Extracting the Proton Longitudinal Structure Function Moments from World Data

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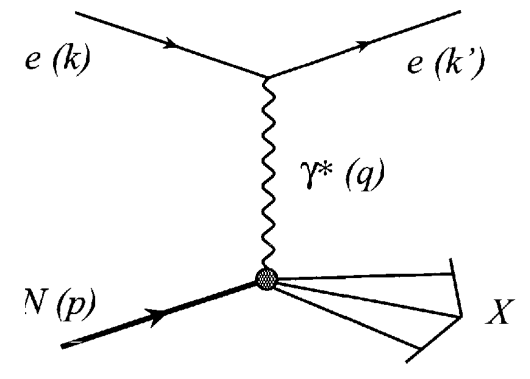
Electron – Nucleon Scattering



- Inclusive cross-section for eN -> eX
- Can be expressed in terms of absorption of transverse and longitudinal photons

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma\left(\sigma_T(x,Q^2) + \epsilon\sigma_L(x,Q^2)\right)$$

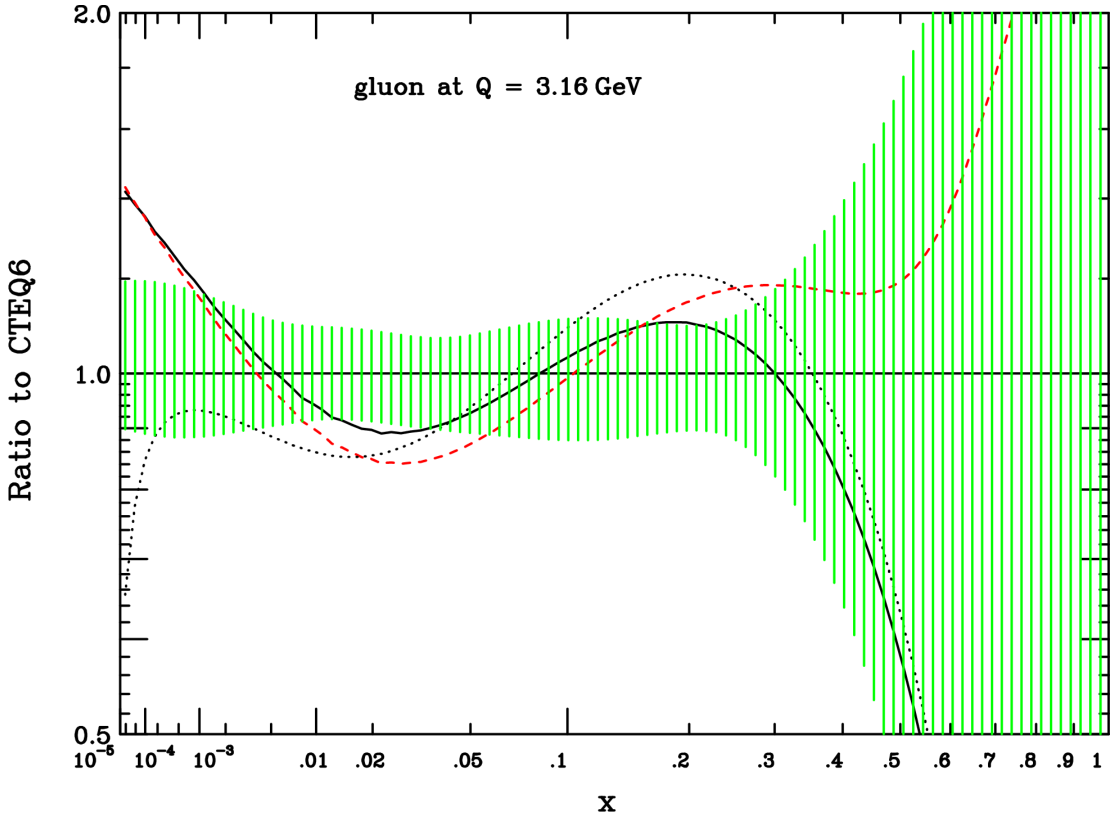
- F1, F2 are structure functions

  ⇒ contain all information about the structure of the nucleon

$$F_1(x,Q^2) = \frac{K}{4\pi^2\alpha} M\sigma_T(x,Q^2)$$

$$F_2(x,Q^2) = \frac{K}{4\pi^2\alpha}\frac{\nu}{(1+\nu^2/Q^2)}\left[\sigma_T(x,Q^2) + \sigma_L(x,Q^2)\right]$$

Gluon Distributions



$$G(x) \approx \frac{d}{d\log Q^2} F_2(x, Q^2)$$

- Gluon distribution sensitive to F_2 through logarithmic evolution in Q^2.
- Large uncertainties in gluon distribution for $x > 0.3$.
- Use F_L instead to directly access the gluon distribution.

Moments of Structure Functions

- Determination of structure function moments allows the transition of QCD from asymptotic to confinement scales to be studied

$$M_{2,L}^{(n)}(Q^2) = \int_0^1 dx\ x^{n-2}\ F_{2,L}(x, Q^2)$$

- Moments of structure functions are their x-weighted integrals

  ⇒ allow Q^2 dependence to be studied

- Higher moments are weighted towards higher x-values

  ⇒ poorly determined

- At large x, cross-sections are small, so resulting extraction of gluon density becomes increasingly difficult

  ⇒ large uncertainties in gluon density

This Analysis of Longitudinal Moments

- F_L sensitive to gluon distribution at Next-to-Leading Order
- F_L also sensitive to power corrections in Q^2
- Previous study by Ricco, Simula and Battaglieri (Nucl. Phys. **B555**, 306-334, 1999)

⇒ little data at low Q^2 and high x

⇒ "... transverse data with better quality at $x > 0.6$ and $Q^2 < 10$ $(GeV/c)^2$ and more precise, systematic determinations of the L/T cross-section ratios are still required ...."

