



# **Hadron Mass Corrections in semi-inclusive deep-inelastic scattering**

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JHEP 1509 (2015) 169  
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# Outline

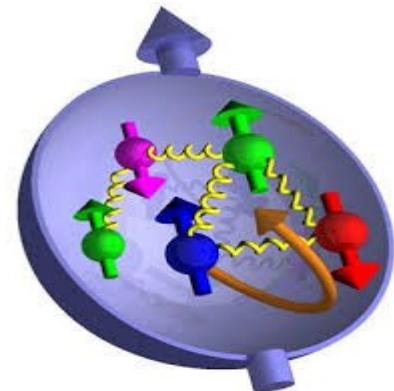
- Introduction
- Kinematics
- Cross sections at finite  $Q^2$
- Phenomenological implications
- Conclusion and Outlook



# Introduction

Hard scattering reactions:

Picture of the nucleon  
(partons)



Examples:

Deep inelastic scattering (DIS):  $e^-p \longrightarrow e^-X$

Semi inclusive Deep inelastic scattering (SIDIS):  $e^-p \longrightarrow e^-hX$

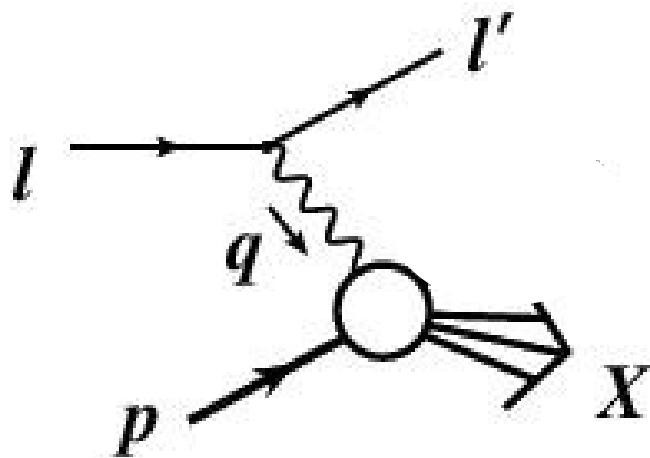
Drell Yan (DY):  $h_A h_B \longrightarrow e^-e^+X$

$e^-e^+$  annihilation:  $e^-e^+ \longrightarrow hX$



# Introduction: DIS

## DIS Diagram



## Theoretical Framework:

- Operator Product Expansion (OPE).
- Collinear factorization (CF)

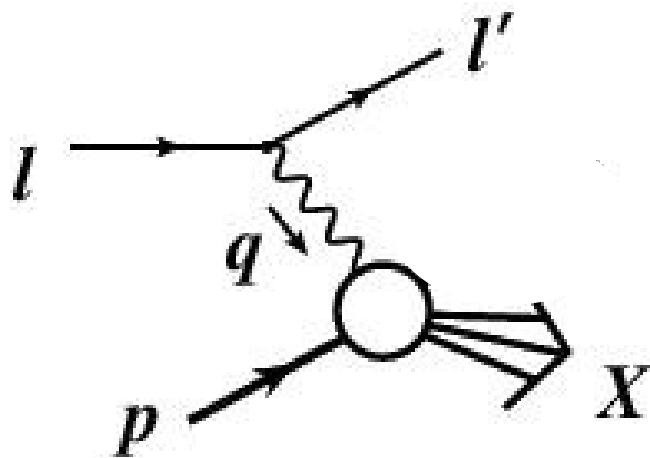
OPE  $\longleftrightarrow$  CF

To be studied:  
massive case



# Introduction: DIS

## DIS Diagram



## Theoretical Framework:

- Operator Product Expansion (OPE).
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OPE  $\longleftrightarrow$  CF

To be studied:  
massive case

## DIS Kinematic invariants

Nucleon Mass

$$m_N^2 = (p_\mu)^2 \quad Q^2 = -(q_\mu)^2$$

Bjorken-x

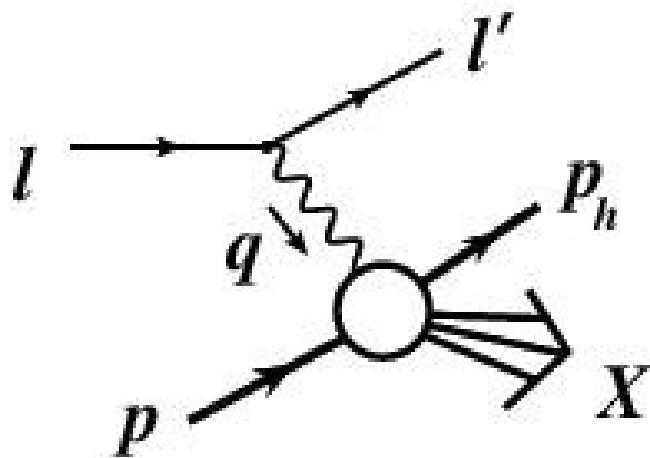
$$x_B = \frac{Q^2}{2p \cdot q} \quad W^2 = (p_\mu + q_\mu)^2$$

Final state

invariant mass

# Introduction: SIDIS

## SIDIS Diagram



## Theoretical Framework:

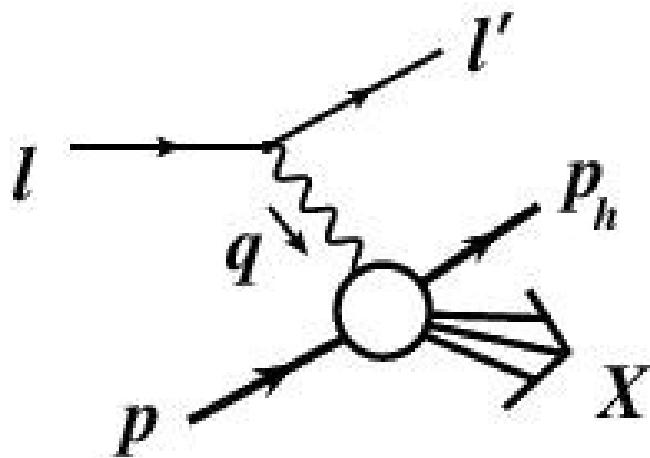
- Operator Product Expansion (OPE). 
- Collinear factorization (CF)

## DIS variables



# Introduction: SIDIS

## SIDIS Diagram



## Theoretical Framework:

- Operator Product Expansion (OPE). 
- Collinear factorization (CF)

