GUF Fit to Global g_1/F_1 Data Set

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Contents

1 Summary

This note documents the details of the phenomenological fit used in the proton paper [4]. In addition to the source data and detailed results, a short FORTRAN routine is included which will provide the value of the function and the (propagated) statistical error for the requested kinematic point (x_{Bj}, Q^2). Besides the basic fit to the global data set, the normalization factors were also determined. All fits used the GUF apparatus developed for the calculation of the radiative corrections [2, 14].

2 Calculation

The fit was determined using the CERNLIB package MINUIT, utilizing the well-established MIGRAD procedure to find the minimum χ^2 [1]. Just as was done in the determination of the A_1 fit for the calculation of the E155 radiative corrections [2], we simultaneously fit proton, neutron and deuteron data.

Specifically, the proton and the neutron data are fitted separately, using

$$\left[\frac{g_1^p}{F_1^p}\right]_{(x,Q^2)} = x^{\alpha_p}\,(a_p + b_p x + c_p x^2)(1 + d_p/Q^2) \tag{1}$$

$$\left[\frac{g_1^n}{F_1^n}\right]_{(x,Q^2)} = x^{\alpha_n}\,(a_n + b_n x + c_n x^2)(1 + d_n/Q^2) \tag{2}$$

while the deuteron data are fitted using the above functions and

$$\frac{g_1^d}{F_1^d} = \frac{1}{2}(1 - \frac{3}{2}\omega_D)\left(\frac{g_1^p}{F_1^p}\,\frac{F_1^p}{F_1^d} + \frac{g_1^n}{F_1^n}\,\frac{F_1^n}{F_1^d}\right) \tag{3}$$

where $\omega_D \approx 0.05$ is the D-state correction. The fit minimizes the total χ^2,

$$\chi^2 = \chi_p^2 + \chi_n^2 + \chi_d^2 \tag{4}$$

A second fit was also done, which applied a multiplicative scale factor to each experiments' data. This scale factor can be interpreted as a correction to the product of beam polarization, target polarization and dilution factor for the experiment in question. These three quantities represent a significant portion of the systematic error of the experiments and it is therefore not unlikely that the data set of a particular experiment is off by a small scale factor.

The scale factor of each data set was constrained through the fit χ^2. We used the quadratic sum of the systematic error in the beam polarization, target polarization and dilution factor of each experiment as a scale to weigh against the normalization factor, using the following procedure:

The basic definition of χ^2 (where y_i is the measurement, σ_i its statistical error and f_i the corresponding fit value)

$$\chi^2 = \sum_{experiments}\sum_{data}\frac{(y_i - f_i)^2}{\sigma_i^2} \tag{5}$$

is modified to reflect the normalization factors λ_j:

$$\chi^2 = \sum_j \left[\sum_i \frac{(\lambda_j y_i - f_i)^2}{\sigma_i^2} + \frac{(\lambda_j - 1)^2}{\delta_j^2}\right] \tag{6}$$

where j is the sum over the experiments, i is the sum over each set's data points and δ_j is the above mentioned multiplicative part of experiment's systematic error.

Thus, should an experiment's normalization factor differ from unity by exactly the systematic error ascribed by the experimenters, the χ^2 for that experiment (everything inside the square [ ] above) is increased by 1.0. Consequently, experiments with a smaller systematic error are less likely to be scaled by any significant amount than an experiment with large systematic uncertainty.

3 Global Data Set

Two separate versions of the data set were used; one for the basic fit, which matches that set used in the calculation of the RCs[1] and the other used in the determination of the normalization factors. The sets differ in their utilization of the experiments' systematic error. In the basic, RC-like fit, the systematic error was added (in quadrature) to the data's statistical error, for the data sets which did NOT have their RCs recalculated (see below). This was done to account for the systematic influence the RCs might have.

Table 1: Systematic Error Weights of the Different Experiments and Source of Data. *Data Sets marked by * had radiative corrections (re)calculated.*

Experiment		*Scale Error*	Source of Data
p	E155*	12.0 %	Phys. Lett. **B** (?) [4] (GM's set "p3")
	E143*	3.7 %	Phys. Rev. **D58**, 112003 (1998) [6]
	E130*	5.0 %	Phys. Rev. Lett. **51**, 1135 (1983) [8]
	E80*	5.0 %	Phys. Rev. Lett. **41**, 70 (1978) [9]
	Hermes	5.7 %	Phys. Lett. **B442**, 484 (1998) [10]
	SMC	5.5 %	Phys. Rev. **D58**, 112001 (1998) [12]
	EMC	11.0 %	Nucl. Phys. **B328**, 1 (1989) [13]
n	E154*	4.1 %	Phys. Rev. Lett. **79**, 26 (1997) [5]
	E142	11.5 %	Phys. Rev. **D54**, 6620 (1996) [7] (syst errors [15])
	Hermes	7.4 %	Phys. Lett. **B404**, 383 (1997) [11]
d	E155*	6.0 %	Phys. Lett. **B463**, 339 (1999) [3] (GM's set "a5")
	E143*	4.9 %	Phys. Rev. **D58**, 112003 (1998) [6]
	SMC	5.9 %	Phys. Rev. **D58**, 112001 (1998) [12]

The data set used to determine the normalization factors did not have all the systematic errors added in. Instead, from each experiment's systematic error was split that part which could well be represented by an overall multiplicative term: the beam polarization, the target polarization and the dilution factor. This part was used as the scale for the normalization factor (see section 2, above). The balance was added to the statistical error, as before.

