Taken from Hall C proposal by X. Zheng and O. Rondon.
The elastic asymmetry is

$$A = \frac{-a\cos\theta^\star - bR\sin\theta^\star\cos\phi^\star}{R^2+c} \quad (1)$$

$$a = 2\tau\tan\frac{\theta}{2}\sqrt{1+\tau+f^2} \qquad \tau = \frac{Q^2}{4M^2} \quad (2)$$

$$b = 2\sqrt{\tau(1+\tau)}\tan\frac{\theta}{2} \quad (3)$$

$$c = \frac{\tau}{\epsilon} \qquad \epsilon = [1+2(1+\tau)\tan^2\frac{\theta}{2}]^{-1} \quad (4)$$

$$f = (1+\tau)\tan\frac{\theta}{2} \quad (5)$$

with the ratio of the proton form factors $R = \frac{G_E}{G_M}$ and $\theta^\star, \phi^\star$ the polar and azimuthal angles of the target proton spin vector relative to the q-vector in the $\vec{k}x\vec{k'}$ plane . $\vec{k}$ is the momentum vector of the incident electron and $\vec{k}$ is the momentum vector of the scattered electron.

For the parallel target field data, the target field was aligned 180° to the beam direction. The beam energy was 5.7526 GeV. The central HMS spectrometer angle was 13.12° and define $\phi = 180°$. The elastic data was taken for HMS central momentum of 4.7 . Averaged over the data in the range $0.9 < \mathrm{W} < 1.0$ the electron polar angle $\theta = 12.97°$ and $\phi = 182.46°$. For the q-vector , $\theta_q = 50.97°$ and ϕ_q = -2.46°. Relative to the q-vector , $\theta^\star = 50.97°$ and $\phi^\star = 183.90°$. Using a fit to $\frac{\mu G_E}{G_M}$ measured by recoil polarization of $\mu\frac{G_E}{G_M} = 0.837$, one predicts A = 0.215 .

The systematics on the prediction. The derivative of A on R is

$$\frac{dA}{dR} = \frac{2XR + Y(R^2-c)}{(R^2+c)^2} \quad (6)$$

$$\frac{dA}{A} = \frac{dR}{R}\frac{R*(2XR - Yc + YR^2)}{(R^2+c)(-X-YR)} = \frac{dR}{R}f_R \quad (7)$$

with $X = a\cos\theta^\star$ and $Y = b\sin\theta^\star\cos\phi^\star$. For parallel target field, $f_R = 0.016$ and so even an overestimate of $\frac{dR}{R} = 10\%$ gives a negligible $\frac{dA}{A}$.

The derivative of A on $\theta^\star$ is

$$\frac{dA}{d\theta^\star} = \frac{a\sin\theta^\star - bR\cos\theta^\star\cos\phi^\star}{R^2+c} \quad (8)$$

For parallel kinematics, $\frac{dA}{d\theta^\star} = 0.0013/\mathrm{deg}$. From the measurement of the target field direction, the angle between the target field direction and the beam line is known to 0.05 degrees. The central spectrometer angle has an error of about 1mr (or 0.057°).

The derivative of A on $\phi^\star$ is

$$\frac{dA}{d\phi^\star} = \frac{bR\sin\theta^\star\sin\phi^\star}{R^2+c} \quad (9)$$

For parallel kinematics, $\frac{dA}{d\phi^\star} = -0.0001/\mathrm{deg}$.

