Gluons, moments and F_L

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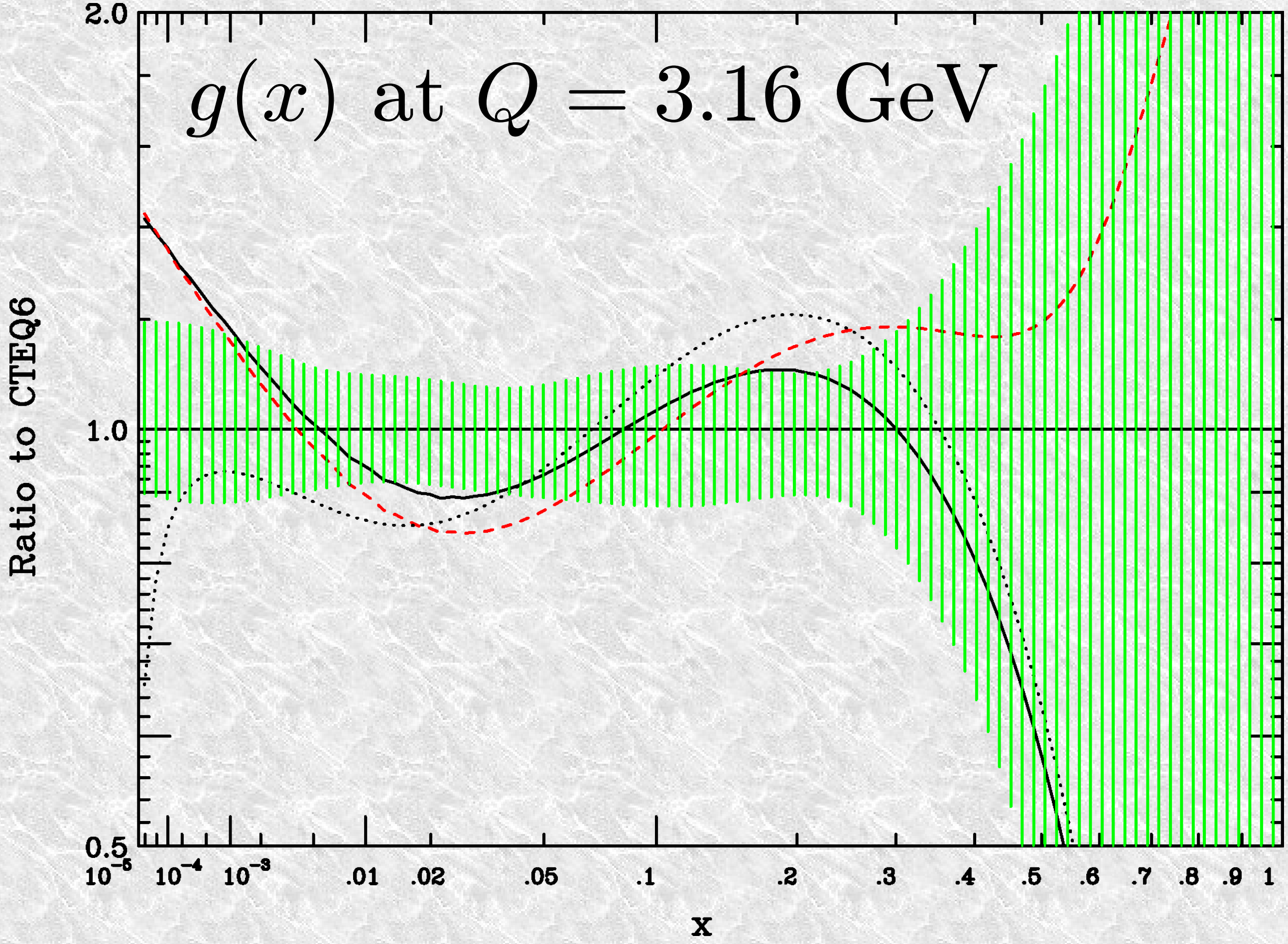
- **Part I: theory and motivation**
  - How to measure gluons
  - Momentum vs. moments
  - Data analysis, sketched
- **Part II: experimental analysis**
  - see Peter's talk