**DIS variables + hadronic variables**

Hadron Mass

$m_h^2 = p_h^2$

Fragmentation invariant

$$z_h = \frac{p_h \cdot p}{q \cdot p}$$



# Introduction: Why Hadron Mass Corrections (HMC's)?

## Jlab experiments



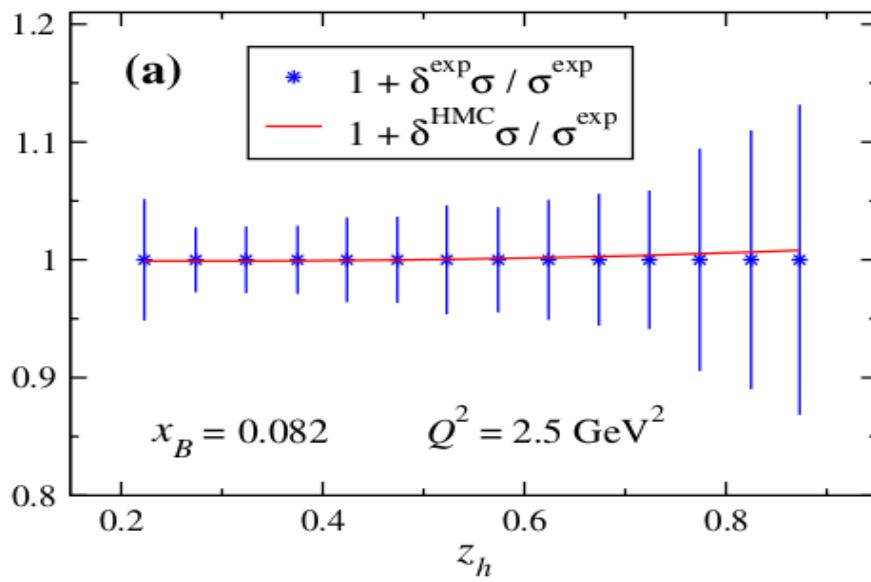
- Usually Low  $Q^2$  ( $\sim 1 \text{ GeV}^2$ )
- $1/Q^2$  corrections have to be controlled

$$O\left(\frac{M^2}{Q^2}\right) \equiv \mathbf{HMC's!!!}$$

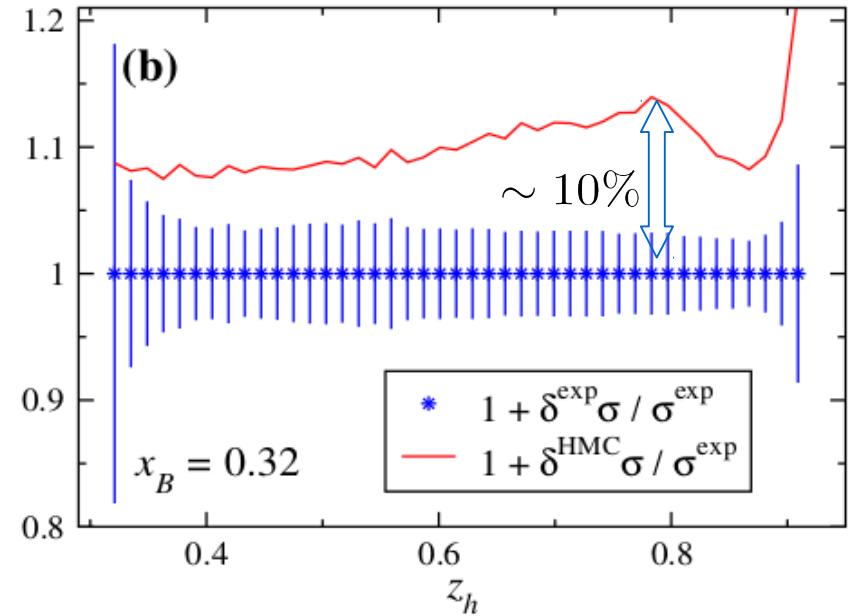


# Introduction: HMC's for charged pion production (unpolarized SIDIS)

HERMES experiment  
(DESY)



Jefferson Lab (E00-108)



Accardi, Hobbs, Melnitchouk  
JHEP 0911, 084 (2009)



# Introduction: Polarized HMC's (Project)

Hadron mass corrections in semi-inclusive deep-inelastic scattering

JHEP 1509 (2015) 169

J. V. Guerrero<sup>1,2</sup>, J. J. Ethier<sup>2,3</sup>, A. Accardi<sup>1,2</sup>, S. W. Casper<sup>2,4</sup>, W. Melnitchouk<sup>2</sup>

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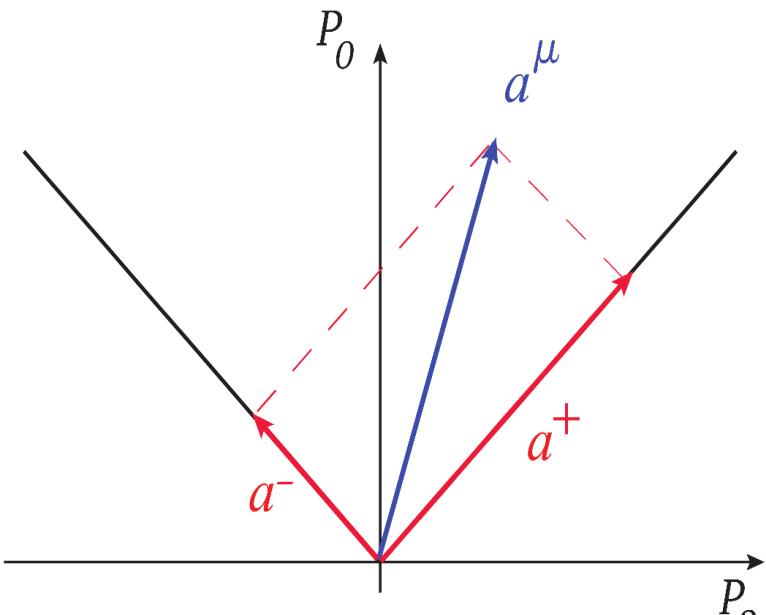
Starting from older JHEP 0911, 084 (2009) we:

- Review all the kinematics.
- Derive the cross section for the unpolarised case and extend to the polarized case.
- Estimate the size of the hadron mass corrections for current and future experiments



# SIDIS: Massive scaling variables

## Light Cone



$$a^- = \frac{a_0 - a_3}{\sqrt{2}} \quad a^+ = \frac{a_0 + a_3}{\sqrt{2}}$$

## Nachtmann Scaling variable

$$\xi = -\frac{q^+}{p^+} = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 M^2 / Q^2}}$$

