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ECS Geometry

Geometry of the EGN Detector system.

Each sector has its local coordinate system with the origin at the 'perpendicular point' P in the middle of the front surface of the first scintillator layer. If the detector is installed perfectly, a perpendicular line from this point will pass through the target. The z axis of the local coordinate system is along this perpendicular and points away from the target. The negative y axis passes through the vertex of the triangular scintillator layer that is closest to the beam line. If the detector is oriented with the y axis pointing upwards, the x axis to the left if one is looking downstream along the beam.

The nominal distance from the point P to target center is L_0. The numerical value is

$$L_0 = 5103.2 \text{ mm}$$

Let the nominal target point at the end of this line be labelled S. Let the actual target be at point T, which has coordinates in the CLAS coordinate system of (0,0,0). From surveying, the absolute position of P can be determined, as can a point V on the local y axis, and a point W on the local x axis. From these data the orientation of the first scintillator layer can be determined. The results of the survey can be represented by the following coordinates in the CLAS frame,

$$P = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}, \qquad V = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \qquad W = \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix}$$

From these we can determine the position of S in the CLAS frame,

$$S = \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}$$

In the local EGN detector frame these points are defined by the following sets of coordinates:

$$P = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \qquad V = \begin{pmatrix} 0 \\ V \\ 0 \end{pmatrix}, \qquad W = \begin{pmatrix} W \\ 0 \\ 0 \end{pmatrix}, \qquad S = \begin{pmatrix} 0 \\ 0 \\ -L \end{pmatrix}$$

Any given point has coordinates (x', y', z') in the local frame, and coordinates (x, y, z) in the CLAS frame. The two sets are related by the transformation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} + R \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

In particular, the coordinates for the point P are related by

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} + R \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

This equation immediately determines the translation vector, so that the transformation becomes

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + R \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

The rotation matrix can be decomposed into three parts, two of which are defined by perfect alignment of the EGN sector and the third of which accounts for misalignments. We write

$$R = R_\phi R_\theta A$$

where R_θ is a rotation around the x axis, and is given

$$R_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

where the angle $\theta = 25°$ is the nominal angle between the EGN perpendicular line and the beam line. The matrix R_ϕ is a rotation around the beam z axis and is defined by the sector. For definiteness I write it as

$$R_\phi = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\phi = -\frac{\pi}{2} + \frac{(n-1)\pi}{6}$, and n is the sector number. The matrix A accounts for misalignments by rotating the coordinates from the local system to the ideal system which has the perpendicular line at an angle of θ to the beam line and has its y axis in the plane defined by the ideal perpendicular line and the beam line. The elements of the matrix A are very small, and we can treat them as infinitesimal. To first order the matrix can be written as

$$A = \begin{pmatrix} 1 & e_3 & -e_2 \\ -e_3 & 1 & e_1 \\ e_2 & -e_1 & 1 \end{pmatrix}$$

where the $e_i << 1$. In the following we work in an absolute frame that has $n = 0$. The correction for $n \neq 0$ is trivial.

To make the infinitesimal nature of A explicit we can define

$$\Delta W = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + R_\theta A \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} - R_\theta \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} - \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = R_\theta (A - 1) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_\theta \begin{pmatrix} 0 & e_3 & -e_2 \\ -e_3 & 0 & e_1 \\ e_2 & -e_1 & 0 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

where Δx, etc. are the differences between the survey values of a point in the absolute coordinate system and the values that would exist if the detector were perfectly aligned. For example, if we measure the point V, we will obtain

$$\begin{pmatrix} v_{x0} \\ v_{y0} \\ v_{z0} \end{pmatrix} = \begin{pmatrix} 0 \\ V \cos\theta \\ -V \sin\theta \end{pmatrix}$$