- New **cross section** data available from JLab (at high x and low Q^2) and HERA (low x)

⇒ high precision measurements, from dedicated experiments

⇒ **DATA driven analysis**

Nachtmann Moments

- Nachtmann moments, defined in terms of ξ, removes target mass corrections ~ M^2/Q^2

$$M_L^{N(n)}(Q^2) = \int_0^1 dx\, \frac{\xi^{n+1}}{x^3} \left\{ F_L(x,Q^2) + \frac{4M^2x^2}{Q^2} \frac{(n+1)\xi/x - 2(n+2)}{(n+2)(n+3)} F_2(x,Q^2) \right\}$$

$$M_2^{N(n)}(Q^2) = \int_0^1 dx\, \frac{\xi^{n+1}}{x^3} \left\{ \frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right\} F_2(x,Q^2)$$

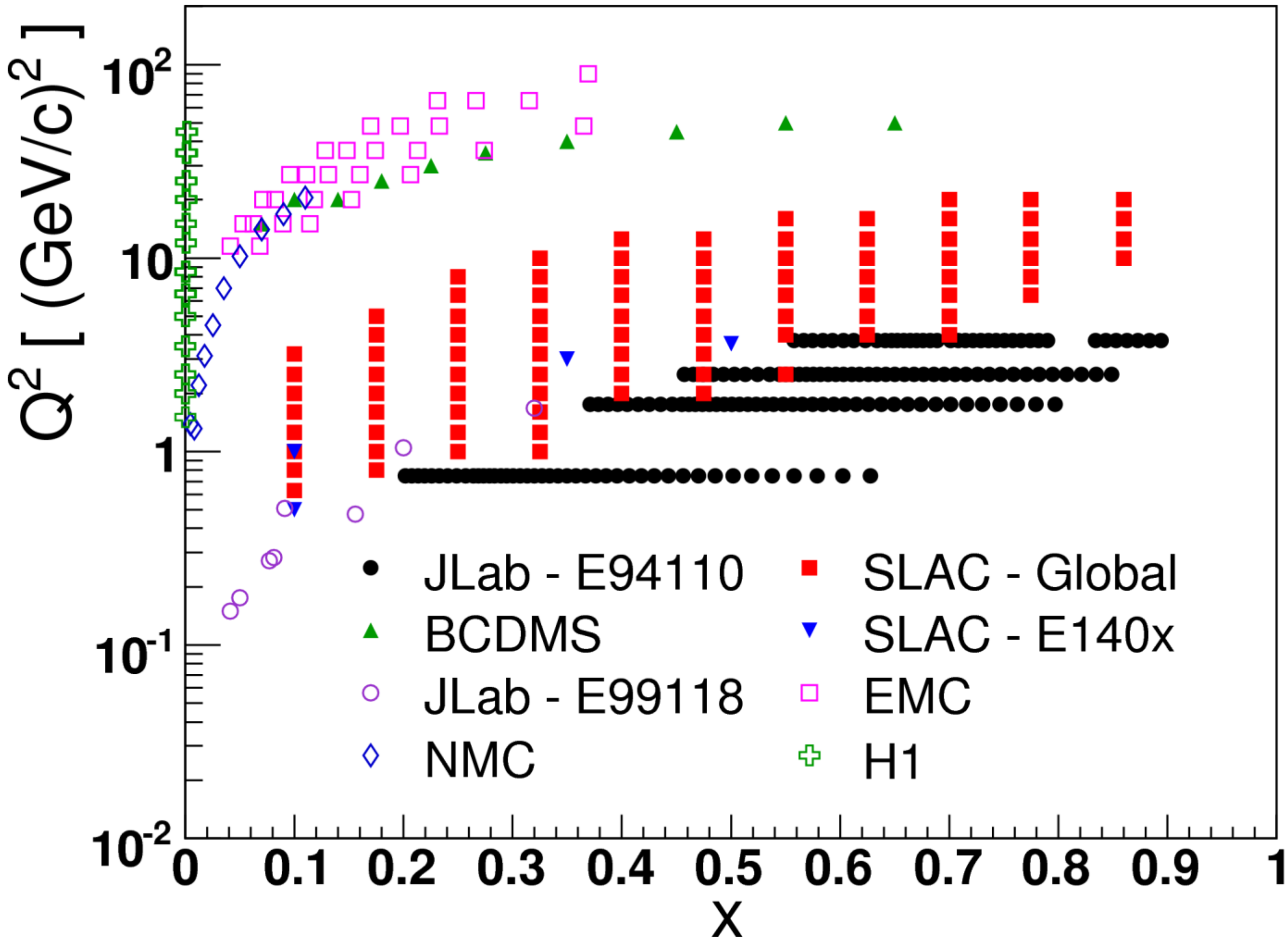
$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}} \qquad r = \sqrt{1+4M^2x^2/Q^2}$$

- Nachtmann moments from experiment are compared to Cornwall-Norton moments from (leading twist, M=0) pQCD calculations

  ⇒ are higher twist components important?

  ⇒ is the gluon contribution in the leading twist calculation sufficient?