Bjorken Limit  $Q^2 \rightarrow \infty \quad \xi \rightarrow x_B$

## Fragmentation Scaling variable

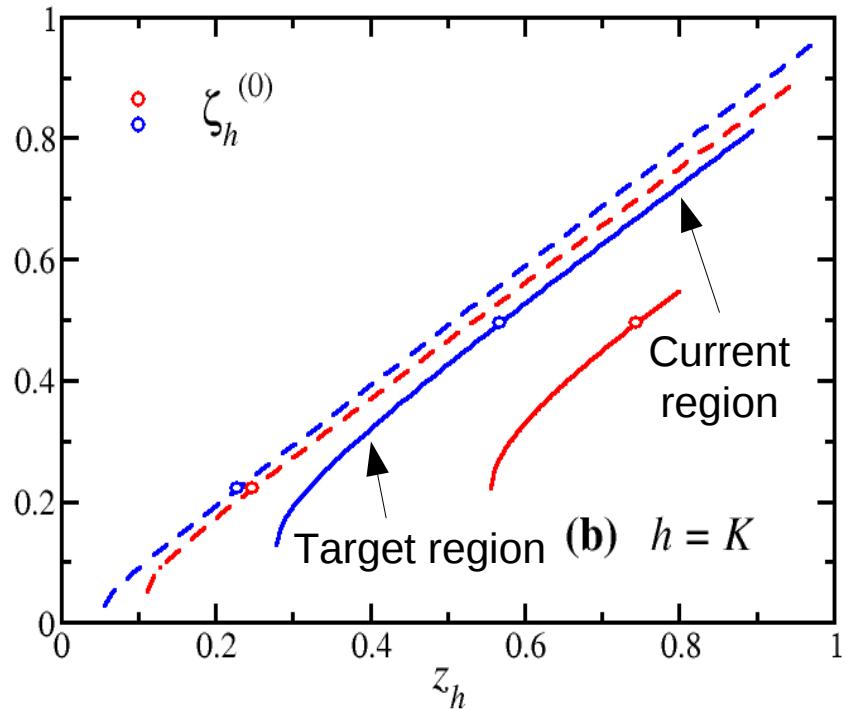
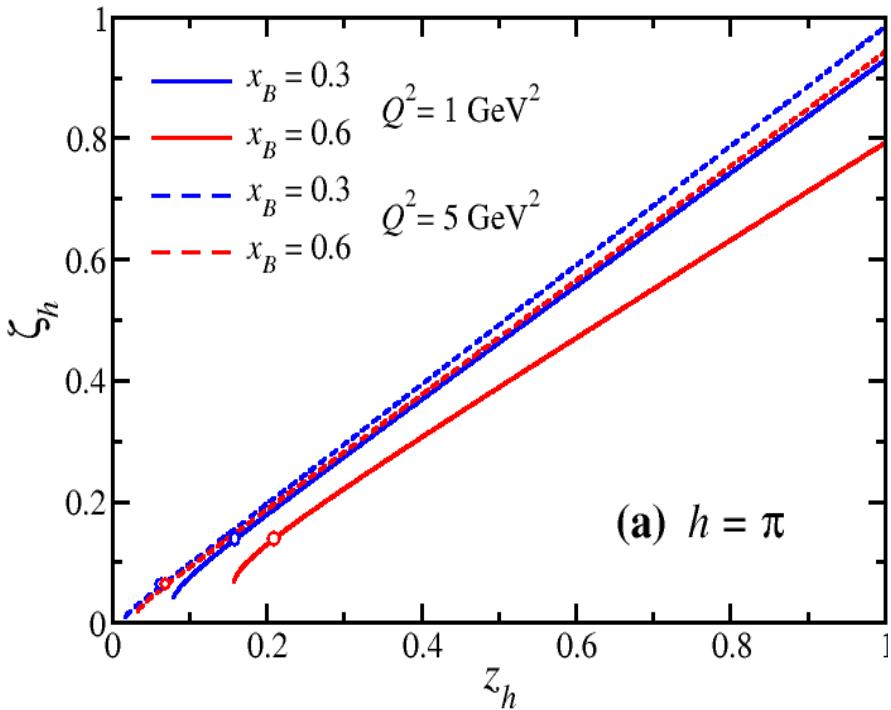
$$\zeta_h = \frac{p_h^-}{q^-} = \frac{z_h}{2} \frac{\xi}{x_B} \left( 1 + \sqrt{1 - \frac{4x_B^2 M^2 m_{h\perp}^2}{z_h^2 Q^4}} \right)$$

with

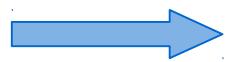
$$m_{h\perp}^2 = m_h^2 + p_{h\perp}^2; \quad \frac{\zeta_h}{Q^2} \rightarrow \infty \quad z_h$$



# SIDIS: External Kinematics



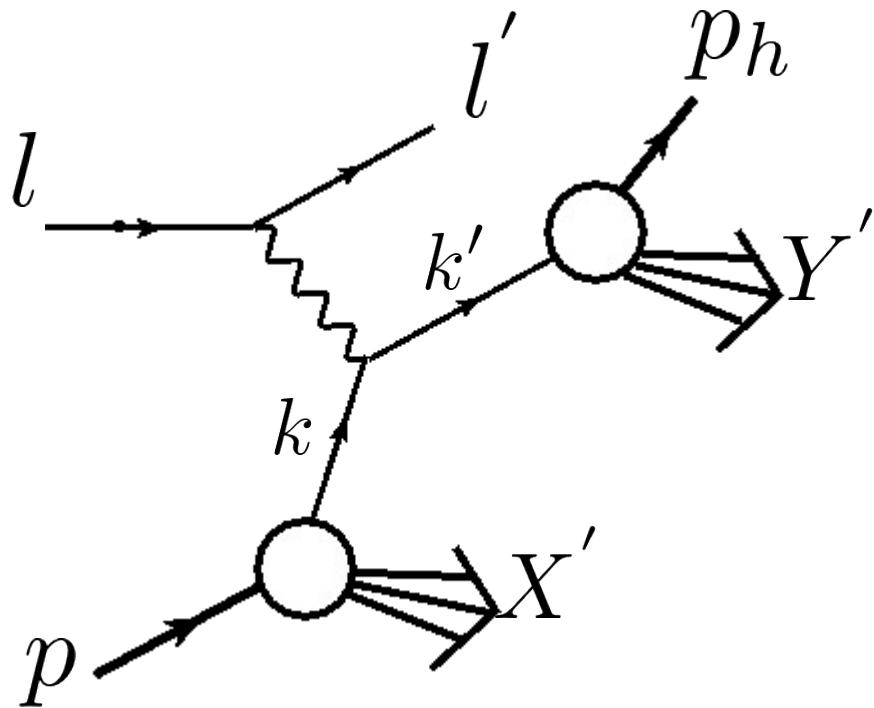
- Target vs. Current Fragmentation region separated by  $\circ$ .
- Limits: 4-momentum + Baryon number conservation



$$z_h^{\min} < z_h < z_h^{\max}$$



# SIDIS: Parton Kinematics



Initial and scattered quarks are off-shell

## Internal (parton) Kinematics

$$x = \frac{k^+}{p^+} \quad (\text{vs. } \xi = -\frac{q^+}{p^+})$$
$$z = \frac{p_h^-}{k'^-} \quad (\text{vs. } \zeta_h = -\frac{p_h^-}{q^-})$$

## “Collinear” Factorization:

Expand around “collinear” momenta  $\tilde{k}_\mu$  and  $\tilde{k}'_\mu$

Virtualities  $\tilde{k}^2$  and  $\tilde{k}'^2$  will be fixed later



# SIDIS: Internal Kinematics

## Virtuality choice:

- Usual Choice:  $\tilde{k}^2 = 0 \quad \tilde{k}'^2 = 0$

at LO

$$x = \xi \xrightarrow{Q^2 \rightarrow \infty} x_B$$
$$z = \zeta_h \xrightarrow{Q^2 \rightarrow \infty} z_h$$

BUT kinematic analysis shows that

$$\tilde{k}^2 \geq x(\zeta_h - 1)Q^2/\xi$$

- Alternative Choice  $\tilde{k}^2 = 0 \quad \tilde{k}'^2 = \frac{m_h^2}{\zeta_h}$

(respects kinematic bound)

at LO

$$x = \xi_h = \xi(1 + \frac{m_h^2}{\zeta_h Q^2})$$
$$z = \zeta_h$$

 NOT UNIQUE: Needs more analysis at NLO



# LO Cross section at finite $Q^2$

## Spin averaged Cross section

$$\sigma_h \equiv \frac{1}{2} \frac{d\sigma_h^{\uparrow\uparrow + \downarrow\downarrow}}{dx_B \, dQ^2 \, dz_h} = \frac{2\pi\alpha^2}{Q^4} \frac{y^2}{1-\varepsilon} J_h \sum_q e_q^2 q(\xi_h, Q^2) D_q^h(\zeta_h, Q^2)$$

Jacobian  $J_h = \frac{d\xi_h}{dz_h}$

Unpolarised PDF

Fragmentation Function



# LO Cross section at finite $Q^2$

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Unpolarised PDF

Fragmentation Function

## Spin dependent Cross section

$$\Delta\sigma_h \equiv \frac{d\sigma_h^{\uparrow\uparrow-\downarrow\downarrow}}{dx_B dQ^2 dz_h} = \frac{4\pi\alpha^2}{Q^4} \frac{y^2 \sqrt{1-\varepsilon^2}}{1-\varepsilon} J_h \sum_q e_q^2 \Delta q(\xi_h, Q^2) D_q^h(\zeta_h, Q^2)$$