[1] Except of course that those fits were to A_1 rather than g_1/F_1 and that the proton data have since been updated.

4 Results of Basic, GUF-style Fit

The basic fit without scale factors is the one (presumably) used to evolve our data. The results are (see table 2 for parameter errors):

$$\frac{g_1^p}{F_1^p} = x_{Bj}^{0.700} \left(0.817 + 1.014 x_{Bj} - 1.489 x_{Bj}^2\right) \left(1 - \frac{0.037}{Q^2}\right) \tag{7}$$

$$\frac{g_1^n}{F_1^n} = x_{Bj}^{-0.335} \left(-0.013 - 0.330 x_{Bj} + 0.761 x_{Bj}^2\right) \left(1 + \frac{0.129}{Q^2}\right) \tag{8}$$

From these the deuteron model is obtained via eq. (3), above. Plots of these fits, as a function of x_{Bj} and of Q^2, are shown in appendix A. Using the error matrix of this fit (table 4) and the code in appendix B, the propagated *statistical* error of the model value can be obtained. Note that we define the reduced χ^2 as χ^2/N, where N is simply the number of data points. The number of fit parameters is *not* considered.

Table 2: Results of Basic Fit – Parameters and Their Errors. *Also given are the parameters' sequence numbers as used in the code.*

Parameter	(p, n)	*Proton*	*Neutron*
constant	(1, 11)	0.817 ± 0.230	-0.013 ± 0.038
linear	(2, 12)	1.014 ± 0.636	-0.330 ± 0.153
quadratic	(3, 13)	-1.489 ± 0.600	0.761 ± 0.590
exponent	(6, 16)	0.700 ± 0.082	-0.335 ± 0.524
$1/Q^2$	(8, 18)	-0.037 ± 0.061	0.129 ± 0.459

Table 3: Results of Basic Fit – χ^2.

	N	χ^2	reduced χ^2
p	247	234	0.948
n	35	18	0.503
d	201	219	1.09
total	483	471	0.976

Table 4: Results of Basic Fit – Error Matrix. *Diagonal entries are in* **boldface**. *See table 2 for parameter identification. Note that this matrix is by definition symmetric.*

	1	2	3	6	8	11	12	13	16	18
1	**5.27e-2**	-1.42e-1	1.18e-1	1.85e-2	1.80e-3	3.57e-4	-1.28e-3	2.39e-3	-2.38e-3	7.81e-3
2	-1.42e-1	**4.04e-1**	-3.63e-1	-4.84e-2	-3.06e-3	-1.30e-3	1.43e-3	-5.79e-4	1.25e-2	-1.56e-2
3	1.18e-1	-3.63e-1	**3.60e-1**	3.91e-2	6.08e-4	1.63e-3	2.25e-3	-1.15e-2	-1.97e-2	7.15e-3
6	1.85e-2	-4.84e-2	3.91e-2	**6.68e-3**	1.21e-3	1.13e-4	-5.41e-4	1.08e-3	-4.94e-4	3.60e-3
8	1.80e-3	-3.06e-3	6.08e-4	1.21e-3	**3.72e-3**	8.60e-6	-2.98e-4	7.01e-4	6.57e-4	4.17e-3
11	3.57e-4	-1.30e-3	1.63e-3	1.13e-4	8.60e-6	**1.45e-3**	4.72e-3	-1.90e-2	-1.96e-2	5.08e-4
12	-1.28e-3	1.43e-3	2.25e-3	-5.41e-4	-2.98e-4	4.72e-3	**2.35e-2**	-8.80e-2	-7.00e-2	-1.42e-2
13	2.39e-3	-5.79e-4	-1.15e-2	1.08e-3	7.01e-4	-1.90e-2	-8.80e-2	**3.48e-1**	2.74e-1	3.29e-2
16	-2.38e-3	1.25e-2	-1.97e-2	-4.94e-4	6.57e-4	-1.96e-2	-7.00e-2	2.74e-1	**2.74e-1**	2.98e-2
18	7.81e-3	-1.56e-2	7.15e-3	3.60e-3	4.17e-3	5.08e-4	-1.42e-2	3.29e-2	2.98e-2	**2.11e-1**

5 Results of Normalized fits

The results of the fits utilizing the normalization factors are summarized in tables 5 and 6. The fits differ by the subset of data which were scaled. This is intended to clarify the significance of the E155 data's scale factor.

The second set of results (table 6) are based on a reduced data set which minimizes Q^2 dependence: only the E155 2.75^o data are used, together with the 29 GeV E143 data and the Hermes data. Also, all data points below $x_{Bj} = 0.06$ were ignored. Note again our definition of the reduced χ^2 as χ^2/N, where N is the number of data points, without considering the number of fit parameters.

Table 5: Results of Scale Fit – Large Data Set.

Total χ^2		478.4	465.9	465.5	463.9
Reduced χ^2					
	Proton	0.971	0.930	0.930	0.927
	Neutron	0.518	0.505	0.504	0.469
	Deuteron	1.097	1.084	1.082	1.079
Scale Factors					
p	E155	1	1.077 ± 0.022	1.075 ± 0.024	1.080 ± 0.030
	E143	1	1	1	0.999 ± 0.026
	E80/E130	1	1	1	1.005 ± 0.039
	SMC	1	1	1	0.996 ± 0.039
	EMC	1	1	1	1.004 ± 0.078
	Hermes	1	1	1	1.022 ± 0.031
n	E154	1	1	1	1.004 ± 0.039
	E142	1	1	1	0.910 ± 0.108
	Hermes	1	1	1	0.984 ± 0.073
d	E155	1	1	0.972 ± 0.045	0.975 ± 0.045
	E143	1	1	1	1.007 ± 0.041
	SMC	1	1	1	0.991 ± 0.054

Table 6: Results of Scale Fit – Reduced Data Set.

