# Measurement of the Elastic Form Factor Ratio $\mu_p G_E/G_M$ using Electron Scattering Spin Asymmetries

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May 2018



U.S. DEPARTMENT OF ENERGY
JSA
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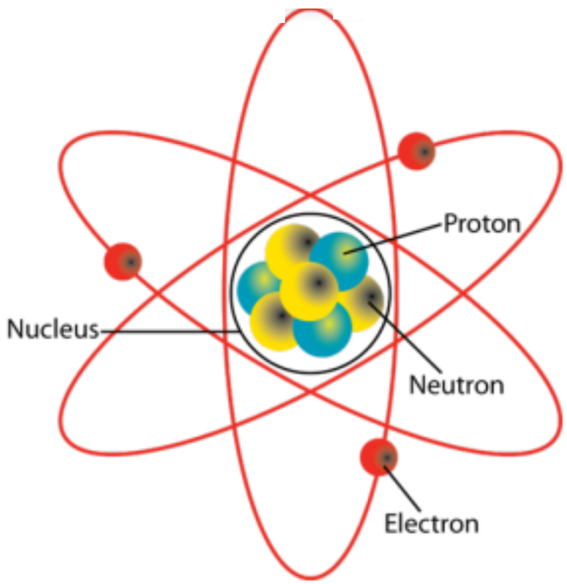
# Background

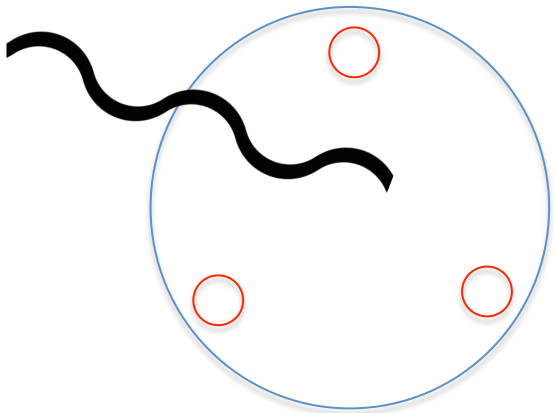
- Modern model of atom by the 1930s
- Nucleons believed to structure-less
- Measurement of $\mu_p = 2.79\mu_N \rightarrow$ proton internal structure



# Background

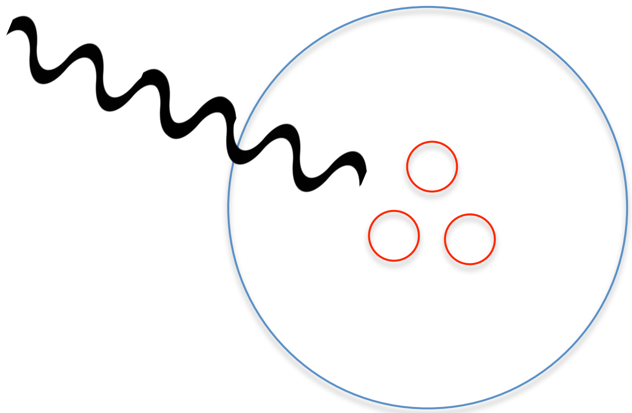
- Nucleons consist of quarks
- QCD at high and low $Q^2$

Low $Q^2$



Quarks in state of confinement
Further apart – sensitive to charge radius
Interaction between quarks is strong
Complicated fields

High $Q^2$



Quarks are asymptotically free
Close together
No force between quarks
Probing small spatial distributions - Probing quarks

- As a consequence…

# Background

- Electron scattering as a tool
  - $e^-$ interacts with nucleus through EM interaction
  - Interaction is weak and is dominated by OPE
  - $e^-p^+$ scattering expressed in terms of $G_E$ and $G_M$

  - What are form factors?
  - How does electron scattering work?
  - How are form factors measured?

# Form Factors

- Form factors are defined by the Fourier transform of the spatial charge and magnetic current densities of the nucleon

$$G_{E,M}(Q^2) = \int \rho(\vec{r})_{E,M}\, e^{i\vec{q}\vec{r}}\, d^3r$$

- Form Factors:
  - $F_1(Q^2)$ – Dirac form factor
  - $F_2(Q^2)$ – Pauli form factor

- Sachs Form Factors
  - $G_E$ – distribution of electric charge
  - $G_M$ – distribution of magnetization

$$G_E = F_1 - \tau\kappa F_2$$

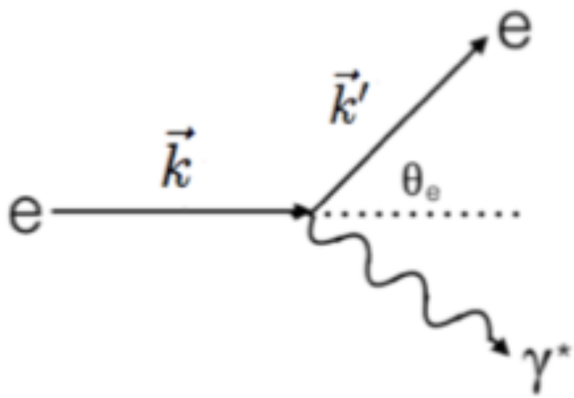
$$G_M = F_1 + \kappa F_2$$

$$Q^2 = 0.$$

$$G_E^p(0) = 1 \quad G_M^p(0) = \mu_p$$

$$G_E^n(0) = 0 \quad G_M^n(0) = \mu_n$$

# Elastic Electron Scattering




$$q^\mu = (\omega, \vec{q})$$
$$\omega = E - E'$$
$$\vec{q} = \vec{k} - \vec{k}'$$

$$q^2 = -4\,E\,E'\,sin^2\,(\theta_e/2)$$

$$Q^2 = -q^2 = -(\omega^2 - \vec{q}^{\,2})$$

- Formula for e-p scattering cross section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \cdot \frac{E'}{E}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

where $G_E$ and $G_M$ form factors take into account the finite size of the proton.

Structure is the variation in the EM form factors

# How Form Factors were Measured?

- Rosenbluth Technique:
  - Elastic scattering cross-section is measured
  - Techniques uses different beam energies and angles for a fixed $Q^2$.
  - How do we extract the form factors from Rosenbluth Method?

$$\sigma_{red} = (1+\tau)\frac{\epsilon}{\tau}\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} = G_M^2 + \frac{\epsilon}{\tau}G_E^2$$

$\varepsilon = \varepsilon(\tau, \theta_e)$ is virtual photon polarization

# Background

- Rosenbluth Technique:
  - Technique shows that both $G_E^P$ and $G_M^P$ follow the dipole parameterization ($G_D$)

The form factors divided by $G_D$ appear to remain close to 1

$$G_D = (a + \frac{Q^2}{\lambda_D^2})^{-2}$$

$$G_E^p \approx G_D, \text{ and } G_M^{p,n} \approx \mu_{p,n} G_D \qquad G_E^n \approx 0 \qquad G_E^p(Q^2) \approx \frac{G_M^p(Q^2)}{\mu_p/\mu_N} \approx \frac{G_M^n(Q^2)}{\mu_n/\mu_N} = G_D(Q^2)$$

- Technique started to show deviations from the dipole formula for the nucleon form factors.

# Background

- Advances in technology usher new generation of experiments
- Depend on spin degrees of freedom
- Distinct advantages over traditional cross-section measurements:
  - Increased sensitivity
  - Systematic errors: luminosity, acceptance, detector efficiency
- What are these techniques?