The deviations due to misalignment are then defined as

$$\begin{pmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y - V \cos\theta \\ v_z + V \sin\theta \end{pmatrix}$$

$$\begin{pmatrix} \Delta v_x - p_x \\ \Delta v_y - p_y \\ \Delta v_z - p_z \end{pmatrix} = R_\theta \begin{pmatrix} 0 & e_3 & -e_2 \\ -e_3 & 0 & e_1 \\ e_2 & -e_1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ V \\ 0 \end{pmatrix}$$

$$R_\theta^{-1} \begin{pmatrix} v_x - p_x \\ v_y - V\cos\theta - p_y \\ v_z + V \sin\theta - p_z \end{pmatrix} = \begin{pmatrix} V e_3 \\ 0 \\ -V e_1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_x - p_x \\ v_y - V\cos\theta - p_y \\ v_z + V\sin\theta - p_z \end{pmatrix} = \begin{pmatrix} Ve_3 \\ 0 \\ -Ve_1 \end{pmatrix}$$

$$\begin{pmatrix} v_x - p_x \\ v_y\cos\theta - p_y\cos\theta - v_z\sin\theta + p_z\sin\theta - V \\ (v_y - p_y)\sin\theta + (v_z - p_z)\cos\theta \end{pmatrix} = \begin{pmatrix} Ve_3 \\ 0 \\ -Ve_1 \end{pmatrix}$$

$$\begin{pmatrix} v_x - p_x \\ \Delta v_y\cos\theta + V\cos^2\theta - p_y\cos\theta - \Delta v_z\sin\theta + V\sin^2\theta + p_z\sin\theta - V \\ (\Delta v_y + V\cos\theta - p_y)\sin\theta + (\Delta v_z - V\sin\theta - p_z)\cos\theta \end{pmatrix} = \begin{pmatrix} Ve_3 \\ 0 \\ -Ve_1 \end{pmatrix}$$

$$\begin{pmatrix} v_x - p_x \\ (\Delta v_y\cos\theta - p_y\cos\theta - \Delta v_z\sin\theta + p_z\sin\theta \\ (\Delta v_y - p_y)\sin\theta + (\Delta v_z - p_z)\cos\theta \end{pmatrix} = \begin{pmatrix} Ve_3 \\ 0 \\ -Ve_1 \end{pmatrix}$$

from which we deduce

$$e_3 = \frac{v_x - p_x}{V}$$

and

$$e_1 = -\frac{(\Delta v_y - p_y)\sin\theta + (\Delta v_z - p_z)\cos\theta}{V}$$

If we measure the point W, we obtain for no misalignments

$$\begin{pmatrix} w_{x0} \\ w_{y0} \\ w_{z0} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} W \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} W \\ 0 \\ 0 \end{pmatrix}$$

so

$$\begin{pmatrix} \Delta w_x \\ \Delta w_y \\ \Delta w_z \end{pmatrix} = \begin{pmatrix} w_x - W \\ w_y \\ w_z \end{pmatrix}$$

$$\begin{pmatrix} \Delta w_x - p_x \\ \Delta w_y - p_y \\ \Delta w_z - p_z \end{pmatrix} = R_\theta \begin{pmatrix} 0 & e_3 & -e_2 \\ -e_3 & 0 & e_1 \\ e_2 & -e_1 & 0 \end{pmatrix} \begin{pmatrix} W \\ 0 \\ 0 \end{pmatrix} = R_\theta \begin{pmatrix} 0 \\ -We_3 \\ We_2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -We_3 \\ We_2 \end{pmatrix} = R_\theta^{-1} \begin{pmatrix} \Delta w_x - p_x \\ \Delta w_y - p_y \\ \Delta w_z - p_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} w_x - W - p_x \\ w_y - p_y \\ w_z - p_z \end{pmatrix} = \begin{pmatrix} w_x \\ (w_y - p_y)\,\mathrm{co} \\ (w_y - p_y)\,\mathrm{si} \end{pmatrix}$$

so that

$$e_3 = \frac{(w_z - p_z)\sin\theta - (w_y - p_y)\cos\theta}{W}$$

and

$$e_2 = \frac{(w_y - p_y)\sin\theta + (w_z - p_z)\cos\theta}{W}$$

These measurements over-determine e_3 and give e_1 and e_2 as well. Other points could be measured to give more over-determination.