Data Coverage in Q^2 and x



- Only using L/T separated data

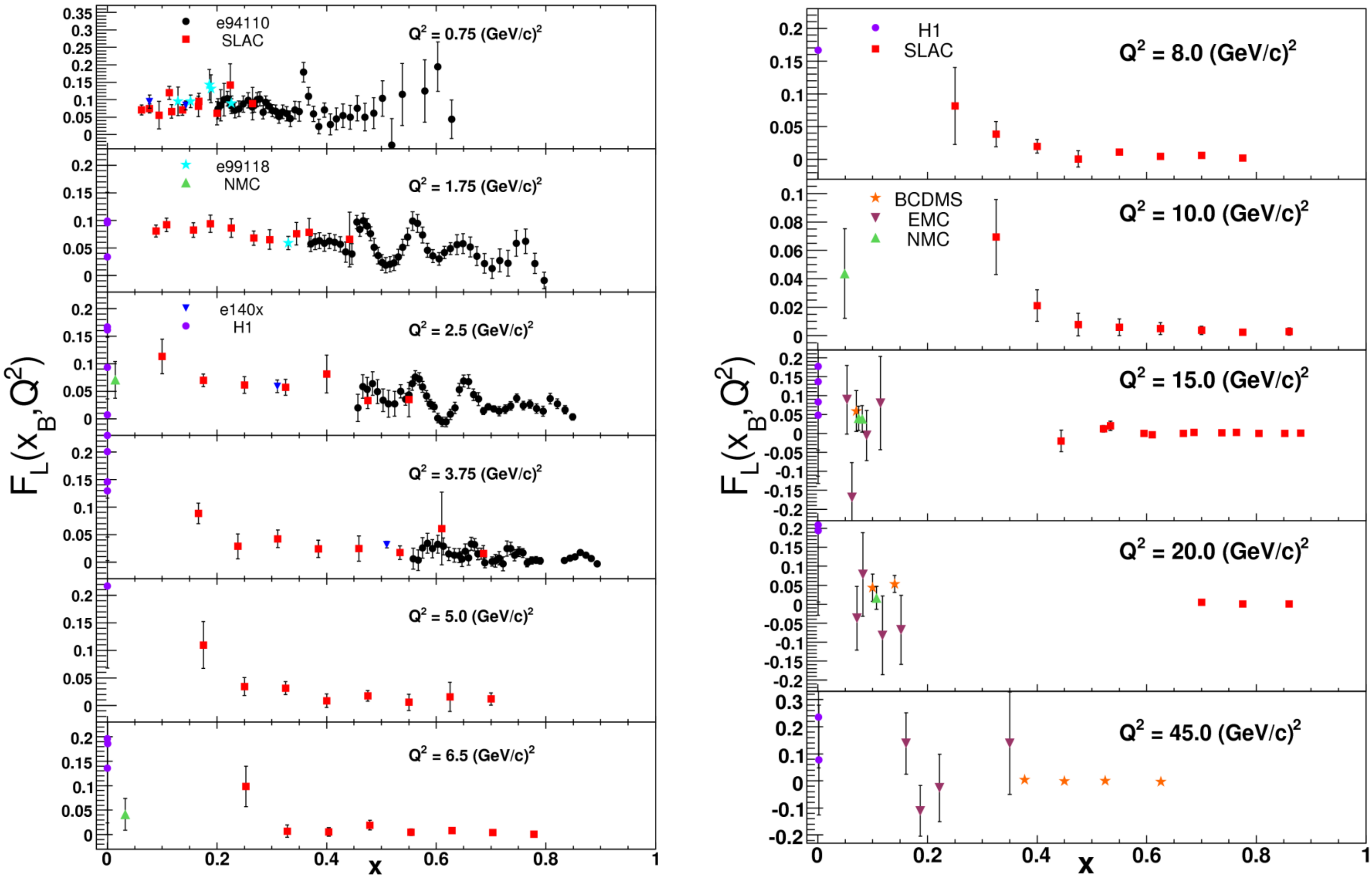
⇒ cross section data

- Proton data only

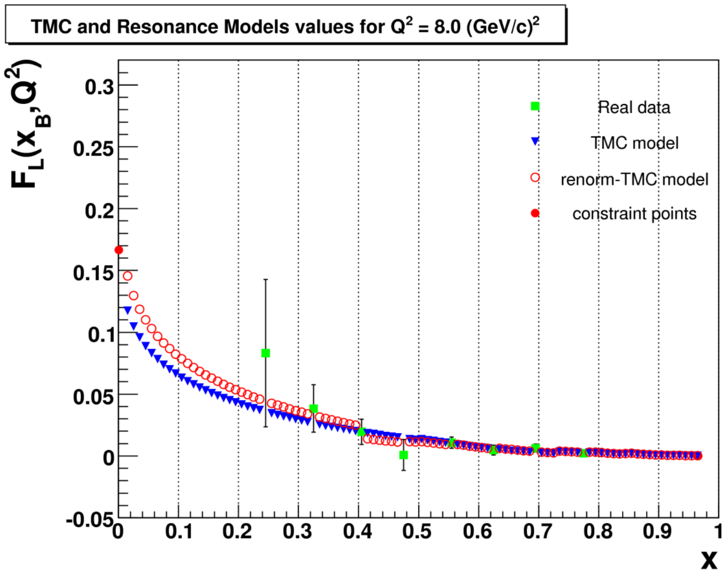
- JLab data covers region with higher x and lower Q^2

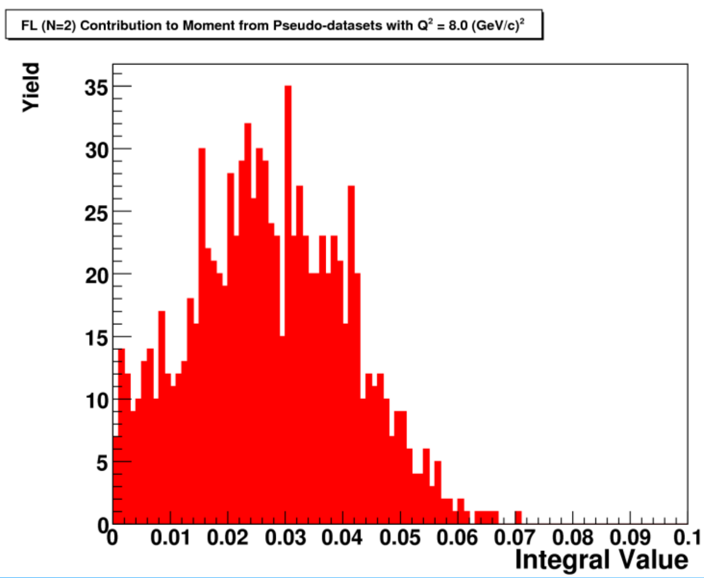
- for $Q^2 < 4$, JLab data covers ~50% of x range

