Ultra-Forward Tagging in Hard Exclusive Reactions on Nuclei

Charles E. Hyde[1]

[1] *Laboratoire de Physique Corpusculaire, Université Blaise Pascal, 63177 Aubière, France, and Old Dominion University, Norfolk, VA 23529, USA.*

(Dated: May 26, 2009)

A high energy electron ion collider offers unique opportunities and challenges for studying coherent deeply virtual exclusive processes on nuclei. If the momentum transfer to the nucleus, $\mathbf{\Delta}_\perp$ transverse to the beam direction, can be resolved to order 10 MeV/c, then it is possible to form spatial images of the quark and gluon distributions of the nucleus. Selecting coherent events requires detecting the final state nucleus. In all reasonable kinematics, the final state nucleus will be contained within the transverse admittance of the accelerator lattice. However, the nucleus will have lost between 0.1% and 10% of its linear momentum. Therefore it should be detectable in a region of high dispersion downstream of the IP optics. Because of the intrinsic transverse momentum spread of the ion beam at the IP, the transverse momentum transfer to the nucleus cannot be resolved by measurement of the scattering angle. Instead, via the exclusivity constraint, high resolution can be achieved in *detl* via a combination of the $(e, e'\text{meson})$ kinematics and a high resolution measurement of the longitudinal momentum of the final state nucleus. In some respects, the nuclear case is simpler than the proton case.

I. DEEPLY VIRTUAL REACTIONS ON NUCLEI

Coherent deeply virtual exclusive reactions, such as

$$\begin{aligned} e + {}^AZ &\to e + \gamma + {}^AZ \\ e + {}^AZ &\to e + \omega + {}^AZ \\ \gamma + {}^AZ &\to J/\Psi + {}^AZ \end{aligned} \tag{1}$$

offer unique opportunities for probing the spatial distributions of quarks and gluons in nuclei. This note will examine some issues regarding the detection of coherent events and resolving the momentum transfer to the nucleus. I will not discuss quasi elastic processes on nuclei. I will focus on nuclei from ^{4}He to ^{20}Ne. Deuterium and ^{3}He are special cases requiring separate study.

Determining exclusivity requires tagging the nucleus in the final state. The typical scale of transverse momentum transfer is given by the rms nuclear radius.

$$P_\perp \approx \frac{\hbar c}{R_A} \approx 0.2\,\text{GeV}A^{-1/3} \tag{2}$$

For nuclei from ^{4}He to ^{20}Ne, this scale ranges from 125 MeV/c to 75 MeV/c, respectively. To understand the challenge of tagging these recoil nuclei, we must examine the intrinsic transverse momentum spread of ion beam at the intersection point.

II. COLLIDER OPTICS

The Interaction Point (IP) optics of the proposed JLAB ELIC collider are summarized in Table I.

A. Ion Beams

For an ion beam of mass number A and fully stripped charge Z, the momentum is $P_A = ZP$, where P is the proton momentum in Table I, which is determined by the radius of the arcs and the bending strengths of the magnets. Note also that the momentum per nucleon is $p = ZP/A$.

The intra-beam forces on an individual ion scale as ZI_i, where I_i is the ion beam current, this force produces an relative impulse which scales only with the ion current:

$$\Delta P_A/P_A \sim I_i/P \tag{3}$$

Therefore, if the beam current I_i is constant as a function of Z, all other emittance, tune-shift, *etcetera*, parameters will be constant. Therefore the ion particle number in the beam scales as $1/Z$ and the luminosity per nucleus scales as $1/Z$, but the luminosity per nucleon scales (relative to the proton) as A/Z .

TABLE I: ELIC Accelerator Parameters, April 2009. Figure eight circumference of electron and ion rings is 560 m. These optics require FF Quads at approximately ±5 m from the IP. moving the quads further away requires increasing β^* and possible lowering the Luminosity, each by roughly a factor of 2. For Nuclei, the beam momentum scales as Z, the current is constant, the particles/bunch scales as $1/Z$, the emittance and β^* parameters are unchanged. Therefore luminosity per nucleus scales as $1/Z$ and luminosity per nucleon scales as A/Z.