With the matrix A and the vector P determined, the absolute coordinates of any point in the calorimeter sector can be determined from its local coordinates using the equation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + R_\phi R_\theta A \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

Ignoring the ϕ rotation, we obtain

$$R_\theta A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & e_3 & -e_2 \\ -e_3 & 1 & e_1 \\ e_2 & -e_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & e_3 \\ -e_3\cos\theta + e_2\sin\theta & \cos\theta - e_1\sin\theta \\ e_3\sin\theta + e_2\cos\theta & -\sin\theta - e_1\cos\theta \end{pmatrix}$$

Coordinates of the strips

In the local coordinate system, the beam vertex of the triangle for layer L is at

$$y_-(L) = -1829.74 - 4.3708(L-1)$$

The top of the triangle (the side at large scattering angle) is

$$y_+(L) = 1899.56 + 4.5419(L-1)$$

The active triangular region is bounded by the three lines:

$$y + x\tan\theta = y_-(L)$$

and

$$y - x\tan\theta = y_-(L)$$

and

$$y = y_+(L) = 1899.56 + 4.5419(L-1)$$

where

$$\tan\theta = 1.95325$$

The y coordinate of the lower edge of a U strip in layer number L is given by

$$y = -1829.74 - 4.3708(L-1) + (U-1)w_u(L)$$

where

$$w_u(L) = 103.655 + 0.2476(L-1)$$

is the strip width for the U strips in layer L. The upper edge of the strip is obtained by setting U=U+1. The edges of the V strips are given by the equation

$$y + x\tan\theta = y_-(L) + w_v(L)(37-n)\sqrt{1+\tan^2\theta}$$

and the edges of the W strips are given by

$$y - x\tan\theta = y_-(L) + w_w(L)(37-n)\sqrt{1+\tan^2\theta}$$

where n is the strip number (1 is the shortest strip and 36 is the longest). The widths of the V and W strips are given by

$$w_v = 94.701 + 0.2256(L-2)$$

and

$$w_w = 94.926 + 0.2256(L - 3)$$

Each strip has a trapezoidal portion on its light collection end that extends beyond the triangular region defined above. The trapezoid has right angles at the end of the strip, and distance from the end to the triangular region is given by two distances

$$d_2, \qquad d_1 = d_2 + \frac{w}{2}\left(\frac{\tan^2\theta - 1}{\tan\theta}\right)$$

where w is the width of the strip. For the first 15 layers

$$d_2 = 36.4, \qquad L \leq 15$$

and for the remaining layers

$$d_2 = 25.4, \qquad L > 15$$

The z spacing between layers is (according to Cassim Riggs)

$$\Delta z = 12.381 \text{ mm}$$

so that the z coordinate at the front face of each layer of scintillator is

$$z(L) = 12.381L$$

Geometry of strips n terms of dimensioned quantities on C. Riggs' drawings::

U layers:

Top:

$$y_+ = C$$

$$y_- = C - A$$

V edge:

$$y = C - A - \frac{A}{D}x$$

W edge:

$$y = C - A + \frac{A}{D}x$$

Width of strips:

$$w = A/36$$

Edges of strips:

$$y = y_- + (n-1)w$$

where $n = 1$ gives the bottom edge of strip 1 and $n = 2$ gives the top edge of strip 1 and the bottom edge of strip 2, etc.

Trapezoidal extensions of U strips on W ends.

$$d_1 = d_2 + \frac{D}{A}w$$

$$d_2 = B - 2D$$

where d_1 is the extension on the bottom side, and d_2 is the extension on the top side of the strip.

V layers:

$$y_+ = F$$

$$y_- = -B$$

V side:

$$y = -B - \frac{B+F}{G-H}x$$

W side:

$$y = -B + \frac{B+F}{G-H}x$$

Width of each strip:

$$w = D/36$$

Boundaries of strips:

$$y = -B - \frac{B+F}{G-H}x + (37-n)w\sqrt{1 + \left(\frac{B+F}{G-H}\right)^2}$$

where for example, $n = 37$, gives the boundary of the active area on the V side.