Bin-center F_L Data in Q^2



Analysis Method and Error Estimation



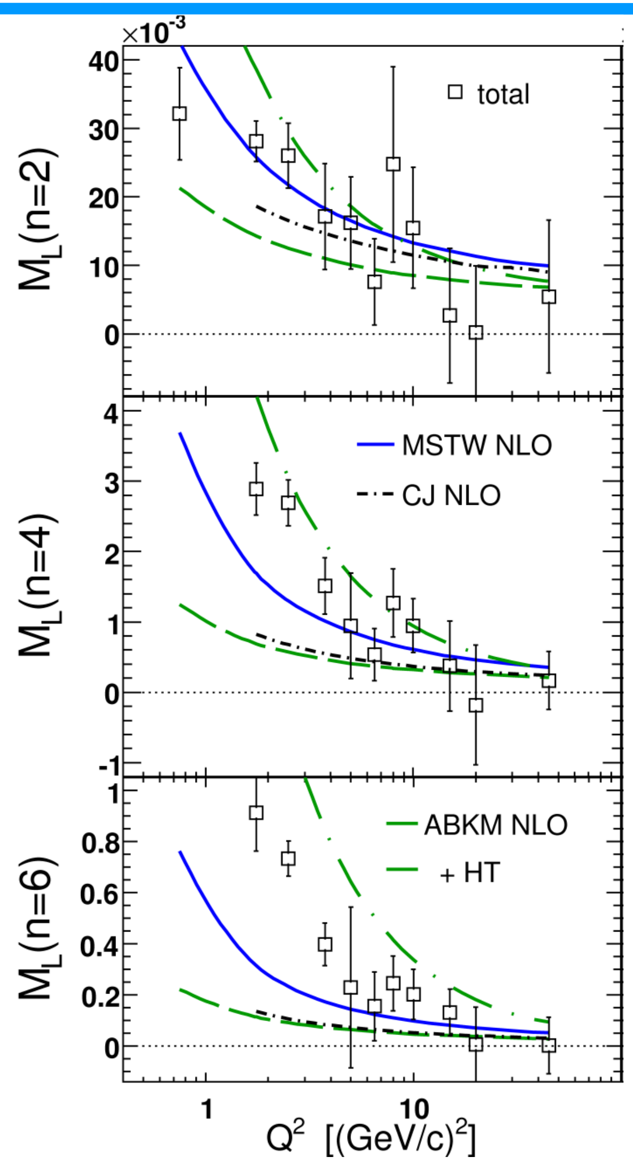


- Use model calculations in empty bins
- Apply rescale factor to model based on error weighted average of adjacent data points
- Integrate to generate moment contribution

- Use Monte Carlo method to estimate uncorrelated errors in data
- Generate pseudo-data via gaussian randomisation of data within error bars
  - ⇒ distribution of moment contributions
  - ⇒ derive statistical error from standard deviation of moment distributions

- Model dependent error estimated via analysis using different models ⇒ **small**

Nachtmann Longitudinal M_L Moments



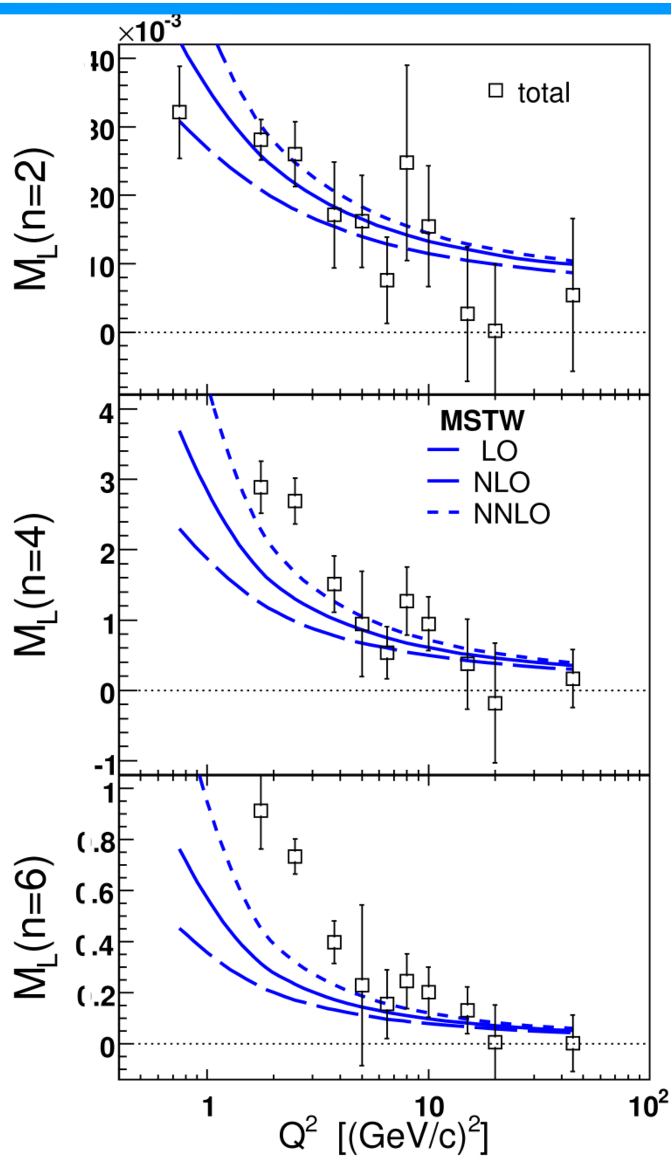
- Comparing data to global PDF fits
- Higher twist appears to improve the fit
- Observe missing strength in highest moment – largest weighting by high x

⇒ require larger gluon contribution at large x?

⇒ higher twist effects?

- MSTW excludes high x data
- CJ includes high x data, but not F_L data directly (HT not available)
- ABKM includes higher twist terms but fits to a subset of the data

Nachtmann Longitudinal M_L Moments



- Comparing different orders of only the MSTW calculation to data
- Higher order calculations in better agreement with data – NNLO best