Proton/Electron	Units		
Beam Momentum	GeV/c	60/5	60/3
CM Energy	GeV	$\sqrt{1200}$	$\sqrt{720}$
Beam Current	Amp	0.6/2.0	2.7/5
Bunch Frequency	GHz	0.5	0.5
Particles/ Bunch	10^{10}	0.7/2.5	3.4/6.3
Norm. Hori. Emittance	μm	1/147	0.8/75
Norm. Vert. Emittance	μm	0.1/15	0.8/75
Optics at High Luminosity IP			
Bunch Length	mm	5	5
Horizontal β^*	mm	5/5	5/5
Vertical β^*	mm	5/5	5/5
RMS Size at IP	μm	8.7/2.7	8/8
Horiz RMS Angle	mr	1.8/1.7	1.6/1.6
Vert RMS Angle	mr	0.6/0.6	1.6/1.6
Luminosity	$10^{34}/(\mathrm{cm}^2\,\mathrm{s})$	3	13

B. Transverse Admittance

We can now calculate the rms transverse momentum in the ion beam at the IP. Using 1.6 mr as a typical number for the rms angular spread of the beam in either the horizontal or vertical direction:

$$\sqrt{\langle P_{A\perp}^2\rangle} = (1.6\,\mathrm{mr})\,Z(60\,GeV/c) \approx Z(100\,\mathrm{MeV/c}) \tag{4}$$

Even if this is reduced by a factor of two with an alternate high luminosity tune, this is a very large spread. Furthermore, the limit for placing any detectors with a geometrical view of the IP (*i.e.* no magnetic analysis) is probably at least 7σ. Therefore, the transverse admittance of the final focus quadrupoles is

$$\Delta P_\perp \le Z(700\,\mathrm{MeV/c}). \tag{5}$$

Any recoil nucleus with transverse momentum in this range is undetectable in front of the final focus quadrupoles (FFQ), but will be cleanly transported through the FFQ.

C. Longitudinal Admittance

The CEBAF beam has an energy spread of less than or equal to a part in 10^4. The CEBAF arcs have a momentum acceptance of 10^{-3}, although alternate tunes are possible that can either increase or decrease this by as much as a factor of ten [1].

The kinematics of the final state nucleus in a deeply virtual exclusive reaction is characterized by the transverse momentum $P_{A\perp}$ and the fraction 2ζ of longitudinal momentum lost by the beam. In terms of the more familiar Bjorken-x variable,

$$2\zeta \approx \frac{2x_{\mathrm{Bj}}/A}{2 - x_{\mathrm{Bj}}/A} \approx x_{\mathrm{Bj}}/A, \tag{6}$$

where x_{Bj} is defined in terms of the momentum per nucleon p:

$$x_{\mathrm{Bj}} = \frac{Q^2}{2q\cdot p} = \frac{Q^2}{2q\cdot P_A/A} \tag{7}$$

with the kinematic bounds

$$0 \leq x_{\rm Bj} \leq A \tag{8}$$

In practice, coherent processes will probably be limited to $x_{\rm Bj} < 1$ for reasons of count rate. The previous section established that the final state nucleus is necessarily within the transverse admittance of the storage ring lattice. What about the the longitudinal admittance? Using a 10σ admittance limit for the whole ring and a 10^{-4} rms energy spread at the IP (which may be affected by some crab schemes), then any exclusive process with $2\zeta > 10^{-3}$ is detectable with tracking detectors strategically located in the first few bends after the IP. These detectors almost certainly have to be of the "Roman Pot" type (named after the Rome group that first implemented this type of detector, and not ancient pottery). These are detectors, usually incorporating Si μstrip trackers that are integrated into the beam-line vacuum, but retract away from the beam-line during beam injection and tuning. In the TOTEM experiment, a pair of Si trackers are installed at 147 and 220 m on either side of the CMS interaction point [2]. With $20\,\mu$m spatial resolution, this will provide μrad resolution for ultra-low t elastic scattering. Additional 8m long trackers are planned at $\pm$420 m for both ATLAS and CMS [3].

Assuming a detectability range of $2\zeta > 0.001$, this corresponds to a A-dependent $x_{\rm Bj}$ range of

$$x_{\rm Bj} > A(10^{-3}) \tag{9}$$

At fixed Q^2, there is a kinematic bound on $x_{\rm Bj}$

$$x_{\rm Bj} > \frac{Q^2}{2k \cdot P(Z/A)} = \frac{A}{Z}\frac{Q^2}{s - M^2} > 0.005. \tag{10}$$

The numerical constraint assumes $Q^2 > 3$, $s = 1200\,{\rm GeV}^2$, and $A/Z = 2$. Thus the kinematic constraint of Eq. 10 is roughly equivalent with the admittance constraint of Eq. 9, for $4 < A < 20$. On the other hand, if $x_{\rm Bj} < 1$ is a practical upper bound, then we do not need to detect nuclei with momentum loss greater than $1/A$.

