Extracting Response Functions

The matrix T transports the spin from the π°-p center-of-mass system to the focal plane of the spectrometer.

$$\begin{pmatrix} P_n \\ P_t \\ P_\ell \end{pmatrix}_{\rm fp} = \begin{pmatrix} T_{nn} & T_{nt} & T_{nl} \\ T_{tn} & T_{tt} & T_{tl} \\ T_{ln} & T_{lt} & T_{ll} \end{pmatrix} \begin{pmatrix} P_n \\ P_t \\ P_\ell \end{pmatrix}_{\rm CM}$$

In the the π°-p center-of-mass the polarizations are related to the response functions:

$$\begin{aligned} P_n\sigma &= \nu_L R_L^n + \nu_T R_T^n - \nu_{LT} R_{LT}^n \cos\phi_p + \nu_{TT} R_{TT}^n \cos 2\phi_p \\ P_m\sigma &= -\nu_{LT} R_{LT}^m \sin\phi_p + \nu_{TT} R_{TT}^m \sin 2\phi_p \text{ with } m = \ell \text{ or } t \\ P'_n\sigma &= -\nu'_{LT} R'^m_{LT} \sin\phi_p \\ P'_m\sigma &= -\nu'_{LT} R'^m_{LT} \cos\phi_p + \nu'_{TT} R'^m_{TT} \text{ with } m = \ell \text{ or } t \\ \phi_p &= \phi_\pi - 180 \end{aligned} \quad (1)$$

with

$$\begin{aligned} \nu_L &= \epsilon \;\; \nu_T = 1 \;\; \nu_{LT} = \sqrt{\epsilon(1+\epsilon)} \;\; \nu_{TT} = \epsilon \\ \nu'_{LT} &= \sqrt{\epsilon(1-\epsilon)} \;\; \nu'_{LT} = \sqrt{1-\epsilon^2} \end{aligned} \quad (2)$$

For the helicity independent polarizations

$$\begin{aligned}
P_n^{fp} &= T_{nn}[\nu_L R_L^n + \nu_T R_T^n - \nu_{LT} R_{LT}^n \cos\phi_p + \nu_{TT} R_{TT}^n \cos 2\phi_p] \\
&+ T_{nt}[-\nu_{LT} R_{LT}^t \sin\phi_p + \nu_{TT} R_{TT}^t \sin 2\phi_p] \\
&+ T_{n\ell}[-\nu_{LT} R_{LT}^\ell \sin\phi_p + \nu_{TT} R_{TT}^\ell \sin 2\phi_p] \\
P_t^{fp} &= T_{tn}[\nu_L R_L^n + \nu_T R_T^n - \nu_{LT} R_{LT}^n \cos\phi_p + \nu_{TT} R_{TT}^n \cos 2\phi_p] \\
&+ T_{tt}[-\nu_{LT} R_{LT}^t \sin\phi_p + \nu_{TT} R_{TT}^t \sin 2\phi_p] \\
&+ T_{t\ell}[-\nu_{LT} R_{LT}^\ell \sin\phi_p + \nu_{TT} R_{TT}^\ell \sin 2\phi_p]
\end{aligned} \tag{3}$$

for helicity dependent polarizations

$$\begin{aligned}
P_n^{fp} &= -T_{nn}\nu'_{LT} R'^n_{LT} \sin\phi_p \\
&+ T_{nt}[-\nu'_{LT} R'^t_{LT} \cos\phi_p + \nu'_{TT} R'^t_{TT}] \\
&+ T_{n\ell}[-\nu'_{LT} R'^\ell_{LT} \cos\phi_p + \nu'_{TT} R'^\ell_{TT}] \\
P_t^{fp} &= -T_{tn}\nu'_{LT} R'^n_{LT} \sin\phi_p \\
&+ T_{tt}[-\nu'_{LT} R'^t_{LT} \cos\phi_p + \nu'_{TT} R'^t_{TT}] \\
&+ T_{t\ell}[-\nu'_{LT} R'^\ell_{LT} \cos\phi_p + \nu'_{TT} R'^\ell_{TT}]
\end{aligned} \tag{4}$$

(5)

$$\begin{pmatrix} T_{nn} & T_{nt} & T_{nl} \\ T_{tn} & T_{tt} & T_{tl} \\ T_{ln} & T_{lt} & T_{ll} \end{pmatrix} = \begin{pmatrix} S_{nn} & S_{nt} & S_{nl} \\ S_{tn} & S_{tt} & S_{tl} \\ S_{ln} & S_{lt} & S_{ll} \end{pmatrix} \begin{pmatrix} R_{nn} & R_{nt} & R_{nl} \\ R_{tn} & R_{tt} & R_{tl} \\ R_{ln} & R_{lt} & R_{ll} \end{pmatrix} \begin{pmatrix} C_{nn} & C_{nt} & C_{nl} \\ C_{nt} & C_{tt} & C_{tl} \\ C_{ln} & C_{lt} & C_{ll} \end{pmatrix}$$

The S-matrix rotates the polarizations from the spectrometer frame at target to the spectrometer frame at focal plane using matrix for COSY magnet code. C-matrix rotates polarizations from center-of-mass frame to lab reaction frame.

$$\begin{pmatrix} C_{nn} & C_{nt} & C_{nl} \\ C_{tn} & C_{tt} & C_{tl} \\ C_{ln} & C_{lt} & C_{ll} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_W & \sin\theta_W \\ 0 & -\sin\theta_W & \cos\theta_W \end{pmatrix}$$

R-matrix rotates from lab reaction frame to spectrometer frame

$$\begin{pmatrix} R_{nn} & R_{nt} & R_{nl} \\ R_{tn} & R_{tt} & R_{tl} \\ R_{ln} & R_{lt} & R_{ll} \end{pmatrix} = \begin{pmatrix} SL_{nn} & SL_{nt} & SL_{nl} \\ SL_{nt} & SL_{tt} & SL_{tl} \\ SL_{ln} & SL_{lt} & SL_{ll} \end{pmatrix} \begin{pmatrix} R1_1 & R1_2 & R1_3 \\ R2_1 & R2_2 & R2_3 \\ R3_1 & R3_2 & R3_2 \end{pmatrix}$$

R1R2R3-matrix rotates from lab reaction frame to beam frame.

$$\begin{aligned} \vec{R1} &= \vec{q}x\vec{p}/|\vec{q}x\vec{p}| \\ \vec{R2} &= -\vec{R1}x\vec{R3} \\ \vec{R3} &= \vec{p}/|\vec{p}| \end{aligned}$$

SL-matrix is the rotation from the beam frame to the fixed spectrometer frame. θ_{sp} is the central spectrometer angle. For proton arm it is less than zero.

$$\begin{pmatrix} SL_{nn} & SL_{nt} & SL_{nl} \\ SL_{nt} & SL_{tt} & SL_{tl} \\ SL_{ln} & SL_{lt} & SL_{ll} \end{pmatrix} = \begin{pmatrix} 0 & -\cos\theta_{sp} & -\sin\theta_{sp} \\ 1 & 0 & 0 \\ 0 & -\sin\theta_{sp} & \cos\theta_{sp} \end{pmatrix}$$