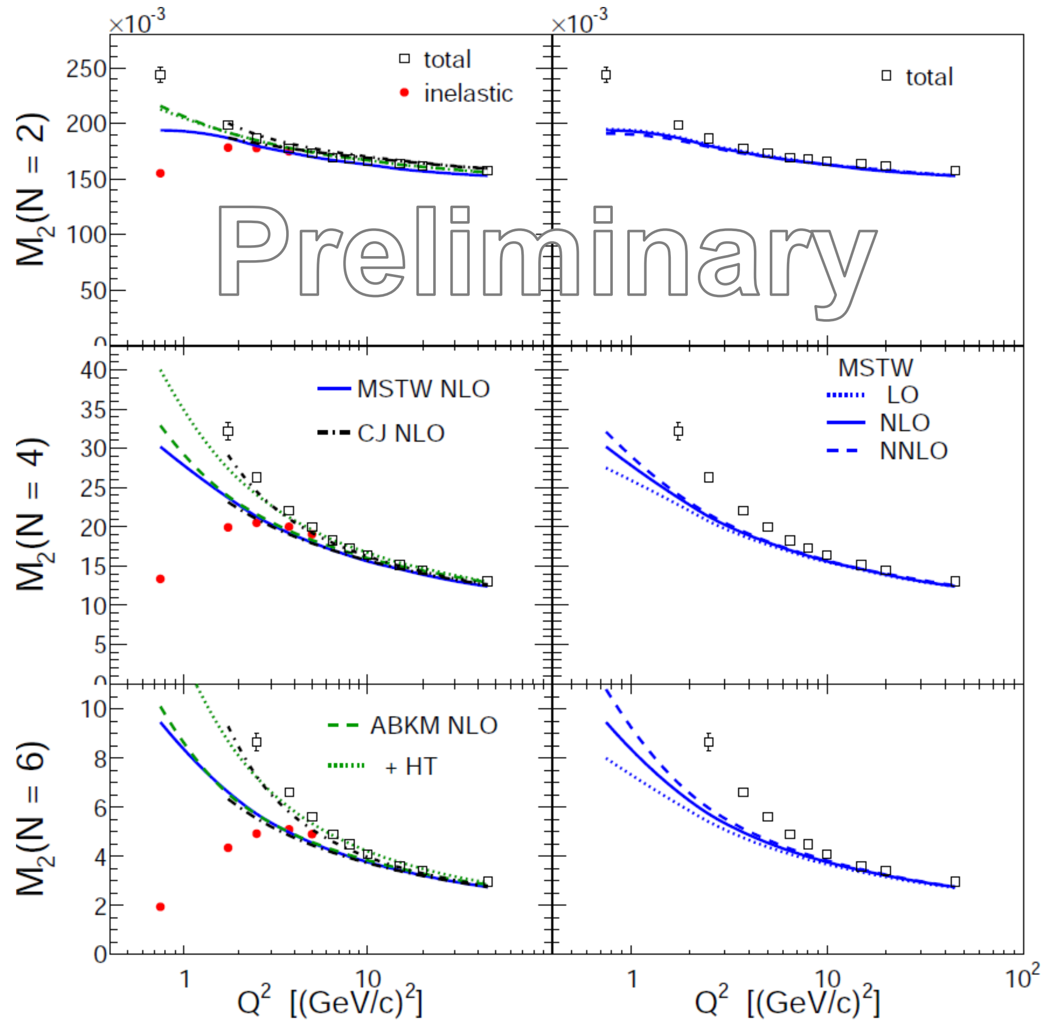
⇒ perhaps no HT contributions needed

- Highest moment curves all undershoot the data

⇒ perhaps a larger gluon contribution at high x

- Need improved global fits to disentangle different effects

Nachtmann M_2 Moments



- Comparison with same PDF calculations as for M_L case
- Including higher twist appears more effective that higher orders

Looking Ahead

- Opportunity to study Gottfried Sum Rule

$$I_G(Q^2, x) = \int_0^1 \frac{dx}{x} \left[F_2^p(x, Q^2) - F_2^n(x, Q^2)\right]$$

- High x neutron data available from JLab
  - BoNuS experiment – Phys. Rev. Lett. **108**, 142001 (2012)
- Available data to cover larger x range than previous evaluations
- Investigate any Q^2 dependence of the sum rule

Summary and Outlook

- First **data driven** extraction of longitudinal Nachtmann moments from data published in **Phys. Rev. Lett. 110, 152002 (2013)**
- Error bars on the data drive larger errors in the extracted moments

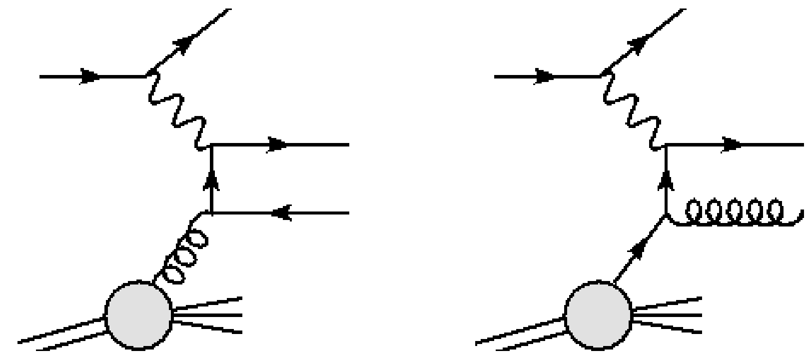
  ⇒ more experimental data will improve the statistics and fill gaps in data!

  ⇒ JLab @ 12 GeV : higher precision data at moderate to high x
- Comparison with global PDF fits shows either higher twist terms becoming more important or a larger gluon contribution at large x or both!
- Intend to include F_L data in the CJ fit to separate the gluon and higher twist contributions.
- Evaluating the M2 moments and comparing with PDF calculations
- With new JLab high x neutron data can study the Gottfried Sum Rule

Extra Slides

Longitudinal Structure Function, F_L

- Next-to-Leading Order (NLO), gluons contribute to both F_2 and F_L.

