Spin - flavor structure of excited baryons

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by

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Jefferson Lab Thomas Jefferson National Accelerator Facility

Motivation

Baryon Summary Table

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the table are not established baryons. The names with masses are of baryons that decay strongly. The spin-parity J^P (when known) is given with each particle. For the strongly decaying particles, the J^P values are considered to be part of the names.

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*	$\Lambda_c(2765)^+$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***	$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$		***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***	$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}^?$	***	$\Sigma_c(2520)$	$3/2^+$	***
$N(1685)$		*	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*	$\Sigma_c(2800)$		***
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**	Ξ_c^+	$1/2^+$	***
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**	Ξ_c^0	$1/2^+$	***
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$		*	$\Xi_c'^+$	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***				$\Xi_c'^0$	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2^+$	****	$\Xi_c(2645)$	$3/2^+$	***
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1775)$	$5/2^-$	****	$\Omega(2250)^-$		***	$\Xi_c(2790)$	$1/2^-$	***
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)^-$		**	$\Xi_c(2815)$	$3/2^-$	***
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)^-$		**	$\Xi_c(2930)$		*
$N(1990)$	$7/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1900)$	$1/2^-$	*				$\Xi_c(2980)$		***
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(1915)$	$5/2^+$	****				$\Xi_c(3055)$		**
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(1940)$	$3/2^+$	*				$\Xi_c(3080)$		***
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(1940)$	$3/2^-$	***				$\Xi_c(3123)$		*
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2000)$	$1/2^-$	*				Ω_c^0	$1/2^+$	***
$N(2120)$	$3/2^-$	**				$\Sigma(2030)$	$7/2^+$	****				$\Omega_c(2770)^0$	$3/2^+$	***
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2070)$	$5/2^+$	*						

Motivation

EDWARDS *et al.* PHYSICAL REVIEW D **84,** 074508 (2011)

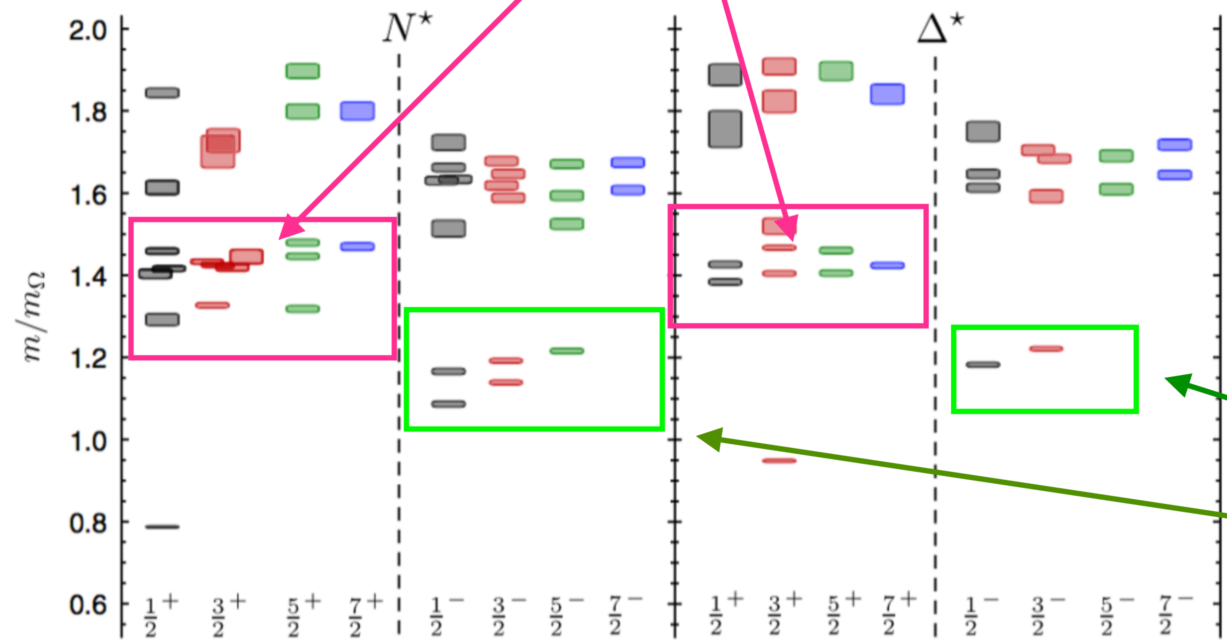

FIG. 11 (color online). Spin-identified spectrum of Nucleons and Deltas from the lattices at $m_\pi = 524$ MeV, in units of the calculated Ω mass.

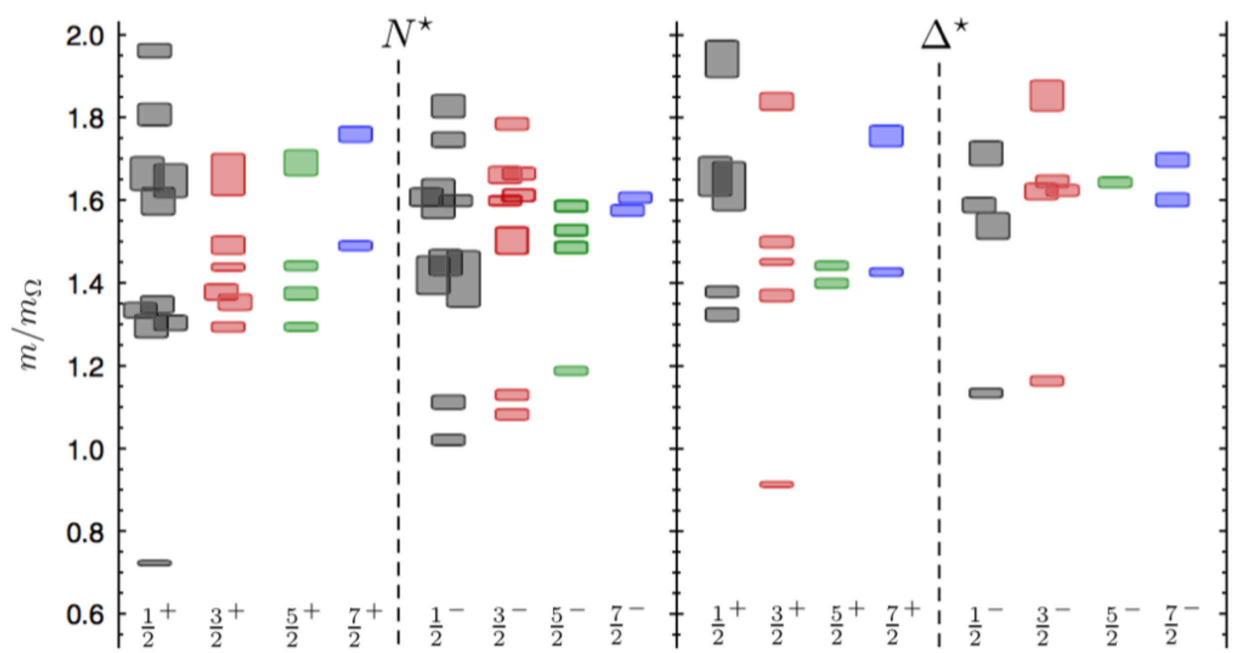

FIG. 12 (color online). Spin-identified spectrum of Nucleons and Deltas from the lattices at $m_\pi = 396$ MeV, in units of the calculated Ω mass.

Motivation

Successful analyses were done on excited baryons with physical data

6 July 2000
Physics Letters B 484 (2000) 260–266
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Predictions for decays of radially excited baryons

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Editor: H. Georgi

Abstract

We consider the strong decays of the lowest-lying radially excited baryons, in SU(6) language the states comprising the first excited **56**-plet. Assuming a single-quark decay approximation, and negligible configuration mixing, we make model-independent predictions for the partial decay widths to final states with a single meson. Masses of unobserved states are predicted using results from large-N_c QCD, and the momentum dependence of the one-body decay amplitude is determined phenomenologically by fitting to observed decays, so that the baryon spatial wave functions are not assumed. We point out that comparison of these predictions to experiment may shed light on whether the Roper resonance can be consistently interpreted as a three-quark state.

PHYSICS LETTERS B
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Physics Letters B 564 (2003) 83–89

Analysis of the [**56**, 2^+] baryon masses in the $1/N_c$ expansion

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Spin-flavor structure of large N_c baryons

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PHYSICS LETTERS B
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PHYSICAL REVIEW D VOLUME 49, NUMBER 9 1 MAY 1994

$1/N_c$ expansion for baryons

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(Received 1 November 1993)

A systematic expansion in $1/N_c$ is constructed for baryons in QCD. Predictions of the $1/N_c$ expansion at leading and subleading order for baryon axial vector current coupling constant ratios such as F/D, baryon masses, and magnetic moments are derived. The baryon sector of QCD has a light quark spin-flavor symmetry at leading order in $1/N_c$. The formalism of induced representations for contracted Lie algebras is introduced to explain the consequences of this symmetry. Relations are first derived for

SU(2 N_f) x O(3)

PHYSICAL REVIEW D 66, 114014 (2002)

Negative parity 70-plet baryon masses in the $1/N_c$ expansion

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The masses of the negative parity $SU(6)$ 70-plet baryons are analyzed in the $1/N_c$ expansion to order $1/N_c$ and to first order in $SU(3)$ breaking. At this level of precision there are 20 predictions. Among them there are the well known Gell-Mann–Okubo and equal spacing relations, and four new relations involving $SU(3)$ breaking splittings in different $SU(3)$ multiplets. Although the breaking of $SU(6)$ symmetry occurs at zeroth order in $1/N_c$, it turns out to be small. The dominant source of the breaking is the hyperfine interaction which is of the order of $1/N_c$. The spin-orbit interaction, of zeroth order in $1/N_c$, is entirely fixed by the splitting between the singlet states $\Lambda(1405)$ and $\Lambda(1520)$, and the spin-orbit puzzle is solved by the presence of other zeroth order operators involving flavor exchange.

DOI: 10.1103/PhysRevD.66.114014 PACS number(s): 14.20.Gk, 12.39.Jh, 12.40.Yx, 14.20.Jn

N_f=3

SU(6) x O(3)

PHYSICAL REVIEW D, VOLUME 59, 114008

Operator analysis of $l=1$ baryon masses in large N_c QCD

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(Received 21 December 1998; published 20 April 1999)

We consider in detail the mass operator analysis for the nonstrange $l=1$ excited baryons in large N_c QCD. We present a straightforward procedure for constructing large N_c baryon wave functions, and provide complete analytic expressions for the matrix elements of all the independent isosinglet mass operators. We discuss the relationship between the old-fashioned operator analyses based on nonrelativistic SU(6) symmetry and the modern large N_c approach, which has a firmer theoretical foundation. We then suggest a possible dynamical interpretation for the subset of operators preferred strongly by the data. [S0556-2821(99)07809-1]

PACS number(s): 14.20.Gk, 12.39.Jh, 12.40.Yx

N_f=2

SU(4) x O(3)

Applications of 1/Nc expansion

Ground State Baryons including the Studies of SU(6) Spin-flavor symmetry

- E. Witten. Nucl. Phys. B160,57(1979)
- R.Dashen and A.V. Manohar phy Lett D 315,425(1993)
- E. Jenkins Phy Lett B315,431(1993)
- R.Dashen E. Jenkins and A.V. Manohar PRD 49,4713(1994)

Baryon Masses (Ground State and Excited State)

- E. Jenkins Phy Lett B 315,441(1993)
- R.Dashen E. Jenkins and A.V. Manohar PRD 51,3697(1995)
- E. Jenkins, R.F. Lebed PRD 52,282(1995)
- C.E. Carlson, C.D. Carone , J.L. Goity, R.F. Lebed PRD 59,114008(1999)
- J.L. Goity and C.L. Schat PRD 66,114014(2002)