Detecting the final state nucleus in a coherent deeply virtual reaction requires a region or regions of moderately high dispersion. Momentum analysis of the final state nucleus requires tracking of the nucleus as well. This means that the high dispersion regions must be in a relatively large drift space. A momentum resolution of 10^{-3} and an angular resolution of 1 mr are the minimum required. However, as described in section IV, a resolution in the longitudinal momentum of 10^{-4} is highly desireable. If Si detectors are placed 1 cm from the beam center, and achieve $100\,\mu$m resolution, then a drift space between 10 and 100 cm should be adequate (this seems surprisingly low, check!!).

III. LATTICE OPTICS

As an example of lattice optics, I use the 60° figure-8 design presented 13 March 2009 by Y. Zhang [4]. The arc consists of super FODO cells, with a 1/2 cell 18.22 m long consisting of

$$(Q/2)DD\overline{Q}DDQDD(\overline{Q}/2) \tag{11}$$

In the center of the final defocussing (in the horizontal plane) $\overline{Q}$ of this half-cell, the dispersion is approximately 1.3 m per 100% and $\beta^* \approx 2$ m. The 10σ size of the ion beam at this point is ≤ 2 mm. Therefore, an ion with momentum loss fraction greater than 0.002 is in principle detectable at this point. A more realistic estimate is that ions with momentum loss fraction greater than 0.01 are detectable with a Si detector 1 cm from the beam line. With a pair of Si trackers on either side of this quadrupole, each ~ 10 cm long, it should be possible to have sufficient drift space in between the two tracking stations to achieve $\sim 10^{-4}$ resolution in the longitudinal momentum. Extending high resolution tracking down to $2\zeta \geq 0.001$ will require either creating a dispersion of ~ 10 m or bringing the detector closer to the beam.

An alternative is to build a chicane spectrometer into the long straight section in between the IP and the crab cavity. This may be necessary also as part of the generation of the crab crossing.

IV. MOMENTUM TRANSFER RESOLUTION

The net four-momentum transfer to the nucleus in the exclusive reactions of Eqs. 1 is denoted by $\Delta^\mu = (q - q')^\mu$, where q and q' are the four-momenta of the initial photon and final photon or meson, respectively.

The spatial distribution of quarks and gluons in a hadronic target (meson, nucleon, nucleus) can be extracted from the Fourier transform with respect to $\boldsymbol{\Delta}_\perp$ of the target Generalized Parton Distributions (GPDs). The deeply virtual cross sections are equal to bilinear combinations of the GPDs (and linear combinations in the case of the

Bethe-Heitler–DVCS interference in $eA \to eA\gamma$). Evaluating this Fourier transform therefore requires a resolution on $\mathbf{\Delta}_\perp$ that is better than the intrinsic scale of Eq. 2. This resolution impossible to achieve *via* resolving the final state nucleus, due to the large intrinsic $P_\perp$ in the beam (Eq. 4). Thus the only strategy for resolving $\mathbf{\Delta}_\perp$ is to determine it from the $(e, e'\gamma)$, $(e, e'\text{meson})$, or $(\gamma, J/\Psi)$ kinematics.

As an initial estimate of the momentum transfer resolution, I will assume a tracking resolution of charged particles of 1% in momentum, 2 mr in polar angle, and 10 mr in azimuthal angle. The resolution can be described as follows:

$$\begin{aligned}\delta(\mathbf{\Delta}_\perp) &= \delta\left(\mathbf{k} - \mathbf{k}' - \mathbf{q}'\right)_\perp \\ &= k10^{-3} \oplus k'\left(2 \cdot 10^{-3} \oplus 10^{-2}\sin\theta_e\right) \oplus \delta\mathbf{q}'_\perp \qquad (12)\end{aligned}$$

