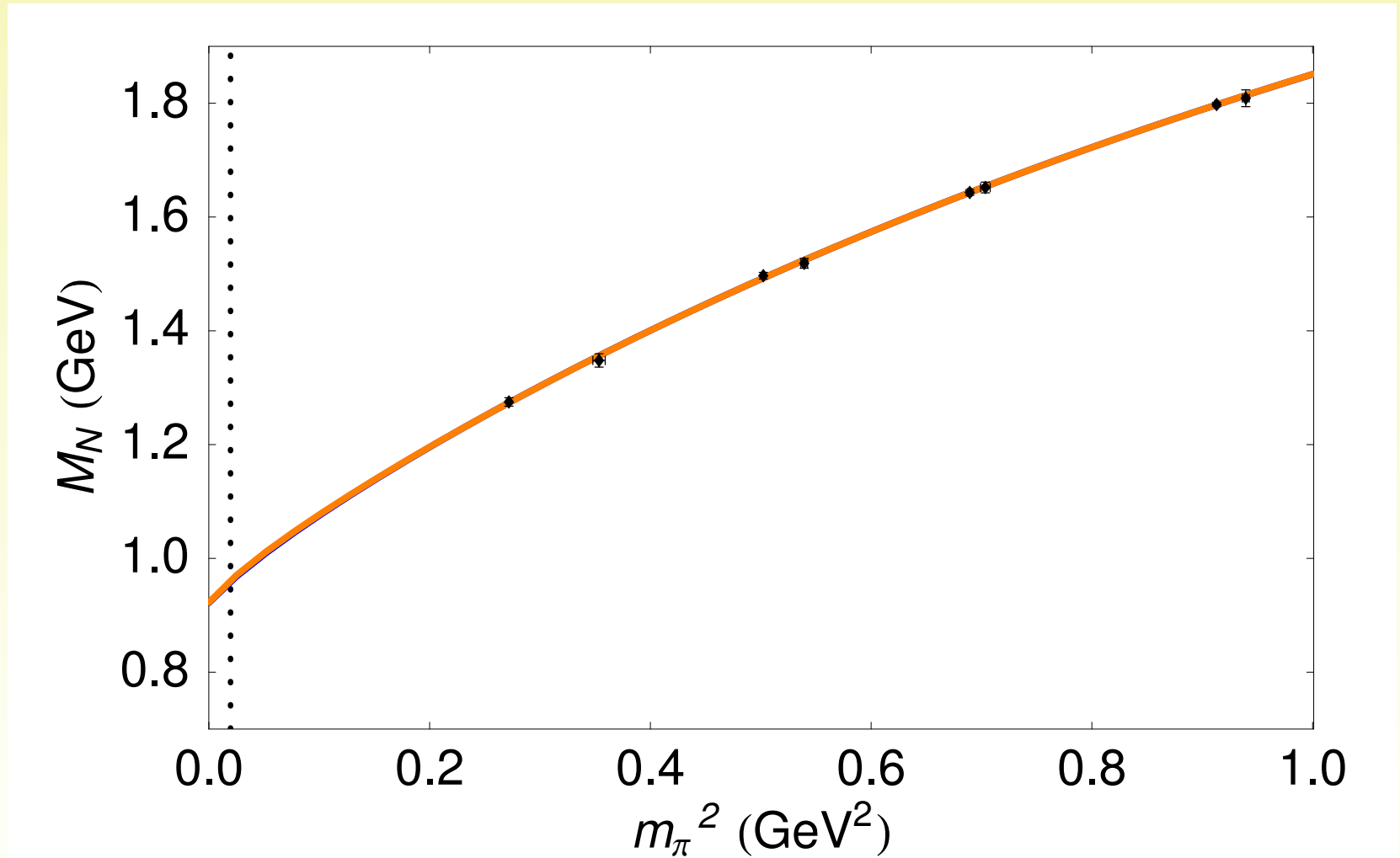
● DIPOLE



Leinweber, Thomas, RDY, PRL92:242002(2004)