Applications and Developments of 1/Nc Expansion

Magnetic Moments

- R.Dashen E. Jenkins and A.V. Manohar PRD 49,4713(1994)
- E. Jenkins and A.V. Manohar Phys Lett B 335,452(1994)
- R.Dashen E. Jenkins and A.V. Manohar PRD 51,3697(1995)
- E. Jenkins, R.F. Lebed PRD 52,282(1995)

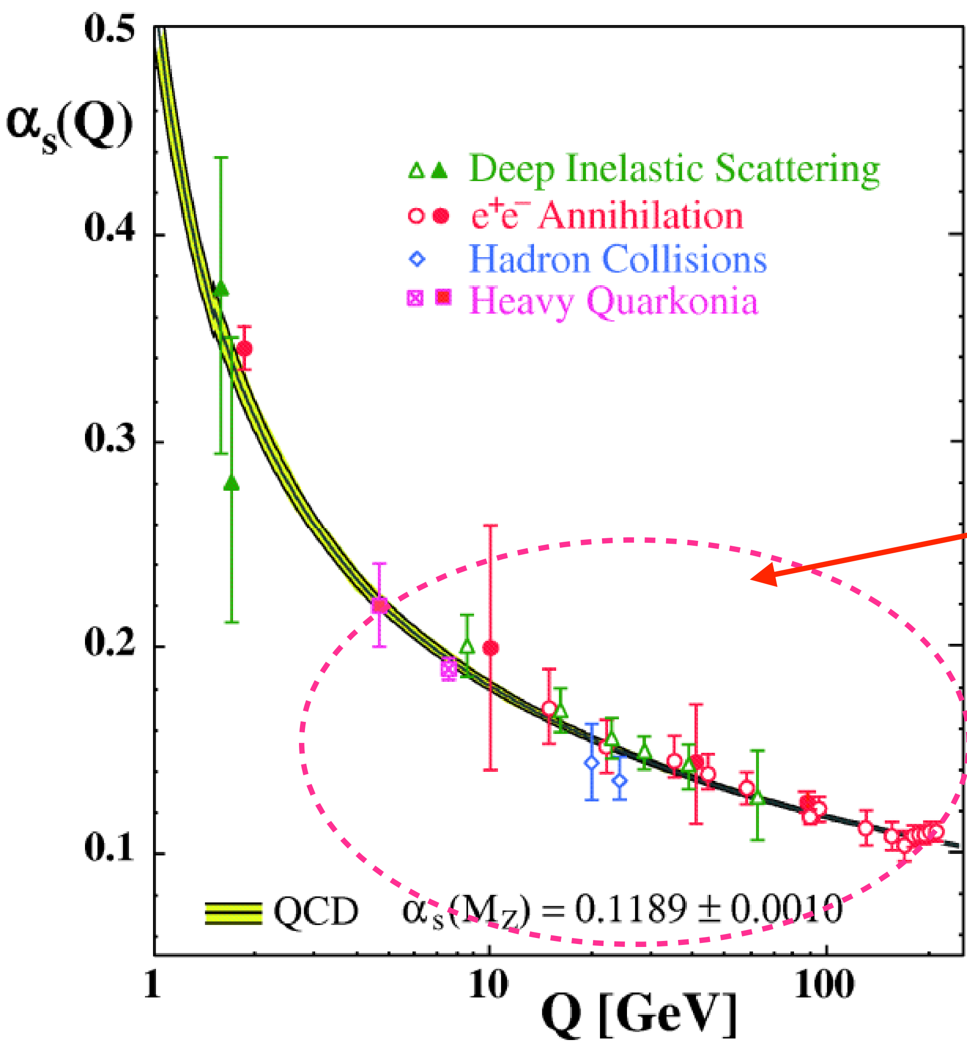
Axial Vector Current Matrix Elements

- R.Dashen and A.V. Manohar phy Lett D 315,425(1993)
- R.Dashen E. Jenkins and A.V. Manohar PRD 49,4713(1994)
- R.Dashen E. Jenkins and A.V. Manohar PRD 51,3697(1995)
- J.Dai, R. Dashen, E. Jenkins, A.V. Manohar PRD 53,273(1996)

A brief Introduction to 1/Nc expansion

QCD is the theory of strong interaction

It is the SU(3) gauge theory of quarks and gluons



Perturbative expansion in QCD gauge coupling is only possible at high energies

$1/N_c$ **expansion framework is perturbative in both low and high energy regimes**

't Hooft suggested the generalization of QCD from SU(3) $\longrightarrow$ SU(N_c)

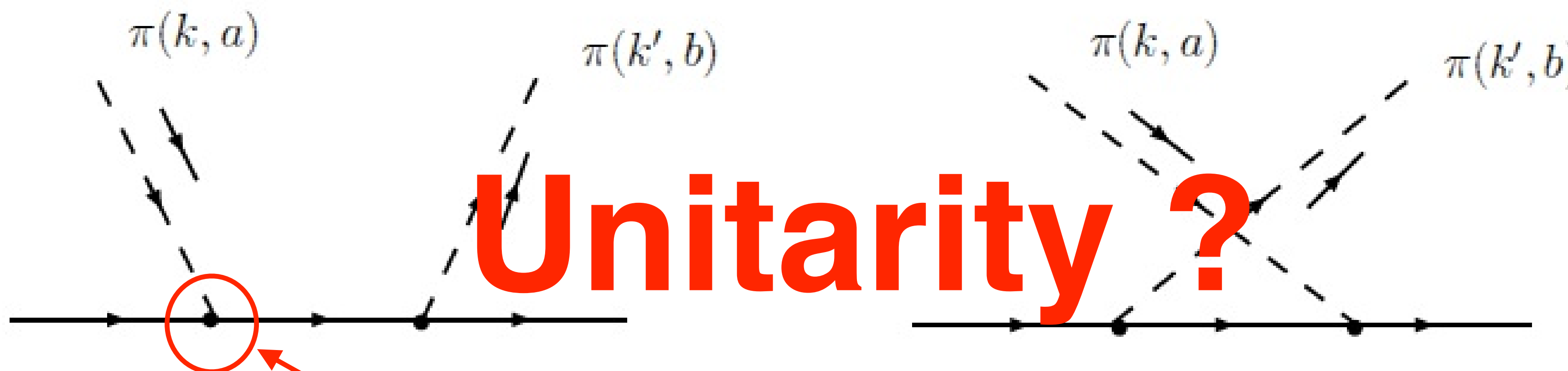
$1/N_c$ is a hidden expansion parameter in QCD

N_c quarks

$N_c^2 - 1$ gluons

A brief Introduction to 1/Nc expansion (Cont.)

Baryon spin-flavor symmetry



$\mathcal{O}(\sqrt{N_c})$

$$\frac{\partial_\mu \pi^a}{f_\pi}\langle B'|\bar{q}\gamma^\mu\gamma_5 T^a q|B\rangle \equiv \frac{\partial_i \pi^a}{f_\pi} g_A N_c \langle B'|X^{ia}|B\rangle \Rightarrow \mathcal{O}(\sqrt{N_c})$$

Since, πN amplitude is $\mathcal{O}(N_c^0)$

$$A = -i\frac{k^i k^j}{k_0}\frac{{N_c}^2 g^2}{f_\pi^2}[X^{ia}, X^{ib}] \Rightarrow [X^{ia}, X^{ib}] \leqslant \mathcal{O}(1/N_c)$$

Large N_c consistency condition

$$[X_0^{ia}, X_0^{ib}] = 0$$

$$X_0^{ia} = \lim_{N_c \to \infty} \frac{G^{ia}}{N_c}$$

At large N_c, QCD has contracted spin-flavor symmetry $SU_c(2N_f)$ in baryon sector

Excited baryon states

$$\text{State} = |S_{core}, I_{core}, m_1, \alpha_1 > \otimes |s, i, m_2, \alpha_2 > \otimes |l, m_l >$$

- In the spin-flavor $\mathbf{SU(6)}$ representation
  $\mathbf{6 \otimes 6 \otimes 6 = 56_S \oplus 70_{MS} \oplus 70_{MA} \oplus 20_A}$
  $Parity = (-1)^l$ Where, $l = 0, 1, 2$

- The multiplets $[\mathbf{56}, 0^+]$,$[\mathbf{56}, 2^+]$,$[\mathbf{70}, 1^-]$ are considered in this analysis.

- $[\mathbf{56}, 0^+] \Longrightarrow \mathbf{10}(\frac{3}{2}) \oplus \mathbf{8}\ (\frac{1}{2})$
- $[\mathbf{56}, 2^+] \Longrightarrow \mathbf{10}\ (\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}) \oplus \mathbf{8}(\frac{3}{2}, \frac{5}{2})$
- $[\mathbf{70}, 1^-] \Longrightarrow \mathbf{10}\ (\frac{1}{2}, \frac{3}{2}) \oplus \mathbf{8}\ (\frac{1}{2}, \frac{3}{2}, \frac{5}{2}) \oplus \mathbf{8}\ (\frac{1}{2}, \frac{3}{2}) \oplus \mathbf{1}\ (\frac{1}{2}, \frac{3}{2})$

Octet-Octet Mixing

$$\begin{pmatrix} \mathbf{8}_S \\ \mathbf{8'}_S \end{pmatrix} = \begin{pmatrix} Cos\theta_S & Sin\theta_S \\ -Sin\theta_S & Cos\theta_S \end{pmatrix} \begin{pmatrix} {}^2\mathbf{8}_S \\ {}^4\mathbf{8}_S \end{pmatrix}$$

Octet-Decuplet and Octet-Singlet Mixings

$$\begin{pmatrix} \mathbf{10}_S/\mathbf{1}_S \\ \mathbf{8}_S \\ \mathbf{8'}_S \end{pmatrix} = \Theta \begin{pmatrix} \mathbf{10}_S/\mathbf{1}_S \\ {}^2\mathbf{8}_S \\ {}^4\mathbf{8}_S \end{pmatrix}$$

Mass Operators

- Mass operators are effective operators which consists of products of $SU(6)$ generators S^i, T^a, G^{ia} and also the $O(3)$ generators l^i

- Negative Parity $(l=1)$ Non-Strange $20-plet$ $SU(4)\otimes O(3)$,

$$M_{20} = \sum_{i=1}^{8} c_i O_i$$

C.E.Carlson,C.D.Carone,J.L.Goity,R.F.Lebed,Phys.Rev.D 59,114008

- Negative Parity $(l=1)$ Strange $70-plet$ $SU(6)\otimes O(3)$,

$$M_{70} = \sum_{i=1}^{11} c_i O_i + \sum_{i=1}^{4} d_i \bar{B}_i$$

J.L.Goity and C.L.Schat,Phys.Rev.D 66,114014

- Positive Parity $(l=2)$ Strange $56-plet$ $SU(6)\otimes O(3)$,

$$M_{56,2+} = \sum_{i=1}^{3} c_i O_i + \sum_{i=1}^{3} b_i \bar{B}_i$$

J.L.Goity,C.L.Schat,N.N.Scoccola Phys.Lett.B 564(2003),83-89

- Positive Parity $(l=0)$ Roper $56-plet$ $SU(6)\otimes O(3)$, Gürsey Radicati formula :

$$\begin{aligned} M_{56,0+} &= c_1 N_c + \frac{c_2}{N_c} S(S+1) + b_1 N_s \\ &+ \frac{b_2}{2\sqrt{12}\,N_c}\Big(3I(I+1) - S(S+1) - \frac{3}{4}N_s(N_s+2)\Big) + \mathcal{O}(1/N_c^2) \end{aligned}$$

Considered up to $\mathcal{O}(\epsilon^0/N_c)$ for the singlets and $\mathcal{O}(\epsilon)$ for the octets where

$\epsilon \equiv m_s - \hat{m}$

$\hat{m} = (m_u + m_d)/2$

Mass Relations

With physical data : Is incomplete due to the lack of the number of identified states with three or more stars

- Gell Mann Okubo (GMO) and Equal Spacing (EQS) relations

$$2(N + \Xi) - (3\Lambda + \Sigma) = 0$$

$$\Sigma - \Delta = \Xi - \Sigma = \Omega - \Xi$$

- Parameter Independent Mass Relations involve baryons of different spins and flavor.