How to measure gluons

Gluons at large x

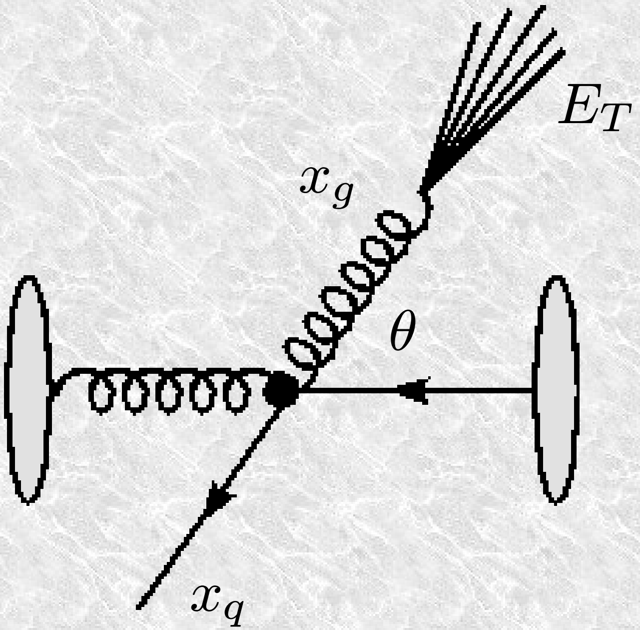
- Large uncertainties gluon PDF at $x > 0.3$ – e.g., CTEQ6



- Jlab to the rescue! (... also SLAC, to be fair ... [Alekhin '01, '03])

Jets in p+p collisions

- Jet production directly sensitive to gluons:



$$x \approx \frac{E_T}{\sqrt{s}} e^y$$

$$y = \frac{1}{2} \log\left(\frac{p^+}{p^-}\right) = -\log[\tan(\theta/2)]$$

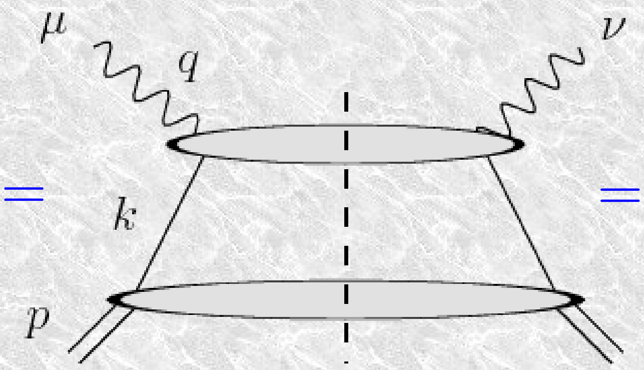
- Large x_g are reached at
  - large E_T – but $\sigma \sim 1/E_T^4$ and precision drops
  - large y – usually less instrumented

  - In both cases, *q(x)* >> *g(x)* : poor constraints on large-*x* gluons

DIS

- Hadronic tensor:

$$W^{\mu\nu}(p,q) =$$



$$= \frac{1}{8\pi}\int d^4z\, e^{-iq\cdot z}\langle p|j^{\dagger\mu}(z)j^{\nu}(0)|p\rangle$$

$$x_B = \frac{-q^2}{2p\cdot q} \qquad Q^2 = -q^2$$

- Structure functions via Lorentz decomposition:

$$W^{\mu\nu}(p,q) = \Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big)F_1(x_B,Q^2) + \Big(p^\mu - q^\mu\frac{p\cdot q}{q^2}\Big)\Big(\mu\leftrightarrow\nu\Big)\frac{F_2(x_B,Q^2)}{p\cdot q}$$

$$W^{\mu\nu}(p,q) = e_T^{\mu\nu}F_T(x_B,Q^2) + e_L^{\mu\nu}F_L(x_B,Q^2)$$

- Helicity vs. invariant structure functions:

$$F_T(x_B,Q^2) = 2F_1(x_B,Q^2)$$

$$F_L(x_B,Q^2) = -2x_BF_1(x_B,Q^2) + \Big(1+4x_B^2\frac{M^2}{Q^2}\Big)F_2(x_B,Q^2)$$

DIS

good review!