Each exclusive reaction (*e.g.* Eq. 1) has different resolution. However, note first that at high Q^2, $|\mathbf{\Delta}| \ll |\mathbf{k}'_\perp|$, therefore $\mathbf{q}'_\perp \approx -\mathbf{k}'_\perp$. If q' represents a meson (ρ, ϕ, J/Ψ) which decays roughly symmetrically into two charged particles, then the net resolution on $\mathbf{q}'_\perp$ will be roughly $1/\sqrt{2}$ that of $\mathbf{k}'_\perp$. For simplicity, consider kinematics with $k' = k = 3$ GeV and $\sin\theta = 0.5$ ($Q^2 = 2.4$ GeV^2 and $x_{\rm Bj} = 0.05$). Then

$$\begin{aligned}\delta(\mathbf{\Delta}_\perp) &= (3\,\text{MeV/c}) \oplus (15\,\text{MeV/c}) \oplus (10\,\text{MeV/c}) \\ &\approx (20\,\text{MeV/c}) \qquad (13)\end{aligned}$$

This resolution is probably adequate up to ^{20}Ne. Note however, that resolution will further degrade at higher Q^2 (because θ_e increases) and at higher $x_{\rm Bj}$ (because k' increases). Electron kinematics for a $3 \otimes 30$ GeV^2 collider are illustrated in Fig. ??. The resolution will likely be considerably worse in the critical $(e, e'\gamma)$ channel. Pb-Glass has a resolution of about $8\%\sqrt{(1\,\text{GeV})\text{E}_\gamma}$, the CLAS PbWO$_4$ calorimeter has a resolution of about $4\%\sqrt{(1\,\text{GeV})\text{E}_\gamma}$, the BABAR [5] CsI calorimeter, although slow readout of 1 μs, functioned at a luminosity of 10^{34} with a resolution of 3%.

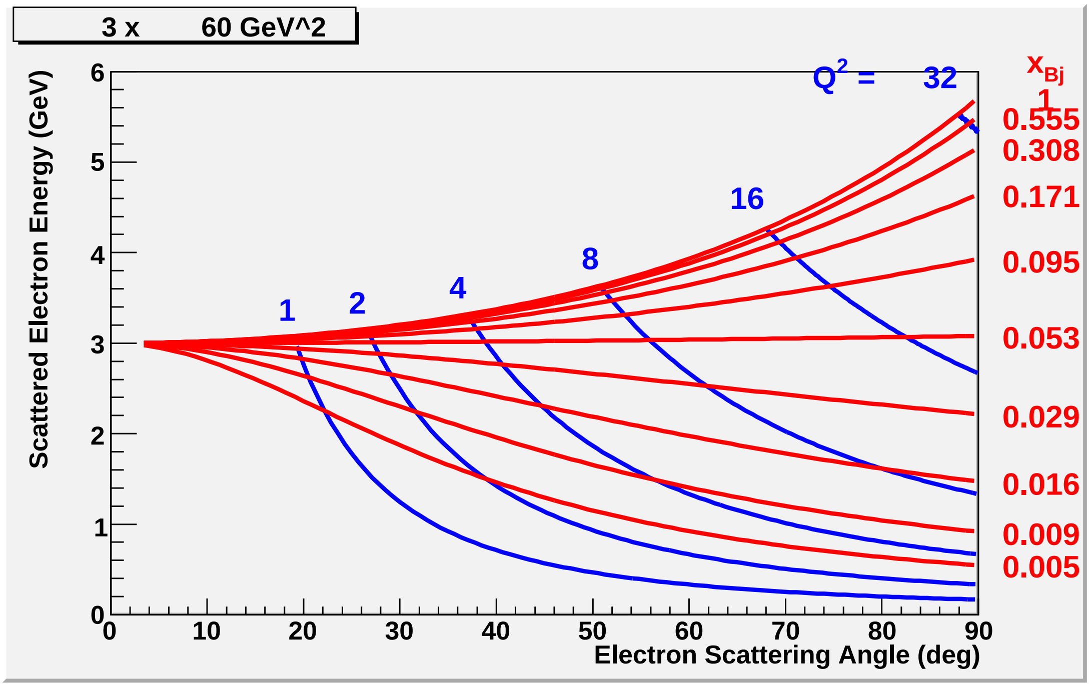

FIG. 1: Electron scattering kinematics for a 3 GeV/c electron $\otimes$ 60Z GeV/c ion collider.