Examples:

$[56, 2^+] multiplet$

$$\Delta_{5/2} - \Delta_{3/2} = N_{5/2} - N_{3/2}$$
$$5(\Delta_{7/2} - \Delta_{5/2}) = 7(N_{5/2} - N_{3/2})$$

With lattice QCD data : We can check all the mass relations

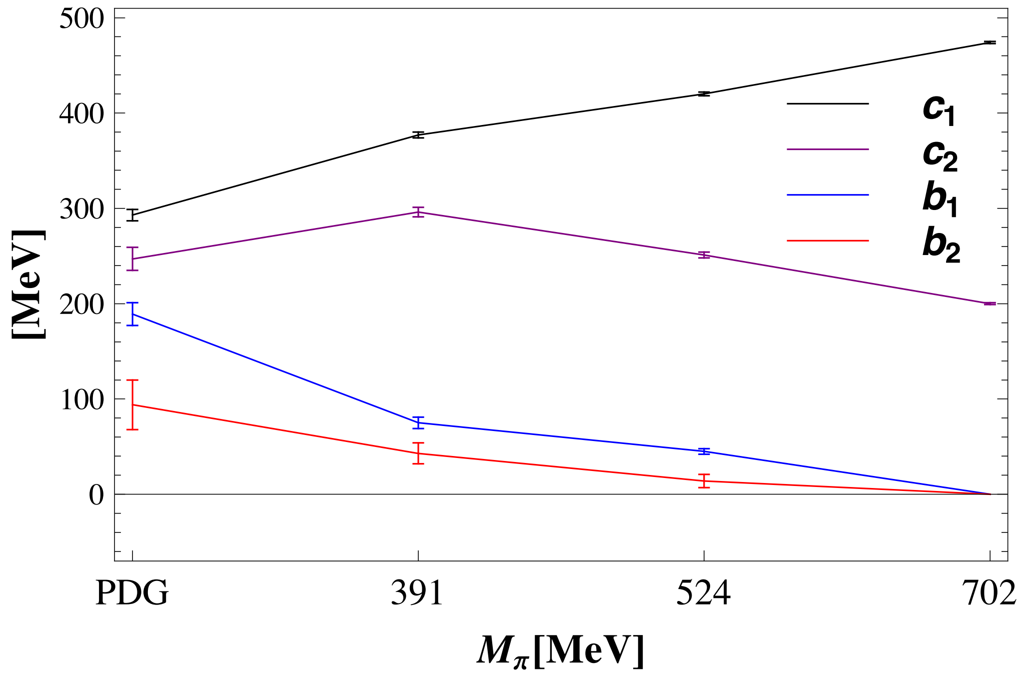
We did it!

$[70, 1^-] multiplet$

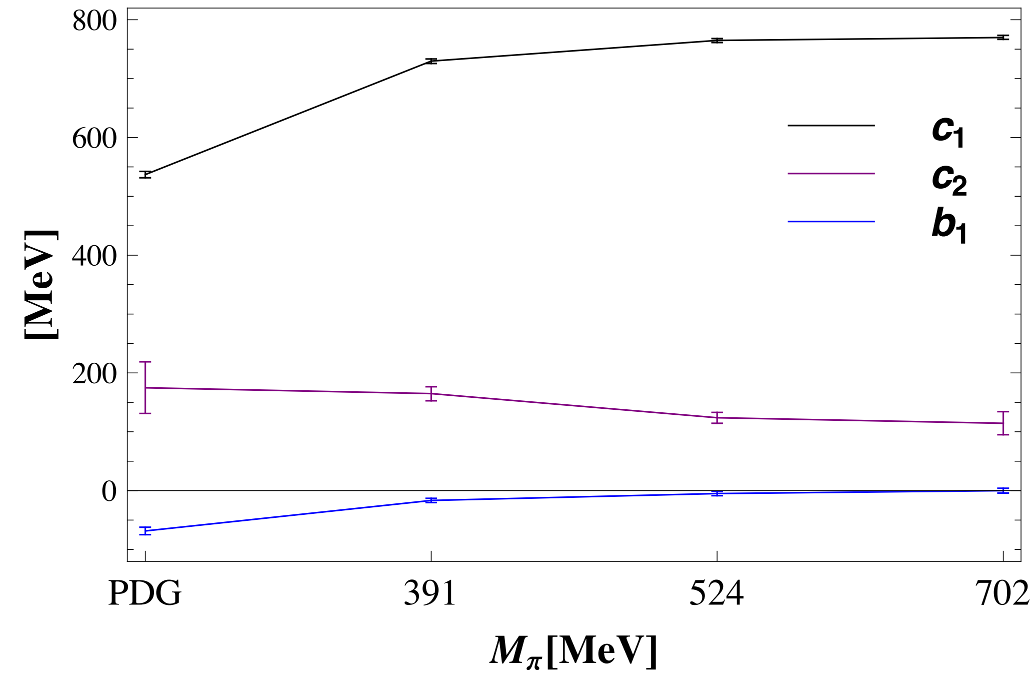
$$14(S_{\Sigma_{3/2}} + S_{\Sigma'_{3/2}}) + 21 S_{\Lambda_{5/2}} - 9 S_{\Sigma_{5/2}} = 18(S_{\Lambda_{1/2}} + S_{\Lambda'_{1/2}}) + 2(S_{\Sigma_{1/2}} + S_{\Sigma'_{1/2}})$$
$$14 S_{\Sigma''_{1/2}} + 49 S_{\Lambda_{5/2}} + 23(S_{\Sigma_{1/2}} + S_{\Sigma'_{1/2}}) = 45(S_{\Lambda_{1/2}} + S_{\Lambda'_{1/2}}) + 19 S_{\Sigma_{5/2}}$$

Fit Results

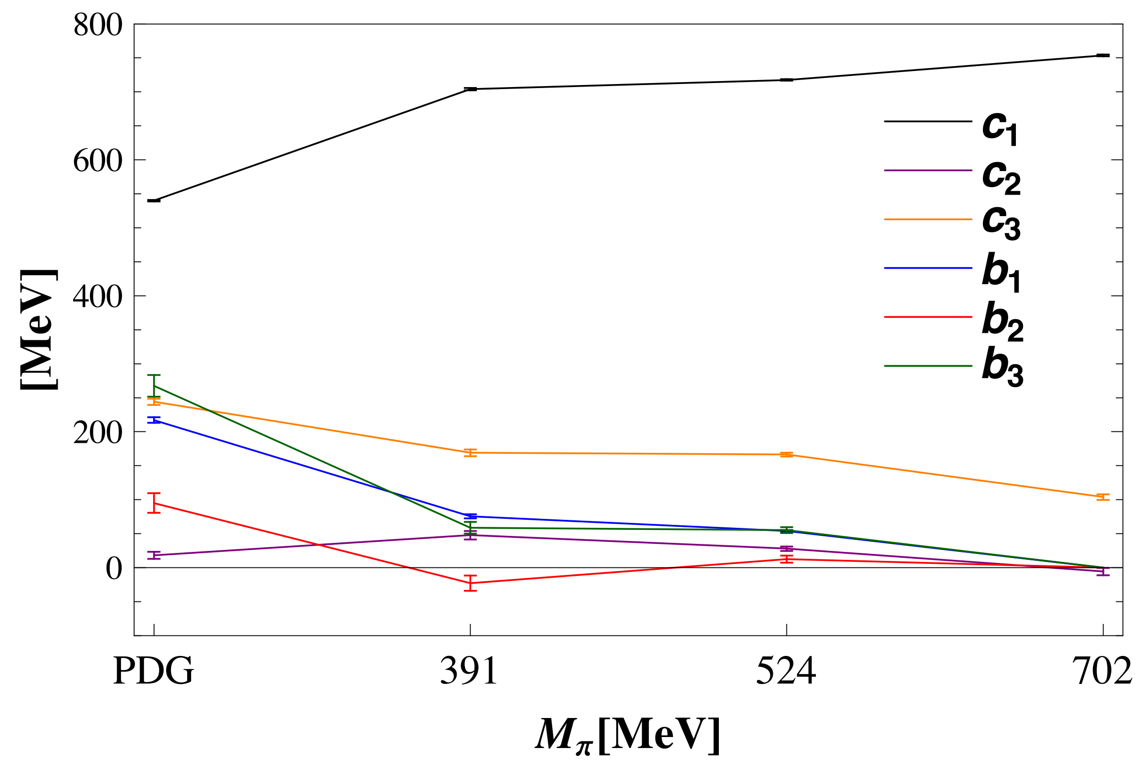
$$M_{56,0^+} = c_1 N_c + \frac{c_2}{N_c} S(S+1) + b_1 N_s + \frac{b_2}{2\sqrt{12}\,N_c}\left(3I(I+1) - S(S+1) - \frac{3}{4}N_s(N_s+2)\right) + \mathcal{O}(1/N_c^2)$$



Ground state operator evolution



$l = 0$, 56-plet minimum operator evolution

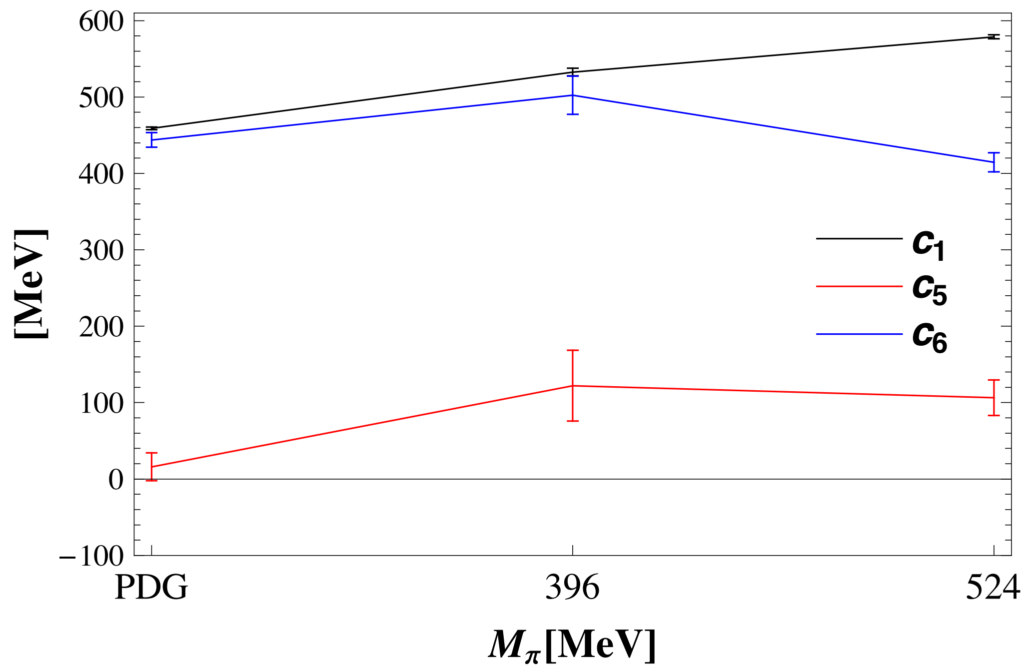


$l = 2$, 56-plet six operator evolution

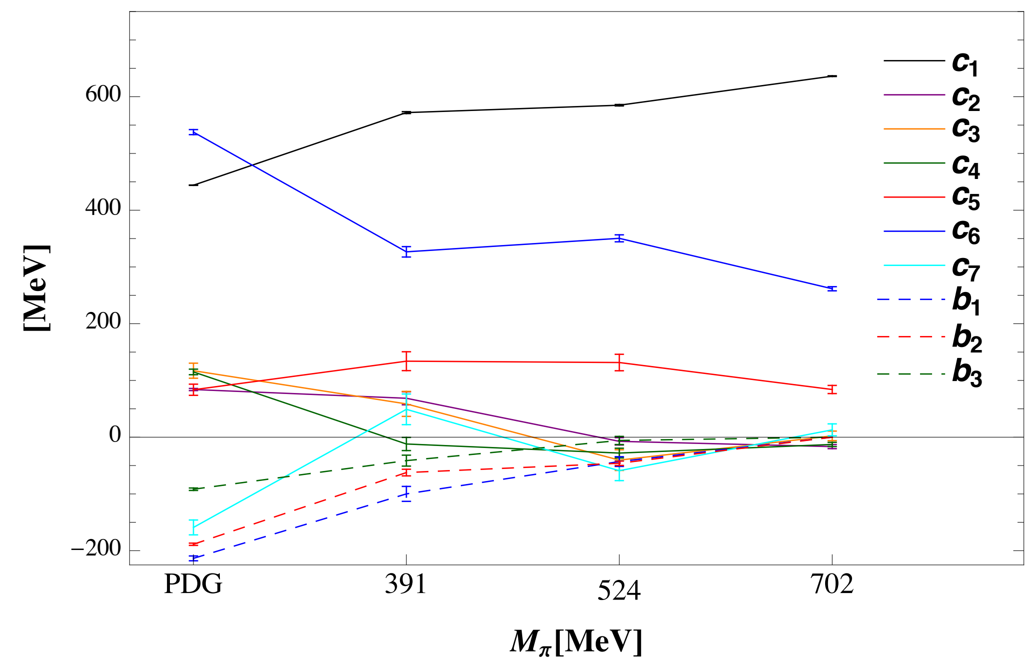
$$M_{56,2^+} = \sum_{i=1}^{3} c_i O_i + \sum_{i=1}^{3} b_i \bar{B}_i$$