Total χ^2	80.69	76.75	76.52
Reduced χ^2	1.105	1.045	1.042
Scale Factors			
E155	1	1.084 ± 0.043	1.073 ± 0.052
E143	1	1	0.989 ± 0.032
Hermes	1	1	1.009 ± 0.044

Appendix A Plots

Figures 1 through 6 show the results of the fit and its statistical error (the shaded band behind the fit curve), together with the data. **Note that these plots show $\mathbf{g_1/F_1}$, contrary to the axis labels indicating $\mathbf{A_1}$.** The fit is shown for proton, neutron and deuteron and separately as a function of x_{Bj} and of Q^2. The plots are subdivided into ranges of the respective other kinematic variable, as indicated on each plot. Also indicated are subset-χ^2 values for each sub-plot, as well the χ^2 for each nucleon type (at the top of each plot) and the actual function plotted (bottom). Additionally, the *approximate* tendency of the function for the four kinematic extremes are given, on the x_{Bj}-dependence plots only.

The data shown in each sub-plot have been evolved to a central value of the respective *other* kinematic variable, using the fit function f. Thus, the plotted asymmetry value (y_{plot}) is related to the actually measured value by

$$y_{plot} = y_{measured} \frac{f(x, Q^2_{plot})}{f(x, Q^2_{measured})} \tag{9}$$

for the x_{Bj}-dependence plots, and in the case of the Q^2-dependence plots:

$$y_{plot} = y_{measured} \frac{f(x_{plot}, Q^2)}{f(x_{measured}, Q^2)} \tag{10}$$

Occasionally, this procedure produces nonsensical results, most likely due to limitations in numerical precision. For example, the lowest Q^2 point of the second sub-plot in figure 6 actually has a much less negative value of g_1/F_1. While these occurrences can be confusing, they do not actually have any impact on any of the calculations or the fit, as these are always based on the non-evolved data.

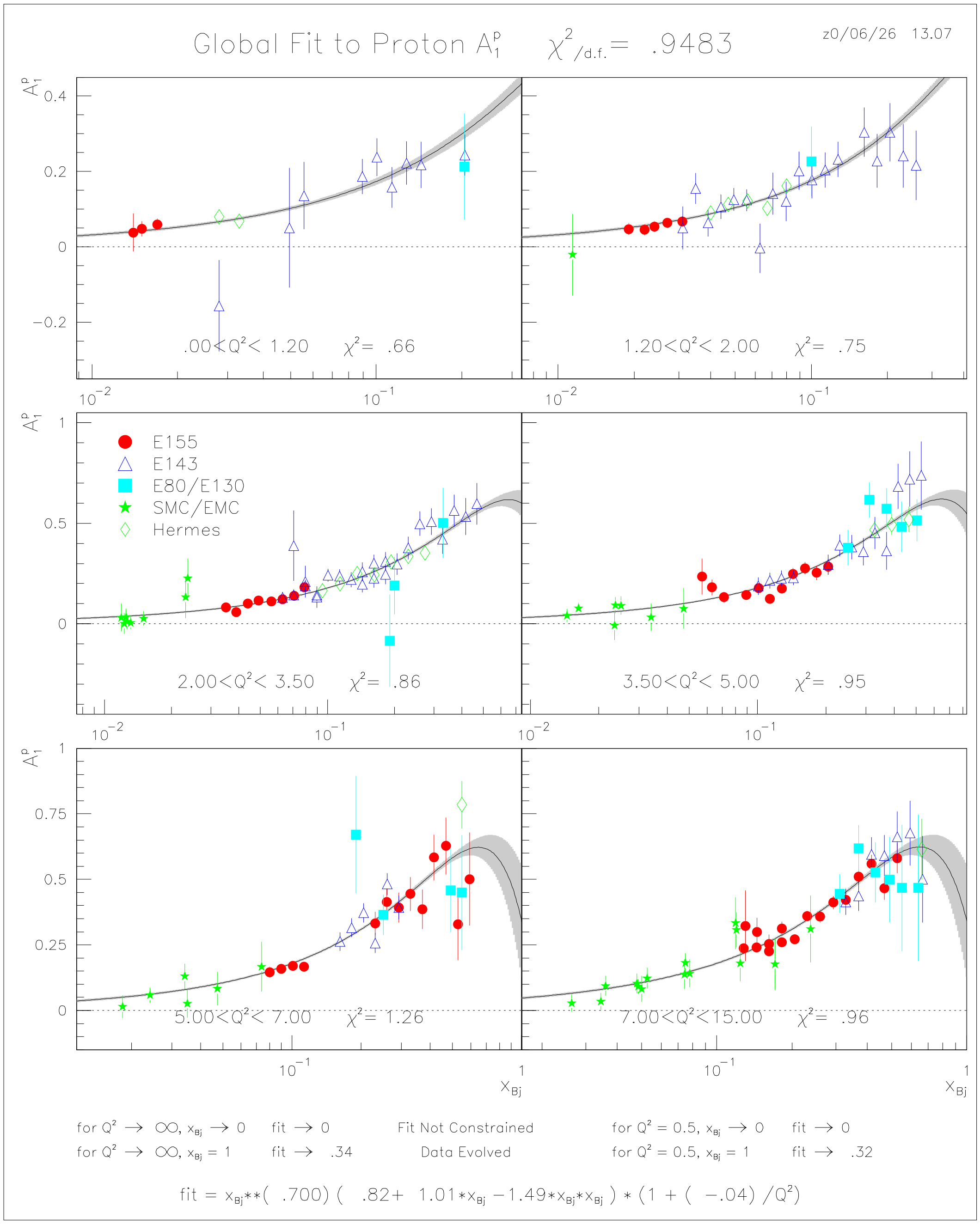

Figure 1: Results of Basic Fit – g_1/F_1 Proton, x_{Bj}-dependence. *The plots show* g_1/F_1 *despite the labels indicating* A_1.