The derivative of A on τ is :

$$\frac{\partial A}{\partial \tau} = \frac{-\frac{\partial a}{\partial \tau}\cos\theta^\star - (\frac{\partial b}{\partial \tau}R + \frac{\partial R}{\partial \tau}b)\sin\theta^\star\cos\phi^\star - A(2R\frac{\partial R}{\partial \tau} + \frac{\partial c}{\partial \tau})}{R^2 + c} \tag{10}$$

For the dependence of R on Q^2, assume that $R = -.14{*}Q^2$ so $\frac{\partial R}{\partial \tau} = -.14{*}4{*}M^2$. For E and E' dependence, τ and θ are independent so:

$$\frac{\partial a}{\partial \tau} = a\{\frac{1}{\tau} + \frac{1 + 2f\frac{\partial f}{\partial \tau}}{2(1 + \tau + f^2)}\} \qquad \frac{\partial f}{\partial \tau} = \tan(\frac{\theta}{2}) \tag{11}$$

$$\frac{\partial b}{\partial \tau} = b\frac{1 + 2\tau}{2\tau(1 + \tau)} \tag{12}$$

$$\frac{\partial c}{\partial \tau} = \frac{\epsilon - \tau\frac{\partial \epsilon}{\partial \tau}}{\epsilon^2} \qquad \frac{\partial \epsilon}{\partial \tau} = -2\epsilon^2\tan^2(\frac{\theta}{2}) \tag{13}$$

$$\tag{14}$$

For derivative of A on E and E' :

$$\frac{\partial A}{\partial E} = \frac{\partial A}{\partial \tau} * (\frac{\partial \tau}{\partial E}) \text{ and } \frac{\partial A}{\partial E'} = \frac{\partial A}{\partial \tau} * (\frac{\partial \tau}{\partial E'}) \tag{15}$$

$$\frac{\partial \tau}{\partial E} = \frac{E'}{M^2}\sin^2(\frac{\theta}{2}) \text{ and } \frac{\partial \tau}{\partial E'} = \frac{E}{M^2}\sin^2(\frac{\theta}{2}) \tag{16}$$

For the parallel field, $\frac{\partial A}{\partial E} = 0.004$ /GeV. For 5.753 GeV beam the uncertainty is 5x10-4 or 0.0006 GeV and the systematic error is negligible. For the scattered electron energy even assuming 1x10-3 uncertainty, this gives 0.005 GeV (for 4.723 GeV) and again the systematic error is negligible.

The derivative of A on θ is

$$\frac{\partial A}{\partial \theta} = \frac{-\frac{\partial a}{\partial \theta}\cos\theta^\star - (\frac{\partial b}{\partial \theta}R + \frac{\partial R}{\partial \theta}b)\sin\theta^\star\cos\phi^\star - A(2R\frac{\partial R}{\partial \theta} + \frac{\partial c}{\partial \theta})}{R^2 + c} \tag{17}$$

$$\frac{\partial a}{\partial \theta} = a[\frac{\partial \tau}{\partial \theta}\frac{1}{\tau} + \frac{1}{\sin\theta} + \frac{\frac{\partial \tau}{\partial \theta} + 2f\frac{\partial f}{\partial \theta}}{2(1 + \tau + f^2)}] \tag{18}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial \tau}{\partial \theta}\tan(\frac{\theta}{2}) + \frac{1 + \tau}{2\cos^2(\frac{\theta}{2})} \tag{19}$$

$$\frac{\partial b}{\partial \theta} = b[\frac{1}{\sin\theta} + \frac{(1 + 2\tau)\frac{\partial \tau}{\partial \theta}}{2\tau(1 + \tau)}] \tag{20}$$

$$\frac{\partial c}{\partial \theta} = c[\frac{\partial \tau}{\partial \theta}\frac{1}{\tau} - \frac{\partial \epsilon}{\partial \theta}\frac{1}{\epsilon}] \qquad \frac{\partial \epsilon}{\partial \theta} = \frac{-2\tan^2(\frac{\theta}{2})}{\epsilon^2}[\frac{\partial \tau}{\partial \theta} + \frac{2(1 + \tau)}{\sin\theta}] \tag{21}$$

$$\frac{\partial \tau}{\partial \theta} = \frac{EE'}{2M^2}\sin\theta \tag{22}$$

For the parallel field, $\frac{\partial A}{\partial \theta} = 0.0235$ /deg or 0.0013 /mr . The systematic error on the HMS angle is about 0.5mr so giving $\delta A = .0007$ or 0.3%.