Properties and applications of GTMDs

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Outline of the talk

Basic properties of GTMDs

- Definition of GTMDs
- Parametrization of GTMDs
- Limits of GTMDs

Relations between GPDs and TMDs

- Trivial relations
- Nontrivial relations
- Relations in terms of GTMDs

GTMDs in physical processes

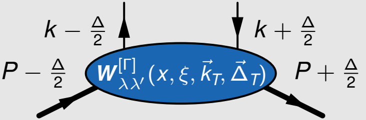
- Gluon GTMDs
- Quark GTMDs

Summary and outlook

Basic properties of GTMDs

Definition of GTMDs

Generalized transverse momentum dependent parton distributions (GTMDs) are defined by the **most general quark-quark correlator**



kinematical variables:

$$x = \frac{k^+}{P^+}, \xi = -\frac{\Delta^+}{2P^+}, \vec{k}_T, \vec{\Delta}_T$$

$$= \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2 \vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p', \lambda' | \bar{\psi}(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}_{\text{GTMD}} \, \psi(\tfrac{z}{2}) \, | p, \lambda \rangle \big|_{z^+=0} \, .$$

Parametrization of GTMDs

The application of **parity**, **hermiticity**, and **time reversal** to the correlator above yields constraints that allow one to parametrize it in terms of GTMDs.

[For spin-0 hadrons see S. Meissner, K. Goeke, A. Metz, M. Schlegel; JHEP **0808** (2008) 038.]

Basic properties of GTMDs

Parity constraint

The parity constraint **reduces the number of independent terms** in the correlator significantly. For $\Gamma = \gamma^+$ one obtains for example **four complex valued GTMDs $F_{1,n}$**

$$
\begin{aligned}
W_{\lambda\lambda'}^{[\gamma^+]}&(x,\xi,\vec{k}_T,\vec{\Delta}_T) \\
&= \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[F_{1,1}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2) \\
&\quad + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2) \\
&\quad + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2) \\
&\quad + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2)\Big]\,u(p,\lambda)\,.
\end{aligned}
$$

Basic properties of GTMDs

Hermiticity constraint

The hermiticity constraint yields the **symmetry relation**

$$F^{*}_{1,n}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2) = \pm F_{1,n}(x,-\xi,\vec{k}_T^2,-\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2)$$

for the GTMDs $F_{1,n}$, where the plus sign holds for $n = 1,3,4$ and the minus sign for $n = 2$.

Time reversal constraint

The time reversal constraint shows that it is reasonable to split each complex valued GTMD $F_{1,n}$ into **two real valued functions** with a different behavior under time reversal,

$$F_{1,n}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2) \\ = F^{e}_{1,n}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2) + i\,F^{o}_{1,n}(x,\xi,\vec{k}_T^2,\vec{k}_T\cdot\vec{\Delta}_T,\vec{\Delta}_T^2)\,,$$

the so-called **T-even** and **T-odd part** of $F_{1,n}$.

Basic properties of GTMDs

GPD limit of GTMDs

The correlator of GTMDs contains the **correlator of GPDs** in the limit

$$F^{q[\gamma^+]}_{\lambda\lambda'}(x,\xi,\vec{\Delta}_T) = \int d^2\vec{k}_T \, W^{q[\gamma^+]}_{\lambda\lambda'}(x,\xi,\vec{k}_T,\vec{\Delta}_T)$$

$$= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\,\langle p',\lambda' |\, \bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathcal{W}_{\text{GPD}}\,\psi(\tfrac{z}{2})\,| p,\lambda\rangle \Big|_{z^+=\vec{z}_T=0}$$

$$= \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E(x,\xi,t)\right] u(p,\lambda)\,.$$

GTMDs can, therefore, be considered as the **mother distributions of GPDs**. Explicitly, one finds

$$H(x,\xi,t) = \int d^2\vec{k}_T \left[F^e_{1,1} + 2\xi^2 \left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, F^e_{1,2} + F^e_{1,3}\right)\right],$$

$$E(x,\xi,t) = \int d^2\vec{k}_T \left[-F^e_{1,1} + 2(1-\xi^2) \left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, F^e_{1,2} + F^e_{1,3}\right)\right].$$