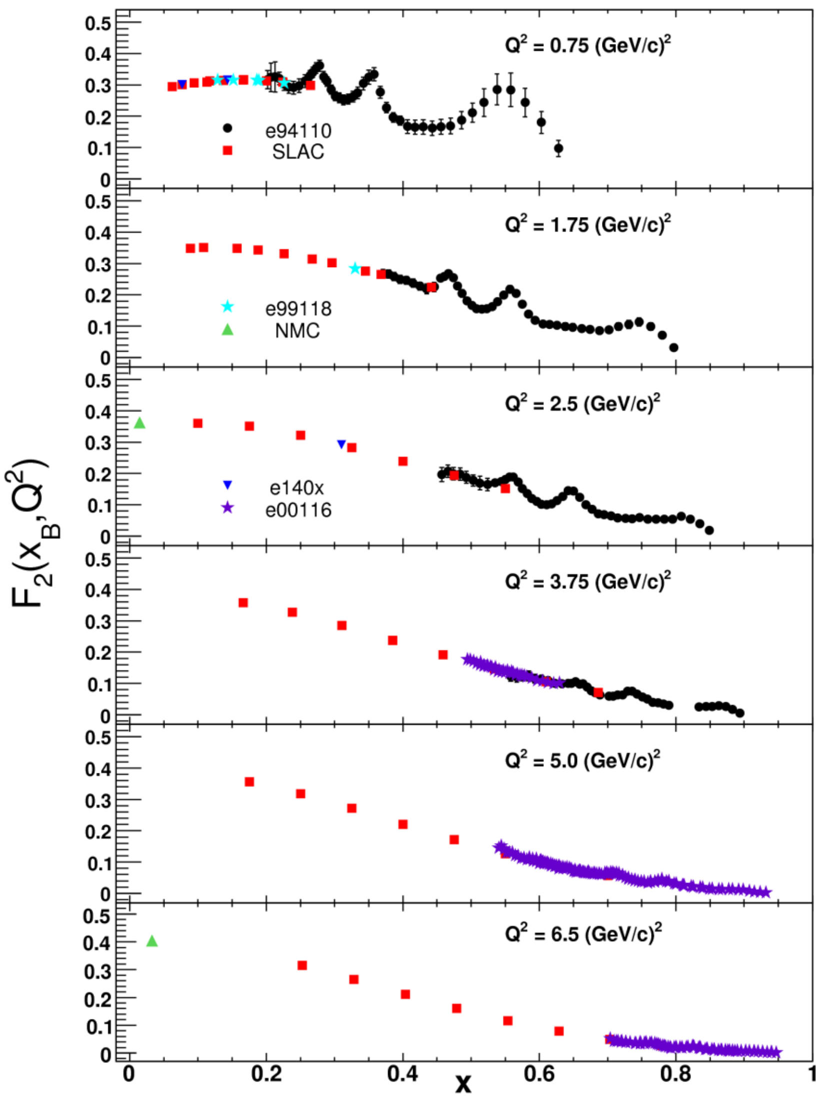


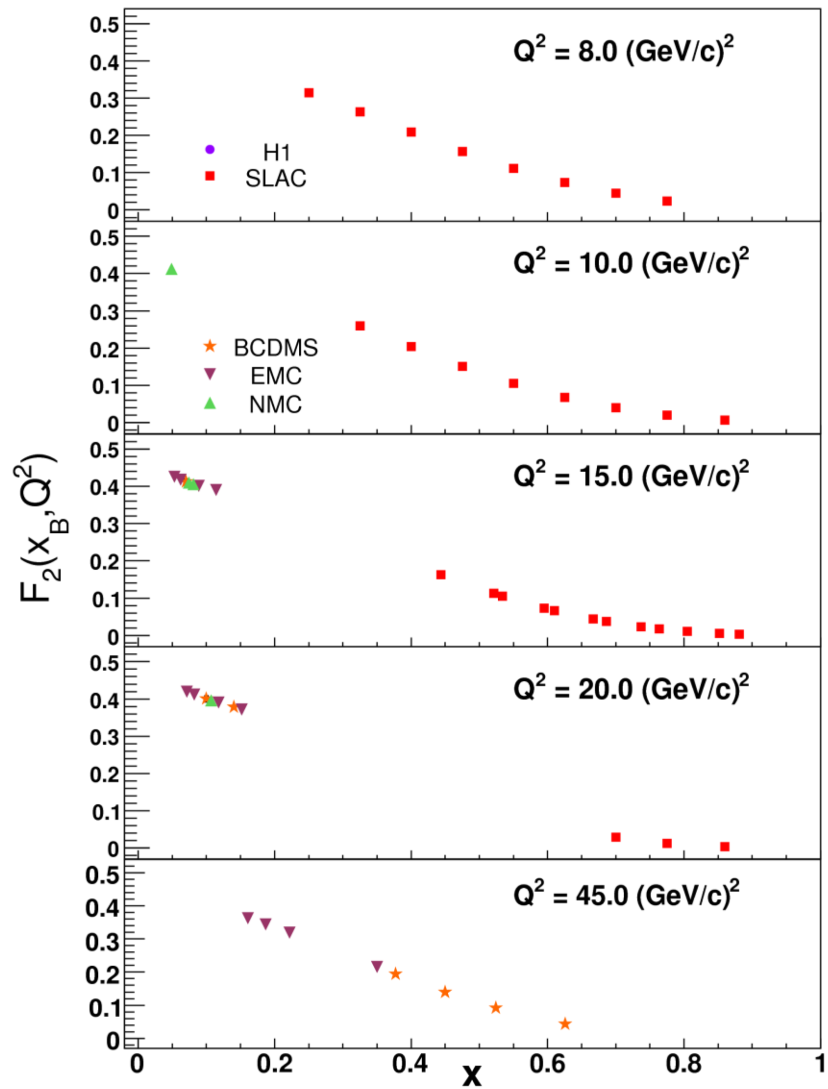
- Obtain a ***gluon sum rule***.

$$F_L(x) = \frac{\alpha_s}{\pi} \int_x^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left\{ \frac{4}{3} F_2(y) + 2c(n_f) \left(1 - \frac{x}{y}\right) yG(y) \right\}$$

- At **leading twist** F_L is directly sensitive to gluons.