W layers:

$$y_+ = A - C$$

$$y_- = -C + J$$

V edge:

$$y = -C + J - \frac{A - J}{G - H}x$$

W edge:

$$y = -C + J + \frac{A - J}{H}x$$

width of strips:

$$w = \frac{D}{36}$$

Edges of strips:

$$y = -C + J + \frac{A - J}{H}x + (n - 37)w\sqrt{1 + \left(\frac{A - J}{H}\right)^2}$$

Dimensions of trapezoid on end of strips:

$$d_1 = \sqrt{F^2 + J^2}$$

$$d_2 = d_1 + w \tan\left(2\theta - \frac{\pi}{2}\right)$$

where

$$\tan\theta = \frac{A - J}{H}$$

Triangular stacks

The calorimeter can be subdivided into triangular stacks, each one directed back towards the target. A convenient labelling for these stacks is provided by the number N, where

$$N = U(U-1) + V - W + 1$$

subject to the subsidiary condition for a valid combination that

$$S = U + V + W = 73 \text{ or } 74$$

With this numbering scheme, the cell #1 is at the vertex near the beam, with $U = 1, V = W = 36$, and $S = 73$. The final cell in this scheme is at $U = 36, V = 36, W = 1$, for which $N = 36^2 = 1296$. In general, if $U + S = 2U + V + W$ is even(odd) the triangular cell points toward(away from) the beam. The center of the cell is at

$$y = -1829.74 - 4.3708(L-1) + (U - .5)w_u(L)$$

$$x \tan\theta = (W - V)w_u(L)$$

or

$$x = .511970(W - V)w_u(L)$$

The lead sheets are on the average 2.387 mm thick. They arecut to cover the active triangular area as well as the extension on the light collection side of the layer defined by the length d_2. Layers 4,7, 10, etc. have the vertex at the beam side clipped off. Their dimensions do not follow a simple formula. On the average, the z coordinate of the front surface of each layer of lead is 100.00 mm greater than the z coordinate of the front surface of the preceding layer of scintillator. In the following table layer 2 of the lead is between scintillator layer 1 and 2, etc. There is no layer 1 of lead.

LAYER	BASE	HEIGHT	TRUNCATION
2	152.096	147.264	0
3	152.454	148.890	0
4	152.948	149.372	1.440
5	153.174	148.636	0
6	153.532	149.943	0
7	154.033	150.433	1.448
8	154.252	150.007	0
9	154.610	150.996	0
10	155.118	151.492	1.455
11	155.328	151.379	0
12	155.688	152.048	0
13	156.203	152.552	1.462
14	156.406	152.750	0
15	156.766	153.101	0
16	156.820	153.153	1.000
17	157.090	153.417	0
18	157.449	153.768	0
19	157.898	154.206	1.000
20	158.168	154.470	0
			0

LAYER	BASE	HEIGHT	TRUNCATION
21	158.527	154.821	0
22	158.975	155.258	1.000
23	159.246	155.523	0
24	159.605	155.873	0
25	160.054	156.311	1.000
26	160.324	156.575	0
27	160.683	156.926	0
28	161.130	157.363	1.000
29	161.400	157.628	0
30	161.760	157.979	0
31	162.208	158.415	1.000
32	162.478	158.681	0
33	162.838	159.032	0
34	163.286	159.468	1.000
35	163.556	159.733	0
36	163.916	160.084	0
37	164.364	160.521	1.000
38	164.634	160.786	
39	164.994	161.137	