Polarised PDF



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**Jacobian**  $J_h = \frac{d\xi_h}{dz_h}$

Unpolarised PDF

Fragmentation Function

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Bjorken Limit:  $\left( \frac{M^2}{Q^2}, \frac{m_h^2}{Q^2} \rightarrow 0 \right)$

$$\sigma_h^{(0)} \equiv \sigma_h(\xi_h \rightarrow x_B, \zeta_h \rightarrow z_h)$$

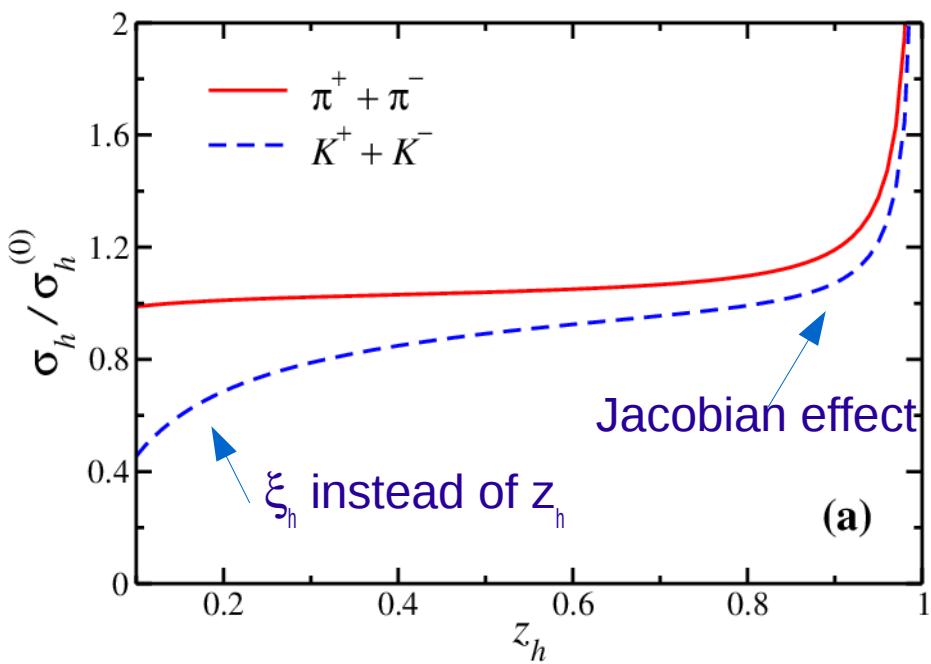
$$\Delta\sigma_h^{(0)} \equiv \Delta\sigma_h(\xi_h \rightarrow x_B, \zeta_h \rightarrow z_h)$$

Polarised PDF

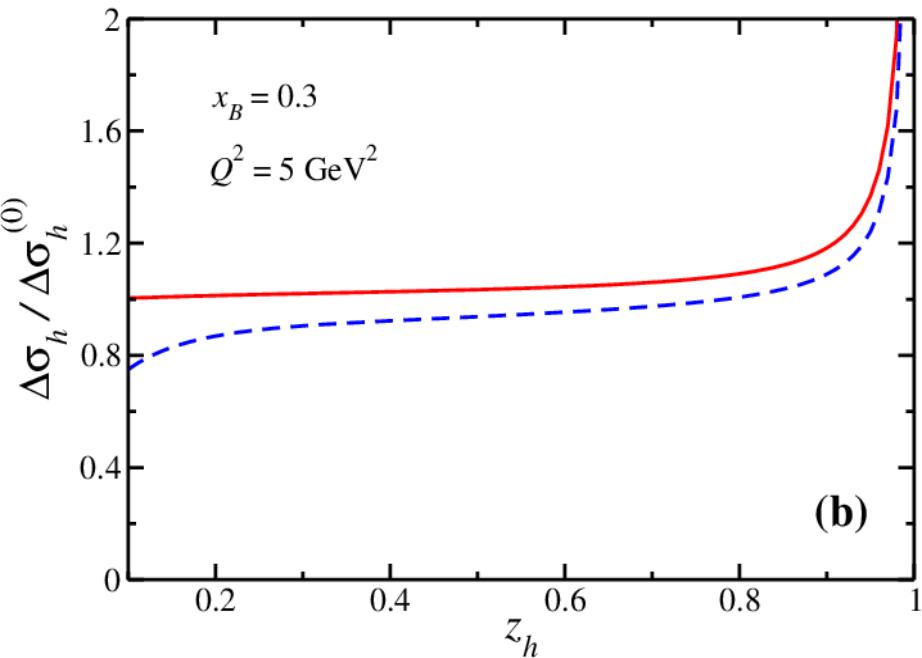
# Phenomenological implications

## Pions vs. Kaons

Unpolarised



Polarised

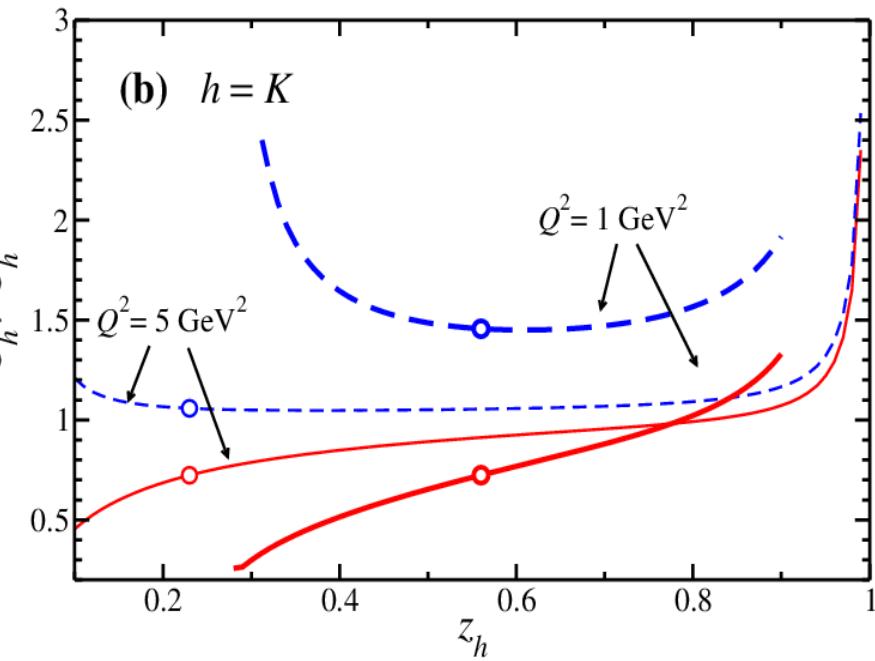
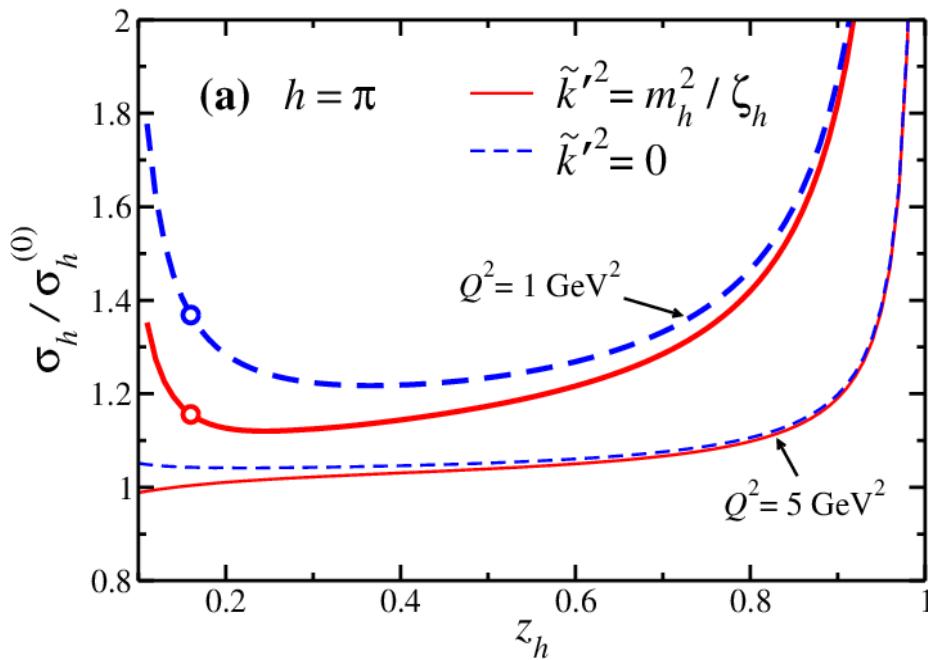