Fit Results (Continued…)

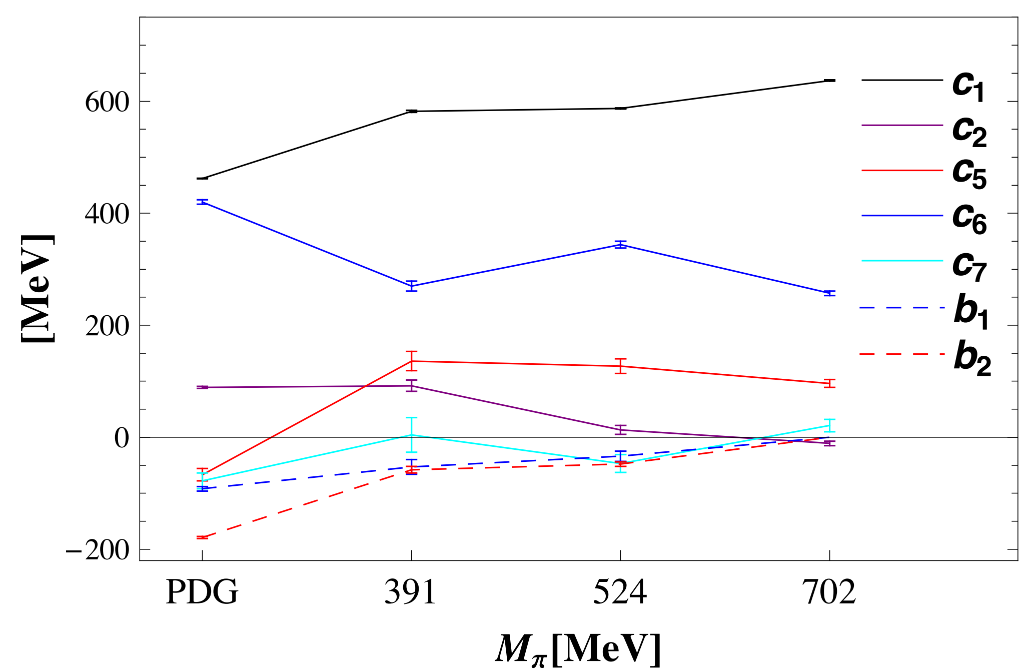
$$M_{20} = \sum_{i=1}^{8} c_i O_i$$



$l = 1$, 20-plet minimum operator evolution



$l = 1$, 70-plet union operator evolution

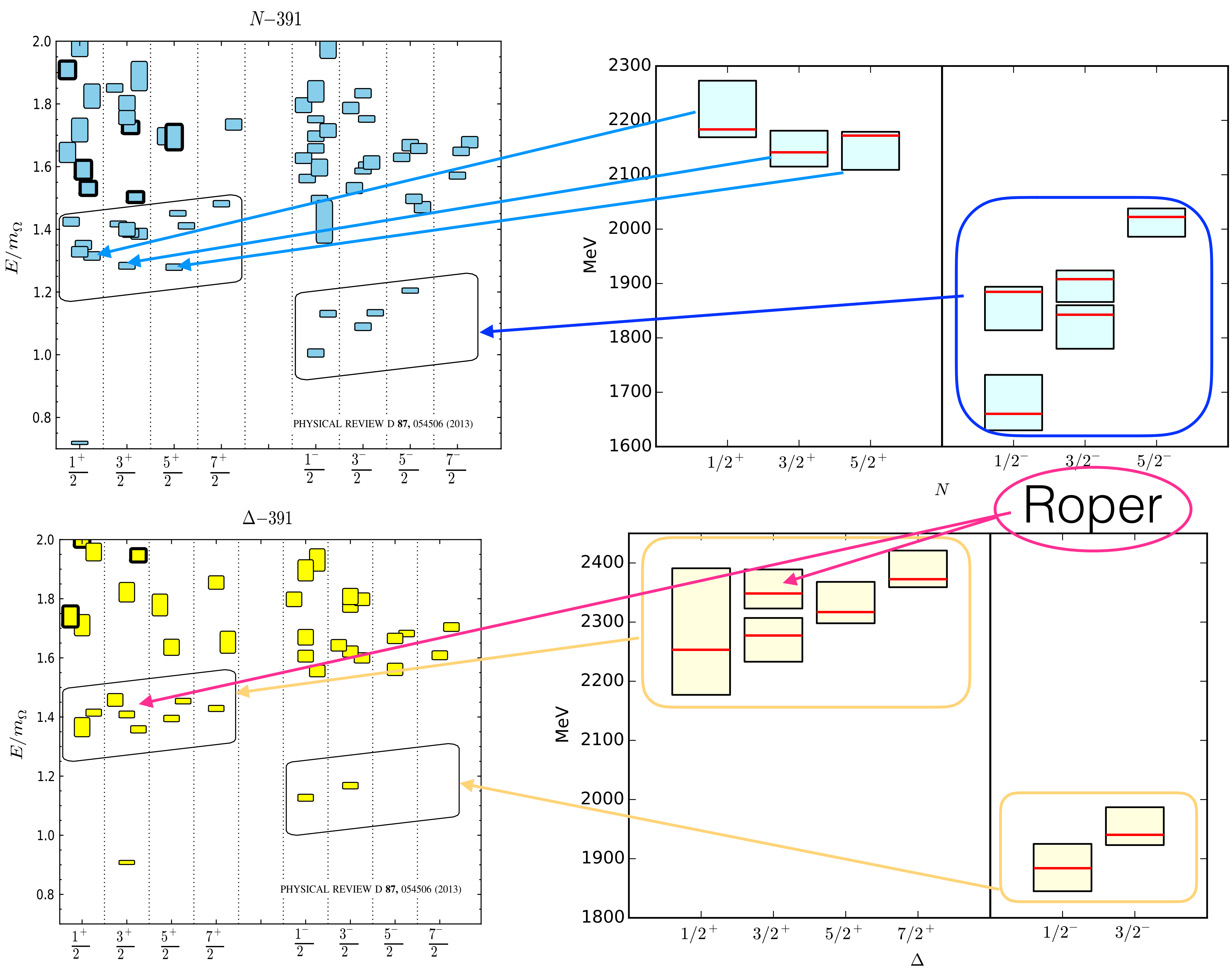


$l = 1$, 70-plet minimum operator evolution

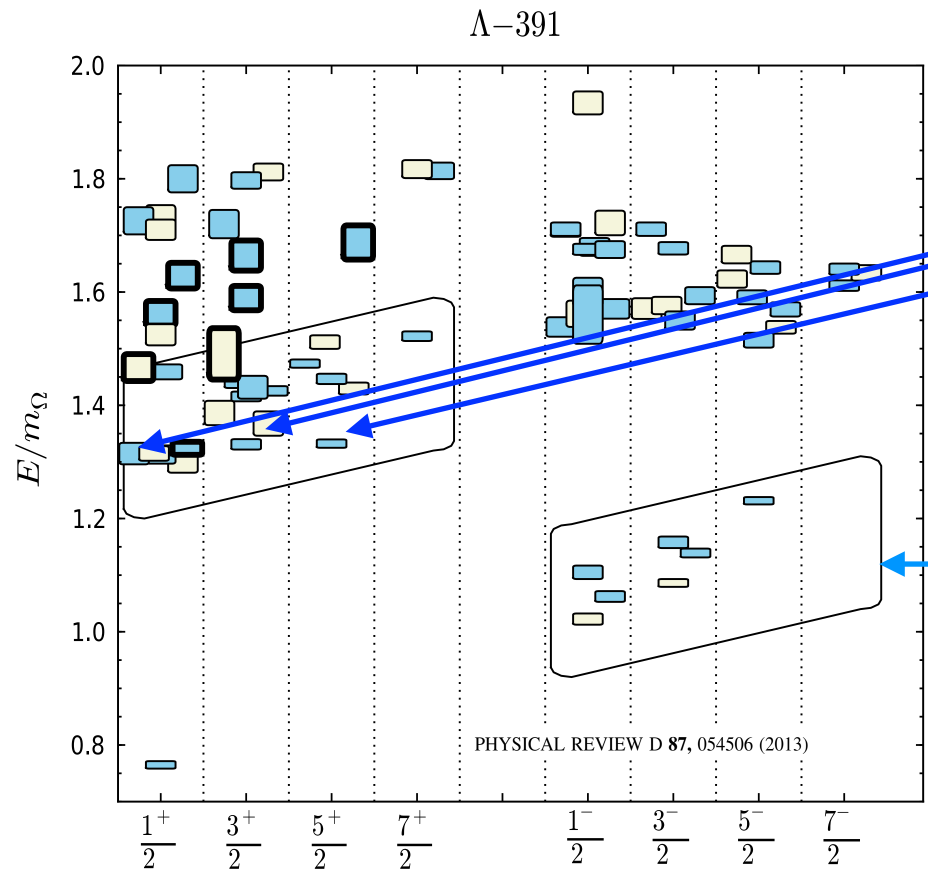
$$M_{70} = \sum_{i=1}^{11} c_i O_i + \sum_{i=1}^{4} d_i \bar{B}_i$$