Basic properties of GTMDs

TMD limit of GTMDs

The correlator of GTMDs contains the **correlator of TMDs** in the limit

$$\Phi^{q[\gamma^+]}_{\lambda\lambda'}(x,\vec{k}_T) = W^{q[\gamma^+]}_{\lambda\lambda'}(x,0,\vec{k}_T,0)$$

$$= \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\,e^{ik\cdot z}\,\langle P,\lambda'|\,\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathcal{W}_{\text{TMD}}\,\psi(\tfrac{z}{2})\,|P,\lambda\rangle\big|_{z^+=0}$$

$$= f_1(x,\vec{k}_T^2) - \frac{\epsilon_T^{ij}k_T^iS_T^j}{M}\,f_{1T}^{\perp}(x,\vec{k}_T^2)\,.$$

GTMDs can, therefore, be considered as the **mother distributions of TMDs**. Explicitly, one finds

$$f_1(x,\vec{k}_T^2) = F^e_{1,1}(x,0,\vec{k}_T^2,0,0)\,,$$

$$f_{1T}^{\perp}(x,\vec{k}_T^2) = -F^o_{1,2}(x,0,\vec{k}_T^2,0,0)\,.$$

Basic properties of GTMDs

Results of the parametrization

		GTMDs		GPDs		TMDs	
		T-even	T-odd	T-even	T-odd	T-even	T-odd
twist-2	$\Gamma = \gamma^+$	4	4	2	0	1	1
	$\Gamma = \gamma^+\gamma_5$	4	4	2	0	2	0
	$\Gamma = i\sigma^{i+}$	8	8	4	0	3	1
twist-3	$\Gamma = 1$	4	4	2	0	1	1
	$\Gamma = \gamma_5$	4	4	2	0	0	2
	$\Gamma = \gamma^i$	8	8	4	0	1	3
	$\Gamma = \gamma^i\gamma_5$	8	8	4	0	3	1
	$\Gamma = i\sigma^{ij}$	4	4	2	0	2	0
	$\Gamma = i\sigma^{+-}$	4	4	2	0	1	1
twist-4	$\Gamma = \gamma^-$	4	4	2	0	1	1
	$\Gamma = \gamma^-\gamma_5$	4	4	2	0	2	0
	$\Gamma = i\sigma^{i-}$	8	8	4	0	3	1

Relations between GPDs and TMDs

Trivial relations

It is well known that trivial relations exist between some GPDs and TMDs as they reduce to the same **forward PDFs**

$$q(x) = H(x,0,0) = \int d^2\vec{k}_T\, f_1(x,\vec{k}_T^2)\,.$$

Nontrivial relations

Model calculations suggest the existence of additional non-trivial relations between GPDs and TMDs. The most prominent example is probably the possible relation **between the GPD *E* and the Sivers function $f_{1T}^{\perp}$**, which reads

$$\int d^2\vec{b}_T\, \vec{\mathcal{I}}(x,\vec{b}_T)\, \frac{\epsilon_T^{ij} b_T^i S_T^j}{M}\, \mathcal{E}'(x,\vec{b}_T^2) = -\int d^2\vec{k}_T\, \vec{k}_T\, \frac{\epsilon_T^{ij} k_T^i S_T^j}{M}\, f_{1T}^{\perp}(x,\vec{k}_T^2)$$

in impact parameter space. However, so far **no model-independent proof** of this relation has been found.

[M. Burkardt, D. S. Hwang; Phys. Rev. D **69** (2004) 074032.]

Relations between GPDs and TMDs

Trivial relations in terms of GTMDs

GTMDs **prove the model-independent existence** of trivial relations between GPDs and TMDs, as the involved GPDs and TMDs have **the same mother distributions**

$$\boldsymbol{q}(x) = \boldsymbol{H}(x,0,0) = \int d^2\vec{k}_T\, \boldsymbol{f_1}(x,\vec{k}_T^2) = \int d^2\vec{k}_T\, \boldsymbol{F^e_{1,1}}(x,0,\vec{k}_T^2,0,0)\,.$$

Nontrivial relations in terms of GTMDs

GTMDs indicate that **no nontrivial model-independent relations exist** between GPDs and TMDs, as the involved GPDs and TMDs have **different mother distributions**

$$\boldsymbol{E}(x,\xi,t) = \int d^2\vec{k}_T \left[-\boldsymbol{F^e_{1,1}} + 2(1-\xi^2)\left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\, \boldsymbol{F^e_{1,2}} + \boldsymbol{F^e_{1,3}} \right) \right],$$

$$\boldsymbol{f^\perp_{1T}}(x,\vec{k}_T^2) = -\boldsymbol{F^o_{1,2}}(x,0,\vec{k}_T^2,0,0)\,.$$