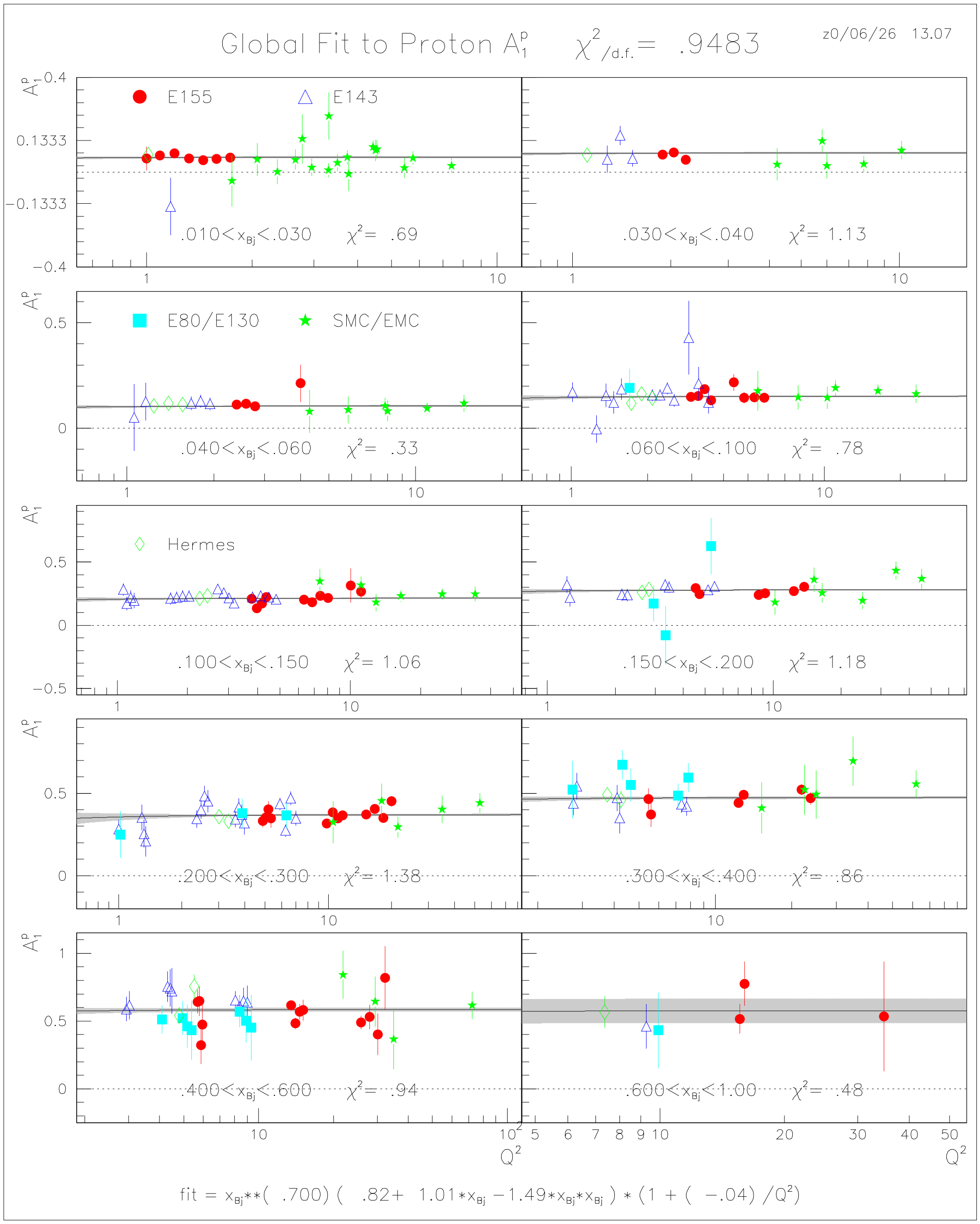

Figure 2: Results of Basic Fit – g_1/F_1 Proton, Q^2-dependence. *The plots show* g_1/F_1 *despite the labels indicating* A_1*.*