Kaons mass effects much larger



# Phenomenological implications

## Dependence on the parton virtuality



Red lines: Our choice  $\tilde{k}'^2 = m_h^2 / \zeta_h$

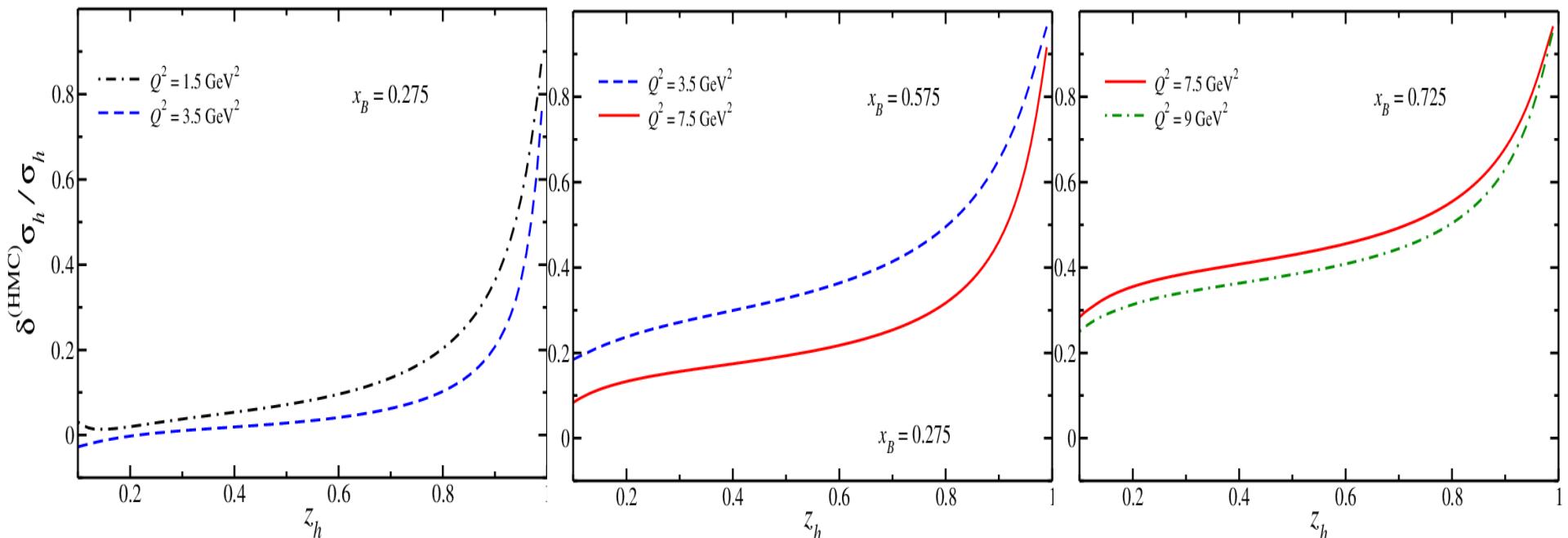
Thin lines: Albino et all. Nucl.Phys B803, 42 (2008)  $\tilde{k}'^2 = 0$



# Mass corrections for specific experiments

Jlab 11 GeV (Unpolarized Pions)

(Experiments: E12-09-007, E12-13-007, E12-06-109)



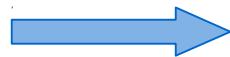
$$\frac{\delta^{(\text{HMC})} \sigma_h}{\sigma_h} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

Relative magnitude of the Mass  
Correction



# Strange quark PDF

• HERMES



- Claim very different s-quark shape compared to CTEQ6L
- Measurements from ATLAS/CMS at LHC also show different s-PDF
- Strange pdf may not be what we think!

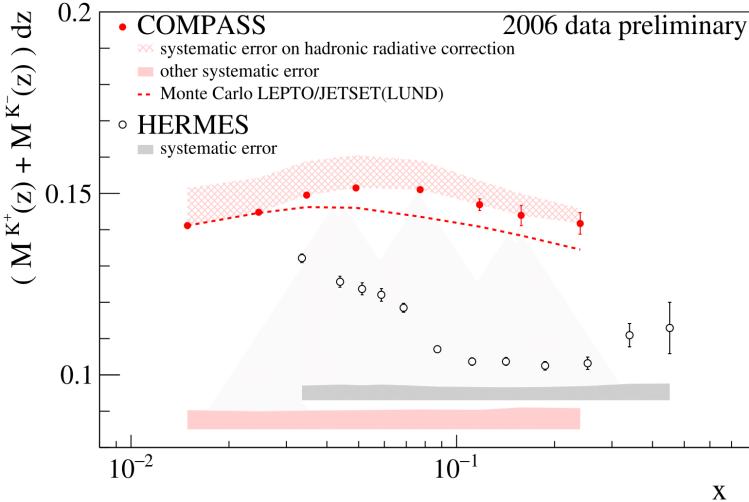


# Strange quark PDF

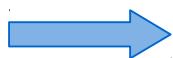
• HERMES



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• But COMPASS



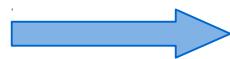
$$\langle Q^2 \rangle_{\text{COMPASS}} \gtrsim \langle Q^2 \rangle_{\text{HERMES}}$$

- Different  $x_B$  dependence
- Overall value higher

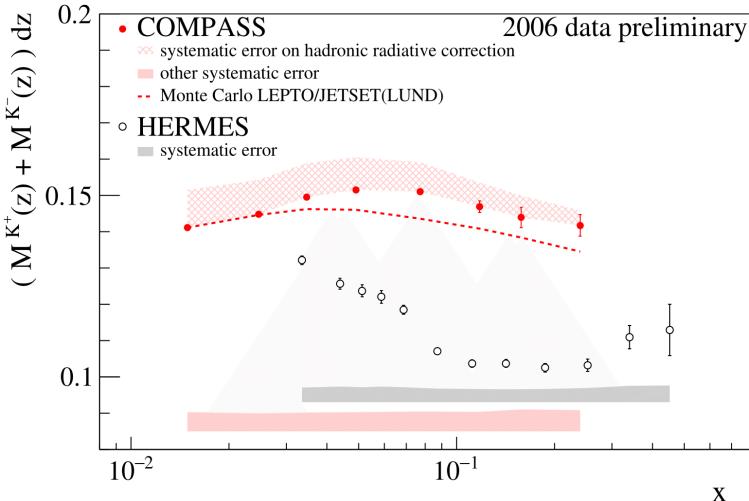


# Strange quark PDF

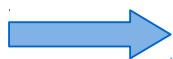
## • HERMES



- Claim very different s-quark shape compared to CTEQ6L
- Measurements from ATLAS/CMS at LHC also show different s-PDF
- Strange pdf may not be what we think!