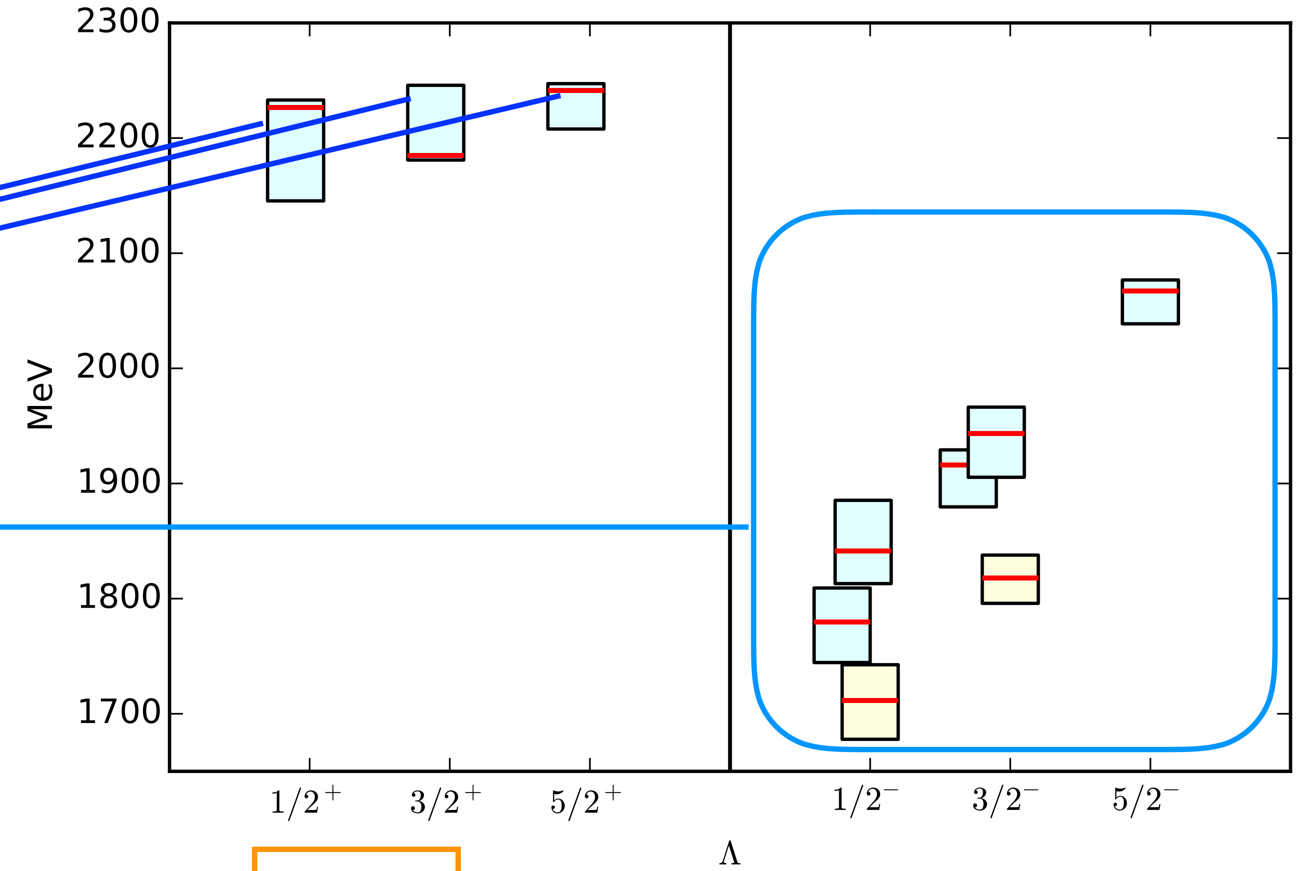
PHYSICAL REVIEW D **91,** 036005 (2015)

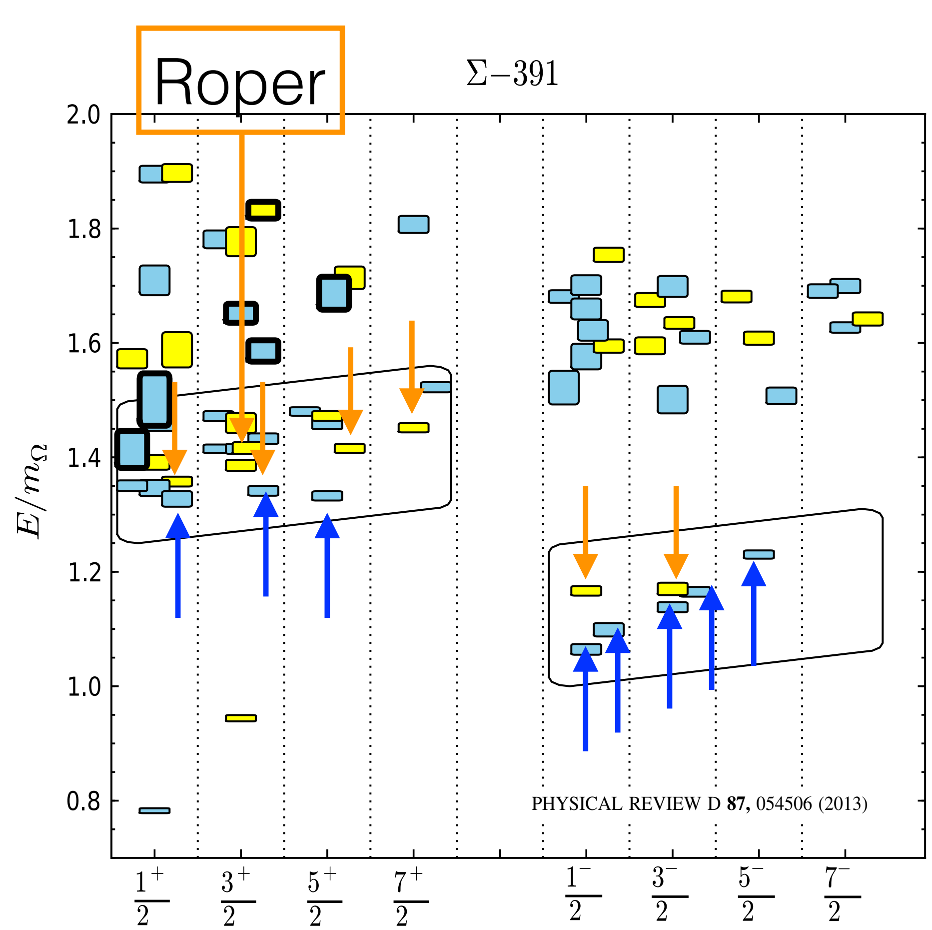
Fit Results (Fitted masses for LQCD)

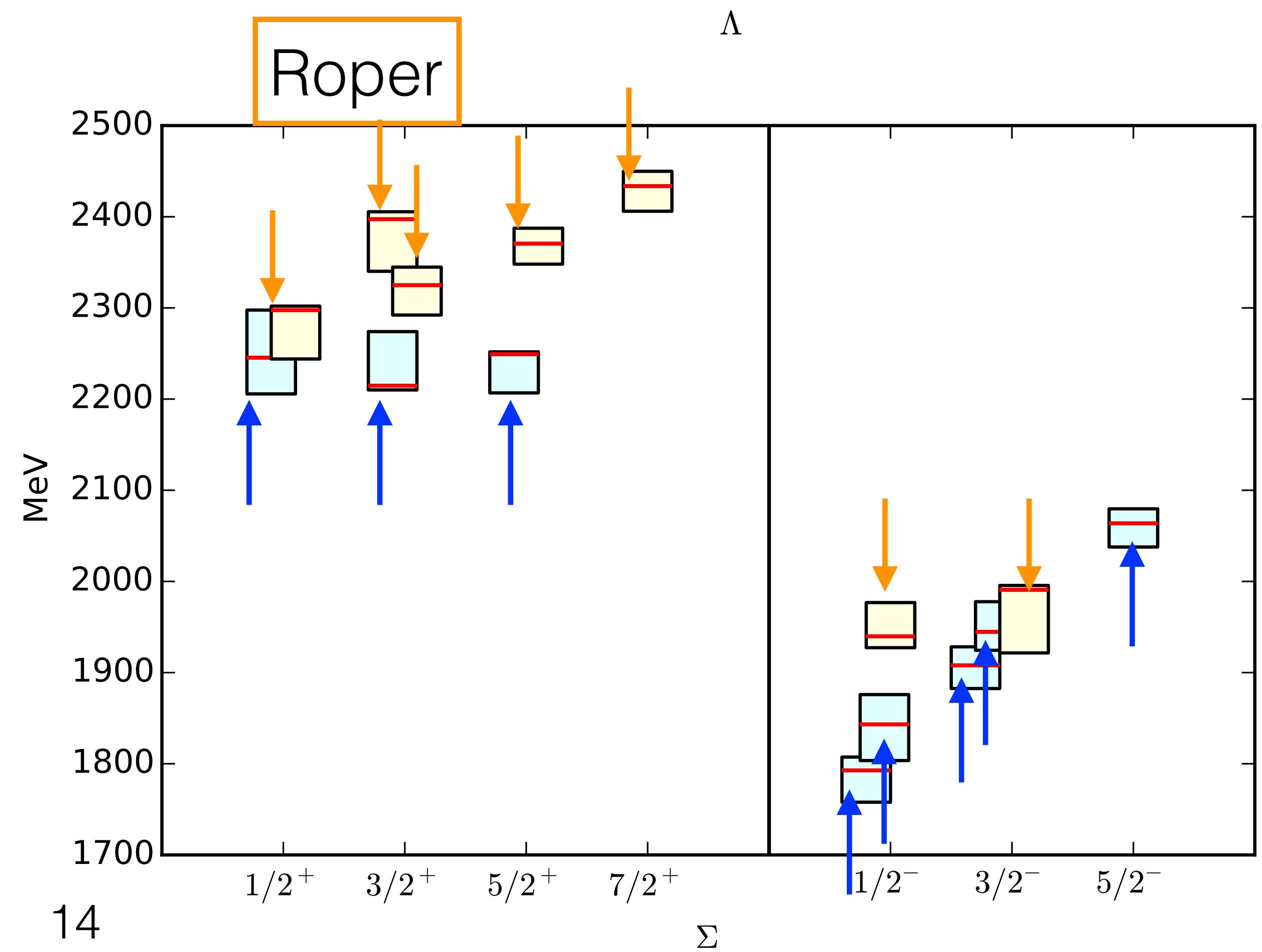


Fit Results (Fitted masses for LQCD)