M_N^{NLNA} Extrapolation

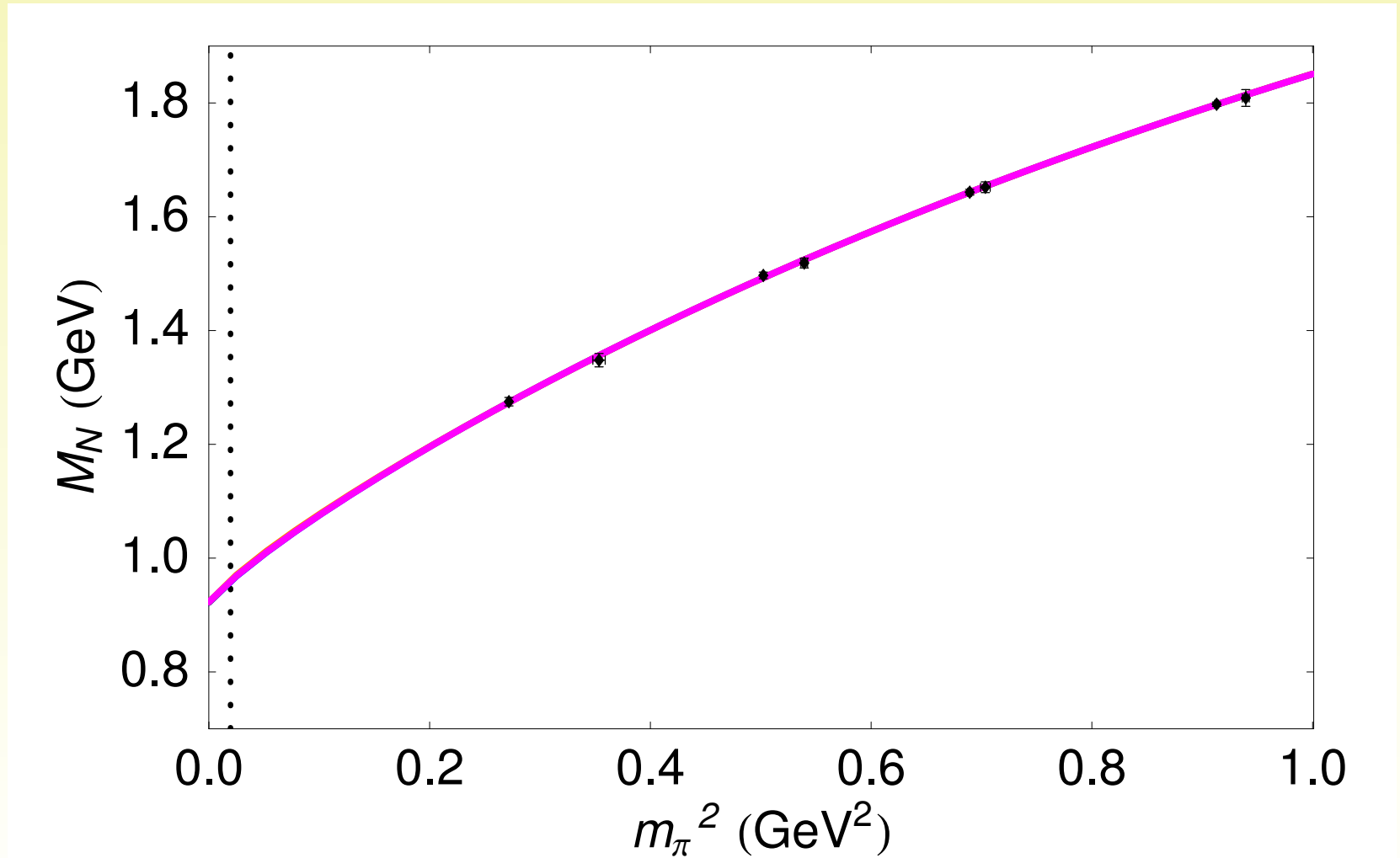
● SHARP CUTOFF



Leinweber, Thomas, RDY, PRL92:242002(2004)

M_N^{NLNA} Extrapolation

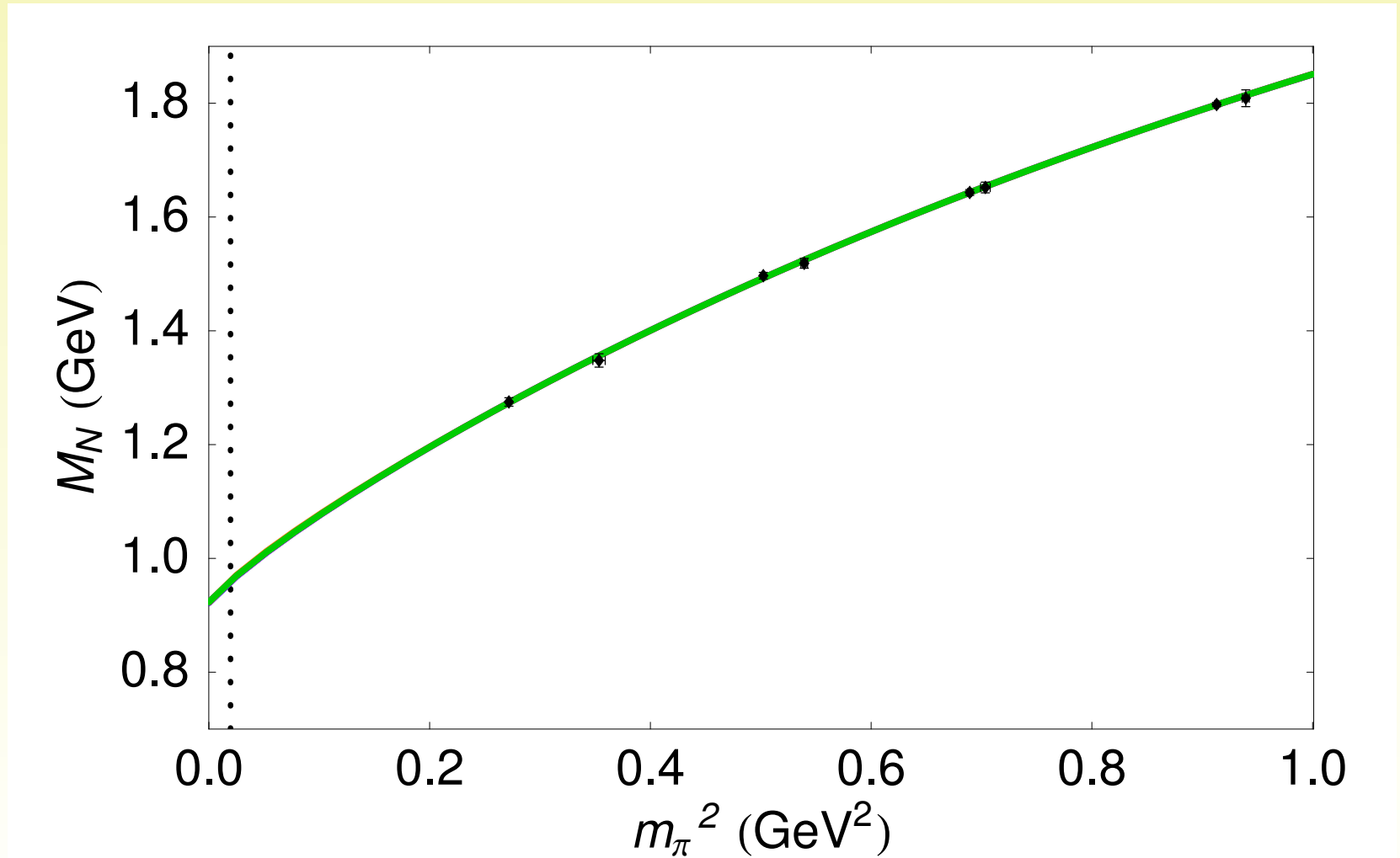
● MONOPOLE



Leinweber, Thomas, RDY, PRL92:242002(2004)