Similarly, bin-center F_2 Data in Q^2



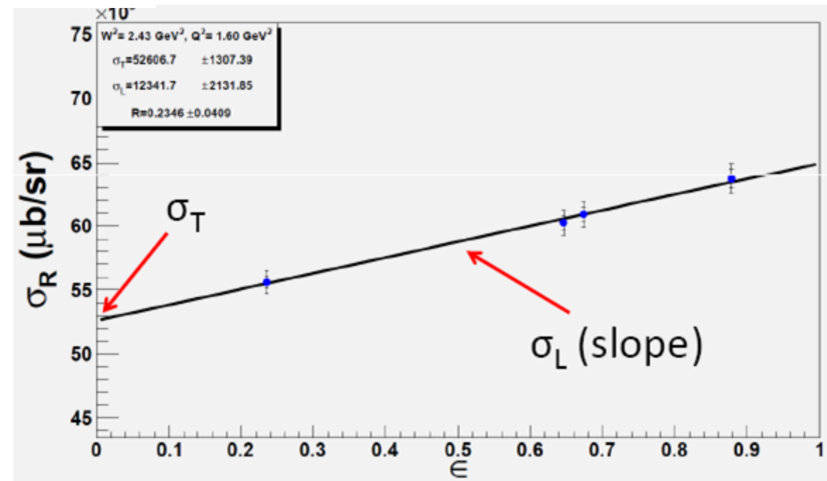


Measuring the Longitudinal Structure Function

$$\sigma_R = \frac{1}{\Gamma}\frac{d^2\sigma}{d\Omega dE'} = \sigma_T(x,Q^2) + \epsilon\sigma_L(x,Q^2)$$

$$\sigma_T \propto F_1 \qquad\qquad \sigma_L \propto F_L$$

$$F_L = \left(1 + \frac{Q^2}{\nu^2}\right) F_2 - 2xF_1$$



- Determine F_L through a Rosenbluth separation of the cross-section
- Require data measured at fixed Q^2 and x, at multiple ε points

  ⇒ need **multiple** beam energies and spectrometer settings
- F_L ~ 25% of cross-section for JLab kinematics σ_T and σ_L

  ⇒ require < 2% uncertainty (pt-to-pt) in ε to extract F_L to ~ 15%

Moments Expansion and *Twist*

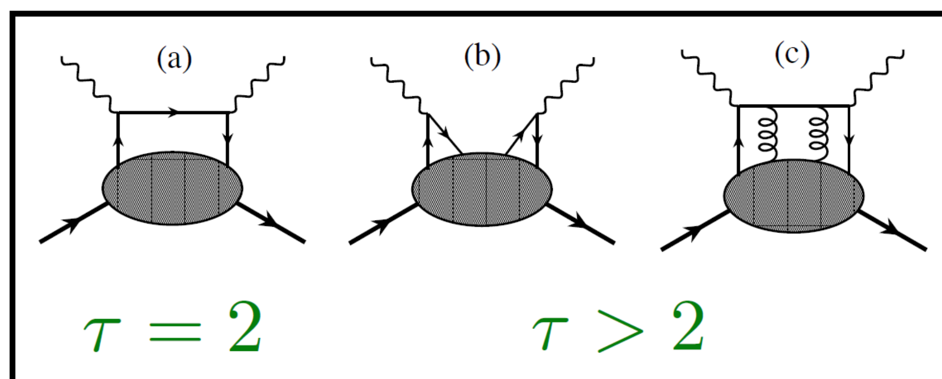
- In the Operator Product Expansion (OPE), moments can be expanded in powers of $1/Q^2$

$$M_L^{(n)}(Q^2) = \sum_\tau \frac{A_\tau^{(n)}(\alpha_s(Q^2))}{Q^{\tau-2}}$$

$$= A_2^{(n)} + \frac{A_4^{(n)}}{Q^2} + \frac{A_6^{(n)}}{Q^4} + \cdots$$

matrix elements of operators with a specific “twist” τ

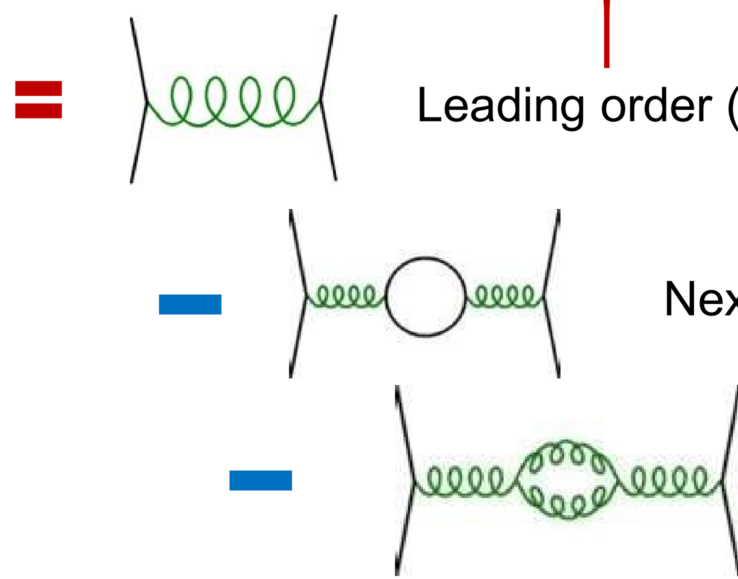
τ = dimension - spin



Strong Coupling Constant, α_s Expansion

- In QCD, α_s is a running coupling constant, dependent on Q^2, number of quark flavors and mass scale Λ

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln[\ln(Q^2/\Lambda^2)]}{\ln(Q^2/\Lambda^2)} + (\cdots) \right\}$$