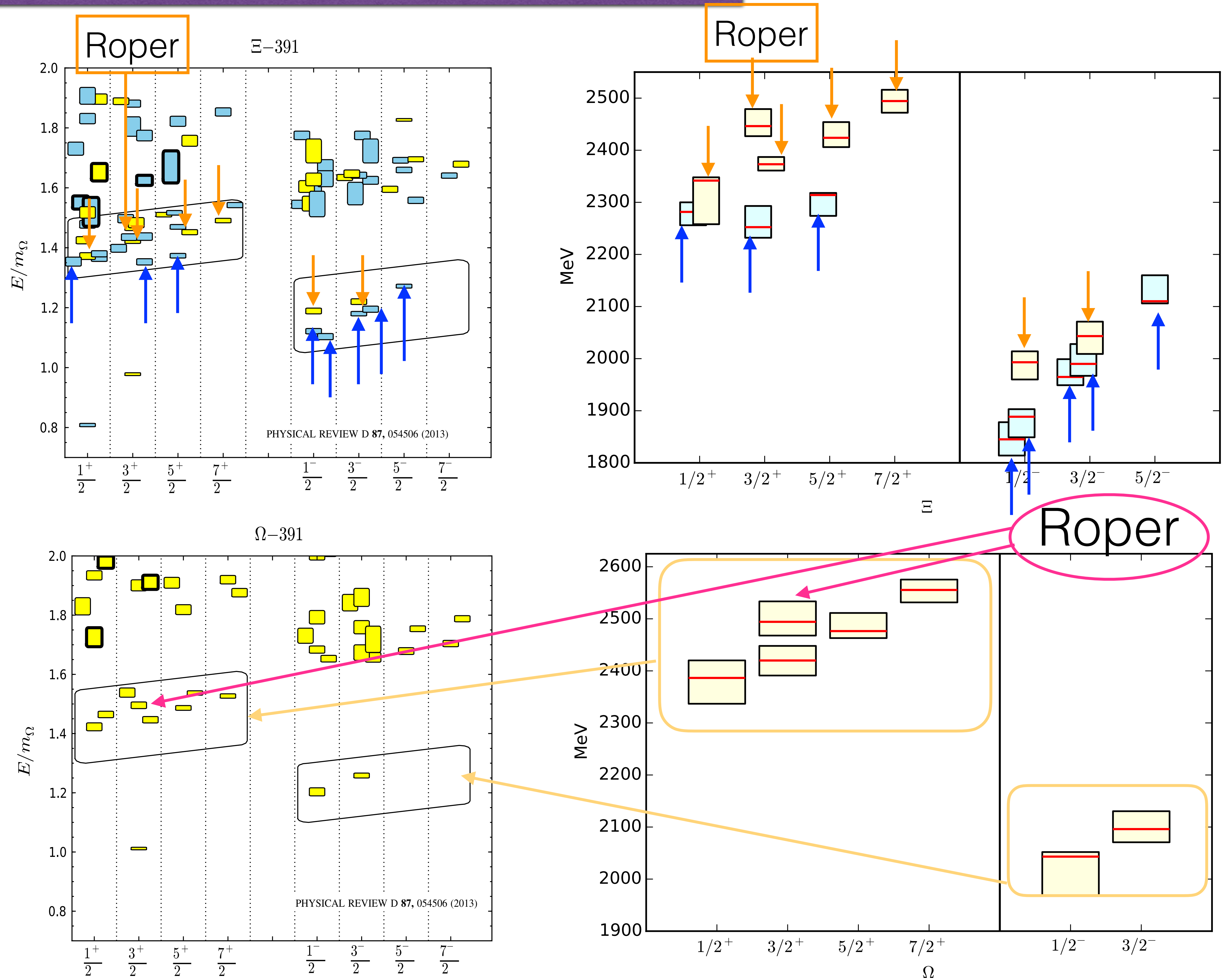




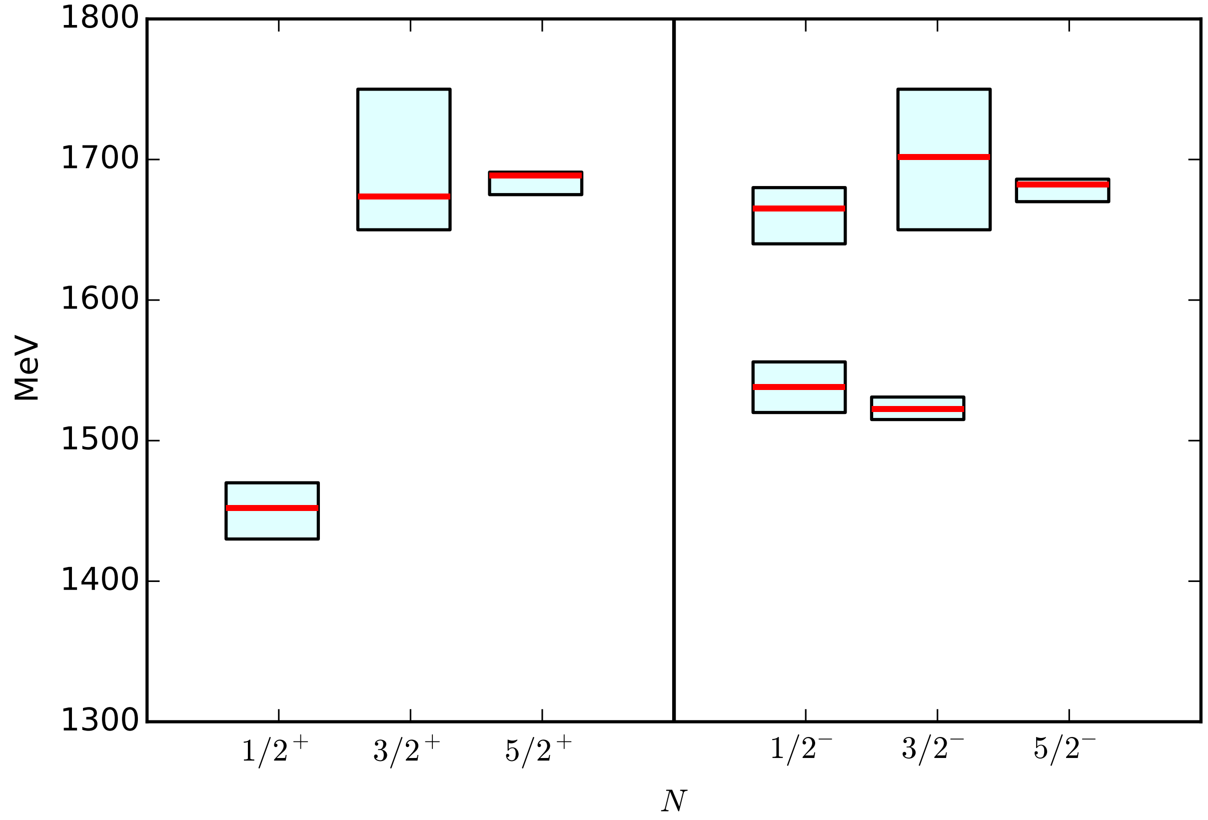


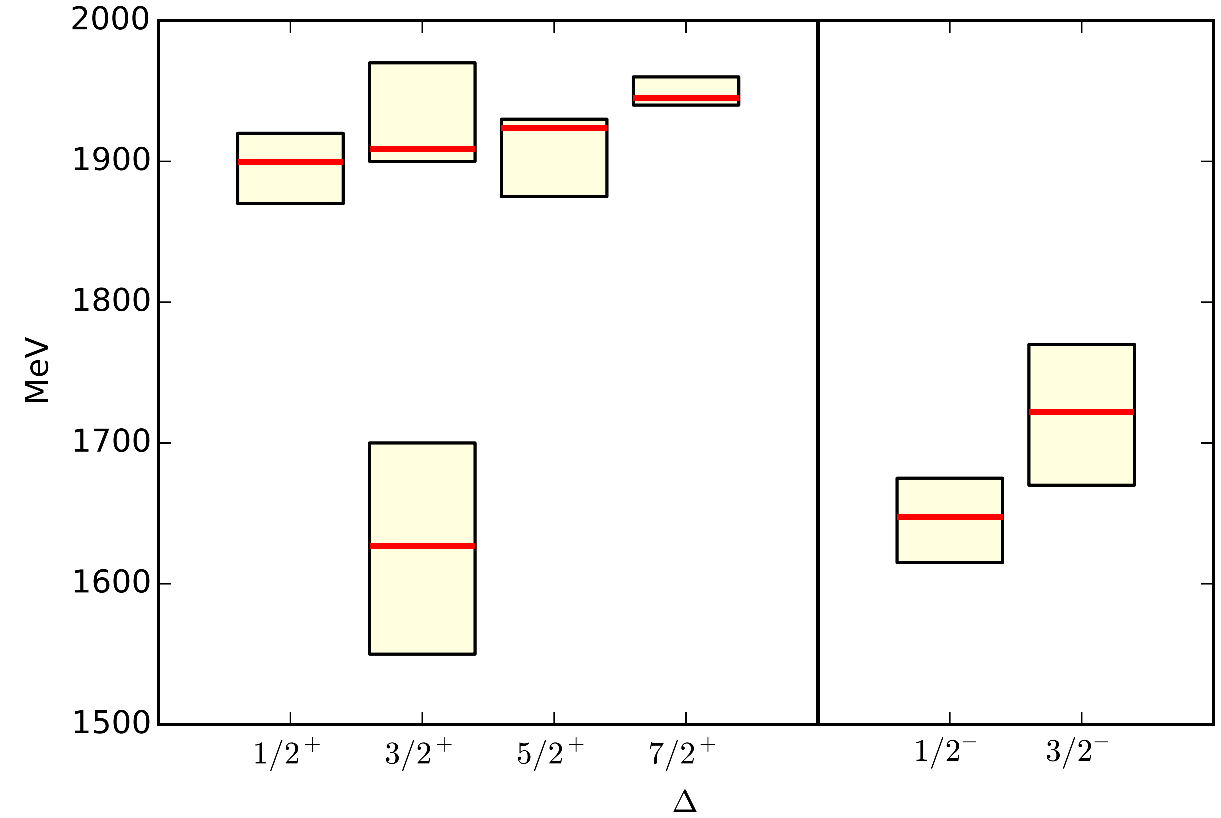


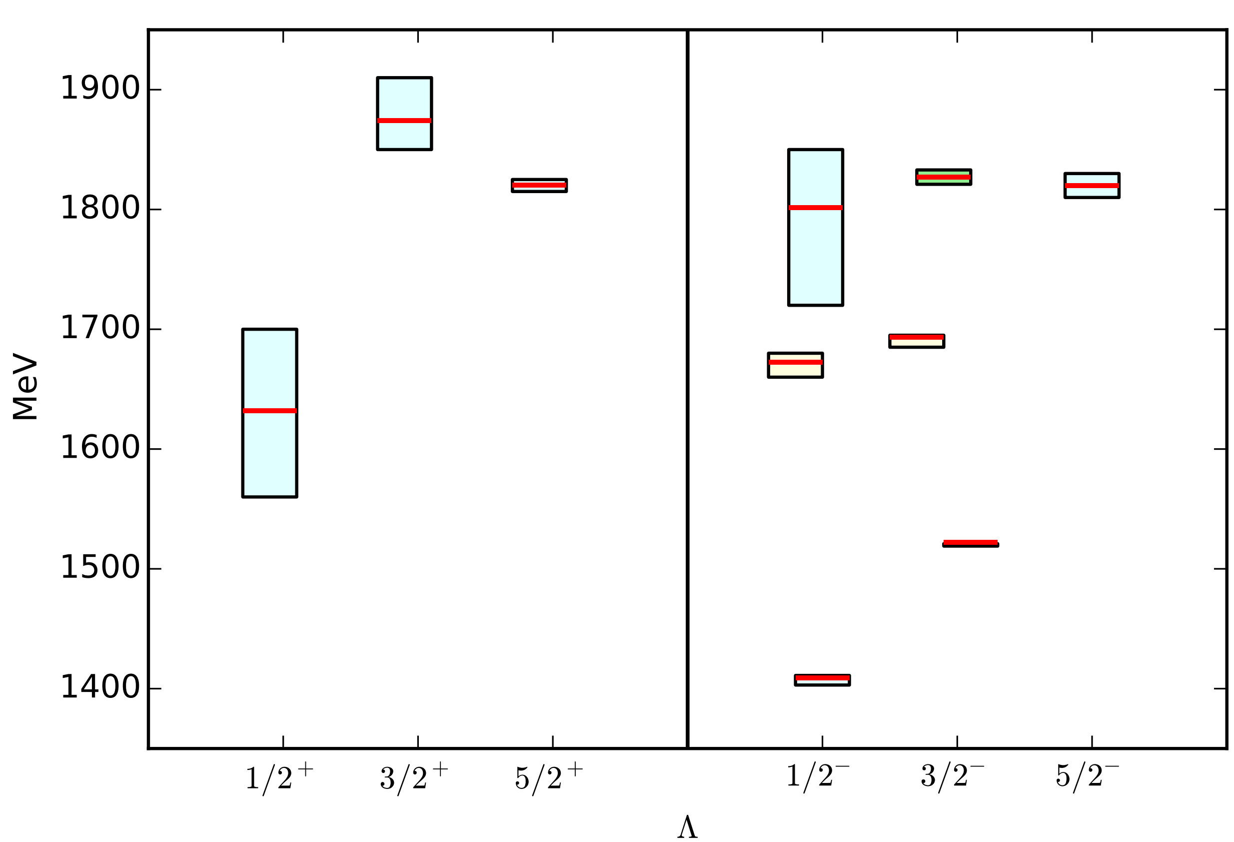
Fit Results (Fitted masses for LQCD)