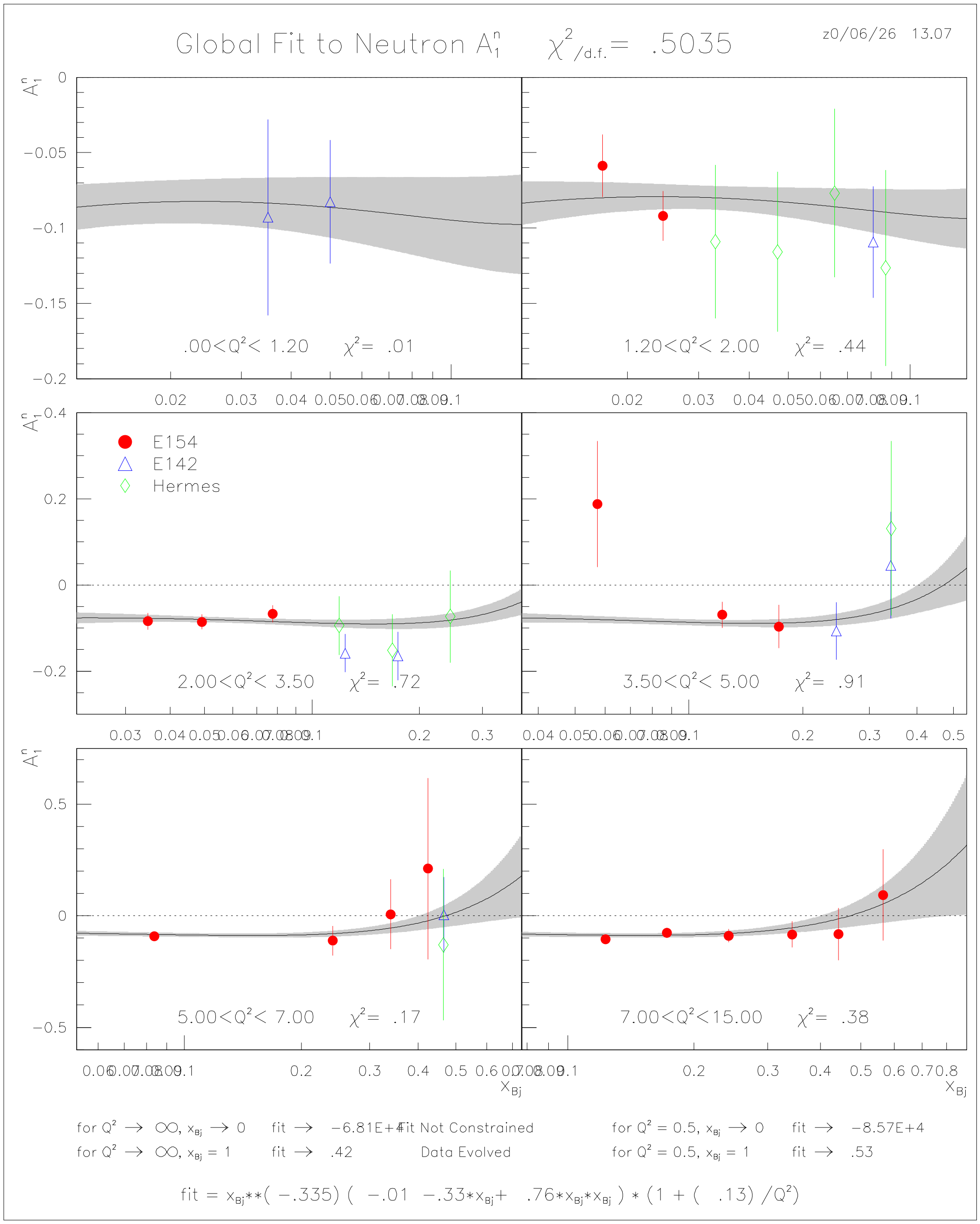

Figure 3: Results of Basic Fit – g_1/F_1 Neutron, x_{Bj}-dependence. *The plots show g_1/F_1 despite the labels indicating A_1.*

