

# Uncertainty, Probability, and Statistics

## Part 1 - Uncertainties

# Further Reading

R Barlow, *Statistics*. Wiley 1989 - Much of these lectures!!!!

G Cowan, *Statistical Data Analysis*. Oxford 1998 - *available on the web!*

L Lyons, *Statistics for Nuclear and Particle Physicists*, Cambridge 1986

B Roe, *Probability and Statistics in Experimental Physics*, Springer 1992

A G Frodesen et al, *Probability and Statistics in Particle Physics*, Bergen-Oslo-Tromso 1979

W T Eadie et al; *Statistical Methods in Experimental Physics*, North Holland 1971

M G Kendall and A Stuart; “*The Advanced Theory of Statistics*”. 3+ volumes, Charles Griffin and Co 1979

Darrel Huff “*How to Lie with Statistics*” Penguin

CERN Workshop on Confidence Limits. Yellow report 2000-005

Proc. Conf. on Adv. Stat. Techniques in Particle Physics, Durham, IPPP/02/39

<http://www.hep.man.ac.uk/~roger>

# Types of Measurement Error

- Blunders - produce grossly inaccurate results, occurrence typically easily detected
  - Incorrect measurement settings, incorrect labeling, etc...
- Systematic Errors - produce results that differ *consistently* from the correct result
  - Can be caused by persistent instrument malfunction, poor calibration, warped ruler
  - Observer bias (clinical study of two population groups with important underlying difference)
  - Inaccurate, but can be highly precise (*repeatable*)
  - Not always easy to detect (typically repeatable) - need to use standards!

# Systematic Effects continued

- Effect does not generally fall as you take more data and average
- Affects all readings the same way ("systematic")
  - Therefore, does NOT show up a poor chi-squared, etc.!
- Can never be sure you have found them all
  - Regard with fear and horror, use caution and conservatism (again, standards where possible, and alternative techniques)

# Systematic Errors

*Systematic Error:*  
reproducible  
inaccuracy  
introduced by  
faulty equipment,  
calibration, or  
technique

Bevington

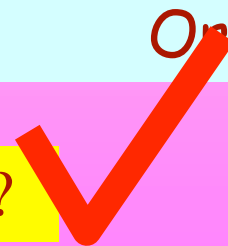
*Systematic effects* is a general category which includes effects such as background, scanning efficiency, energy resolution, angle resolution, variation of counter efficiency with beam position and energy, dead time, etc. The uncertainty in the estimation of such as systematic effect is called a *systematic error*

Error=mistake?



Onear

Error=uncertainty?



# Systematic Effects vs. Systematic Errors

- Example - make measurements with a steel ruler, but forget thermal expansion
- Bevington - this is a systematic error. No! This is a systematic mistake.
- $I = a \times L + b$ , where  $I$  = actual length and  $L$  = measured length, assumed  $a=1$ ,  $b=0$
- But, realizing they exist, can change them and ascribe error to them

# Also - theoretical uncertainties

An uncertainty which does not change when repeated does not match a Frequency definition of probability.

*Statement of the obvious*

Theoretical parameters:

B mass in CKM determinations

Strong coupling constant in  $M_W$

All the Pythia/Jetset parameters in just about everything

*High order corrections in electroweak precision measurements*

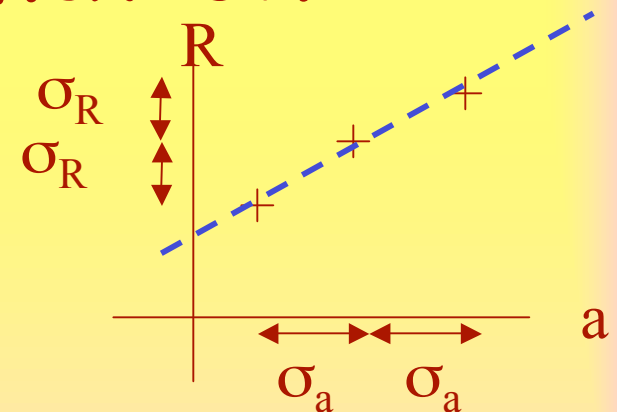
etcetera etcetera etcetera.....

No alternative to subjective probabilities

But worry about robustness with changes of prior!

# Numerical Estimation

Theory(?) parameter  
 $a$  affects your  
result  $R$

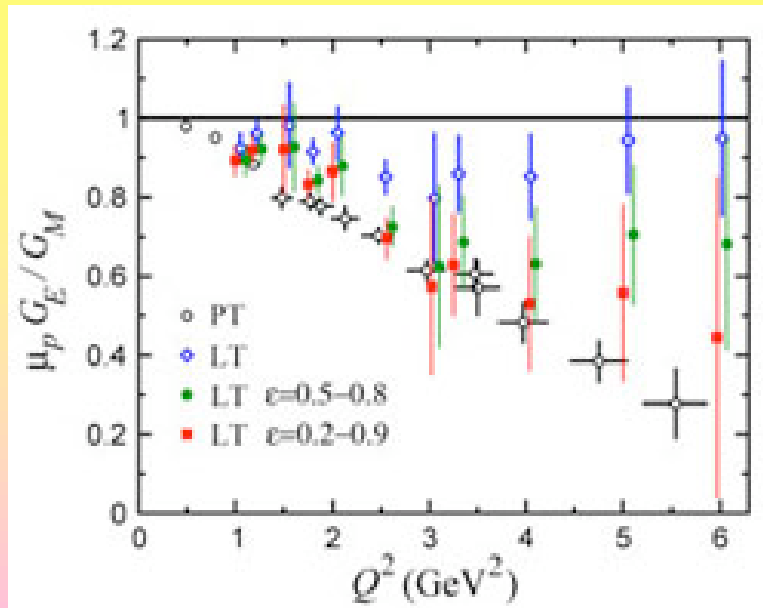


$a$  is known only with some precision  $\sigma_a$   
Propagation of errors impractical as no  
algebraic form for  $R(a)$   
Use data to find  $dR/da$  and  $\sigma_a dR/da$   
Generally combined into one step

# Random (Statistical) Errors

- Variations in result from one measurement to the next
  - Arise from physical limitations in the measurement system (ruler repositioning, eye angle, etc.), or
  - Actual random variations of the measured quantity itself (patient variation in medical applications)
    - *Always present in radiation counting measurements*
- Affect measurement reproducibility and ability to detect real differences in measured data
- Impossible to eliminate completely, always present, "uncertainty"

# Precision, accuracy, and mistakes - examples



- Are these measurements precise?
- Are they accurate?

Here are some readings of the *same* quantity:

1.23, 1.25, 1.24, 1.25, 1.21, 1.52, 1.22, 1.27

Note the error and the mistake.

Slide 10

Can you say anything about the systematic error?