- Yet another decomposition (sorry...) but useful today

[Buras, Rev.Mod.Phys. 52(1980)199]

$$W^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_L + \left(\frac{p^\mu q^\nu + q^\mu p^\nu}{p \cdot q} - p^\mu p^\nu \frac{q^2}{p \cdot q^2} - g^{\mu\nu}\right) F_2$$

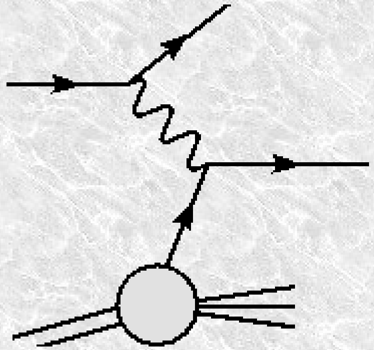
- More concretely (but I found it only for massless targets, sorry)

$$\frac{d\sigma}{dx_B dQ^2} = \frac{2\pi\alpha^2}{Q^4 x_B} \left\{ \left(1 + (1-y)^2\right) F_2 - y^2 F_L \right\}$$

- need a range in $y=\nu/E$ to separate F_2 and F_L

DIS – scaling violation of F_2

- Leading Order (LO), no gluons!



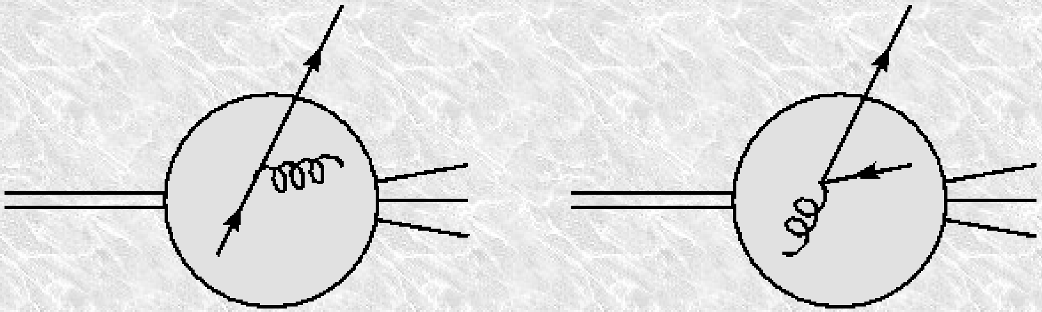
$$F_2 = x_B \sum_q e_q^2 q(x_B, Q^2)$$

$$F_L = 0$$

  - gluons subdominant also at Next-to-Leading Order (NLO)

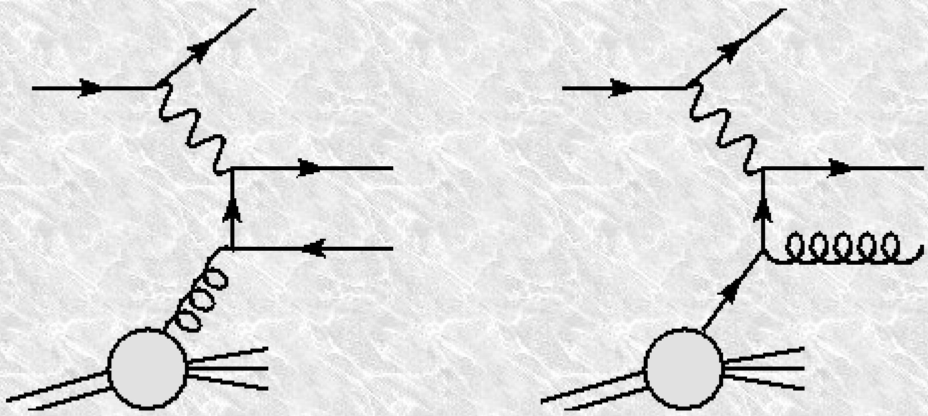
- Gluons accessed via DGLAP evolution in Q^2:

$$G(x) \approx \frac{d}{d \log Q^2} F_2(x, Q^2)$$



DIS – longitudinal structure function F_L

- Next-to-Leading Order: gluons contribute to both F_2 and F_L
  - subdominant in F_2
  - contribute to F_L



- One obtains a **"gluon sum rule"** in the massless target limit

  (and suppressing Q^2 for convenience)

$$F_L(x_B) = \frac{\alpha_s}{\pi} \int_{x_B}^{1} \frac{dy}{y} \left(\frac{x_B}{y}\right)^2 \left\{ \frac{4}{3} F_2(y) + 2c(n_f) \Big(1 - \frac{x_B}{y}\Big) y G(y) \right\}$$

  - This we want to use to study the gluons
  - Either take a derivative, or go to moment space

Operator Product Expansion

- Q^2 evolution is particularly simple for Cornwall-Norton moments of structure functions:

$$M_k^{[n]}(Q^2) = \int_0^1 dx x^{n-2} F_k(x, Q^2) = \sum_i A_n^i(\mu^2) C_{k,n}^i(Q^2/\mu^2)$$

n-th moment — any str. fns — non-perturbative info on partons — Wilson coefficients (perturbatively calculable)

  - in math terms, *M* are Mellin transforms of str. fns *F*
    The str. functions can be obtained as inverse Mellin transforms of *M*

- The **gluon sum rule** reads

$$F_L^{[n]}(Q^2) = \frac{\alpha_s(Q^2)}{\pi}\left[\frac{4}{3(n+1)} F_2^{[n]}(Q^2) + \frac{c(n_f)}{(n+1)(n+2)} (xG)^{[n]}(Q^2)\right]$$

Momentum vs. moments

Momentum vs. moments

- Give the same information on the gluons

- **Momentum space (*x*-space)**
  - fit F_2, F_L and take derivatives
  - parametrize $G(x)$, fit to gluon sum rule
  - in practice, encoded in a "global fit"

- **Moment space (*Mellin* space)**
  - Obtains $G^{[n]}(Q^2)$ directly
  - but... needs Mellin inverse transform (tricky) to obtain $G(x)$

- In both cases,
  - needs to interpolate $F_{2,L}$ where no data exist **(see Peter's talk)**

Target Mass Corrections: moment space

[Nachtmann, 1974]

- Use *Nachtmann moments* to eliminate target mass effects. E.g.,

TMC subtracted:
comparable to M=0 pQCD computations;
Can be used in gluon sum rule

experimental structure function

$$M_2^{[n]}(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} K_2(n, x, Q^2) F_2(x, Q^2)$$

where

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2 m_N^2/Q^2}} = \frac{2x_B}{1+\gamma^2} \qquad \text{Nachtmann variable}$$

$$K_2(n, x, Q^2) = \frac{n^2+2n+3+3(n+1)(1+\gamma^2)^{1/2}+n(n+2)\gamma^2}{(n+2)(n+3)}$$

Target Mass Corrections: momentum space

- **The OPE way:** [Georgi, Politzer 1976; see review of Schienbein et al. 2007]
  Mellin transform, sum O(M2/Q2) operators, transform back:

$$F_2^{OPE}(x_B,Q^2) = \frac{x_B^2}{\rho_B^3}F(\xi) + 6\frac{m_N^2}{Q^2}\frac{x_B^3}{\rho_B^4}\int_\xi^1 d\xi' F(\xi') + 12\frac{m_N^4}{Q^4}\frac{x^4}{\rho_B^5}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

  - Threshold problem: $x_B \leq 1$ implies $0 \leq \xi \leq \xi_{th} \stackrel{\text{def}}{=} \xi(x_B=1)$

    [Johnson, Tung 1979]

    $F_2^{OPE} \neq 0$ at $x_B=1$

  - Unphysical region:

    $F_2^{OPE} > 0$ also for $x_B>1$!!