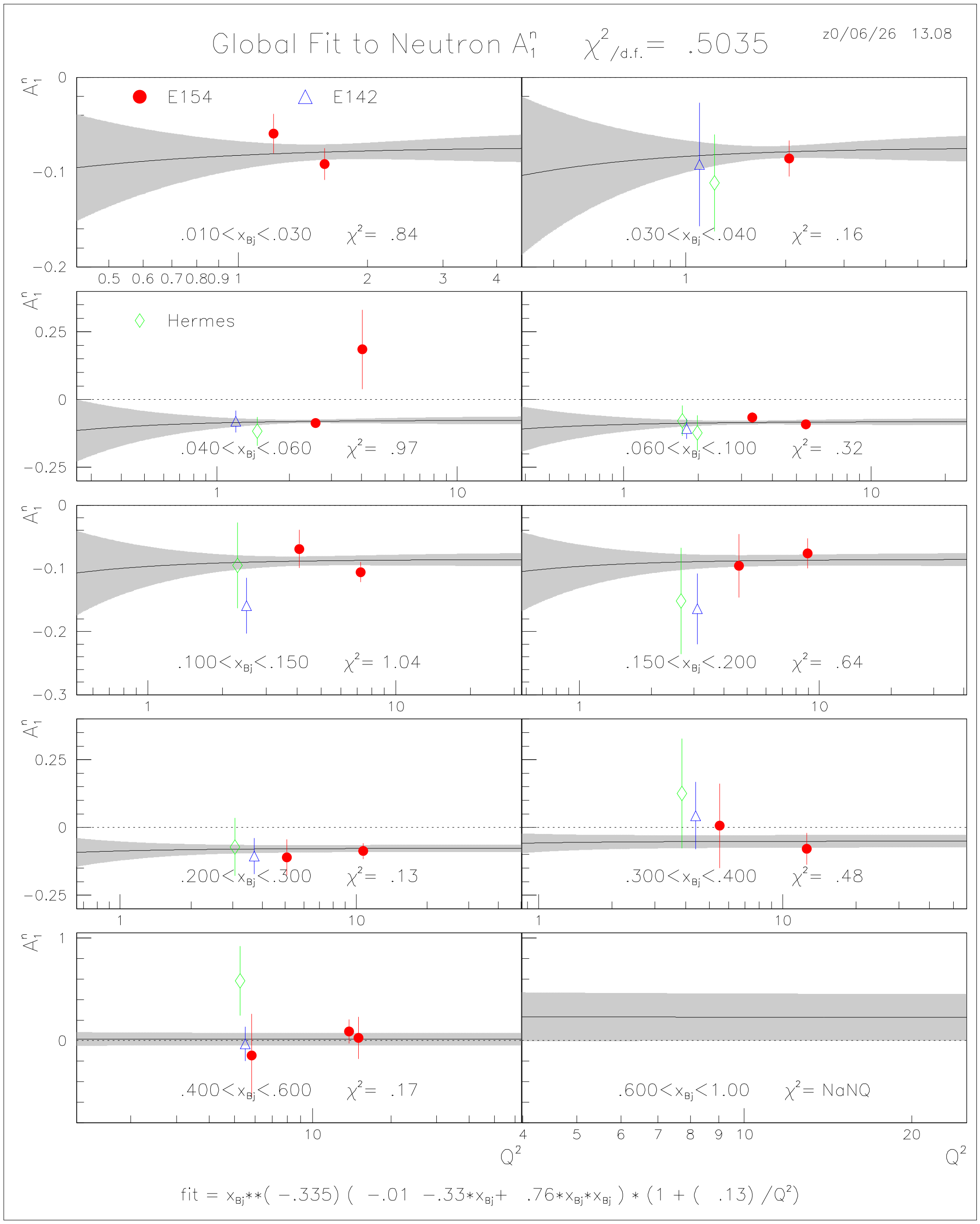

Figure 4: Results of Basic Fit – g_1/F_1 Neutron, Q^2-dependence. *The plots show g_1/F_1 despite the labels indicating A_1.*

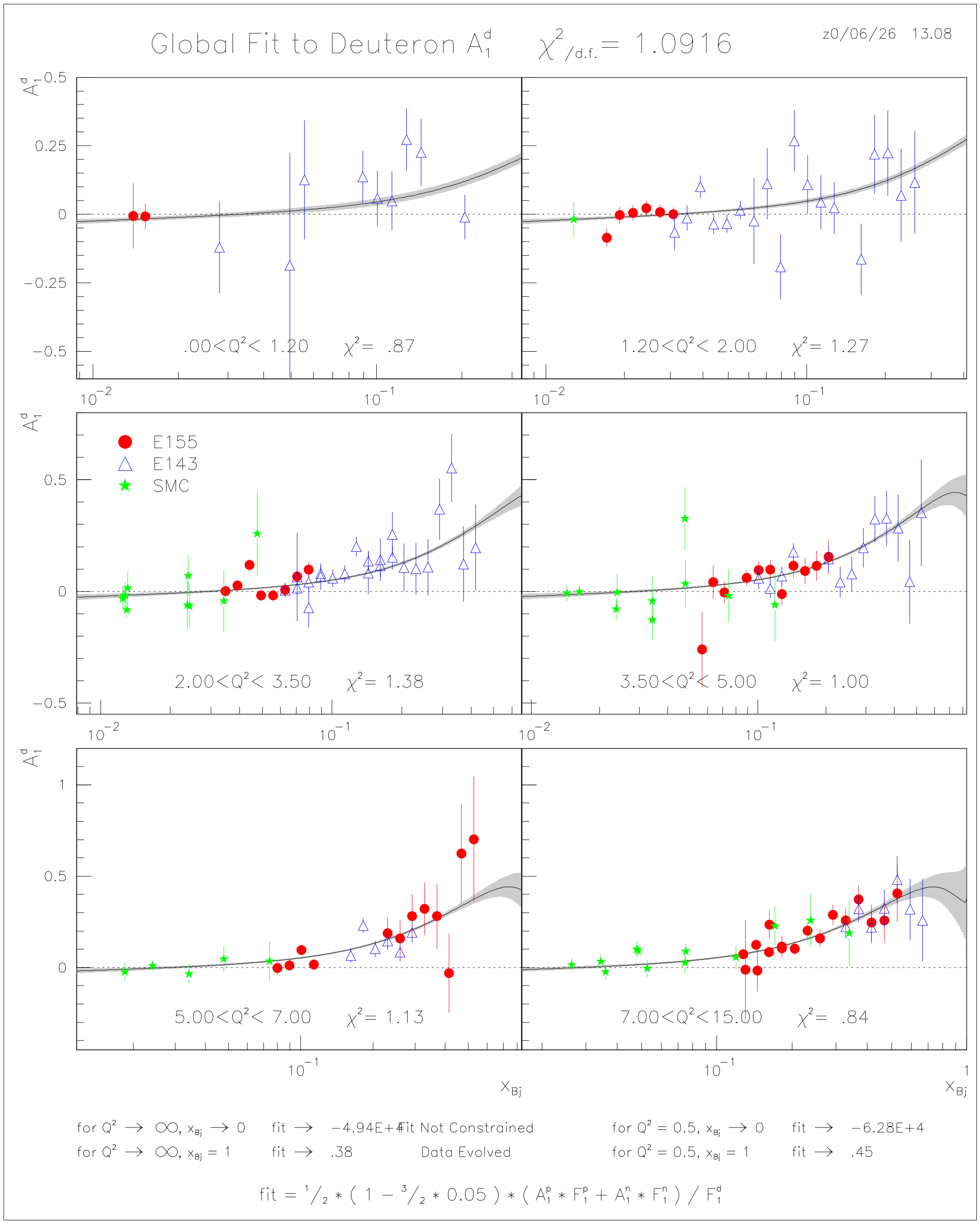

Figure 5: Results of Basic Fit – g_1/F_1 Deuteron, x_{Bj}-dependence. *The plots show g_1/F_1 despite the labels indicating A_1.*