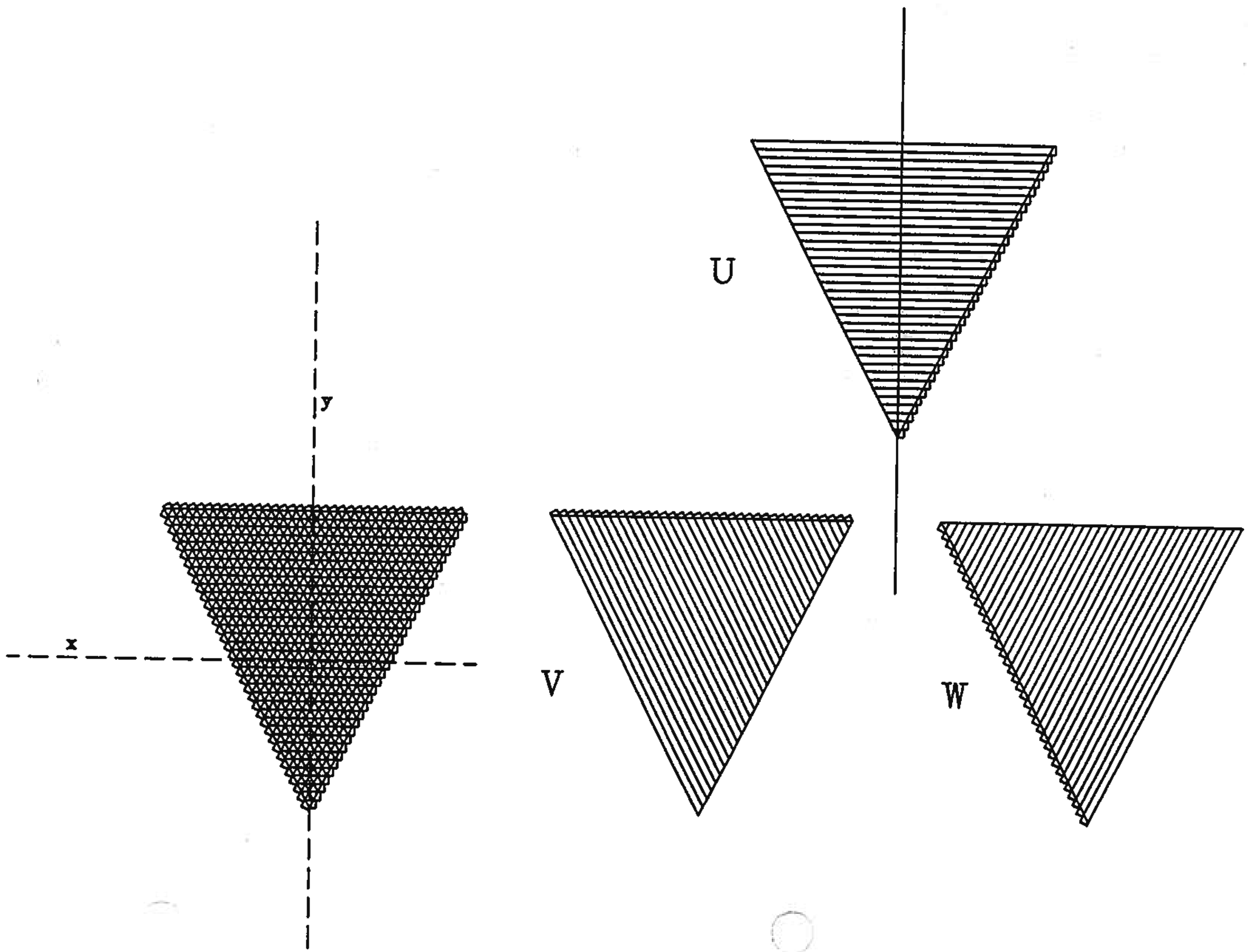
Walls of the Containment Box

The side walls of the containment box can be represented by six planes of 1.5" thick aluminum. The vertices of the planes can be represented by 12 points at the front of the box and by 12 points at the rear of the box, six for the outside and six for the insides surfaces. The labelling of these points is indicated in the figure. The coordinates of the points in the local detector frame are

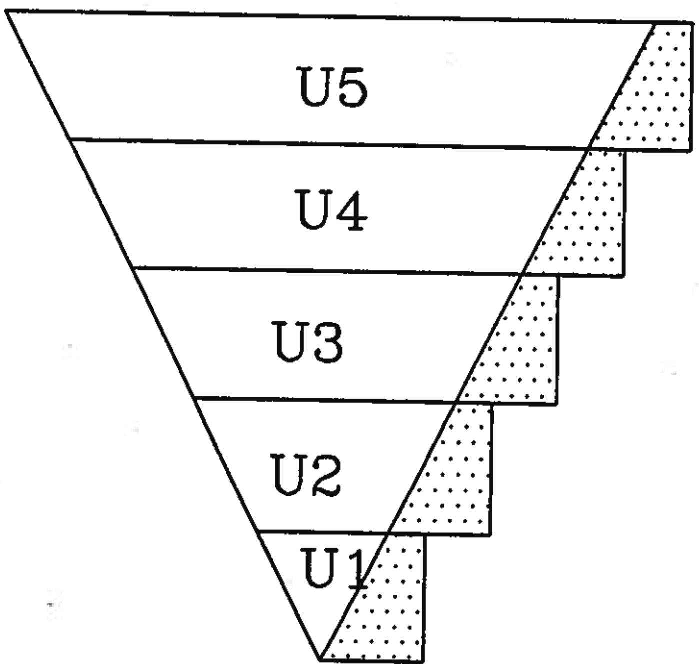
P1	2079.19	2076.73	0.	P7	2259.10	2255.70	480.72
P2	-2079.19	2076.73	0.	P8	-2259.10	2255.70	480.72
P3	-2139.49	1959.89	0.	P9	-2319.43	2138.86	480.72
P4	-65.02	-2092.05	0.	P10	-65.02	-2264.50	480.72
P5	65.02	-2092.05	0.	P11	65.02	-2264.50	480.72
P6	2139.49	1959.89	0.	P12	2319.43	2138.86	480.72
P1'	2043.41	2036.09	0.	P7'	2223.31	2215.06	480.72
P2'	-2043.41	2036.09	0.	P8'	-2223.31	2215.06	480.72
P3'	-2089.43	1946.89	0.	P9'	-2269.36	2125.85	480.72
P4'	-42.34	-2051.61	0.	P10'	-42.34	-2224.05	480.72
P5'	42.34	-2051.61	0.	P11'	42.34	-2224.05	480.72
P6'	2089.43	1946.89	0.	P12'	2269.36	2125.85	480.72