# Measurement Techniques

- Recoil Polarization
- Double Spin Asymmetry

$$\sigma(h) = \Sigma + h\Delta \qquad h = \pm 1.$$

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{\Delta}{\Sigma} \longrightarrow A_{raw} = P_b P_t f A_{phys} \longrightarrow A_{phys} = \frac{1}{P_b P_t f} \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{1}{P_b P_t f} \frac{\Delta}{\Sigma}$$

How do we find the FFR?

$$\mu_P \frac{G_E^P}{G_M^P} = -\mu_P \frac{a(\tau_1, \theta_1) \cos[\theta_1^*] - \frac{f_2}{f_1} \Lambda a(\tau_2, \theta_2) \cos[\theta_2^*]}{\cos[\phi_1^*] \sin[\theta_1^*] - \frac{f_2}{f_1} \Lambda \cos[\phi_2^*] \sin[\theta_2^*]}$$

1:RHRS
2:LHRS

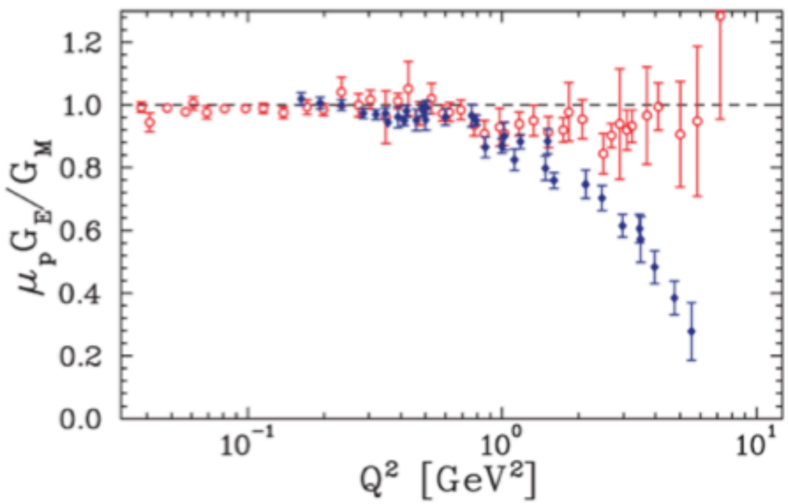
$$a(\tau_n, \theta_n) = \sqrt{\tau_n (1 + (1 + \tau_n) \tan^2[\theta_n/2])} \qquad \Lambda = R = \frac{A_1}{A_2} \qquad \tau = \frac{Q^2}{4M_p^2}$$

- Single Spin Asymmetry

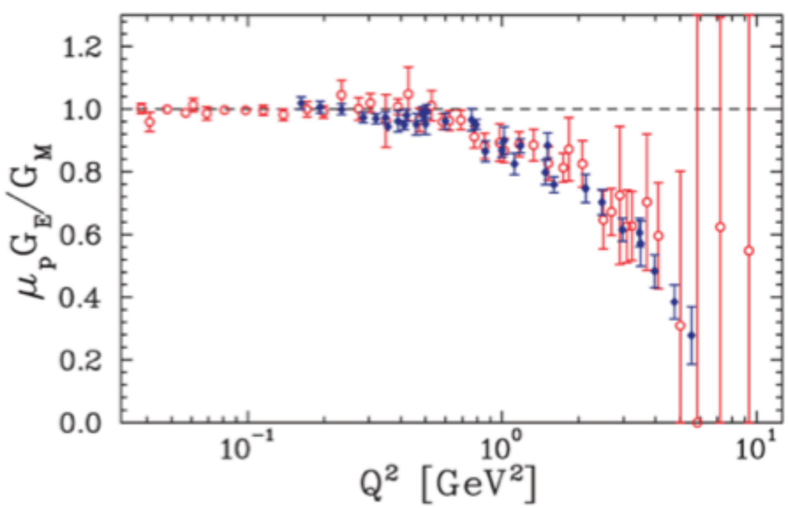
$$\frac{G_E}{G_M} = -\frac{b}{2A_p} \sin[\theta^*] \cos[\phi^*] + \sqrt{\frac{b^2}{4A_p^2} \sin^2[\theta^*] \cos^2[\phi^*] - \frac{a}{A_p} \cos[\theta^*] - c}$$

# Background

Recent measurements of $\mu G^P_E/G^P_M$ using recoil polarization at high $Q^2$ can deviate dramatically from the un-polarized data




Now generally accepted that two-photon-exchange processes (TPE) mostly account for the discrepancy at high $Q^2$ using the Rosenbluth extraction of $\mu G^P_E/G^P_M$



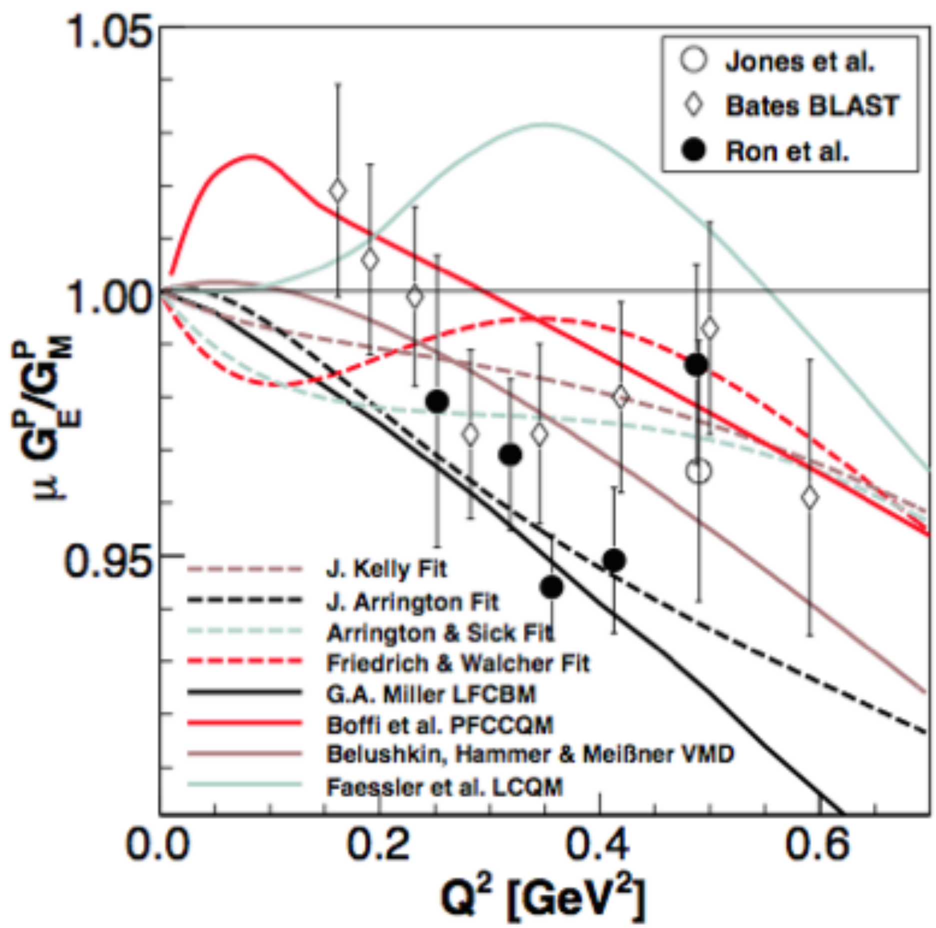

Rosenbluth Polarization

# Background

- Interest in low $Q^2$
  - Semi-phenomenological fits
  - Recent experiments with discrepancies
- Experiments:
  - BLAST
  - LEDEX
- The Result: Need to carry out new high precision measurements
- Thesis focuses on DSA and Single arm measurement

# Motivation

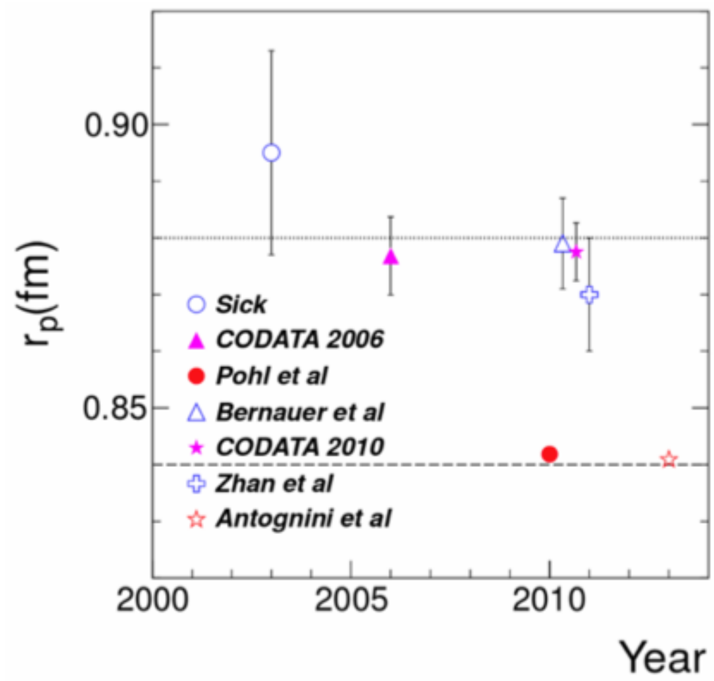
- High $Q^2$ accepted that $p^+$ FFR decreases smoothly as $Q^2$ increases
- $Q^2 < 1$ $GeV^2$ less conclusive
- Key: Slope of FF as $Q^2 \rightarrow 0$ is related to size of proton