M_N^{NLNA} Extrapolation

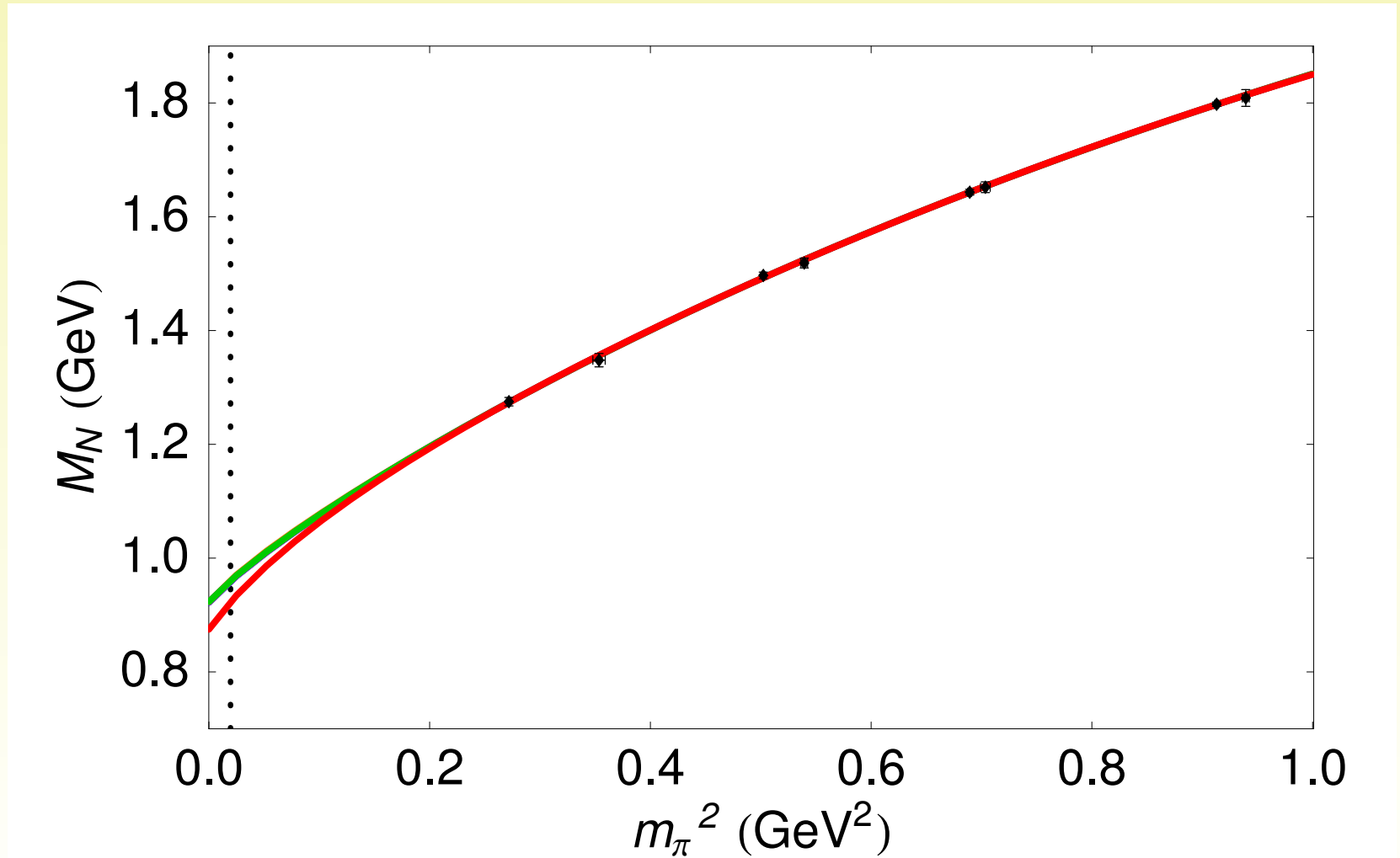
● GAUSSIAN



Leinweber, Thomas, RDY, PRL92:242002(2004)

M_N^{NLNA} Extrapolation

● DIMENSIONAL REGULARISATION



Leinweber, Thomas, RDY, PRL92:242002(2004)

Partially-Quenched QCD

- Valence and sea quark-masses differ

$$m_{\text{sea}} \neq m_{\text{val}}$$

- Rigorous connection to unitary theory

Sharpe & Shoresh, PRD62:094503(2000)