## • But COMPASS



$$\langle Q^2 \rangle_{\text{COMPASS}} \gtrsim \langle Q^2 \rangle_{\text{HERMES}}$$

- Different  $x_B$  dependence
- Overall value higher

Can HMCs reduce the discrepancy HERMES vs. COMPASS?



# Strange quark PDF

Hadron Multiplicity(Kaon)  Strange quark “Tagging”

Experimental definition:

$$M^k(x_B, z_h, Q^2) = \frac{\frac{dN^K}{dx_B dQ^2 dz_h}}{\frac{dN^{DIS}}{dx_B dQ^2}}$$

Parton model:

$$M^K(x_B, z_h, Q^2) = \frac{\sum_q e_q^2 q(x_B, Q^2) D_q^h(z_h, Q^2)}{\sum_q e_q^2 q(x_B, Q^2)}$$

Integrated Multiplicity:

$$M^k(x_B, Q^2) = \int_{0.2}^{0.8(0.85)} dz_h M^k(x_B, Q^2, z_h)$$

HERMES (COMPASS)



Sensitive to s-quark



# Strange quark PDF

## SIDIS on Deuteron (HERMES and COMPASS )

Parton model:  $M^K(x_B, Q^2) = \frac{Q(x) \int \mathcal{D}_Q^K(z) dz + S(x) \int \mathcal{D}_S^K(z) dz}{5Q(x) + 2S(x)}$

$$Q(x) \equiv u(x) + \bar{u}(x) + d(x) + \bar{d}(x)$$

$$S(x) \equiv s(x) + \bar{s}(x)$$

$$\mathcal{D}_Q^K(z) \equiv 4D_u^K(z) + D_d^K(z)$$

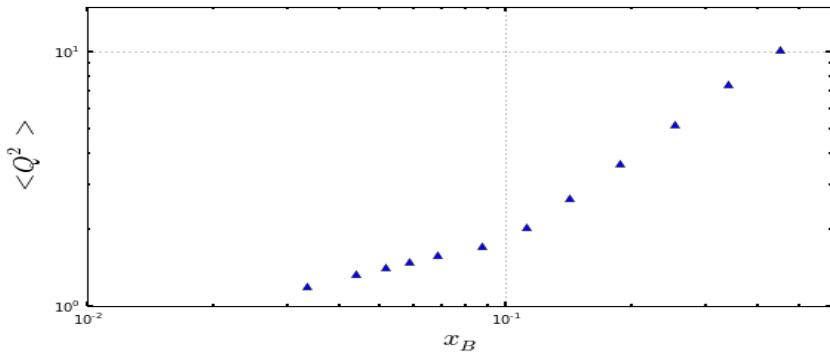
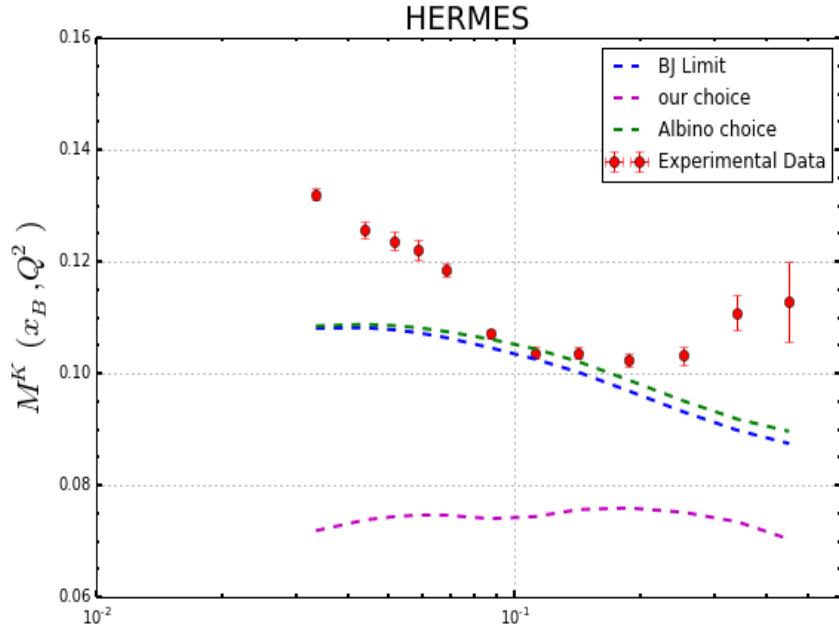
$$\mathcal{D}_s^K(z) \equiv 2D_s^K(z)$$