Relations between GPDs and TMDs

Results for all possible relations

The knowledge of all GTMDs allows one to check the model-independent existence of all relations between GPDs and TMDs that have been found in model calculations so far.

type of relation	model relations	model independent
1st type (trivial)	$H \leftrightarrow f_1$ $\tilde{H} \leftrightarrow g_1$ $H_T \leftrightarrow h_1$	GPDs and TMDs have the same mother distributions. $\Rightarrow$ **Relations are valid!**
2nd type (nontrivial)	$E \leftrightarrow -f_{1T}^{\perp}$ $\bar{E}_T \leftrightarrow -h_1^{\perp}$	GPDs and TMDs have different mother distributions. $\Rightarrow$ **It is very likely that relations are not valid!**
3rd type (nontrivial)	$\tilde{H}_T \leftrightarrow \frac{1}{2} h_{1T}^{\perp}$	

[For a review on model relations see S. Meissner, A. Metz, K. Goeke; Phys. Rev. D **76** (2007) 034002.]

Relations between GPDs and TMDs

Limitations of model results

So far all nontrivial relations have been confirmed using **lowest order calculations** in simple spectator models. However, GTMDs indicate that the relations are **not valid in general**.

Even in model calculations they will **probably break down**

- if one takes into account **higher orders** or
- if one uses **more complicated models**.

Evidence for possible approximate relations

Although the relations are probably not valid in general, it is still possible that they are **at least approximately valid**.

Evidence for this is that the relations

- provide an **intuitive picture of the Sivers effect** and
  [M. Burkardt; Phys. Rev. D **66** (2002) 114005.]
- are **compatible with data and lattice calculations**.
  [M. Göckeler *et al.* (QCDSF and UKQCD Collaborations); Phys. Rev. Lett. **98** (2007) 222001.]

GTMDs in physical processes

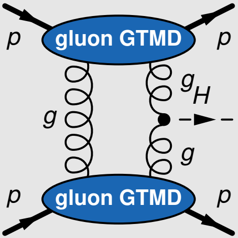
Gluon GTMDs

Gluon GTMDs appear **already at leading twist** in the description of diffractive Higgs production and **at subleading twist** in diffractive vector meson production.

[A. D. Martin *et al.*; Phys. Rev. D **62** (2000) 014022. V. A. Khoze *et al.*; Eur. Phys. J. C **14** (2000) 525.]

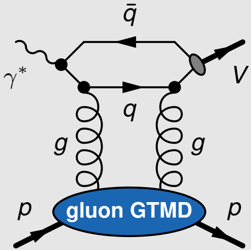
Higgs production

$$p + p \to p + H + p$$



Vector meson production

$$\gamma^* + p \to V + p$$



GTMDs in physical processes

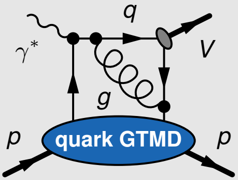
Quark GTMDs

So far no process is known, where quark GTMDs appear at leading twist. It is expected, however, that they generate **significant subleading twist corrections** in hard exclusive reactions like for example diffractive vector meson production and deeply virtual Compton scattering (DVCS).

[M. Vanderhaeghen *et al.*; Phys. Rev. D **60** (1999) 094017. M. Diehl, W. Kugler; Eur. Phys. J. C **52** (2007) 933.]

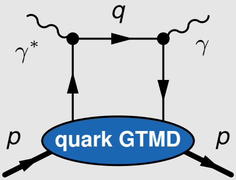
Vector meson production

$$\gamma^* + p \rightarrow V + p$$



DVCS

$$\gamma^* + p \rightarrow \gamma + p$$



Summary and outlook

Summary

- We performed the **first complete parametrization** of the most general quark-quark correlator in terms of GTMDs and obtained the first complete parametrization of the **nucleon GPDs beyond leading twist**.
- We were able to express all GPDs and TMDs through their respective **mother distributions**.
- Our results indicate that **no nontrivial model-independent relations exist** between GPDs and TMDs.

Outlook

- It is still under investigation how exactly GTMDs enter in the **description of physical processes**.
- Up to now only quark GTMDs and no **gluon GTMDs** have been studied.