# Motivation

- The Proton Radius Puzzle

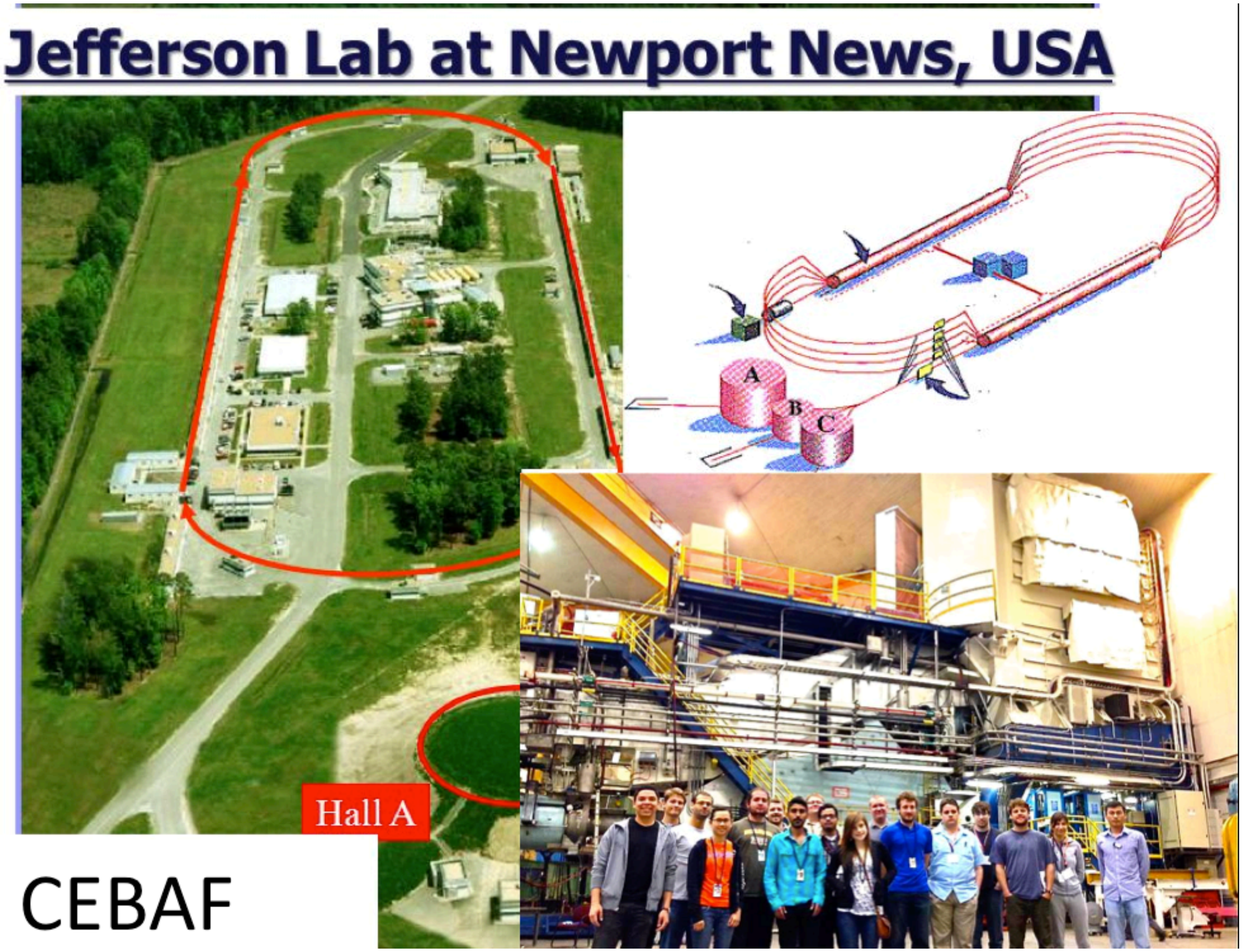
- Techniques to determine proton radius:
  - Elastic Electron Scattering
  - Muonic Hydrogen Lamb Shift
  - Atomic Hydrogen Lamb Shift




- Proton Radius Puzzle
  - the inconsistency between the proton charge radius was determined from muonic hydrogen and electron-proton systems: atomic hydrogen and e-p elastic scattering
  - Results from Muonic Hydrogen are smaller
  - Want high precision results at low $Q^2$

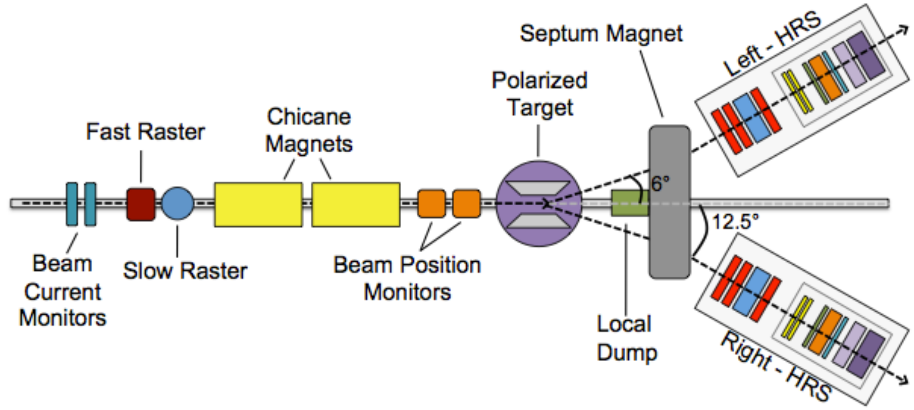
# Experimental Set-up and Kinematics

- Measure $p^+$ elastic FFR: $Q^2 \rightarrow 0.01\text{-}0.08\ GeV^2$
- The experiment utilized: Two High Resolution Spectrometers (HRS) used to detect θ ~ 6.0°
- Beam energies were: 1.1,1.7, & 2.2 GeV.
- CEBAF: Continuous Electron Beam Accelerator Facility



# Hall A Equipment

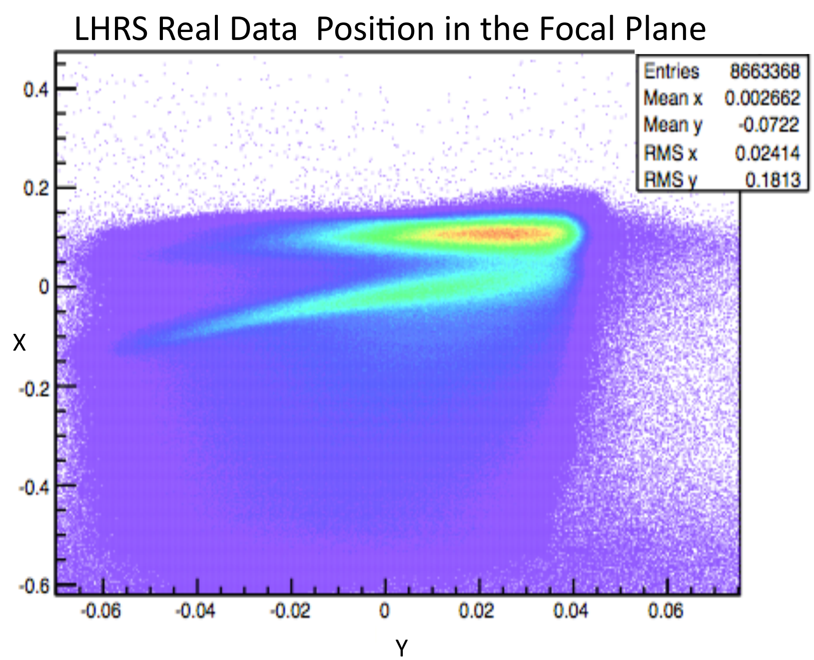
- Polarized beam passes through fast/slow raster's
- Two Chicane Magnets
- Electrons scattered off the polarized $NH_3$ target
- Bent by the Septum Magnets
- Enters the two HRS arms
- Detected by the detector package – Similar to Mass Spectroscopy:
  - QQDQ Magnet
  - Vertical Drift Chambers
  - Scintillators
- Between runs Moller measurements are taken to determine the beam polarization