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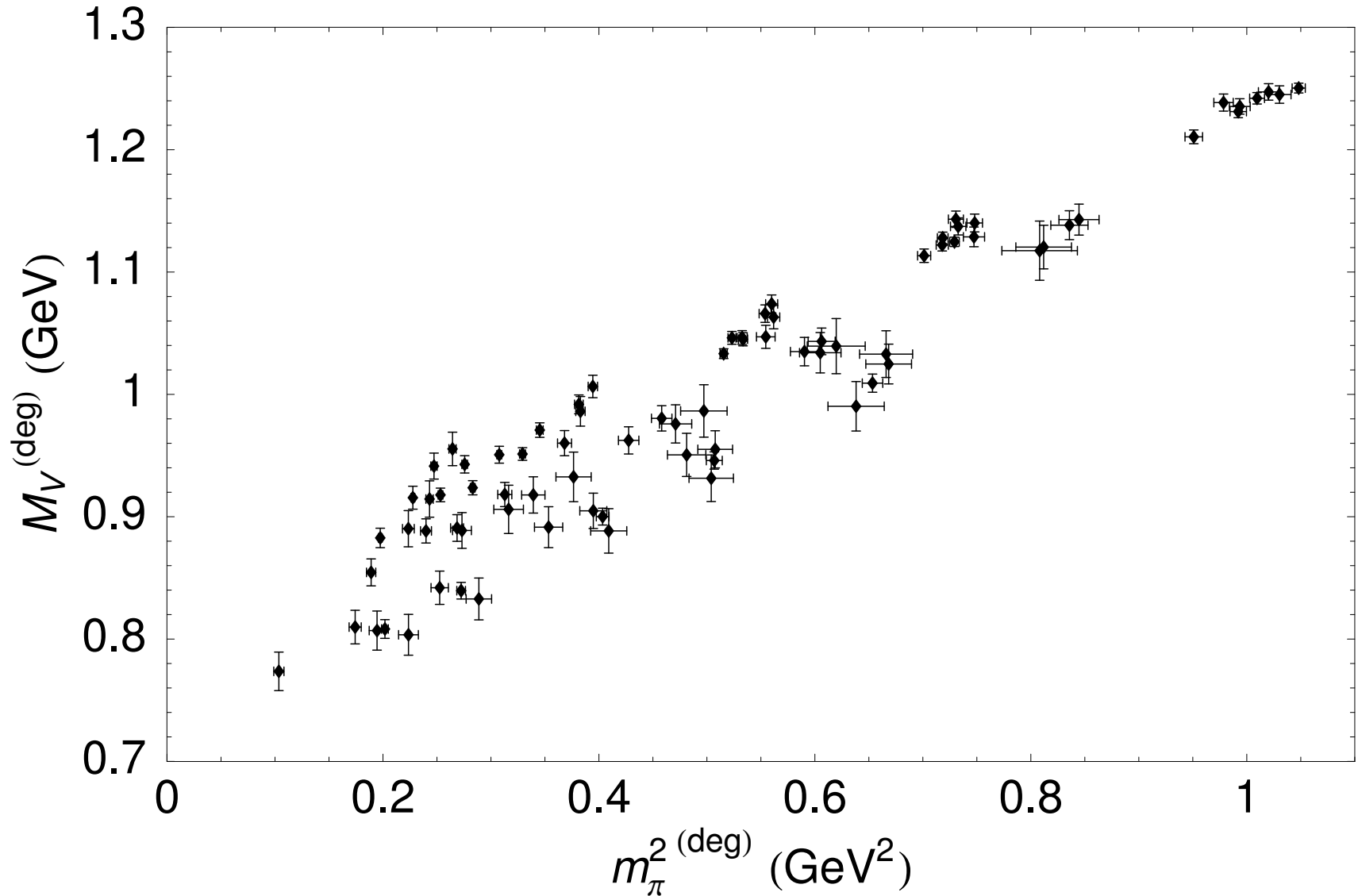
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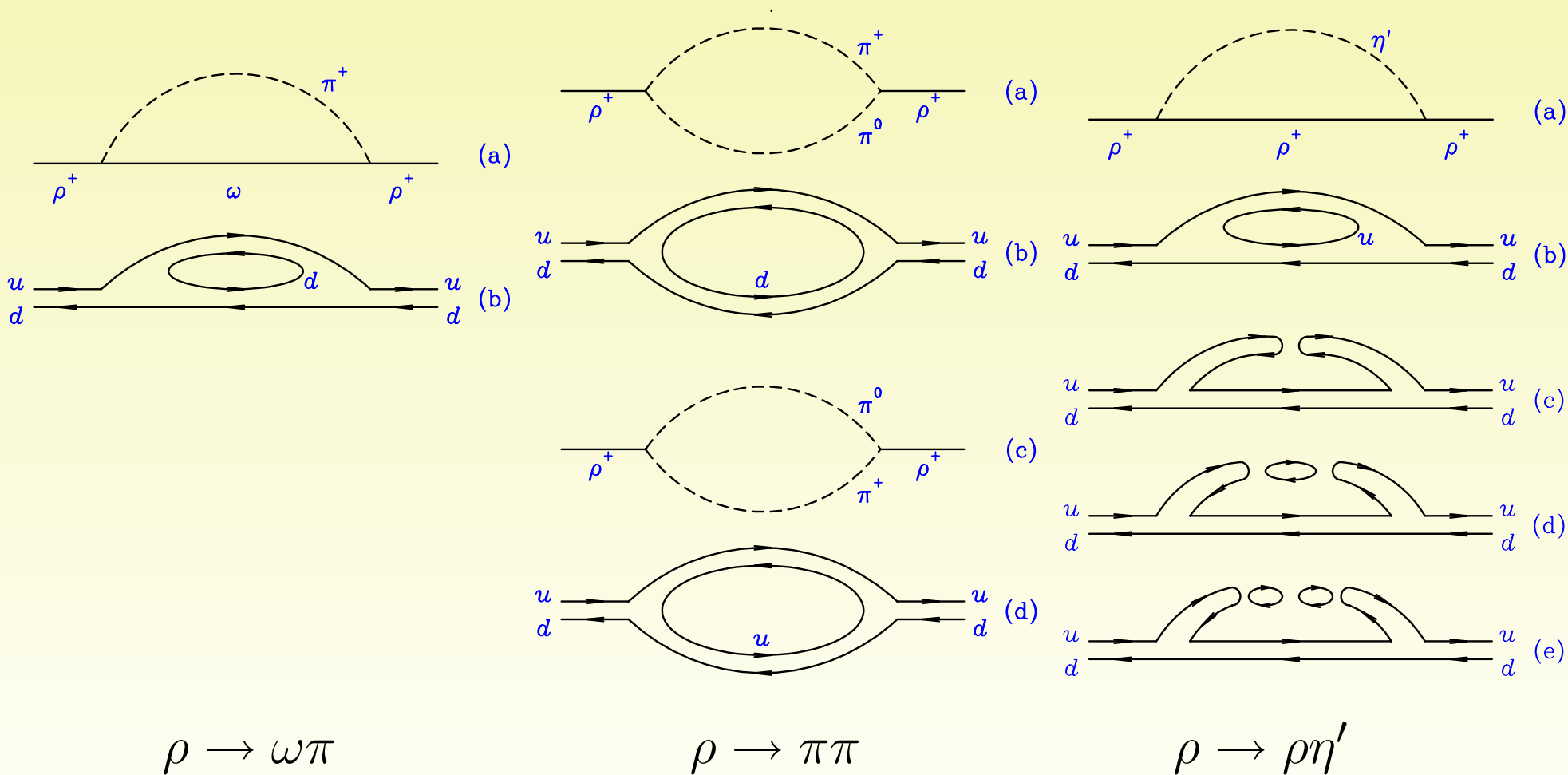
- Use Effective Field Theory to fit PQ lattice results

Partially-Quenched Vector Meson

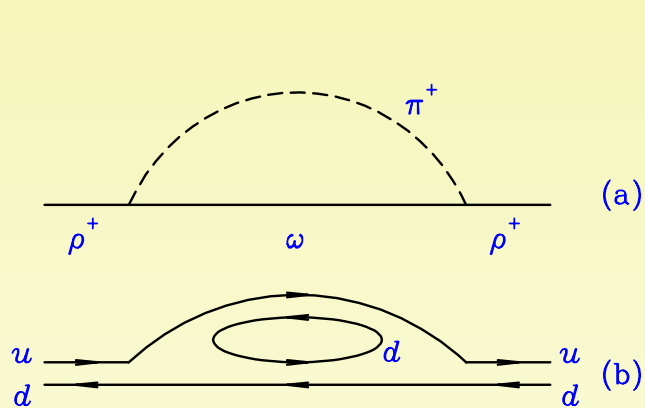
● CP-PACS Collaboration, PRD65:054505(2002)