$$K = K^+ + K^-$$



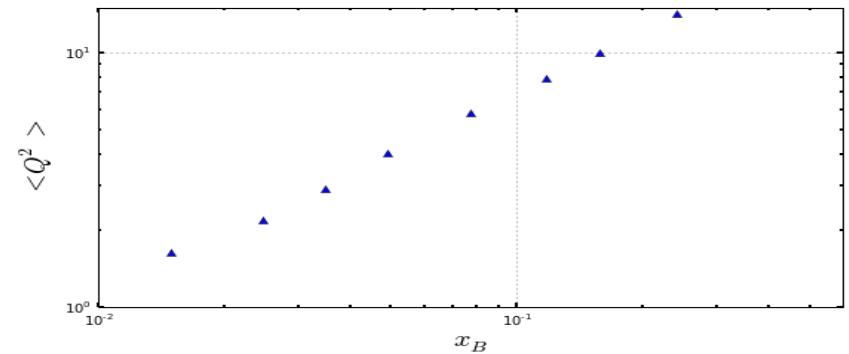
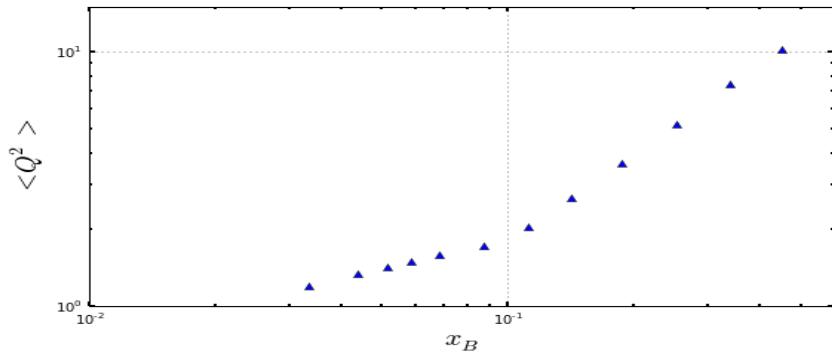
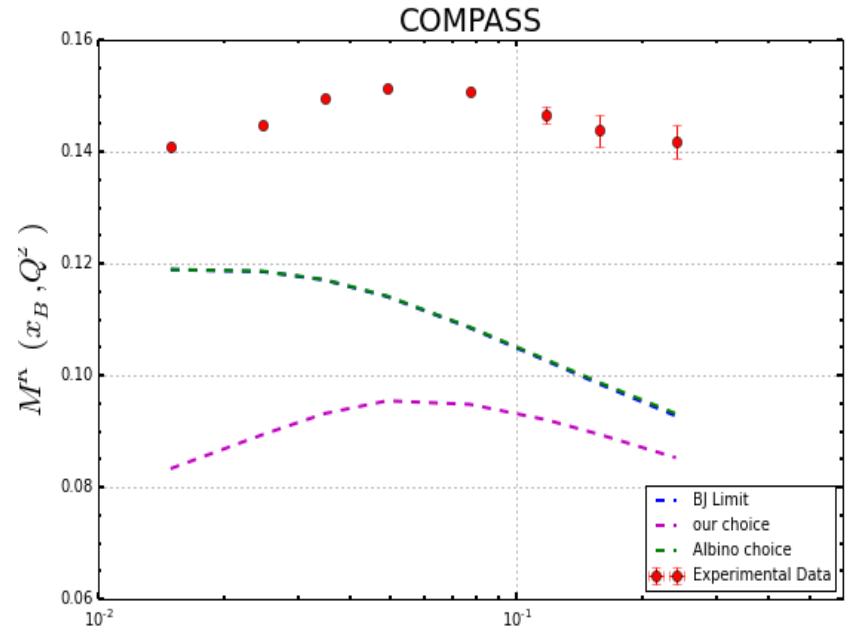
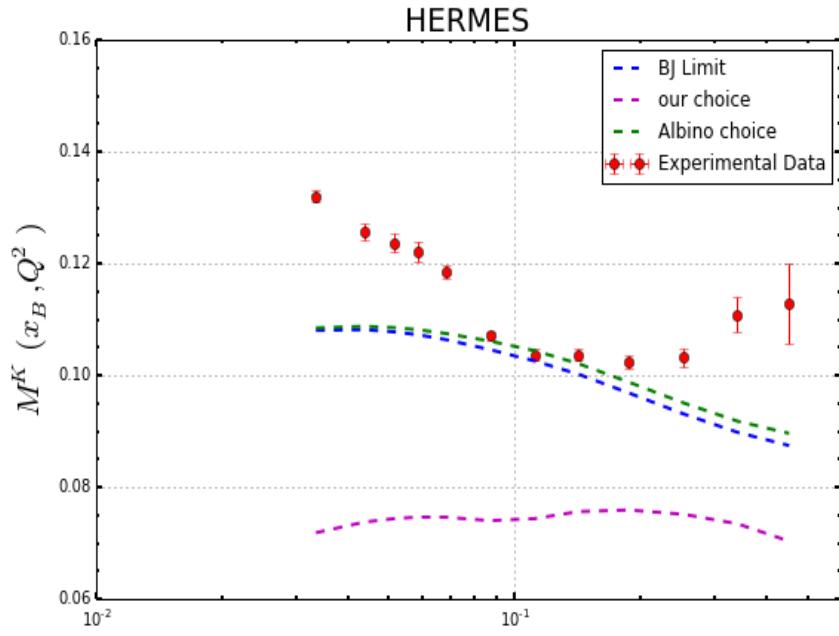
# Strange quark PDF

PDFs: CTEQ6L, FFs: DSSV07

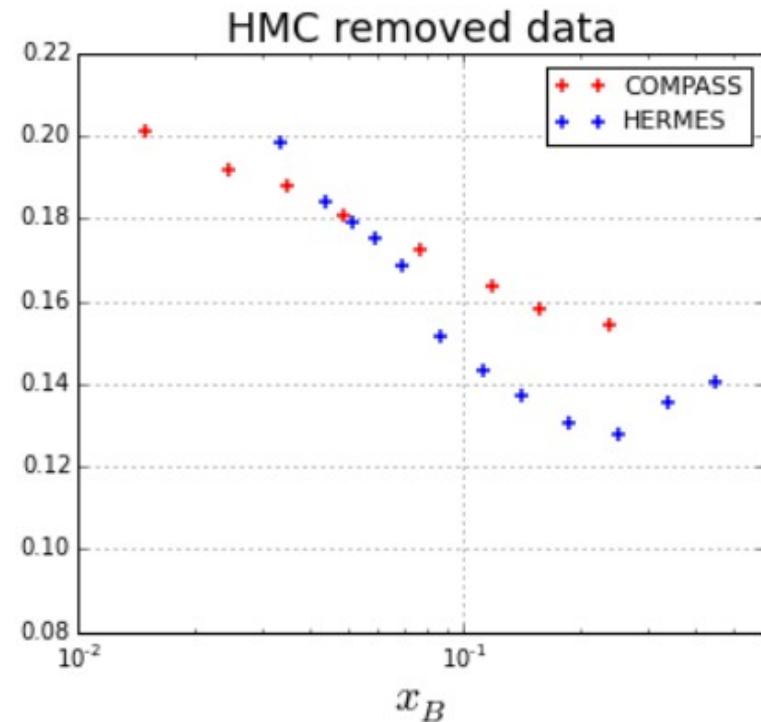
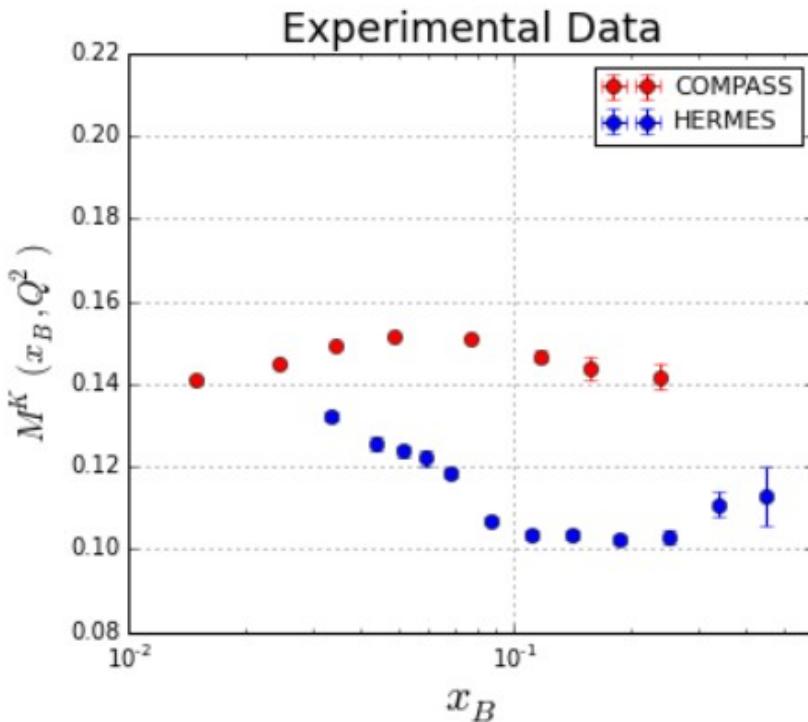


# Strange quark PDF

PDFs: CTEQ6L, FFs: DSSV07



# COMPASS vs. HERMES



Data without HMCs:  $D_{w/oHMCs} = D_{exp} \frac{M^{K^{(0)}}}{M^K} \Big|_{Theory}$



# Conclusion and outlook

- HMC's at LO are captured by new scaling variables  $\xi_h$  and  $\zeta_h$ .
  - Matching “internal” and external kinematics requires a massive fragmenting parton with  $\tilde{k}'^2 \geq m_h^2/\zeta_h$ .
  - $x_B$  and  $z_h$  mixed in the new scaling variables  $\xi_h$  and  $\zeta_h$
- We have quantified HMC's effects numerically: stronger for large as well very small values of  $z_h$ , stronger effects at large  $x_B$  and low  $Q^2$  (as DIS).
- More dramatic effects for Kaons compared to pions.
- Jefferson Lab Pions: corrections up to ~40 %- 50 % even at moderately large  $Q^2$  (small effects for HERMES and COMPASS)
- HMCs reduce the discrepancy between HERMES and COMPASS data for averaged Kaon integrated multiplicities
- Future development: prove factorization at NLO in the presence of massive fragmenting partons with non-zero virtuality, and “correct” choice of  $\tilde{k}'^2$ .