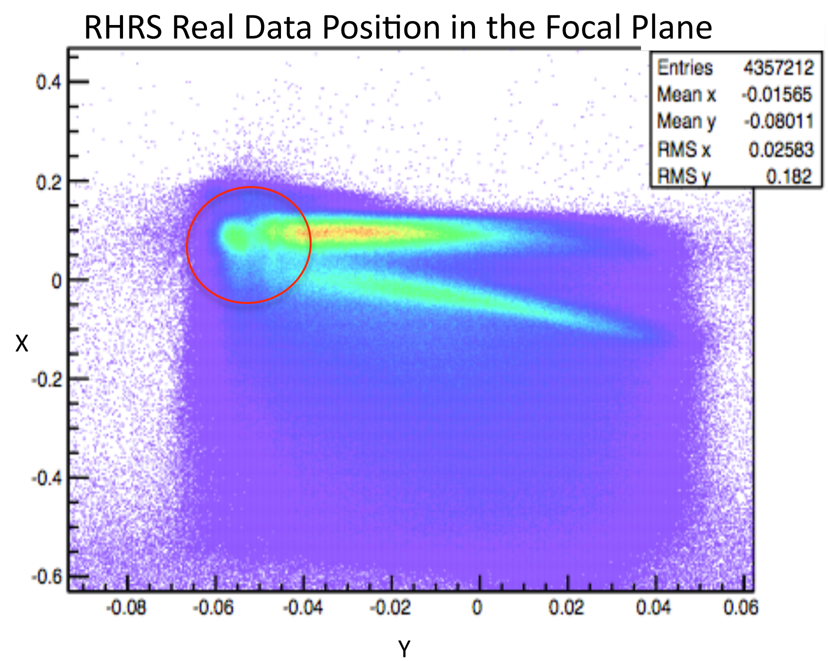


# Analysis

- Standard Approach:
  - Reconstruction of particle trajectories
  - Does not include target field which complicates the process
  - Solution:
    - Simulation used for trajectories between target and septum magnet
    - Optics matrix used for trajectories between septum magnet to focal plane
- Reconstruction Approach RHRS:
  - Could not use standard approach:
    - Loss of BPM data
    - Issues with magnets (especially septum magnet)

# Analysis




What is the focal plane?
What are the reaction components?



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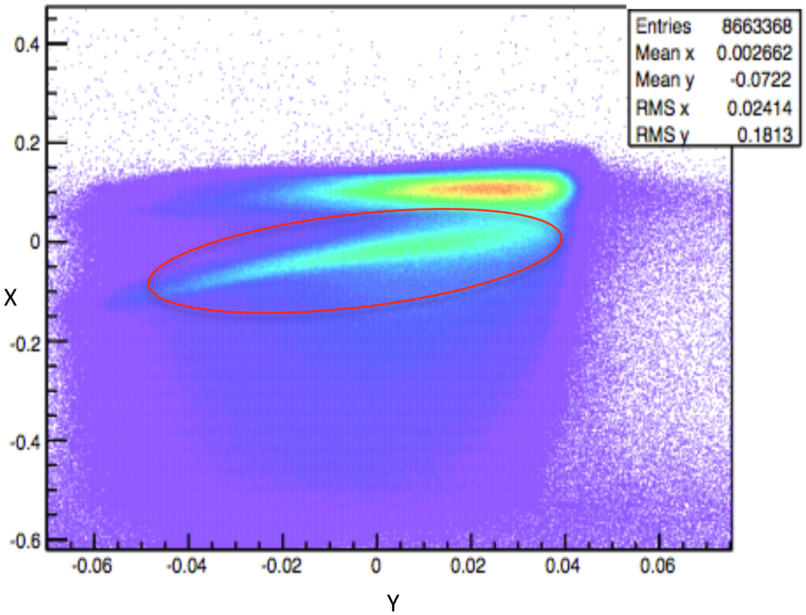
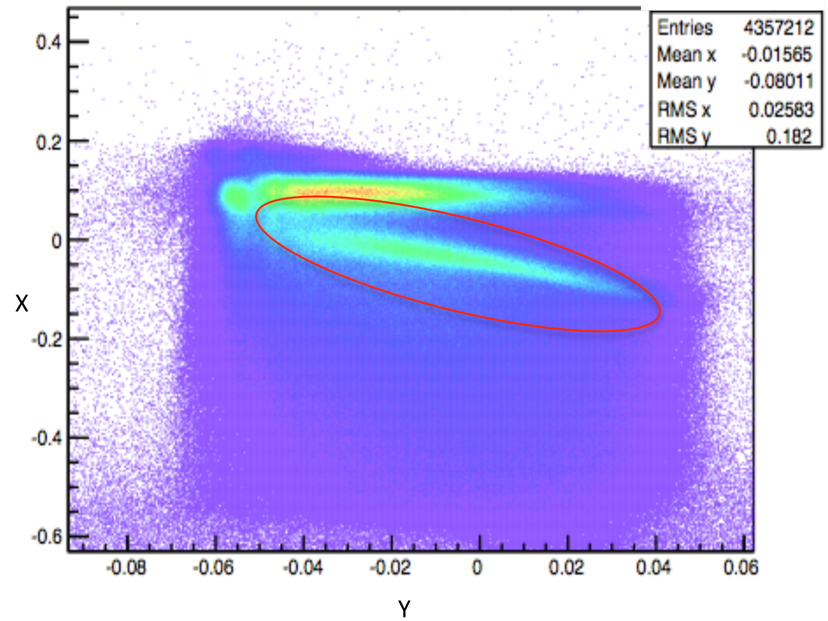
# Analysis

Predominately the hydrogen elastic reaction components

LHRS Real Data Position in the Focal Plane



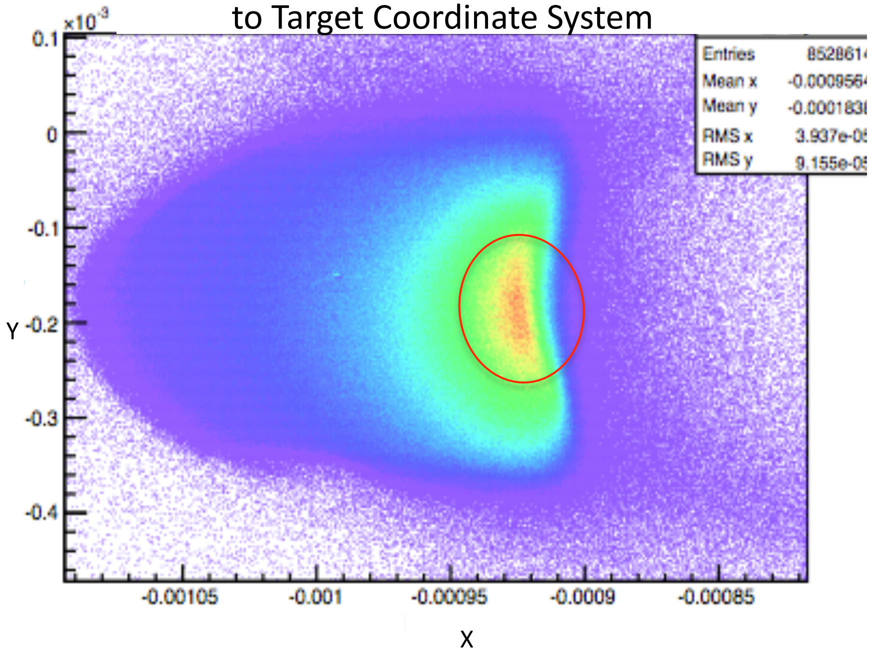

RHRS Real Data Position in the Focal Plane



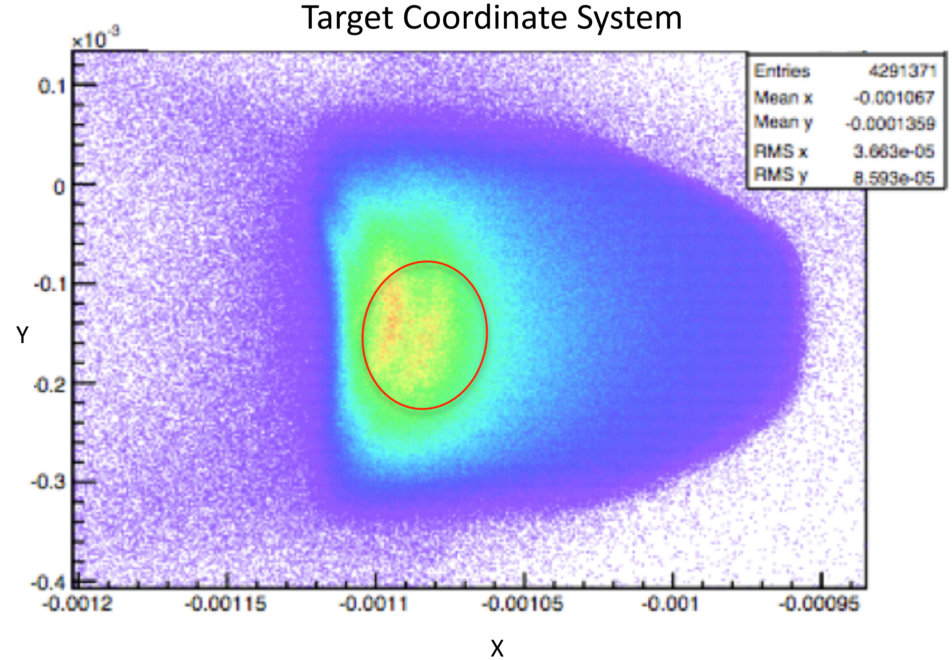

What is the focal plane?
What are the reaction components?