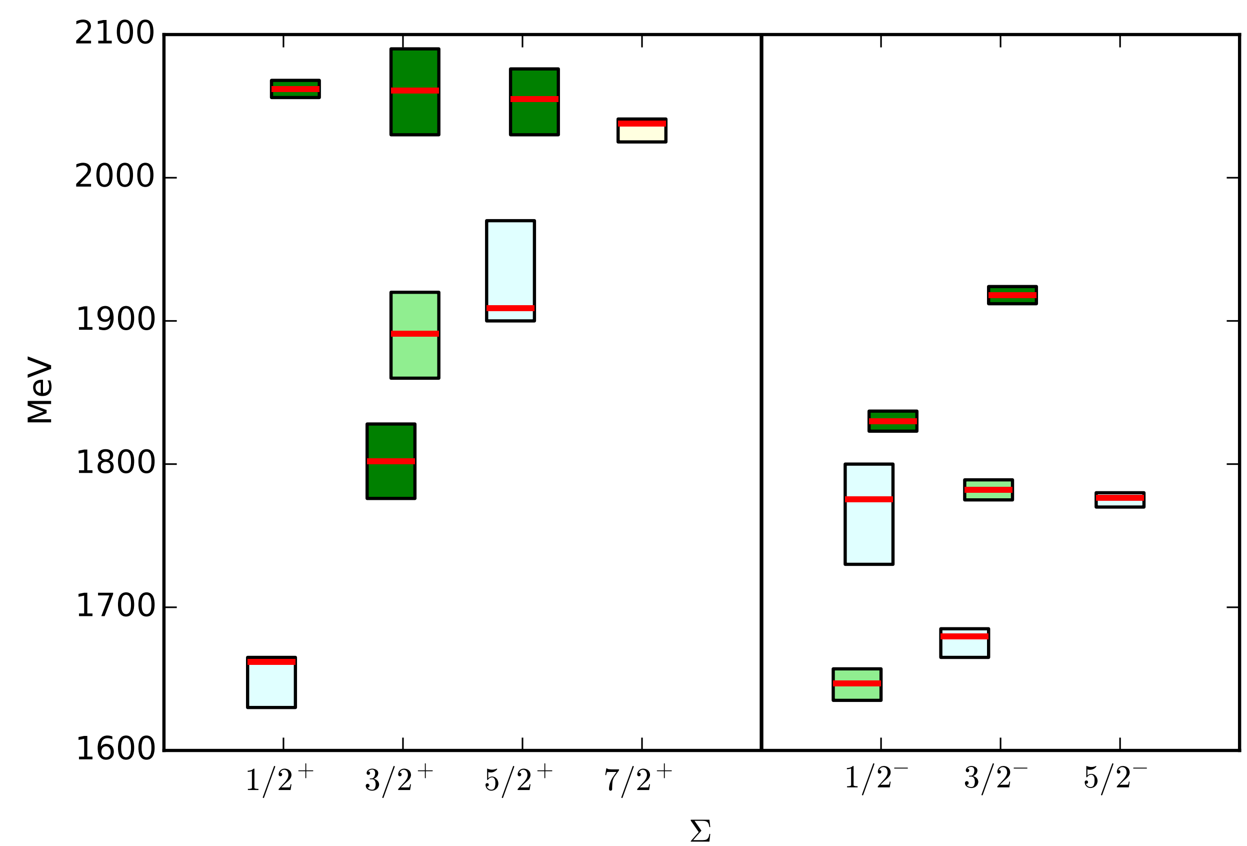


Fit Results (Fitted Physical masses)

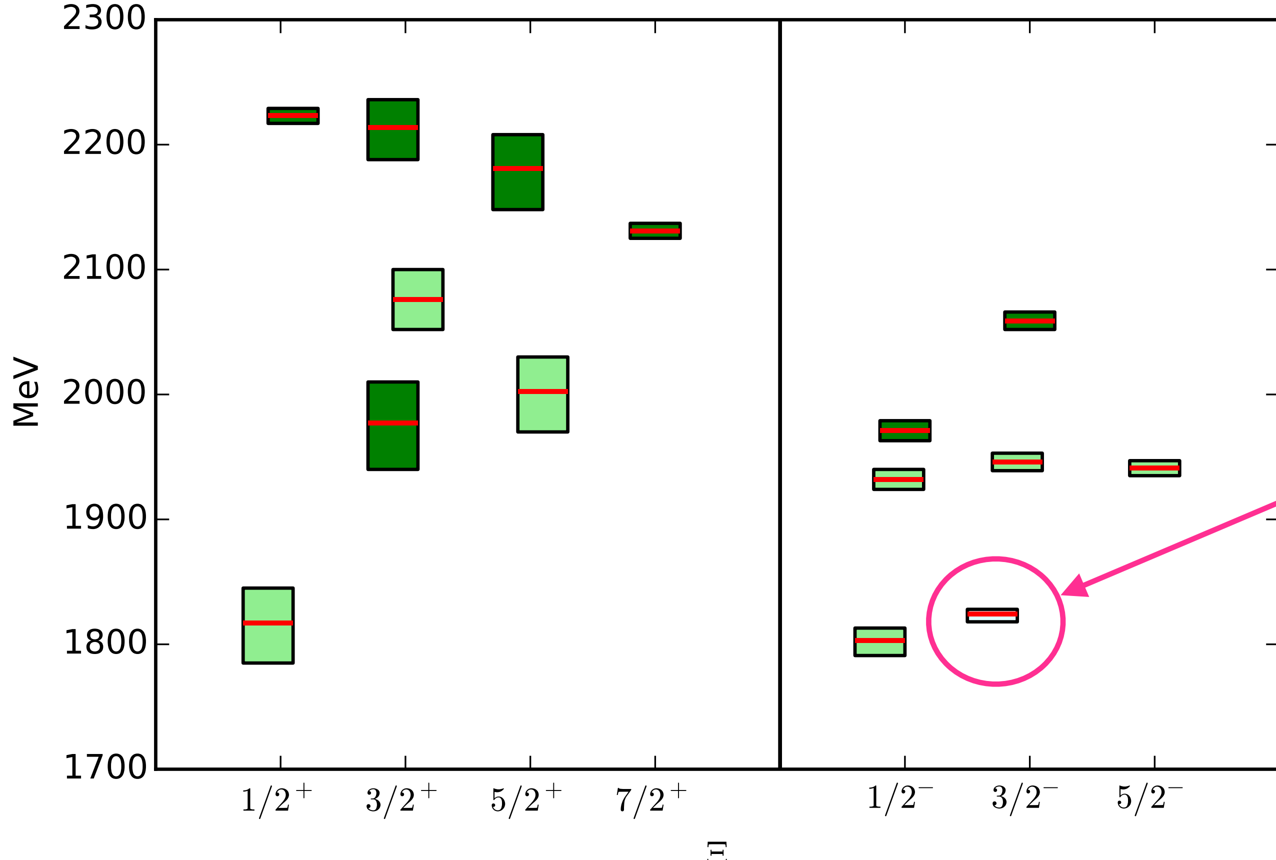






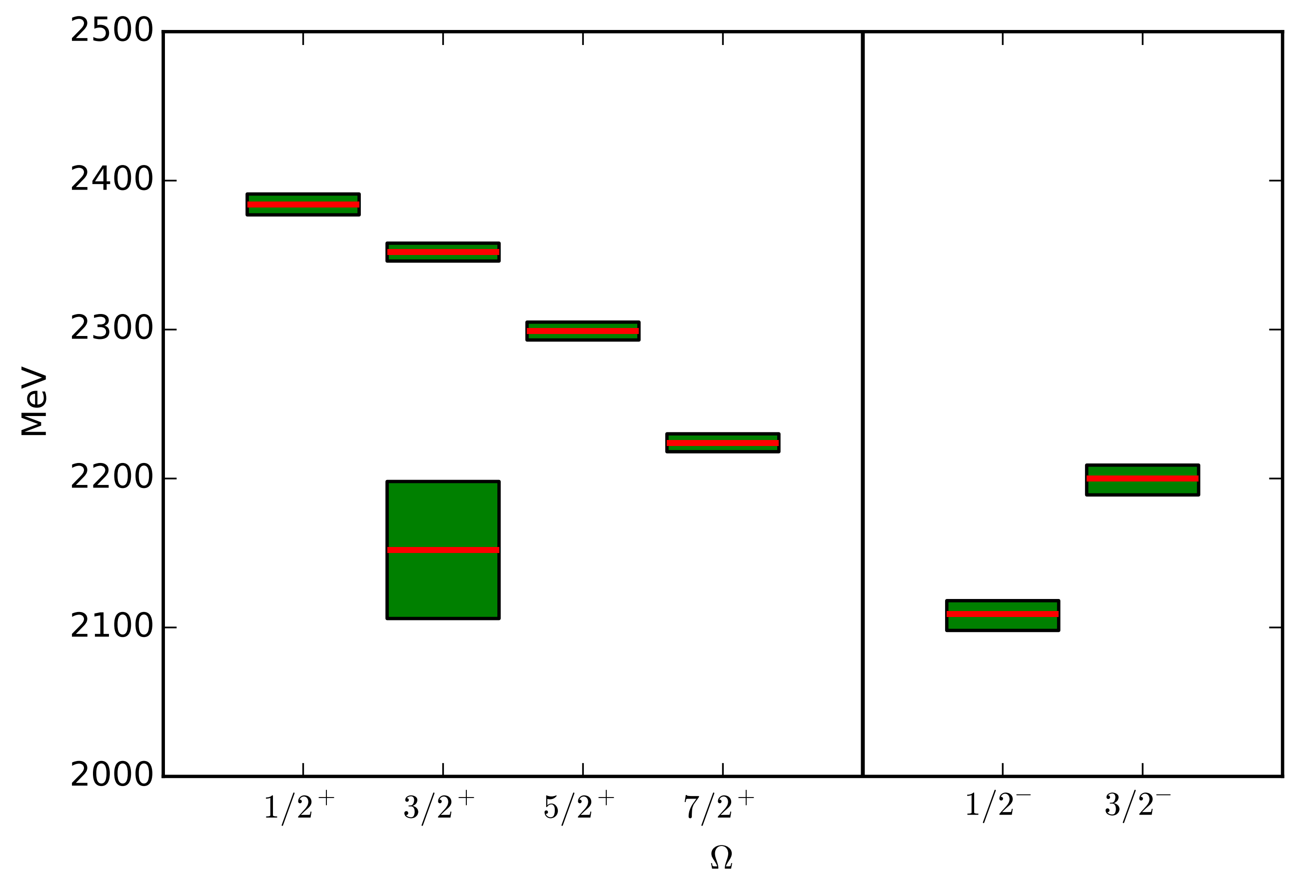


Fit Results (Fitted Physical masses)



Experimentally identified state in PDG

Rest of all are our predictions from fits



Mass Predictions (Physical)

For the Physical baryons

Multiplet	Missing states	Fitted mass [MeV]	Mass listed in PDG[MeV]	From mass relations[MeV]
$[\mathbf{56}, 0^+]$	$\Sigma''_{3/2}$	1800±26	$\Sigma(1840)(3/2^+)^*$ with mass $\sim$ 1840	1790 ± 131
	$\Xi_{1/2}$	1815±31	$\cdots$	1825 ± 108
	$\Xi''_{3/2}$	1975±35	$\Xi(1950)(?^?)^{***}$ with mass $\sim 1950 \pm 15$	1955 ± 171
	$\Omega_{3/2}$	2150±46	$\cdots$	2120 ± 219
$[\mathbf{56}, 2^+]$	$\Sigma_{3/2}$	1889±30	$\Sigma(1840)(3/2^+)^*$ with mass $\sim$ 1840	1920±70
	$\Xi_{3/2}$	2074±24	$\Xi(2120)^*(?^?)$: 2130±7	2080±75
	$\Xi_{5/2}$	2000±31	$\Xi(2030)^{***}(S \geqslant 5/2^+)$ with 2025±5	2006±14
	$\Sigma''_{1/2}$	2060±6	$\cdots$	2127±120
	$\Xi''_{1/2}$	2221±6	$\Xi(2250)^{**}(?^?)$: 2214±5	$\cdots$
	$\Omega_{1/2}$	2382±7	$\cdots$	$\cdots$
	$\Sigma''_{3/2}$	2059±29	$\Sigma(2080)^{**}(3/2^+)$: 2120±40	2109±96
	$\Xi''_{3/2}$	2212±24	$\cdots$	$\cdots$
	$\Omega_{3/2}$	2350±6	$\cdots$	$\cdots$
	$\Sigma''_{5/2}$	2053±23	$\Sigma(2070)^*(5/2^+)$: 2070±10	2077±56
	$\Xi''_{5/2}$	2178±31	$\cdots$	$\cdots$
	$\Omega_{5/2}$	2297±6	$\cdots$	$\cdots$
	$\Xi''_{7/2}$	2129±6	$\Xi(2120)^*(?^?)$: 2130±7	$\cdots$
	$\Omega_{7/2}$	2222±6		

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Mass Predictions (Physical) (Continued…..)