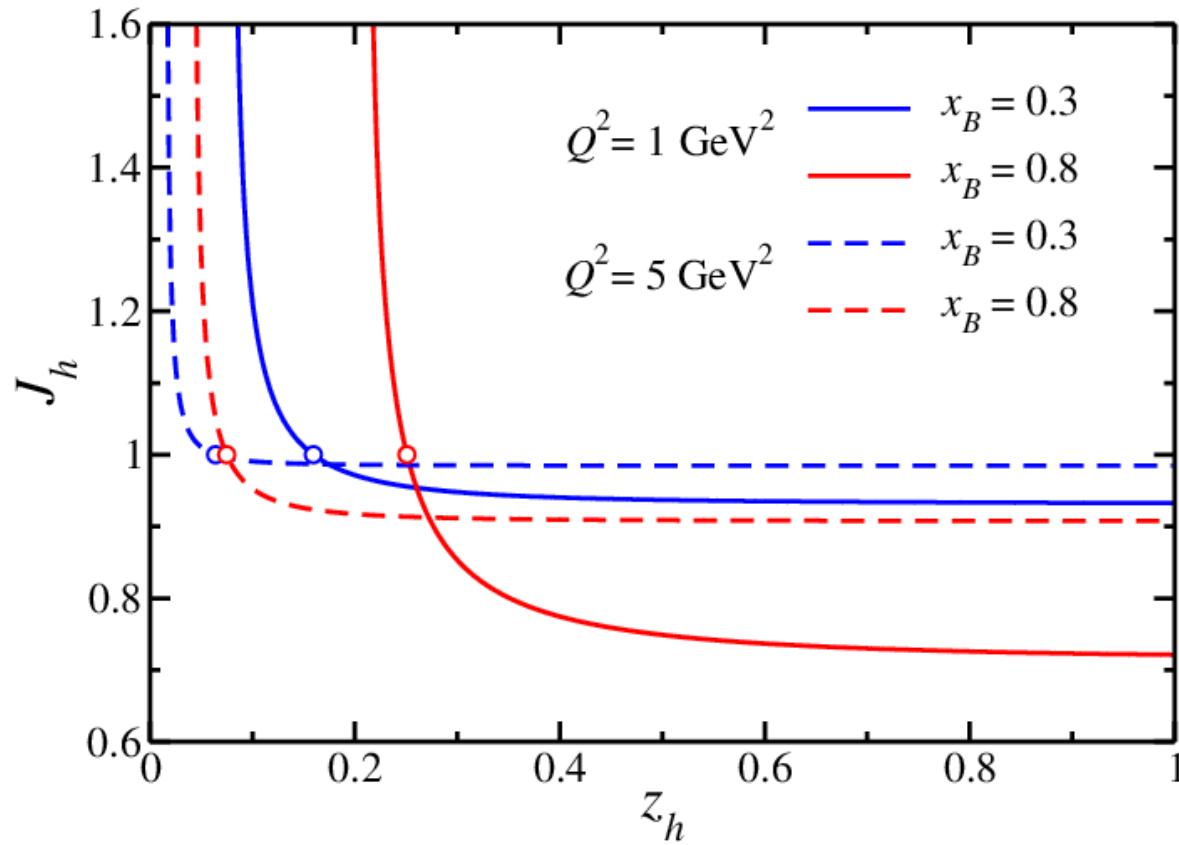


# Thank you!



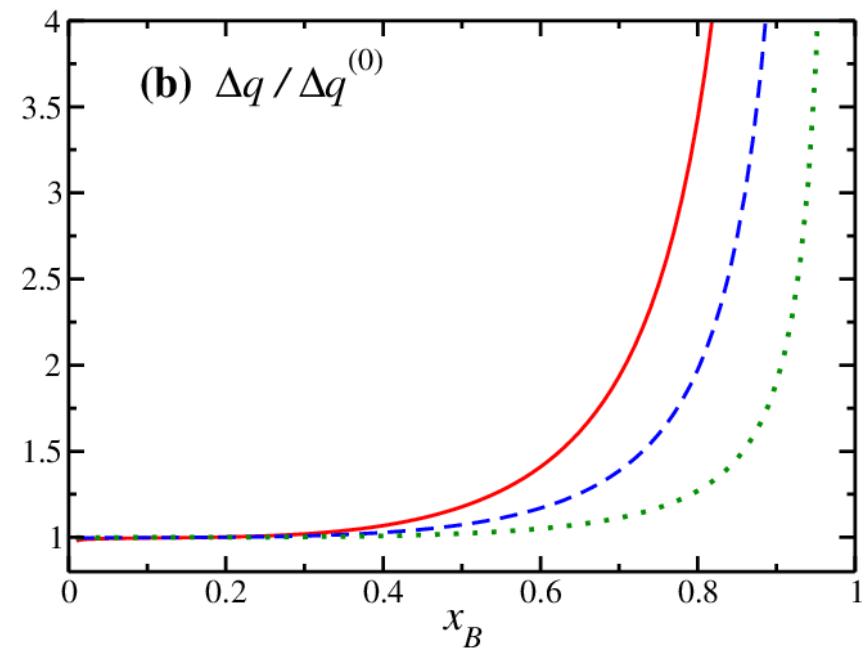
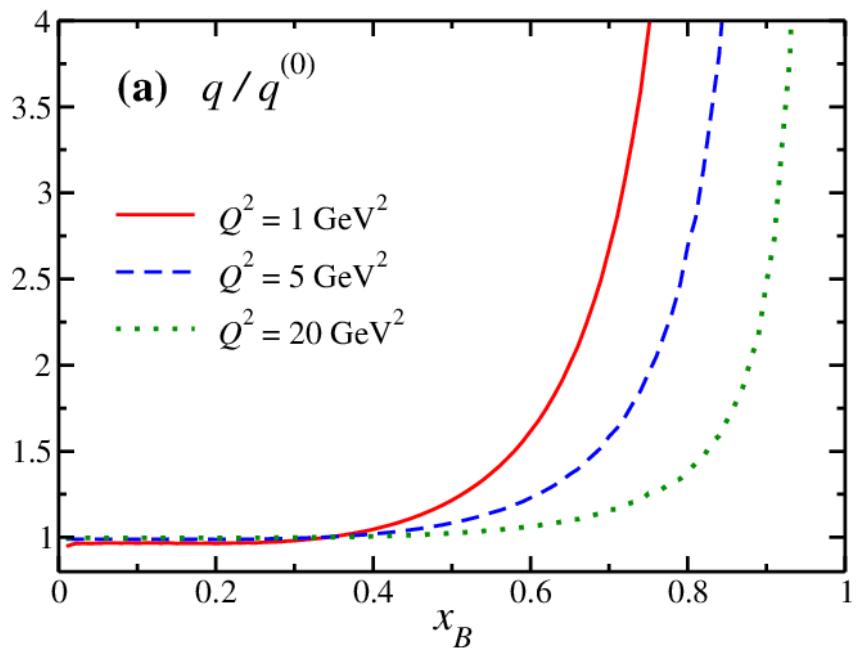
# Backup slides

## Jacobian



# Backup slides

## PDF's



# Backup slides

## Fragmentation function