Chiral Contributions — Σ

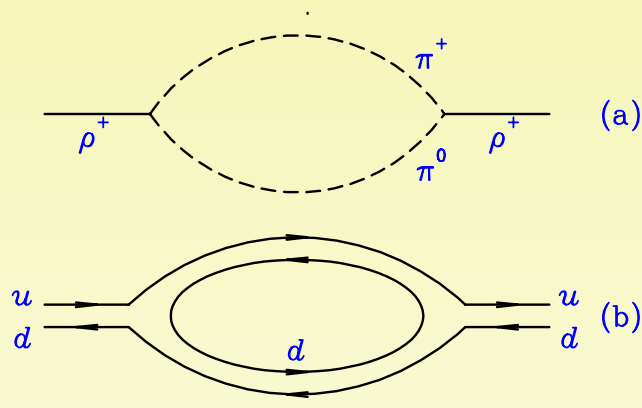


Chiral Contributions — Σ



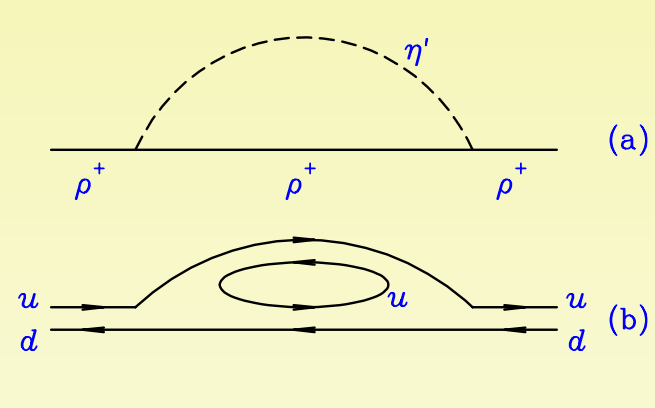
$$\rho \rightarrow \omega \pi$$

$$\sim m_\pi^3$$



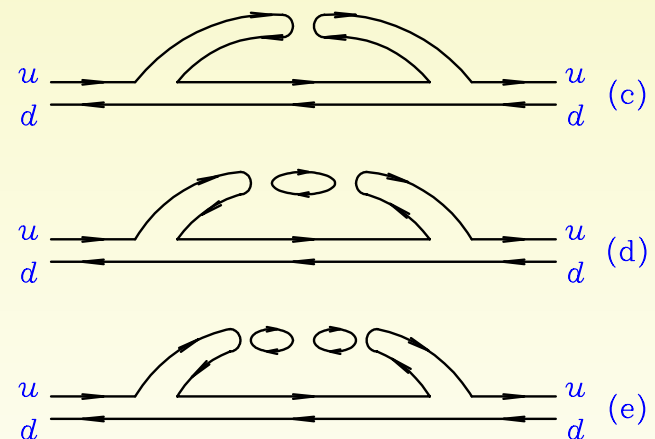
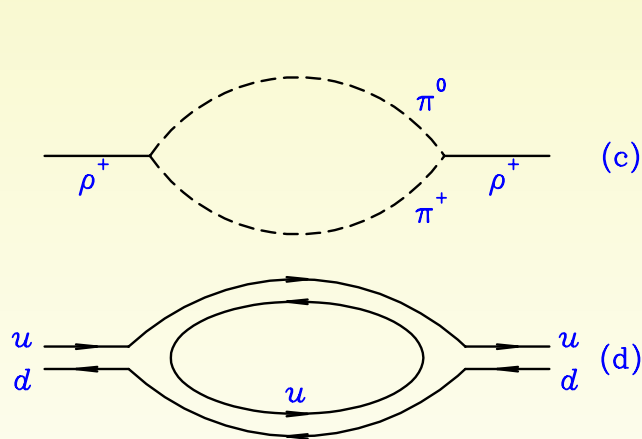
$$\rho \rightarrow \pi \pi$$

$$\sim m_\pi^4 \log m_\pi$$



$$\rho \rightarrow \rho \eta'$$

$$\sim (m_{\text{val}} - m_{\text{sea}}) m_\pi$$



Lattice Artefacts

- Quark-mass dependence:

$$M_V^2 = (\alpha_0 + \alpha_2 m_\pi^2 + \alpha_4 m_\pi^4)^2 + \Sigma$$

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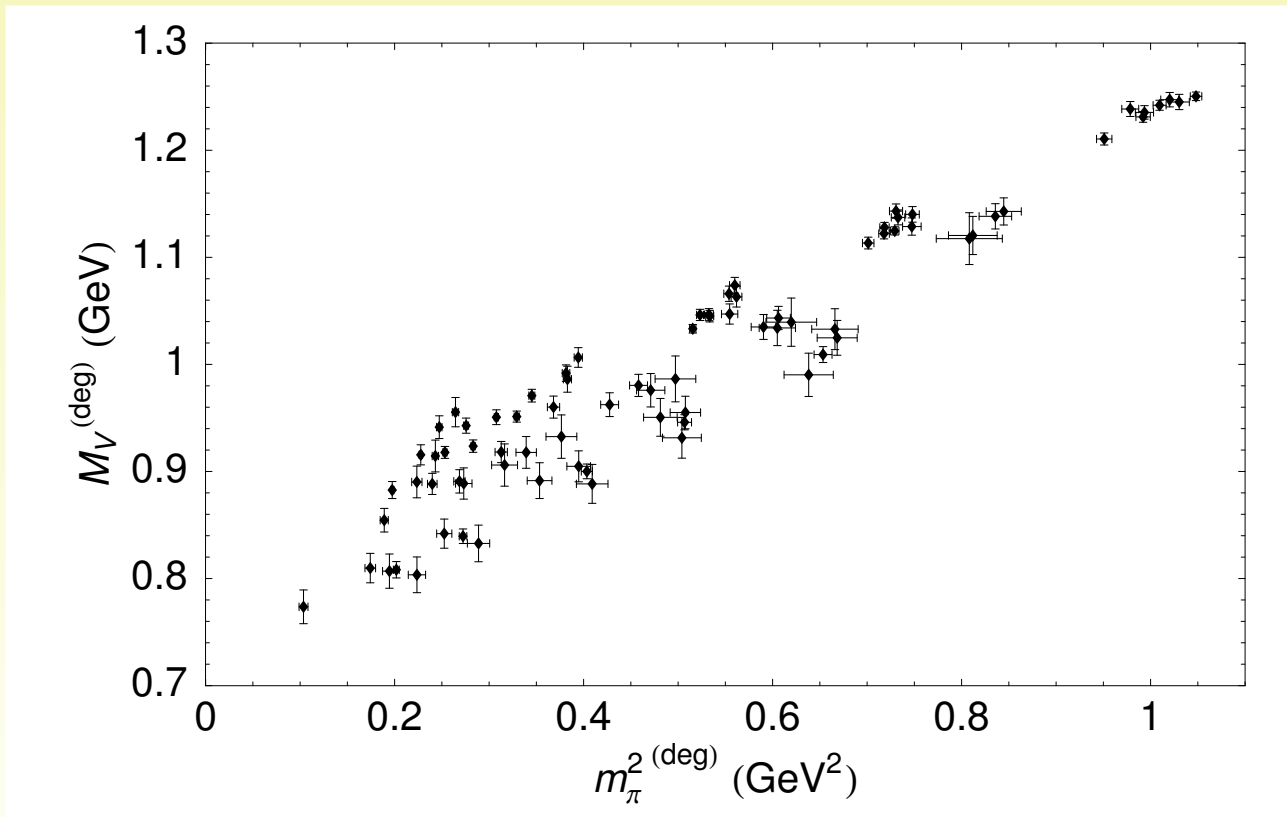
- Finite volume:

$$M_V^2 = (\alpha_0 + X_2 a^2 + \alpha_2 m_\pi^2 + \alpha_4 m_\pi^4)^2 + \Sigma(L)$$

$$\Sigma(L) : \quad \int d^3 k \rightarrow \left(\frac{2\pi}{L}\right)^3 \sum_{\vec{k}}$$