- Does it satisfy the gluon sum rule ??? (Really, I don't know)

Target Mass Corrections: momentum space

- The **Collinear Factorization** way: [Accardi, Qiu, JHEP 2008]

$$F_k(x_B, Q^2) = \sum_f \int_{\xi}^{\frac{\xi}{x_B}} \frac{dx}{x} h_k^f\Big(\frac{\xi}{x}, Q^2\Big) \varphi_f(x, Q^2)$$

  - **<u>no unphysical region</u>**: $F_k^{CF} = 0$ for $x_B > 1$
  - Threshold problem solved introducing "Jet Functions"

- Should satisfy a **"target mass corrected" gluon sum rule** (my guess, still to be proven)

$$F_L(x_B) = \frac{\alpha_s}{\pi} \int_{x_B}^{1} \frac{dy}{y} \left(\frac{x_B}{y}\right)^2 \frac{4}{3} F_2(y) + \frac{\alpha_s}{\pi} \int_{\xi}^{\frac{\xi}{x_B}} \frac{dy}{y} 2c(n_f) \Big(1 - \frac{\xi}{y}\Big) y G(y)$$

  - how sensitive to (absence of) jet mass corrections ???

Data analysis - ideas

Moment analysis

- **Procedure**, sketched:
  1) Take Nachtmann moments of $F_{2,L}$(exp) [Peter's talk]
  2) Extract $G^{[n]}$(exp) via massless sum rule
  3) compare with existing gluon parametrizations, $G^{[n]}$(PDF)

- **Issues**:
  - $G^{[n]}$(exp) include
    1) Higher-Twist $1/Q^2$ contrbutions
    2) non-perturbative effects which may be large
  - G(PDF) is rather unconstrained at large x:
    don't expect meaningful comparison for higher moments

- **Advantages**:
  - "experimentally" feasible, highlights data impact on large-x gluons