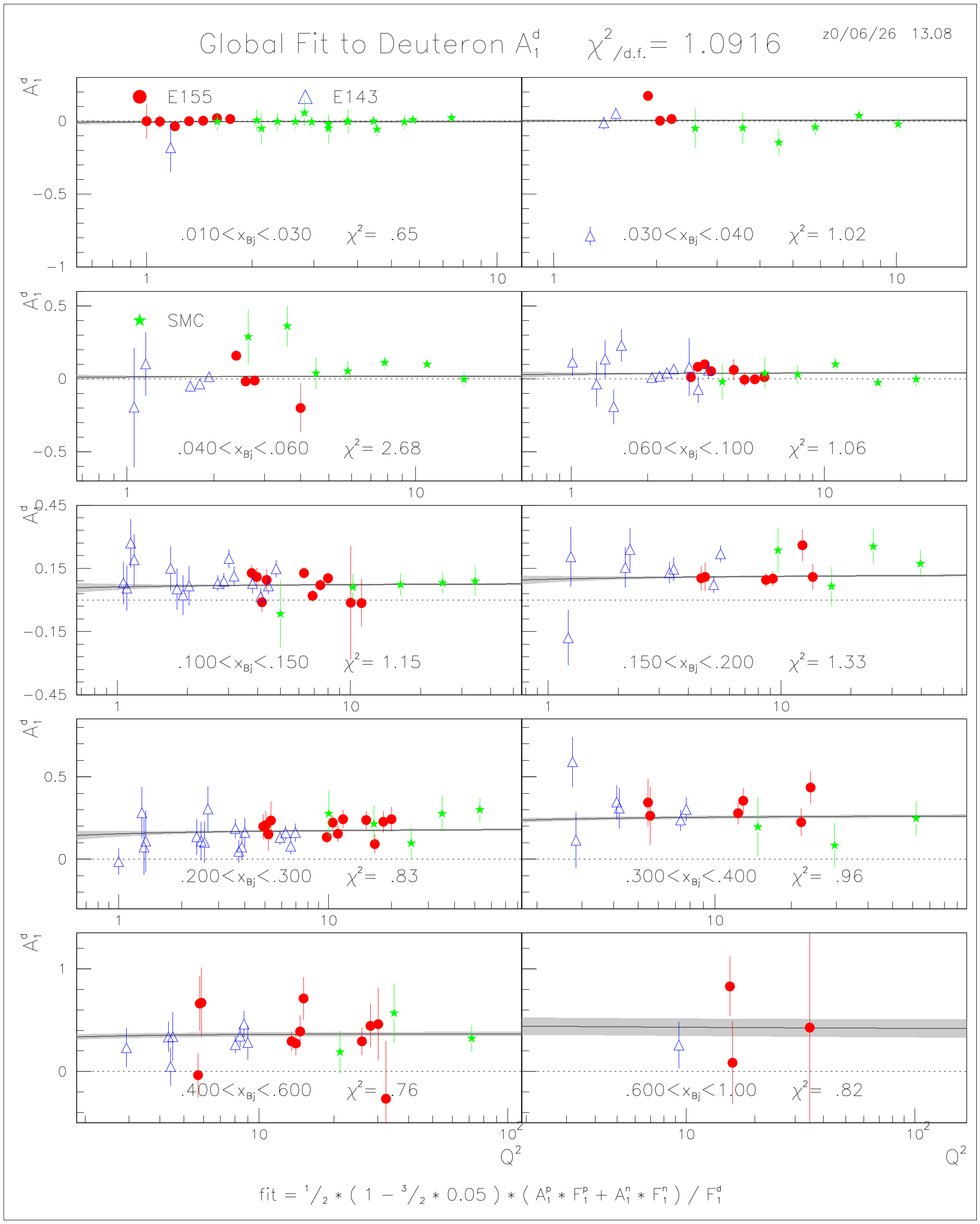

Figure 6: Results of Basic Fit – g_1/F_1 Deuteron, Q^2-dependence. *The plots show g_1/F_1 despite the labels indicating A_1.*

Appendix B   Sample FORTRAN Code

The listing below is a compressed version of the FORTRAN code `A1fit_error.f` which can also be found under `~frw/e155/radcor/new/utils` in the SLAC AFS space. This code calculates the fit model value and, optionally, the propagated statistical fit error for the desired kinematics and target type. The fit parameter and, if needed, error matrix are read from files. This example requires the parameter to have sequential file positions as indicated in table 2 and a header line (see listing for formatting details). Note that the code applies to the A_1 and the g_1/F_1 models equally, despite the A_1 notation.