# Analysis

LHRS Real Data  Position Reconstructed to Target Coordinate System




RHRS Real Data  Position Reconstructed to Target Coordinate System




Indication that events map back to two interaction points → Physically Impossible



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# Analysis

- The approach:
  - Process focal plane momenta distributions
  - Why choose momentum?
    - HRS p-res = $10^{-4}$
  - Use a MC simulation to simulate events for the reaction components to fit the to data
  - If done properly:
    - Should account for overall background
    - Select elastic H events for asymmetry measurements

# Analysis

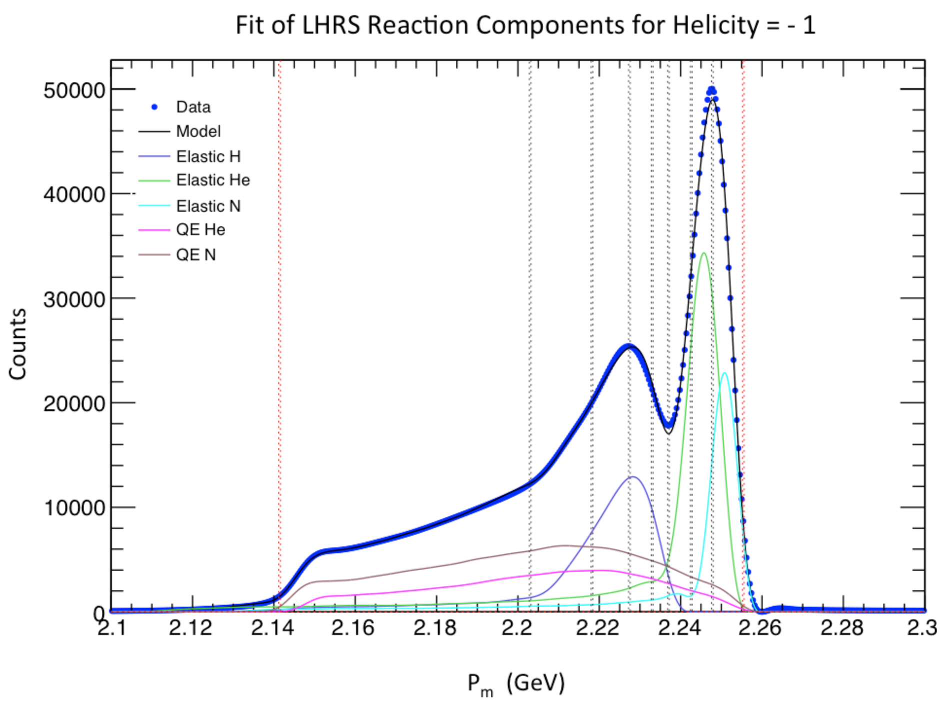
- Fitting process:
  - Momentum distributions of measured data
  - Simulate reaction components: elastic H, He, N, and Inelastic He, N $h_k[P_m]$
  - Each RC is transformed as a function of the momentum by individually shifting $\beta_k$, scaling $\gamma_k$, and skewing $\alpha_k$ the RC: $H_k[P_m] = \gamma_k\, h_k[\alpha_k(P_m - \beta_k)]$
  - Final fit model: $H[P_m] = \sum_k H_k[P_m]$

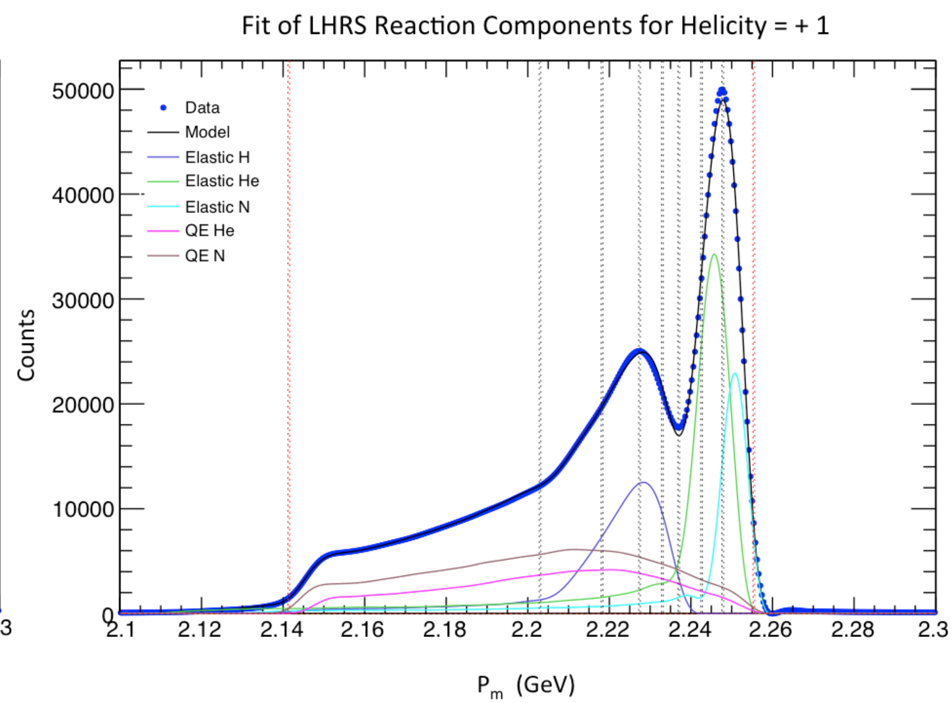
# Analysis

- Fit Process:
  - initial placement of each RC prior to fitting a run list
- How do we define a run list?

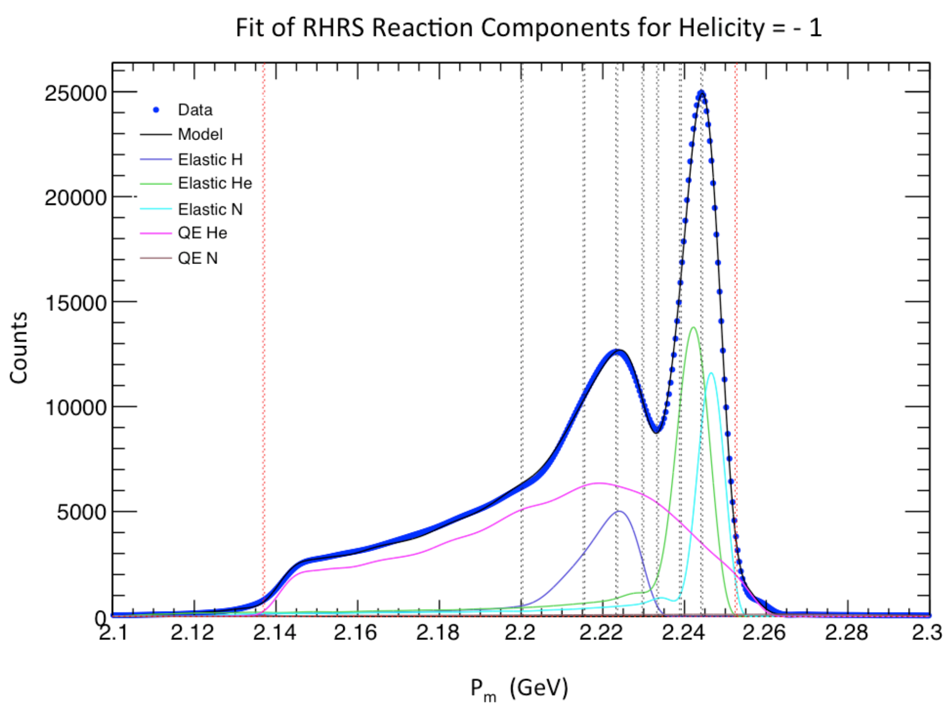
| Run List | Target Polarization | HWP Status |
|---|---|---|
| NI | Negative | In |
| NO | Negative | Out |
| PI | Positive | In |
| PO | Positive | Out |
| S | Negative | Out |