If the final state nucleus is tagged, and especially if its longitudinal momentum can be measured to high precision, then the the exclusivity constraint can be applied to the kinematics to improve the resolution on $\mathbf{\Delta}_\perp$. Exclusivity can be used to calculate the energy of *e.g.* q' from just a knowledge of its angle. In the case of DVCS, this removes the photon energy resolution from $\delta(\mathbf{\Delta}_\perp)$. In this case the contribution from $\delta k'$ dominates:

$$\begin{aligned}\delta(\mathbf{\Delta}_\perp)_{\rm Exc} &\approx 10^{-2}k'\sin\theta \\ &= (15\,\text{MeV/c}) \qquad \text{at 3 GeV and } 30^\circ; \\ &= (50\,\text{MeV/c}) \qquad \text{at 5 GeV and } 90^\circ. \qquad (14)\end{aligned}$$

If the final state nucleus can be tracked with 100 μm resolution over a $\geq$ 1m drift space (and with a separation from of beam of 1cm), then it should be possible to resolve the P'_A longitudinal momentum to the intrinsic beam spread of 10^{-4}. Then the measured value of $(P_A - P'_A)_{||}$ replaces $(q - q')_{\|}$. A resolution $10^{-4} \cdot 60\,\mathrm{GeV/c} = 6$ MeV/c is equivalent to a 10^{-3} resolution on k'. Thus with high performance tracking on the final state nucleus, resolution in the transverse momentum transfer equal or better than 10 MeV/c is feasible. This is in spite of an intrinsic transverse momentum spread of the ion beam at the IP of hundreds of MeV/c. This result is a key motivation for a lower energy collider. Lower beam energies improves the resolution dramatically for coherent nuclear processes.

V. CONCLUSIONS

Over a very broad kinematic region of interest, the final state nucleus in coherent deeply virtual processes will be fully contained in the admittance of the final focus beam elements. As a consequence, these coherent processes can be tagged by detection of the nucleus far downstream after magnetic analysis by the dipoles of the lattice. For He, the typical range of final momenta (relative to the beam momentum) is 0.975 to 0.8. For Ne, the range is 0.995 to 0.96. The feasibility of tracking these final state nuclei to a longitudinal momentum transfer resolution of $\sim 10^{-4}$ should be explored in more detail with the lattice design, as this will greatly enhance the coherent physics program.

Appendix A: DVCS Kinematics

The momentum four-vectors of all particles in a deeply virtual exclusive reaction can be expressed in terms of "pseudo" light-cone vectors k, P_A, and components perpendicular to the beam axis. Define 2ζ as the light cone transfer fraction

$$\Delta^+ = \left[(P'_A - P_A)^0 + (P'_A - P_A)^z\right]/\sqrt{2} = 2\zeta P_A^+ \tag{A1}$$

The invariant momentum transfer squared is:

$$\Delta^2 = -\frac{4\zeta^2 M_A^2 + \mathbf{\Delta}_\perp^2}{1 - 2\zeta} \tag{A2}$$

The exclusivity constraint is

$$\begin{aligned}(q - q' + P_A)^2 &= P_A'^2 = M_A^2 \\ \Delta^2 + 2(q - q') \cdot P_A &= 0 \\ \Delta^2 = (q - q')^2 &= (P'_A - P_A)^2 = 2M^2 - 2P'_A \cdot P\end{aligned} \tag{A3}$$

The final nuclear four-momentum can be written as

$$P_A'^\mu = (1 - 2\zeta)P_A^\mu + \frac{2\zeta M_A^2 + \mathbf{\Delta}_\perp^2/(1 - 2\zeta)}{2k \cdot P_A} k^\mu + \Delta_\perp^\mu \tag{A4}$$

Discuss exclusivity constraint in more detail...

[1] D. Douglas, private communication, 2009.
[2] Total Elastic and diffractive cross section Measurement (TOTEM) experiment at the LHC `http://totem.web.cern.ch/Totem/`.
[3] A. De Roeck, *The FP420 project: The physics potential*, Nucl. Phys. Proc. Suppl. **179-180**, 181 (2008); and `http://www.fp420.com`.
[4] Y. Zhang, *What Peak Luminosity a 500 m Figure-8 Ring Can Offer? MEIC 5th Iteration*, ELIC Study Group, 13 March 2009.
[5] B. Aubert *et al.* [BABAR Collaboration], Nucl. Instrum. Meth. A **479**, 1 (2002) [arXiv:hep-ex/0105044].