Calculation of the deuteron value requires a model for F_1 of the proton, neutron and deuteron. This implementation makes use of the subroutine `F1F2`, which is part of the RC code. Care must be taken to ensure the calculations are treating the definition of F_1^d, per nucle*on* or per nucle*us*, consistently. To obtain a model value for ^{3}He, certain lines of the sample code need to be uncommented ("`cHe3`") and a value for `signp`, the ratio of proton to neutron cross sections, must be provided. This can also be found in the RC code. Lastly, note that this code depends on the local variables to be *static*; depending on the particular compiler used, corresponding declarations may need to be added.

```
      SUBROUTINE A1fit_error(xBj, Q2, A1, error, targ, calc_error)

cHe3  SUBROUTINE A1fit_error(xBj, Q2, A1, error, signp, targ, calc_error)

      IMPLICIT NONE

      REAL*8 xBj, Q2, A1
cHe3  REAL*8 signp
      real*8 A1p, A1n
      real*8 F1p, F1n, F1d
      real*8 errmat(20,20)
      real*8 param(20), grad(20)
      real*8 wd/0.05D0/
      real*8 M2/0.880354345984D0/
      real*8 scale, error
      real*8 dummy1, dummy2     !  for subroutine F1F2
      real*8 temp
      real*8 part1, part2, part3, part4     ! intermediate values

      integer i,j

      CHARACTER*1 targ          ! one of P, N, D, 3

      logical isused(20)
      logical firstcall/.TRUE./
      logical calc_error
      logical suppress          !  for subroutine F1F2
```

```
c ******** INITIALIZE *************

      if (firstcall) then
         isused = .FALSE.
         open(unit=1,file='fit.params',status='old')
         read(1,*)    ! skip header
         do i=1, 20
           read(1,*) param(i)
           if (param(i).ne.0.D0) isused(i) = .TRUE.
         enddo
         close(1)
         firstcall = .FALSE.
         scale = 1.D0 - 1.5D0*wd   ! F1F2 puts F2d as per nucleus!

         if (calc_error) then      ! read in errmat
           open(unit=1,file='fit.errmat',status='old')
           read(1,*)
10         read(1,*,end=20) i,j,temp
           if ((i.ge.1).and.(i.le.20).and.(j.ge.1).and.(j.le.20)) then
              errmat(i,j) = temp
           else
              write(6,*) ' Problem with Error Matrix!!!'
              stop
           endif
           goto 10
20         close(1)
         endif
      endif

c ********* calculate model value and error ***************

      grad = 0.D0

      if ( (targ=='P').or.(targ=='D').or.(targ=='3') ) then

         part1 = param(1) +param(2)*xBj +param(3)*xBj*xBj
         part2 = 1.D0
         part3 = xBj**param(6)
         part4 = 1.D0 + param(8)/Q2

         A1p = part1 * part3 * part4  / part2

         if (calc_error) then
           grad(1) =           part3 * part4  / part2
           grad(2) =     xBj * part3 * part4  / part2
           grad(3) = xBj*xBj * part3 * part4  / part2
           grad(6) = dlog(xBj) * A1p
           grad(8) = part1 * part3  / part2 / Q2
         endif
```

```
endif

if ( (targ=='N').or.(targ=='D').or.(targ=='3') ) then

   part1 = param(11) +param(12)*xBj +param(13)*xBj*xBj
   part2 = 1.D0
   part3 = xBj**param(16)
   part4 = 1.D0 + param(18)/Q2

   A1n = part1 * part3 * part4  / part2

   if (calc_error) then
     grad(11) =           part3 * part4  / part2
     grad(12) =     xBj * part3 * part4  / part2
     grad(13) = xBj*xBj * part3 * part4  / part2
     grad(16) = dlog(xBj) * A1n
     grad(18) = part1 * part3  / part2 / Q2
   endif
endif

if (targ=='P') then
   A1 = A1p
elseif (targ=='N') then
   A1 = A1n
elseif (targ=='D') then

   suppress = .false.
   if ((M2 - Q2 + Q2/xBj) < 4.D0) suppress = .true.
   call F1F2(xBj, Q2, 'P', F1p, dummy1, dummy2, suppress)
   call F1F2(xBj, Q2, 'N', F1n, dummy1, dummy2, suppress)
   call F1F2(xBj, Q2, 'D', F1d, dummy1, dummy2, suppress)

   if (F1d .eq. 0.D0) then
     A1 = 0.D
     if ((F1p .ne. 0.D0).or.(F1n .ne. 0.D0)) then
       write(6,*) ' PROBLEM!!!   F1d = 0  in GUF FIT'
     endif
   else

     A1 = scale * (F1p*A1p + F1n*A1n) / F1d

     if (calc_error) then
       do i=1,9
         grad(i) = grad(i) * F1p * scale / F1d
       enddo
       do i=11,19
         grad(i) = grad(i) * F1n * scale / F1d
       enddo
     endif
   endif
```

```
cHe3  elseif (targ=='3') then

cHe3     A1 = (0.87D0*A1n*signp - 2.D0*0.027D0*A1p) / (signp + 2.D0)

cHe3     if (calc_error) then
cHe3      do i=1,9
cHe3        grad(i) = grad(i) * 2.D0*0.027D0 / (signp + 2.D0)
cHe3      enddo
cHe3      do i=11,19
cHe3        grad(i) = grad(i) * 0.87D0*signp / (signp + 2.D0)
cHe3      enddo
cHe3     endif

      else
         write(6,*) '%%%%% illegal Target in GUF fit !!!'
         A1 = 0.D0
      endif


c ******** Now figure the fit error ********

      error = 0.D0
      if (calc_error) then
       do i=1, 20
         if (isused(i)) then
           temp = 0.D0
           do j=1, 20
             if (isused(j)) then
               temp = temp + errmat(i,j)*grad(j)
             endif
           enddo
           error = error + temp*grad(i)
         endif
       enddo
       error = dsqrt(error)
      endif

      return
      end
```

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