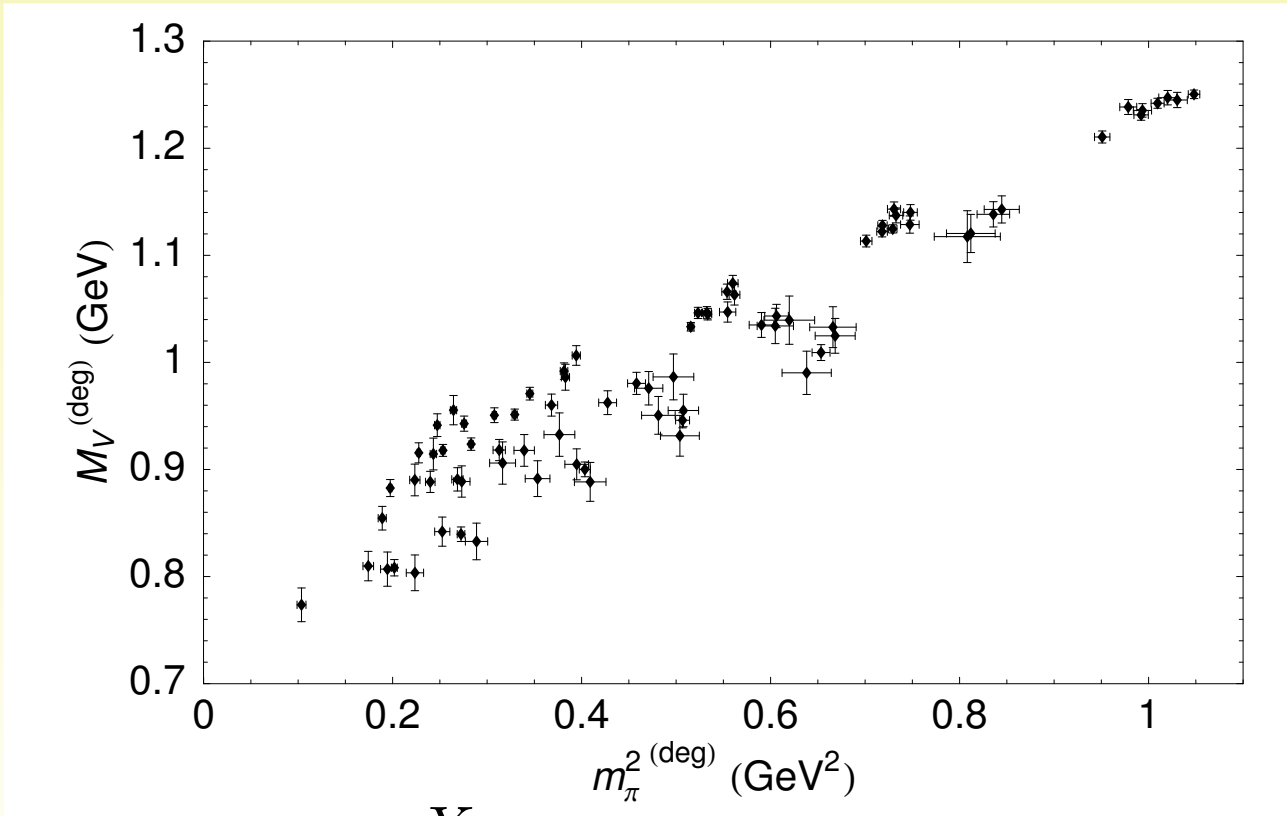
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α_0

X_2

α_2

α_4

χ_{dof}^2

0.832(4)

-1.40(4)

0.494(12)

-0.061(9)

39/76

Remove Lattice Artefacts

- Restore unitary limit, $m_{\text{val}} \rightarrow m_{\text{sea}}$

Remove Lattice Artefacts

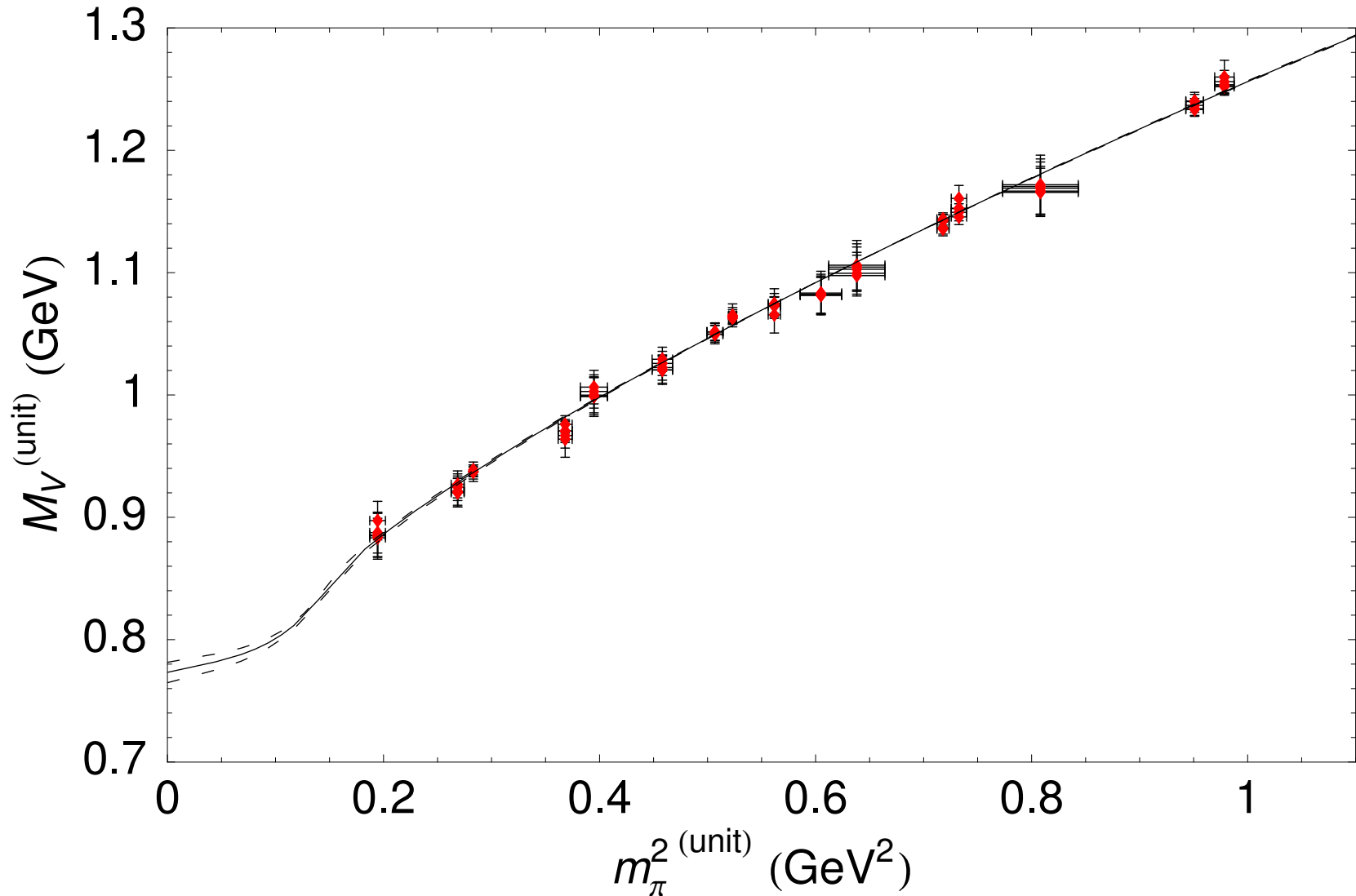
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- Remove discretisation, $a \rightarrow 0$