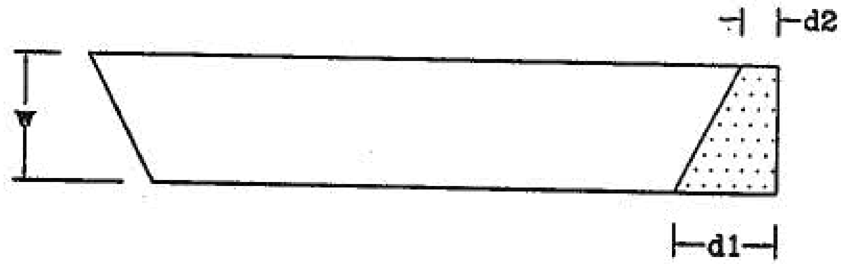
The six surfaces can be characterized by the following outward pointing normal vectors:

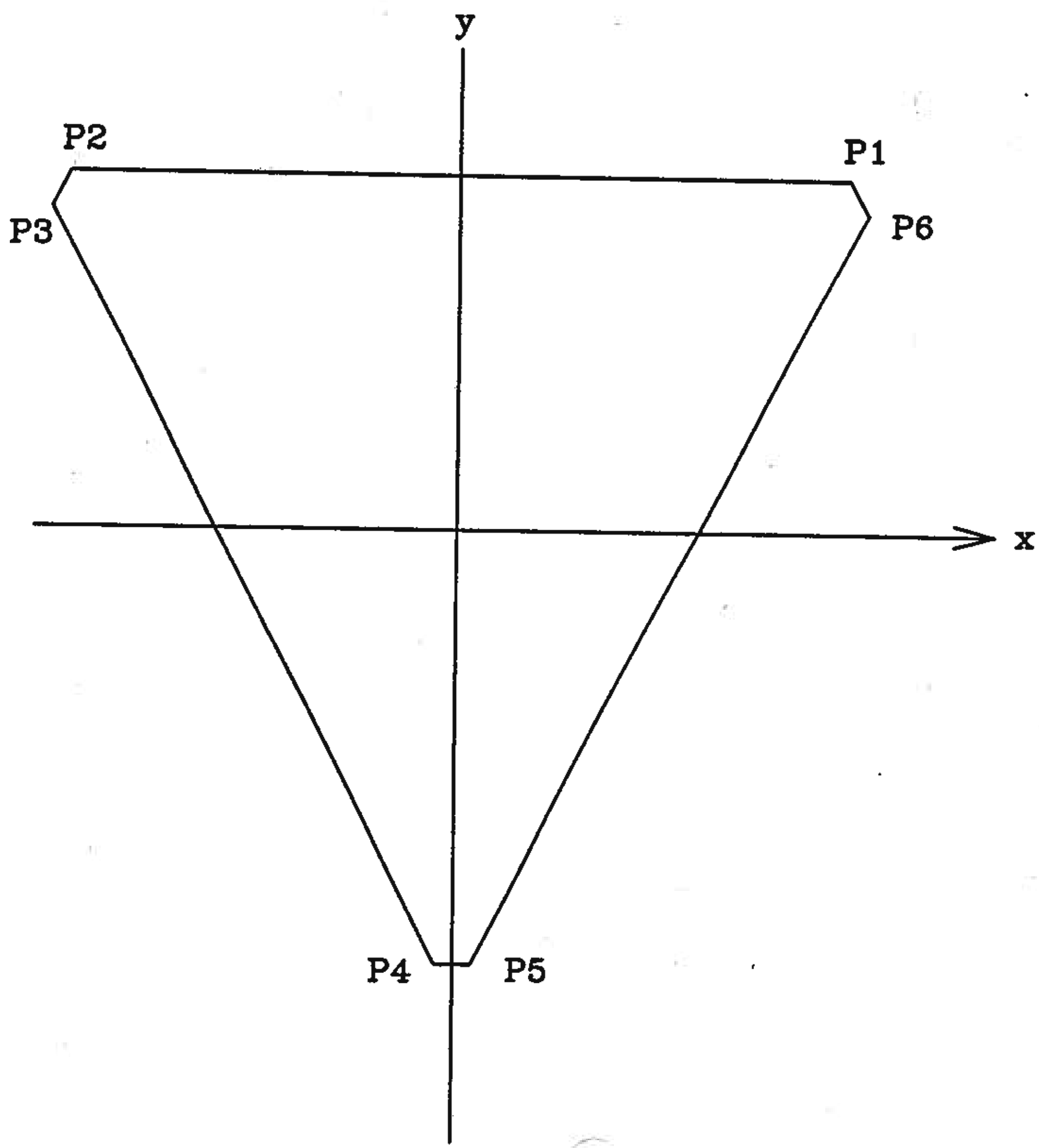
	x	y	z
n_1	0	cos(20.42)	-sin(20.42)
n_2	.87848	-.44975	-.161259
n_3	-.87848	-.44975	-.161259
n_4	.79367	.40978	-.44963
n_5	-.79367	.40978	-.44963
n_6	0	-.94127	-.33765