Multiplet	Missing states	Fitted mass [MeV]	Mass listed in PDG [MeV]	From mass relations [MeV]
$[\mathbf{70}, 1^-]$	$\Sigma_{1/2}$	1645±11	$\Sigma(1620)1/2^{-**}$=1620±10	$\cdots$
	$\Xi_{1/2}$	1801±11	$\cdots$	$\cdots$
	$\Xi'_{1/2}$	1930±8	$\cdots$	$\cdots$
	$\Lambda'_{3/2}$	1825±6	$\cdots$	$\cdots$
	$\Sigma'_{3/2}$	1780±7	$\cdots$	$\cdots$
	$\Xi'_{3/2}$	1944±7	$\Xi(1950)(?^{?})^{***}$=1950±15	$\cdots$
	$\Xi_{5/2}$	1939±6	$\Xi(1950)(?^{?})^{***}$=1950±15	$\cdots$
	$\Sigma''_{1/2}$	1828±7	$\cdots$	$\cdots$
	$\Xi''_{1/2}$	1969±8	$\Xi(1950)(?^{?})^{***}$=1950±15	$\cdots$
	$\Omega_{1/2}$	2107±10	$\cdots$	$\cdots$
	$\Sigma''_{3/2}$	1916±6	$\Sigma(1940)3/2^{-***}$=1950±30	$\cdots$
	$\Xi''_{3/2}$	2057±7	$\cdots$	$\cdots$
	$\Omega_{3/2}$	2198±9	$\cdots$	$\cdots$

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Mass Relations (with LQCD masses)

TABLE III. Mass relations for the ground-state octet and decuplet. The relations are valid up to corrections $\mathcal{O}(\epsilon^{\frac{3}{2}4}/N_c)$ in the case of the GMO and EQS relations which stem from the one-loop chiral corrections [36], and up to $\mathcal{O}(1/N_c^2)$ for the rest of the relations.

Relation	M_π [MeV]		
	PDG	391	524
$2(N+\Xi)-(3\Lambda+\Sigma)=0$	30.2 ± 0.4	38 ± 75	32 ± 32
$\Sigma''-\Delta=\Xi''-\Sigma''=\Omega''-\Xi''$	155 ± 2	64 ± 25	40 ± 11
	149.0 ± 0.5	55 ± 19	33 ± 13
	140.7 ± 0.5	54 ± 17	40 ± 14
$\frac{1}{3}(\Sigma+2\Sigma'')-\Lambda-\frac{2}{3}(\Delta-N)=0$	9 ± 1	1 ± 28	14 ± 12
$\Sigma''-\Sigma-(\Xi''-\Xi)=0$	23.5 ± 0.5	12 ± 25	12 ± 15
$3\Lambda+\Sigma-2(N+\Xi)+(\Omega-\Xi''-\Sigma''+\Delta)=0$	16 ± 2	29 ± 81	32 ± 36
$\Sigma''-\Delta+\Omega-\Xi''-2(\Xi''-\Sigma'')=0$	2.5 ± 2.4	8 ± 51	14 ± 37

Well Satisfied !!!

TABLE IV. Mass relations for the Roper multiplet. The relations hold at the same orders as in the case of the ground-state baryons.

Relation	M_π [MeV]	
	391	524
$2(N+\Xi)-(3\Lambda+\Sigma)=0$	179 ± 180	106 ± 155
$\Sigma''-\Delta=\Xi''-\Sigma''=\Omega''-\Xi''$	13 ± 45	-27 ± 26
	84 ± 40	41 ± 49
	48 ± 42	41 ± 57
$\frac{1}{3}(\Sigma+2\Sigma'')-\Lambda-\frac{2}{3}(\Delta-N)=0$	51 ± 65	29 ± 41
$\Sigma''-\Sigma-(\Xi''-\Xi)=0$	58 ± 63	77 ± 80
$3\Lambda+\Sigma-2(N+\Xi)+(\Omega''-\Xi''-\Sigma''+\Delta)=0$	144 ± 189	174 ± 170
$\Sigma''-\Delta+\Omega''-\Xi''-2(\Xi^*-\Sigma'')=0$	107 ± 110	67 ± 147

Mass Relations (with LQCD masses)

SU(3) Symmetric limit

TABLE VII. Mass relations for the [**56**, 2^+] multiplet. The relations hold at the same orders as in the case of the ground-state baryons. The last column corresponds to the SU(3) symmetric limit.

Relation	M_π [MeV] 391	524	702
$2(N_{3/2}+\Xi_{3/2})-(3\Lambda_{3/2}+\Sigma_{3/2})=0$	98 ± 126	49 ± 173	0
$2(N_{5/2}+\Xi_{5/2})-(3\Lambda_{5/2}+\Sigma_{5/2})=0$	40 ± 98	55 ± 65	0
$\Sigma''_{1/2}-\Delta_{1/2}=\Xi''_{1/2}-\Sigma''_{1/2}=\Omega_{1/2}-\Xi''_{1/2}$	-13 ± 110	36 ± 33	0
	23 ± 44	43 ± 22	0
	85 ± 54	35 ± 19	0
$\Sigma''_{3/2}-\Delta_{3/2}=\Xi''_{3/2}-\Sigma''_{3/2}=\Omega_{3/2}-\Xi''_{1/2}$	48 ± 46	36 ± 23	0
	56 ± 29	30 ± 16	0
	45 ± 31	41 ± 15	0
$\Sigma''_{5/2}-\Delta_{5/2}=\Xi''_{5/2}-\Sigma''_{5/2}=\Omega_{5/2}-\Xi''_{5/2}$	35 ± 40	34 ± 26	0
	62 ± 31	26 ± 23	0
	57 ± 34	52 ± 18	0
$\Sigma''_{7/2}-\Delta_{7/2}=\Xi''_{7/2}-\Sigma''_{7/2}=\Omega_{7/2}-\Xi''_{7/2}$	38 ± 38	35 ± 25	0
	67 ± 31	36 ± 20	0
	59 ± 31	22 ± 18	0
$\Delta_{5/2}-\Delta_{3/2}-(N_{5/2}-N_{3/2})=0$	70 ± 68	4 ± 68	44 ± 33
$(\Delta_{7/2}-\Delta_{5/2})-\frac{7}{5}(N_{5/2}-N_{3/2})=0$	68 ± 78	2.5 ± 92	75 ± 41
$\Delta_{7/2}-\Delta_{1/2}-3(N_{5/2}-N_{3/2})=0$	129 ± 175	13 ± 192	133 ± 74
$\frac{8}{15}(\Lambda_{3/2}-N_{3/2})+\frac{22}{15}(\Lambda_{5/2}-N_{5/2})-(\Sigma_{5/2}-\Lambda_{5/2})-2(\Sigma_{7/2}''-\Delta_{7/2})=0$	91 ± 100	29 ± 75	0
$\Lambda_{5/2}-\Lambda_{3/2}+3(\Sigma_{5/2}-\Sigma_{3/2})-4(N_{5/2}-N_{3/2})=0$	10 ± 207	10 ± 272	0
$\Lambda_{5/2}-\Lambda_{3/2}+\Sigma_{5/2}-\Sigma_{3/2}-2(\Sigma_{5/2}''-\Sigma_{3/2}'')=0$	111 ± 81	12 ± 72	87 ± 59
$7(\Sigma''_{3/2}-\Sigma''_{7/2})-12(\Sigma''_{5/2}-\Sigma''_{7/2})=0$	44 ± 319	39 ± 268	67 ± 266
$4(\Sigma''_{1/2}-\Sigma''_{7/2})-5(\Sigma''_{3/2}-\Sigma''_{7/2})=0$	83 ± 170	87 ± 104	58 ± 161

Well Satisfied !!!

Deviations are within the expected magnitude of higher order corrections

Mass Relations (with LQCD masses)

TABLE XIII. GMO and EQS relations for the $[\mathbf{70}, 1^-]$ multiplet. Due to the insufficient number of known states with three or more stars, the mass relations cannot be checked for the physical case.

Relation	PDG	M_π [MeV] 391	524
$2(N_{1/2} + \Xi_{1/2}) - (3\Lambda_{1/2} + \Sigma_{1/2}) = 0$	...	59 ± 156	17 ± 125
$2(N_{3/2} + \Xi_{3/2}) - (3\Lambda_{3/2} + \Sigma_{3/2}) = 0$	...	31 ± 121	13 ± 74
$2(N_{5/2} + \Xi_{5/2}) - (3\Lambda_{5/2} + \Sigma_{5/2}) = 0$	...	46 ± 91	6 ± 64
$\Sigma''_{1/12} - \Delta_{1/2} = \Xi''_{1/2} - \Sigma''_{1/2} = \Omega_{1/2} - \Xi''_{1/2}$	...	67 ± 47	35 ± 56
	...	34 ± 36	40 ± 41
	...	24 ± 49	22 ± 26
$\Sigma''_{3/2} - \Delta_{3/2} = \Xi''_{3/2} - \Sigma''_{3/2} = \Omega_{3/2} - \Xi''_{3/2}$	...	2 ± 49	39 ± 23
	...	82 ± 47	37 ± 21
	...	61 ± 43	31 ± 21

Well Satisfied !!!

TABLE XIV. Octet-decuplet mass relations for the $[\mathbf{70}, 1^-]$ multiplet. S_B is the mass splitting between the state B and the nonstrange states in the SU(3) multiplet to which it belongs. The results shown correspond to the relation divided by the sum of the positive coefficients in the relation (e.g., 163 for the first relation).

Relation	M_π [MeV] 391	524
$14(S_{\Lambda_{3/2}} + S_{\Lambda'_{3/2}}) + 63S_{\Lambda_{5/2}} + 36(S_{\Sigma_{1/2}} + S_{\Sigma'_{1/2}})$ $-68(S_{\Lambda_{1/2}} + S_{\Lambda'_{1/2}}) - 27S_{\Sigma_{5/2}} = 0$	9.4 ± 40	0.96 ± 34
$14(S_{\Sigma_{3/2}} + S_{\Sigma'_{3/2}}) + 21S_{\Lambda_{5/2}} - 9S_{\Sigma_{5/2}}$ $-18(S_{\Lambda_{1/2}} + S_{\Lambda'_{1/2}}) - 2(S_{\Sigma_{1/2}} + S_{\Sigma'_{1/2}}) = 0$	37 ± 45	5.4 ± 38
$14S_{\Sigma''_{1/2}} + 49S_{\Lambda_{5/2}} + 23(S_{\Sigma_{1/2}} + S_{\Sigma'_{1/2}})$ $-45(S_{\Lambda_{1/2}} + S_{\Lambda'_{1/2}}) - 19S_{\Sigma_{5/2}} = 0$	9.4 ± 40	0.7 ± 34
$14S_{\Sigma''_{3/2}} + 28S_{\Lambda_{5/2}} + 11(S_{\Sigma_{1/2}} + S_{\Sigma'_{1/2}})$ $-27(S_{\Lambda_{1/2}} + S_{\Lambda'_{1/2}}) - 10S_{\Sigma_{5/2}} = 0$	0.8 ± 40	0.1 ± 33

Conclusions

- A picture of spin-flavor composition of excited baryons is derived from a description of masses calculated in LQCD using the $1/N_c$ expansion.
- All the predicted mass relations are well satisfied by the lattice QCD data.
- Missing masses in PDG are predicted by means of model independent mass relations and also by the fits.
- LQCD masses show a weaker breaking of $SU(6) \times O(3)$ symmetry limit than the physical ones.
- The spin-flavor singlet operator, hyper-fine and spin-orbit interaction play the most important role in the fits to physical and Lattice QCD masses.
- The minimum number of operators which can be used to fit the lattice QCD masses is smaller than for physical masses.

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Thank you