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- Restore infinite volume, $L \rightarrow \infty$

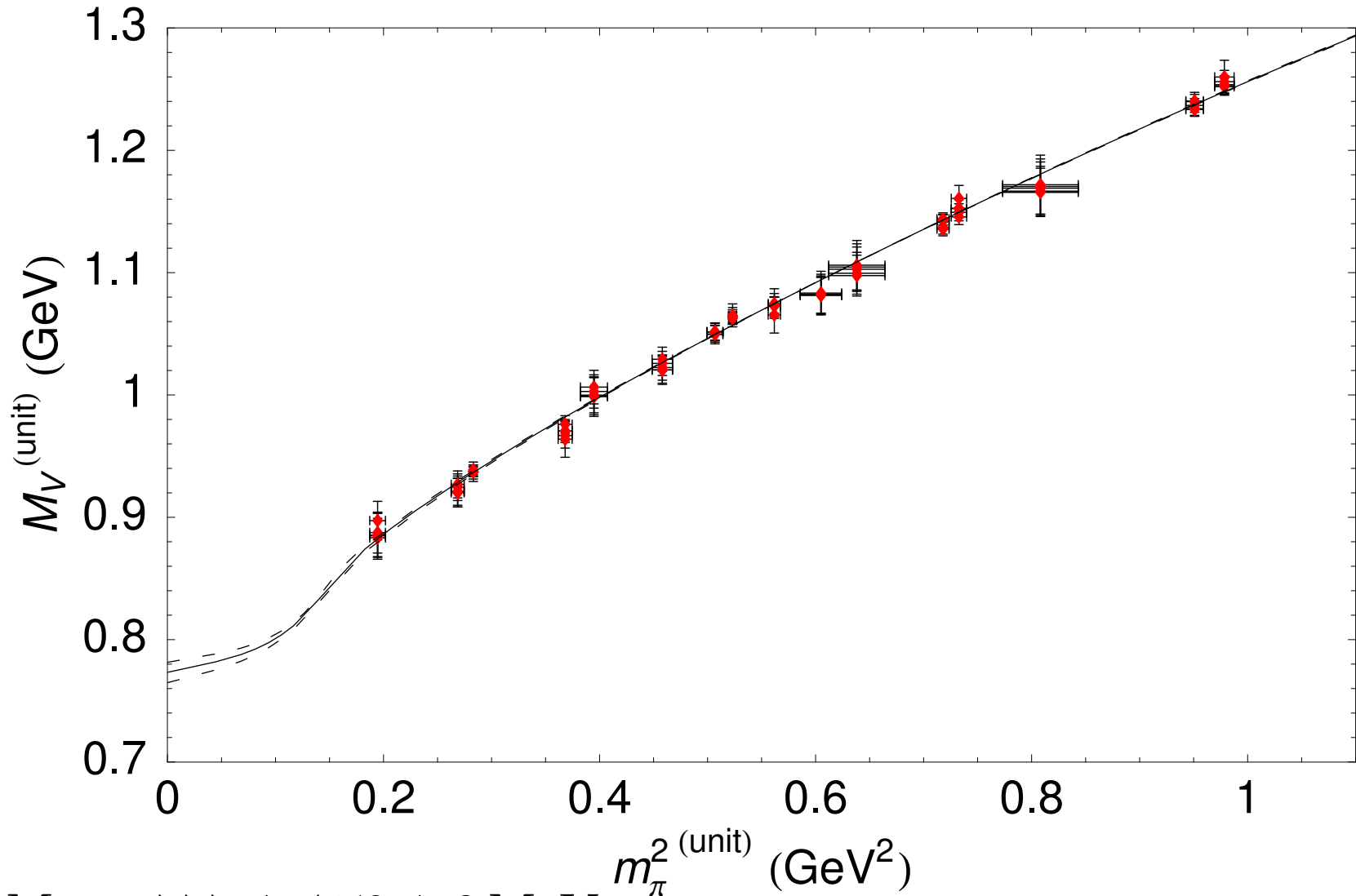
Restoring Unitary Continuum

● Allton et al., hep-lat/0504022



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$$M_V = 777 \pm 4_{-0}^{+13} \pm 8 \text{ MeV}$$

Strangeness or Strange?

- Latest G0 result, combined with earlier result
- $Q^2 = 0.1 \text{ GeV}^2$
 - ▶ $G_E^s = -0.013 \pm 0.028$
 - ▶ $G_M^s = +0.62 \pm 0.31$