y
x
U
V
W


U5
U4
U3
U2
U1


d2
d1


y
x
P1
P2
P3
P4
P5
P6

CLAS ELECTROMAGNETIC CALORIMETER (WBS 6.6.5) 8/13/96

Item	Activity	Cost(K$)	% completed	value (K$)	est. complet.(C
Lead	Procurement	90	100	90	6/94
	Processing	60	100	60	3/96
Box 1-6	Walls & plates	660	100	660	5/95
	Pushers	216	100	216	5/95
	PMT support	100	100	100	12/95
	Assembly	40	100	40	10/95
Devices	Wall removal	10	100	10	9/94
	Bending	20	100	20	7/94
	Gluing	30	100	30	7/94
	Surface milling	30	100	30	9/94
Scintillator	Procurement	2000	100	2000	9/95
	Testing	100	100	100	10/95
Lead/Scint.	Stacking	100	100	100	3/96
	Teflon	30	100	30	6/94
PMTs	Procurement	400	100	400	9/95
	Testing	15	100	15	12/95
	Bases	130	100	130	12/95
	Laser calibr.	60	95	57	9/96
Calorimeter	Final Assembly	50	90	45	8/96
	testing	60	85	51	9/96
Fiber Readout	Fibers	225	100	225	9/95
	Parts	50	100	50	7/94
	Assembly	300	100	300	5/96
	Machining	40	100	40	6/96
	Testing	20	100	20	6/96
	Adapt. guides	55	100	55	6/96
HV-System	Procurement	150	100	150	12/95
ADCs, TDCs	Procurement	225	100	225	9/95
	Testing	10	100	10	9/95
Timing-Disc.	Procurement	100	100	100	9/95
	Testing	10	100	10	12/95
Linear Electr.	Energy sums	60	80	48	9/96
	Splitters	20	100	20	9/96
Cables	Delay	200	80	160	10/96
	HV	20	100	20	12/95
	Trigger	20	100	20	6/94
	Ribbon ax	50	50	25	10/96
	Assembly	50	20	10	10/96
Crates & PS	Fastbus	60	100	60	7/96
	Camac & NIM	50	100	50	5/96
Forward carriage	Procurement	150	100	150	5/95
Misc. Items		200	90	180	6/96
Eng., Install.		400	90	360	9/96
Grand Total		6666		6472 (97.1%)	