# Fit Results

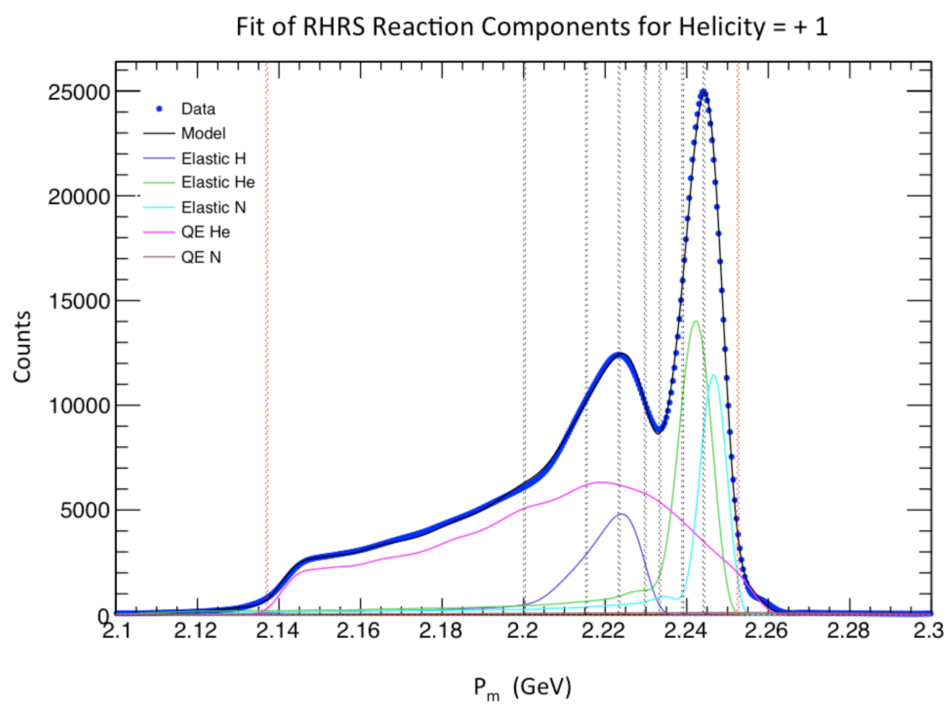

Fit of LHRS Reaction Components for Helicity = - 1
Data
Model
Elastic H
Elastic He
Elastic N
QE He
QE N
Counts
$P_m$ (GeV)
50000
40000
30000
20000
10000
0
2.1
2.12
2.14
2.16
2.18
2.2
2.22
2.24
2.26
2.28
2.3


Fit of LHRS Reaction Components for Helicity = + 1
Data
Model
Elastic H
Elastic He
Elastic N
QE He
QE N
Counts
$P_m$ (GeV)
50000
40000
30000
20000
10000
0
2.1
2.12
2.14
2.16
2.18
2.2
2.22
2.24
2.26
2.28
2.3

# Fit Results

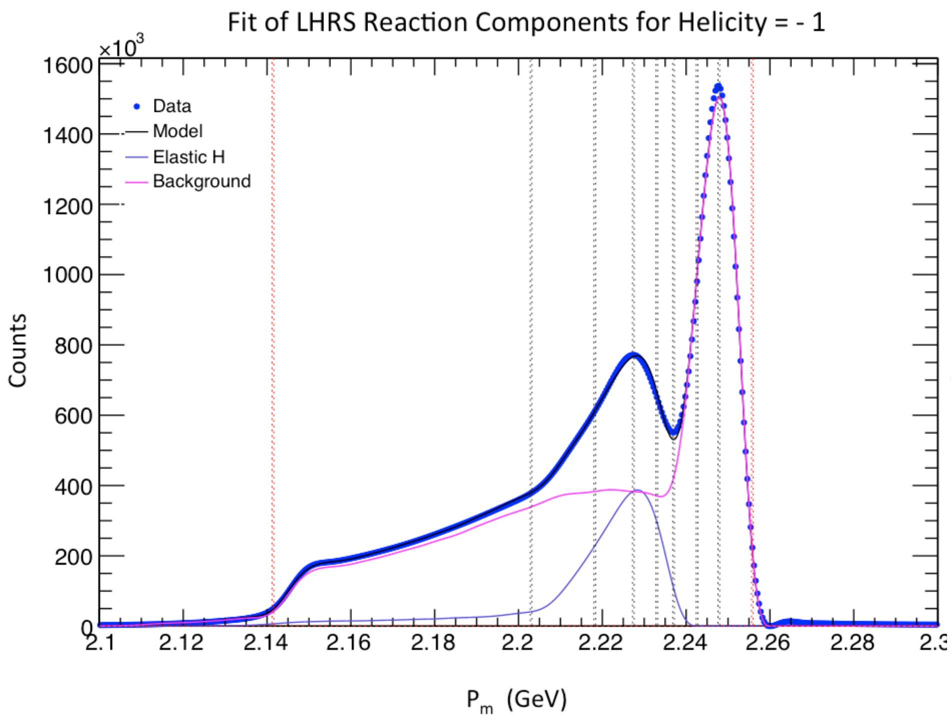


Fit of RHRS Reaction Components for Helicity = - 1
Data
Model
Elastic H
Elastic He
Elastic N
QE He
QE N
Counts
$P_m$ (GeV)

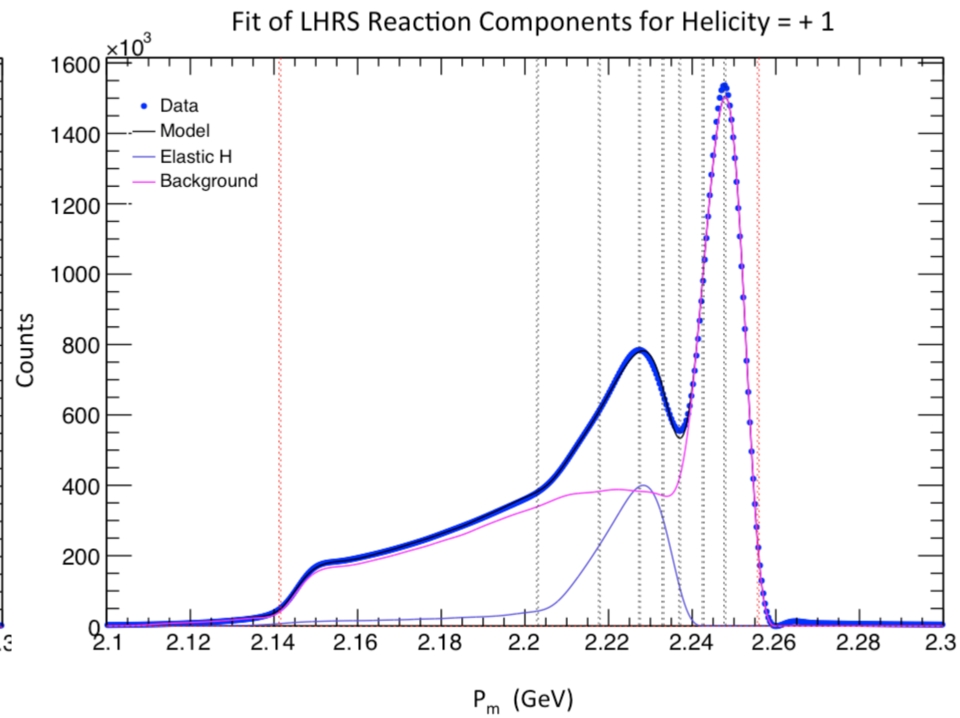



Fit of RHRS Reaction Components for Helicity = + 1
Data
Model
Elastic H
Elastic He
Elastic N
QE He
QE N
Counts
$P_m$ (GeV)

# Fit Results

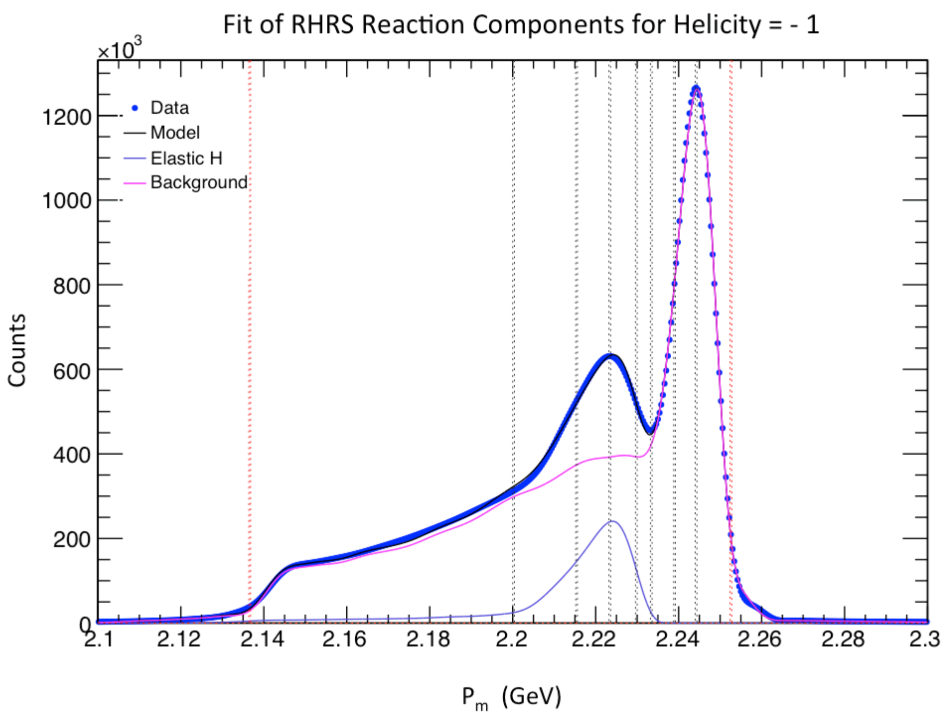


Fit of LHRS Reaction Components for Helicity = - 1
×10^3
Counts
Data
Model
Elastic H
Background
P_m (GeV)