Comment on G_M^s

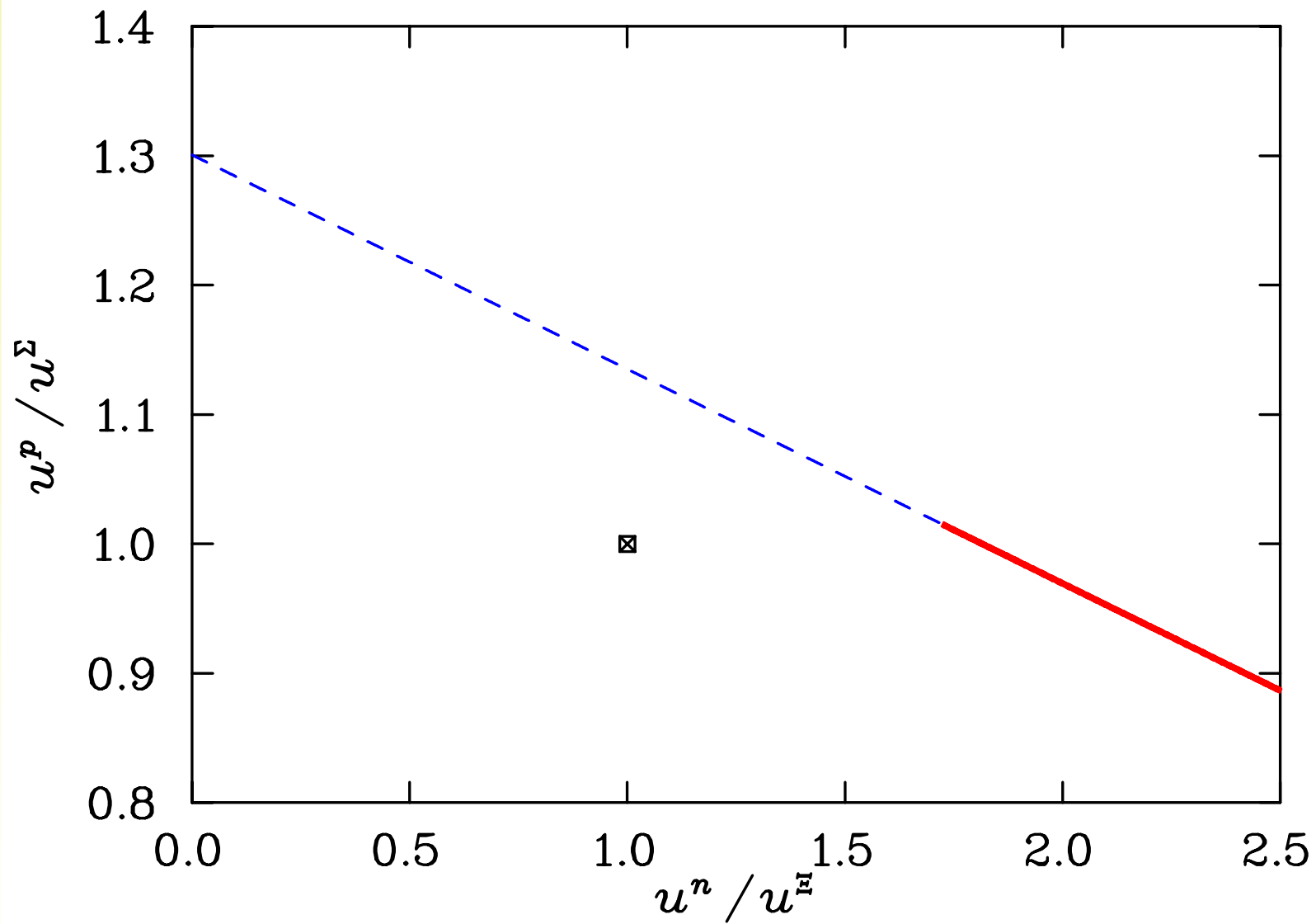
- Strangeness magnetic moment

$$G_M^s = \left(\frac{{}^\ell R_d^s}{1 - {}^\ell R_d^s} \right) \left[2p + n - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \right]$$

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- ${}^\ell R_d^s = {}^\ell G_M^s / {}^\ell G_M^d$ ($0 \leftrightarrow 1$)
- Environment sensitivity, u^p / u^Σ , u^n / u^Ξ

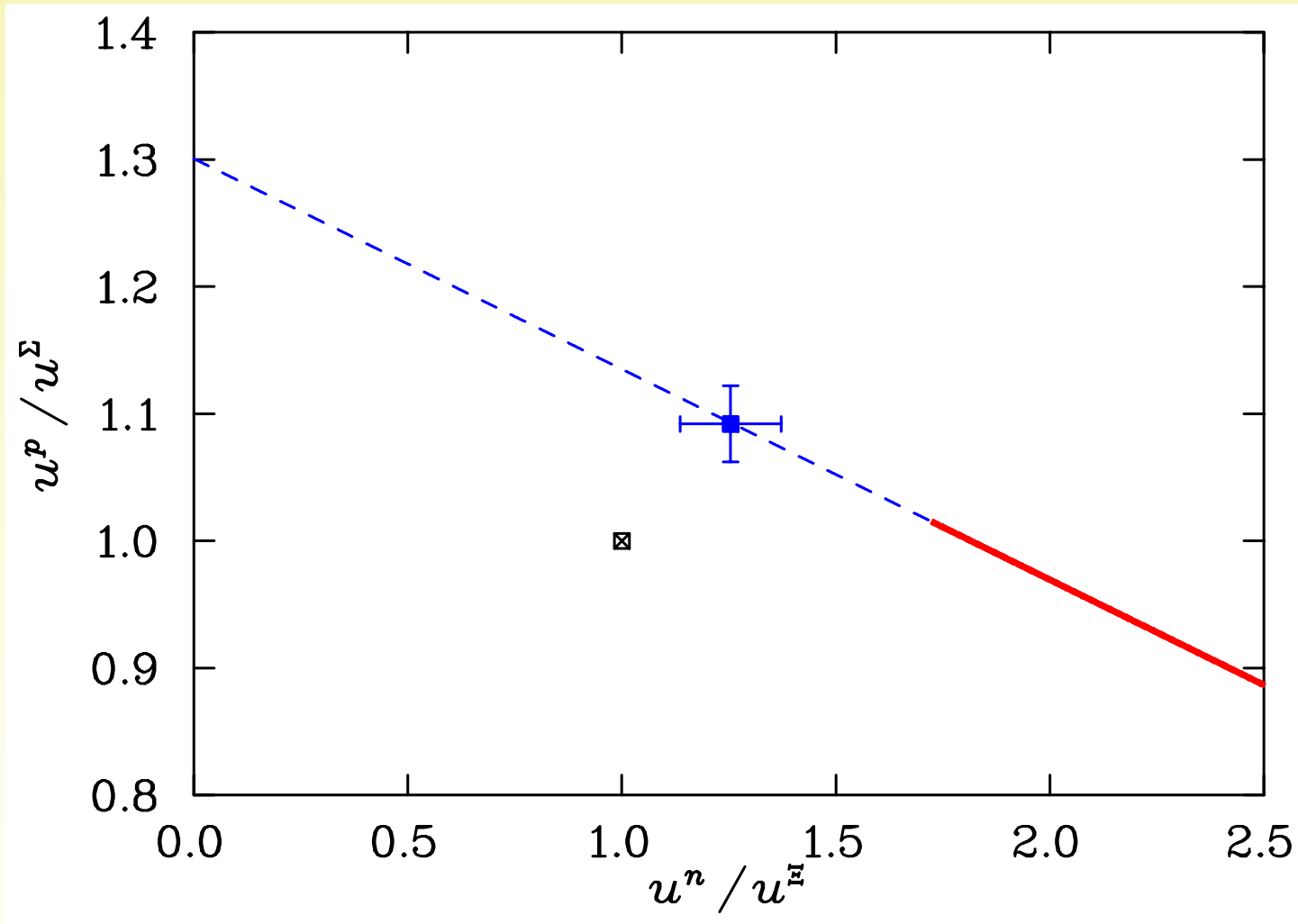
Comment on G_M^s



$G_M^s < 0$

$G_M^s > 0$

Final Result



$$G_M^s = -0.046 \pm 0.019 \mu_N \quad \text{Leinweber et al., PRL94:212001(2005)}$$

Conclusions

- Chiral extrapolation solved with FRR
- Use of PQ results dramatically increases statistics
 - ▶ Excellent description of ρ -mass
- “Strange” things are happening
- QCD Expectations:
 - ▶ $G_M^s < 0$
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