Moment analysis

- (Old) example for F_2

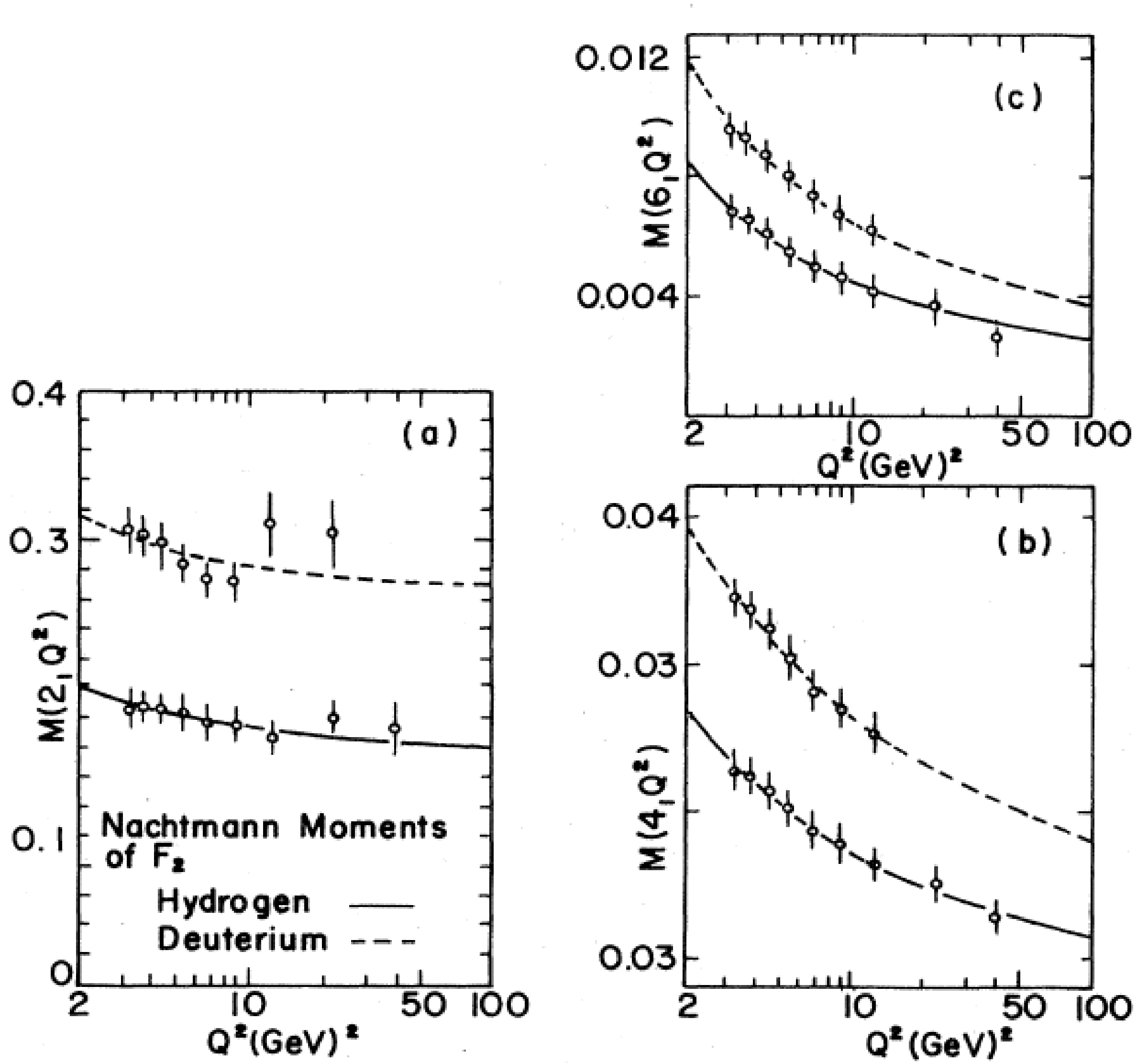


FIG. 14. Comparison of the QCD predictions for the moments of F_2 with the *ep*, *ed*, *μp*, and *μd* data (Anderson, Matis, and Myrianthopoulos, 1978).

Question:
how different are the Cornwall-Norton moments ??

Momentum space analysis: global QCD fits

- **Procedure**:
  1) Use F_2 and F_L data (plus many others) in a global QCD fit
  2) parametrize quark, gluons; use pQCD to compute
     Note: the <u>gluon sum rule is built-in</u>
  3) consider TMC + HT (compensates OPE vs CF differences)

$$F_2(data) = F_2(TMC) \times \left(1 + \frac{C(x_B)}{Q^2}\right)$$

- **Answer the questions:**
  - is there a DGLAP evolving gluon fitting well F_L over wide Q^2 range?
  - conversely, any sign for non-$1/Q^2$ non-perturbative / other effects ?? (deviations from F_L data)

- **Disadvantage:**
  - needs a dedicated effort (but we have the people and expertise for it)

Momentum space analysis: simplified

- **Procedure**:
  1) Extract massless F_2 and F_L from world data
     [Eric's procedure, OPE or ξ-scaling CF]
  2) Take appropriate derivative of sum rule
  3) Extract $G(x)$

- **Advantage**:
  - "feasible"

- **Disadvantages**:
  - Exact CF-TMC cannot be implemented; not sure about HT
  - Hard to detect non-perturbative / other effects (having been included in the fitted F_L)
  - Other disadvantages ??