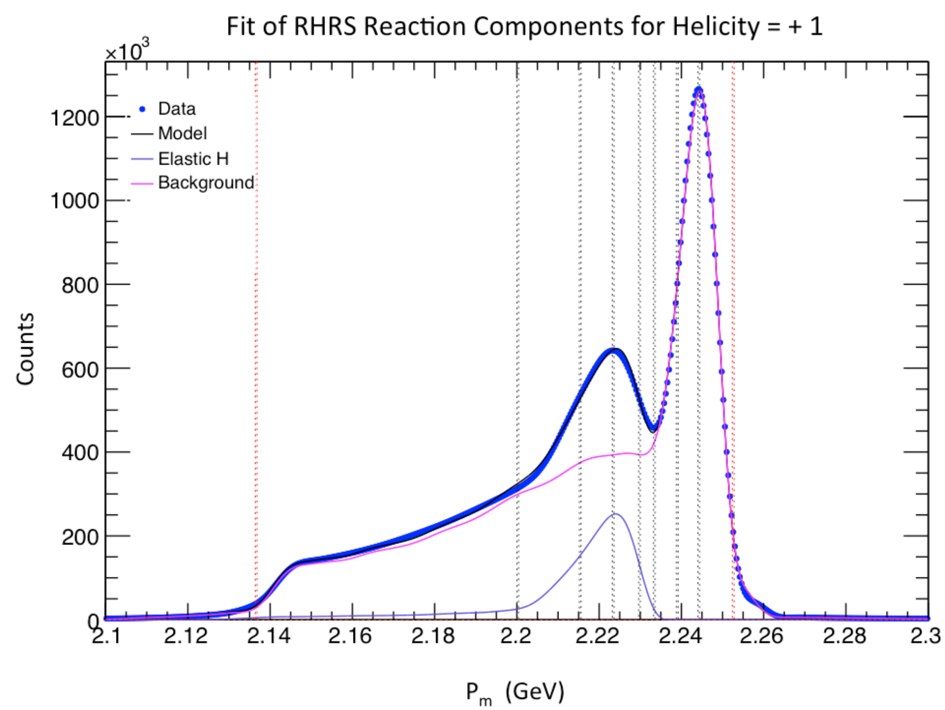



Fit of LHRS Reaction Components for Helicity = + 1
×10^3
Counts
Data
Model
Elastic H
Background
P_m (GeV)

# Fit Results



Fit of RHRS Reaction Components for Helicity = - 1
×10^3
Data
Model
Elastic H
Background
Counts
0
200
400
600
800
1000
1200
2.1
2.12
2.14
2.16
2.18
2.2
2.22
2.24
2.26
2.28
2.3
P_m (GeV)

Fit of RHRS Reaction Components for Helicity = + 1
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Data
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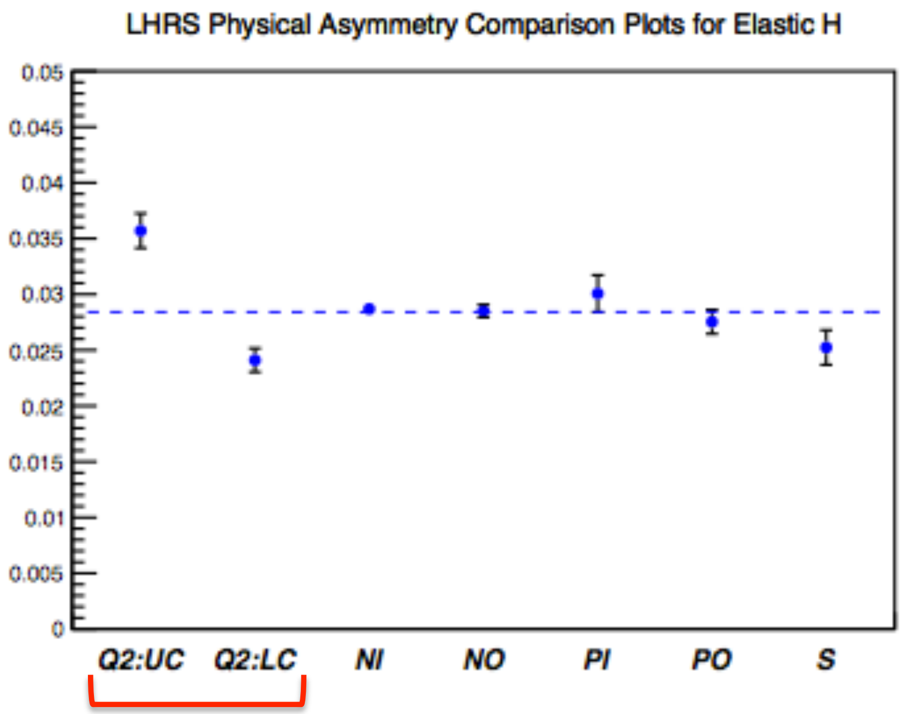
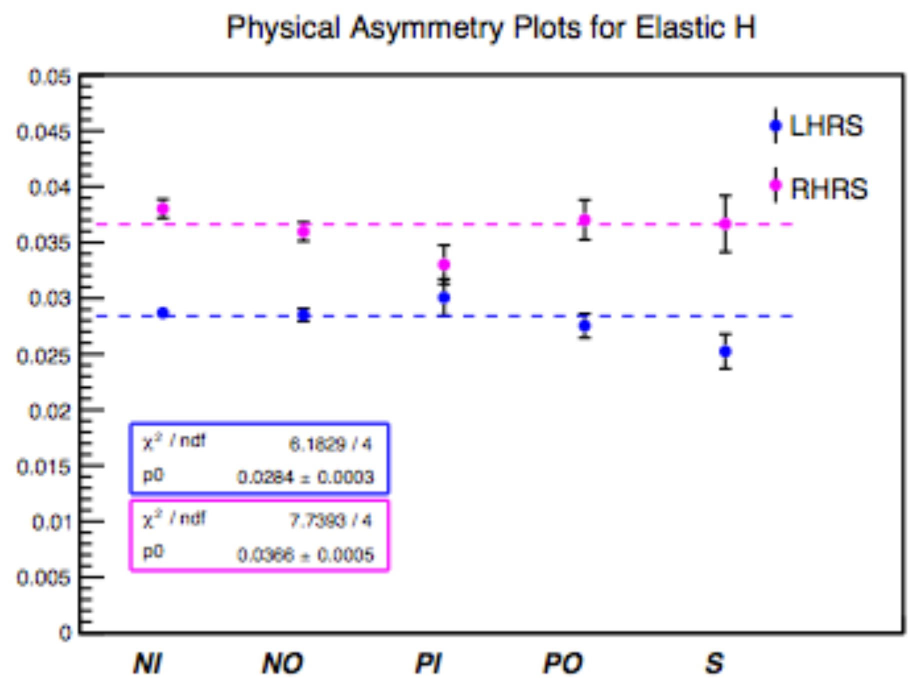
2.3
P_m (GeV)