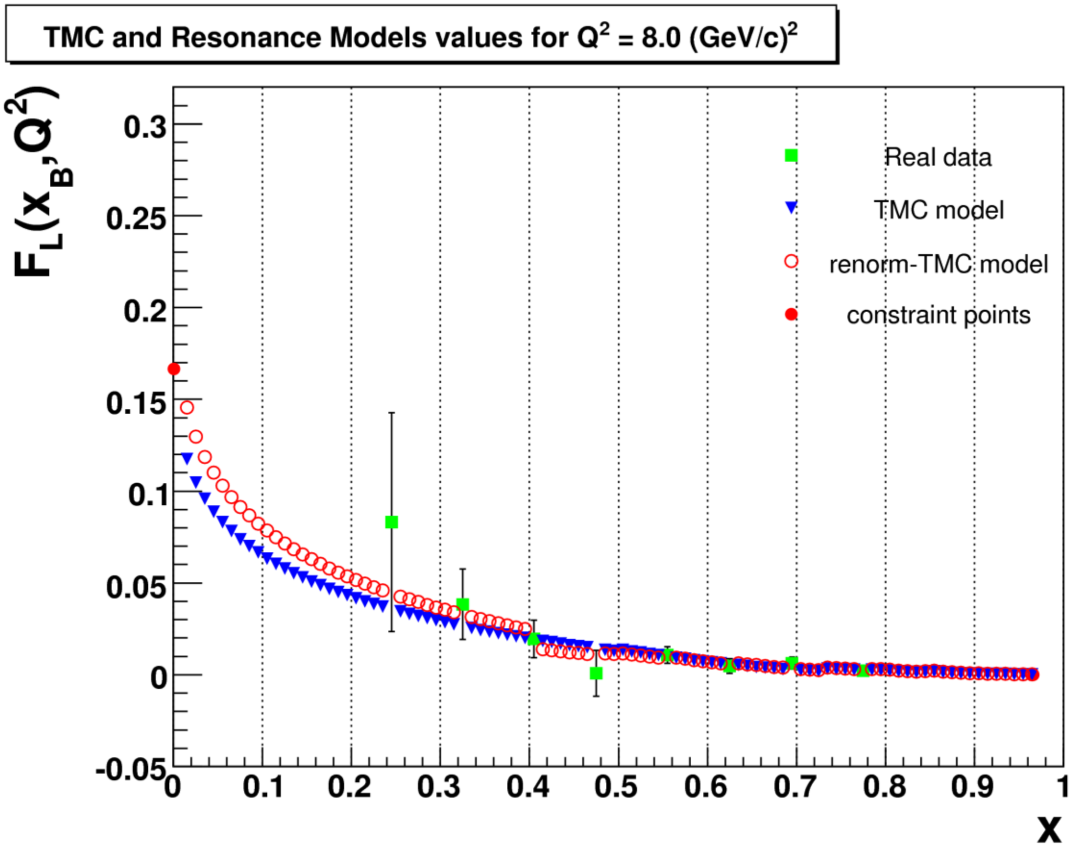


Leading order (LO)

Next-to-Leading order (NLO)

Next-to-Next-to-Leading order (NNLO)

Filling the Gaps in the Data



- Some bins with no data
- Use model calculations in empty bins

  DIS : $W^2 > 3.9$ GeV^2

  Resonance : $W^2 < 3.9$ GeV^2

⇒ apply rescale factor based on the error weighted average of adjacent data points

⇒ for x<0.4, use all data points to determine the rescale factor

DIS model : M. E. Christy, J. Blumlein and H. Bottcher (2012), hep-ph/1201.0576 ⇒ “TMC model”
Resonance model : Y. Liang, Ph. D. thesis, The American University (2003) ⇒ “Liang model”

Error Estimation using Monte Carlo Technique

- Calculate moment by integrating data from x = 0.01 – pion threshold
- For each data point, generate a random number within its error bar

  ⇒ generate a complete pseudo-dataset
- Fill in gaps in the pseudo-dataset with the same models
- Integrate to generate moment for that pseudo-dataset
- Repeat 1000 times

  ⇒ obtain a distribution of moments from the pseudo-datasets
- Repeat process for F_2
- Obtain the mean and standard deviation of each distribution of moments

Define data point : $$M_L^{N(n)}(Q^2) = \overline{i_{F_L}^{(n)}} + \overline{i_{F_2}^{(n)}}$$

Define error bar : $$\delta M_L^{N(n)} = \sqrt{(\delta i_{F_L})^2 + (\delta i_{F_2})^2}$$

Model Dependent Error Estimate

- Other DIS and resonance region models available
  - ⇒ DIS: R1990 and ALLM parameterisation

    see references: H. Abramowicz & A. Levy (1997), hep-ph/9712415
    L. W. Whitlow, Ph. D. Thesis, Stanford University (1990), SLAC-0357
  - ⇒ Resonance model: C-B fit

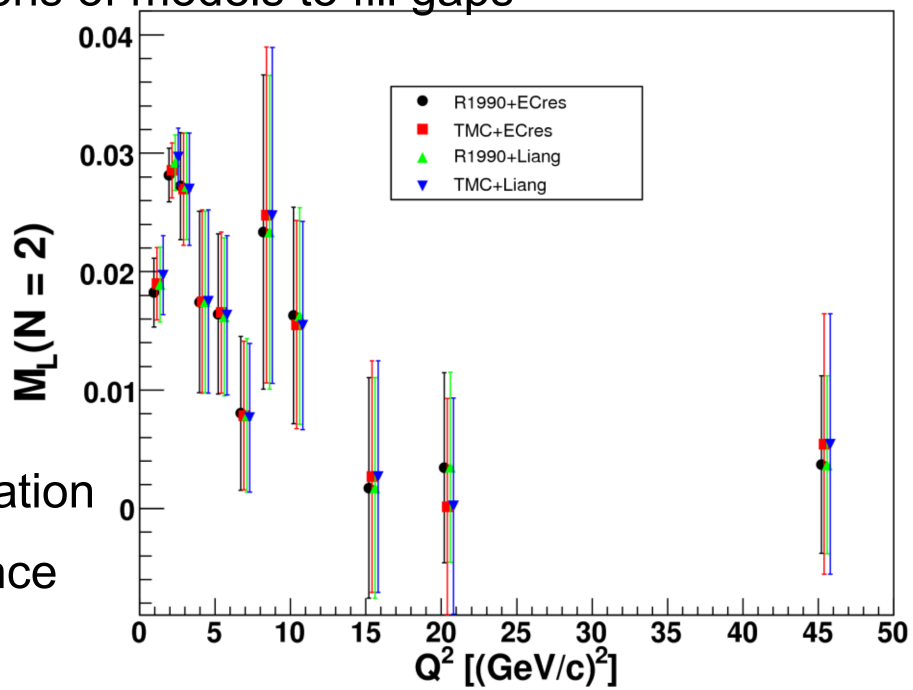
    see reference: M. E. Christy & P. E. Bosted, Phys. Rev. C **81**, 055213 (2010)
- Evaluate four possible combinations of models to fill gaps

  i. TMC + Liang ⇒ ideal case

  ii. TMC + C-B

  iii. R1990 + Liang

  iv. R1990 + C-B



- Repeat analysis for each combination
- Define error as maximum difference from the ideal case