# Asymmetries




Independent Analysis Asymmetry Results



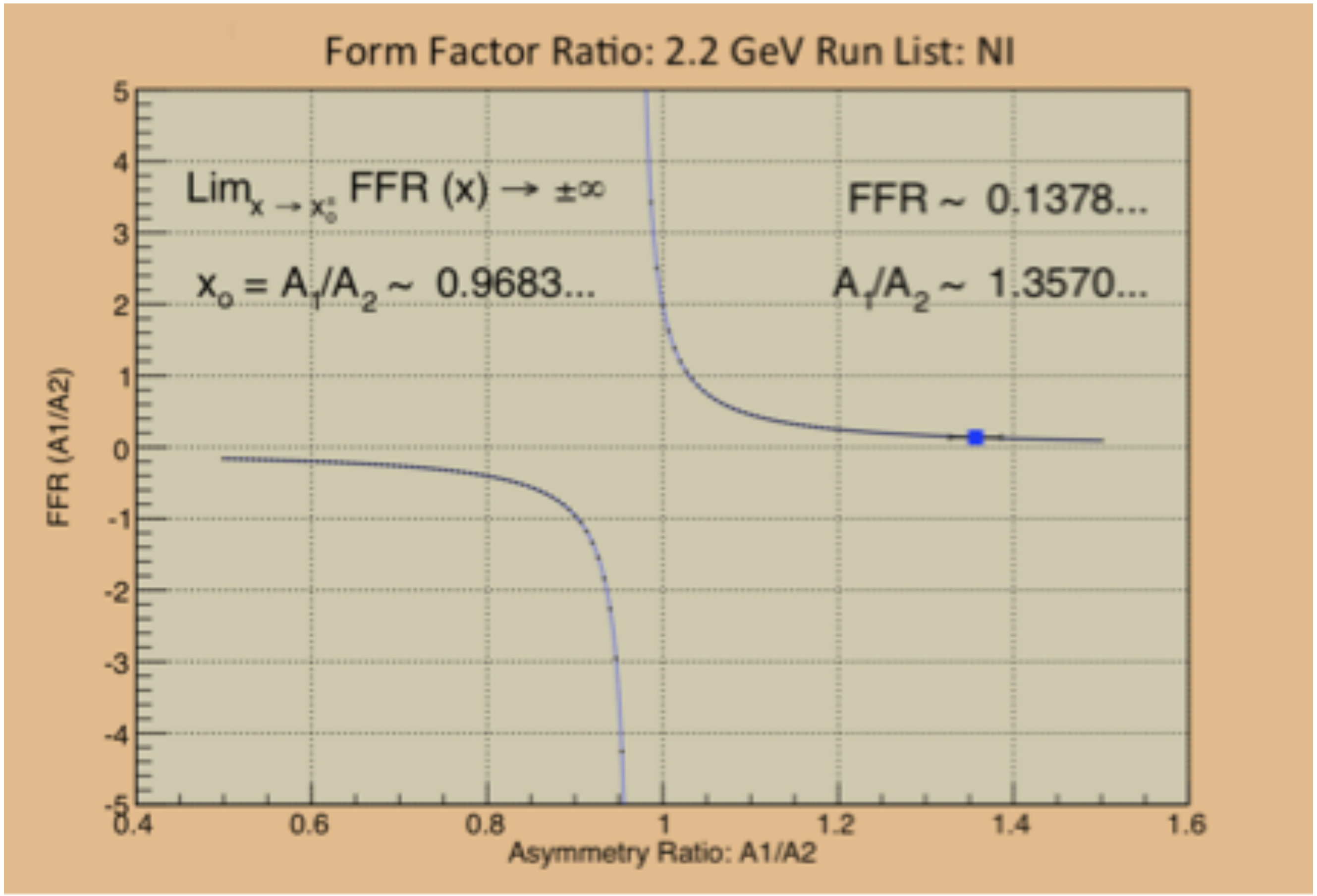
# Results

Able to extract results for 2.2 GeV for LHRS and RHRS NI and NO run lists

- LHRS: self consistent within experimental uncertainty
- RHRS: not self consistent within uncertainty
- DSA: discrepant results

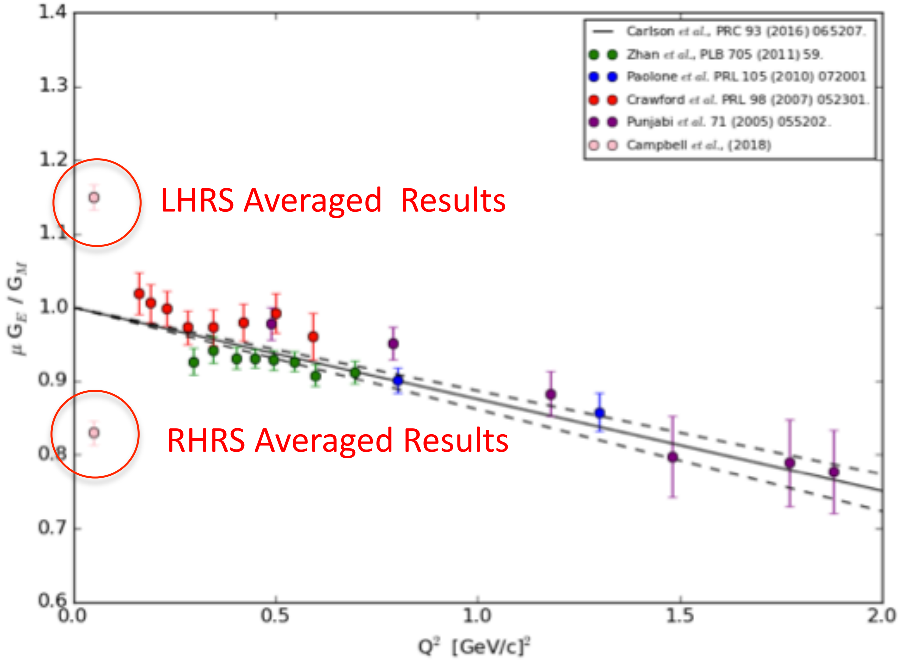
| Energy (GeV) | Method | Run List | FFR | $\Delta$ FFR |
|---|---|---|---|---|
| 2.2 | DSA | NI | 0.138 | 0.012 |
| | | NO | 0.169 | 0.019 |
| | LHRS | NI | 1.142 | 0.023 |
| | | NO | 1.153 | 0.025 |
| | | Avg | 1.147 | 0.017 |
| | RHRS | NI | 0.802 | 0.022 |
| | | NO | 0.860 | 0.025 |
| | | Avg | 0.831 | 0.017 |

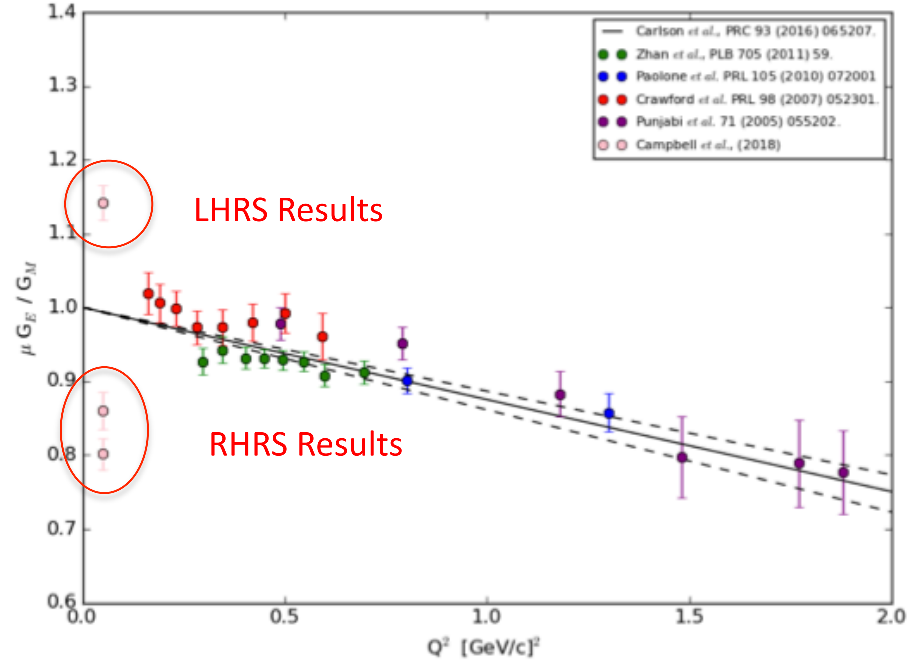
# Results




Kinematics kept constants and ratio of asymmetries is varied to generate this plot

# Results




| Energy (GeV) | Method | Run List | FFR | Δ FFR |
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| | RHRS | NI | 0.802 | 0.022 |
| | | NO | 0.860 | 0.025 |
| | | Avg | 0.831 | 0.017 |

Errors on order of LEDEX and BLAST Experiments

LHRS confirmed with independent analysis

One reliable FFR measurement

# Results

- Conclusions:
  - Able to extract reliable asymmetries using this fitting method
  - Single independent result at lowest attempted $Q^2$
  - Single Arm LHRS and RHRS FFR not in agreement → points to a problem in the results, either:
    - 1. Uncertainties are under estimated or
    - 2. RHRS results unreliable
  - Option 1 less likely than Option 2

- Final Conclusion:
  - Able to produce one reliable FFR result using LHRS where asymmetries were confirmed through an independent analysis
  - New Technique for extracting FFR

# Acknowledgements

Dr. Adam Sarty, Doug Higinbotham, and Moshe Friedman

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