If There's a Unique Neutral Axis, How Do We Find It?

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Abstract

The expressions Jim Birchall recently obtained (in a 2nd order approximation of the Qweak experimental response including limited symmetry breaking) suggest a prescription for quickly finding the neutral axis, $(x_0,\Theta_0,y_0,\Phi_0)$. Severe symmetry breaking can make the neutral axis inaccessible if it lies beyond the range that we can safely steer the beam when rastered to 4mm x 4mm. However, in that case, analysis of the data taken during the search for the neutral axis may guide us in improving the symmetry of the experiment and so push the neutral axis back into an accessible region of beam position and angle.

1. Introduction

Omitting the terms related to beam size changes, Jim's expression for the false asymmetry in 2nd order for only the x-like variables is[1]

$$\varepsilon = (2bx_0 + d^*\Theta_0 + e)\delta x + (2c\Theta_0 + d^*x_0 + f)\delta\Theta$$

where the *e* and *f* terms implicitly contain one form of symmetry breaking, a departure from a perfectly azimuthal acceptance. If we want to make both the position and angle sensitivities vanish, then we have the independent constraints

$$2bx_0 + d^*\Theta_0 + e = 0$$

and

$$2c\Theta_0 + d^*x_0 + f = 0\ .$$

Mathematically, these are just the formulas for lines in the (Θ_0,x_0) plane:

$$x_0 = -d/(2b)^*\Theta_0 - e/(2b)$$

which is the locus of points yielding a vanishing sensitivity to δx, and

$$x_0 = -2c/d^*\Theta_0 - f/d$$

[1] See Jim's report for notation, "Updated Requirements on Beam Properties for Qweak", J. Birchall, June 2008. It should be understood that these expressions need to be generalized from x→y and Θ→Φ.

which is the locus of points with a vanishing sensitivity to $\delta\Theta$. Fortunately, we don't have to choose between having zero position sensitivity and zero angle sensitivity. As long as the slopes of the two lines aren't identical, they will intersect at a unique neutral axis given by

$$\Theta_0^{neut} = (2bf - ed)/(d^2 - 4bc)$$

$$X_0^{neut} = (2ce - fd)/(d^2 - 4bc)$$

The above formula is one main result of this report. Hence, to determine the x-like neutral axis we effectively have to determine Jim's 5 parameters *b, c, d, e*, and *f*. If the symmetry breaking parameters *e* and *f* are both 0, then the x-like neutral axis in this model will be at (Θ_0 = 0, X_0 = 0). If the symmetry breaking parameters *e* and *f* are *not* 0, then the x-like neutral axis will still exist. However, if *e* and *f* are too large, the intersection point may be outside the range that we can safely steer the beam. Then we would be faced with living with the resulting sensitivities, or making changes to the detector positions or collimators to improve the symmetry. For example, when I naively plug in the central values from Jim's report setting the azimuthal distortion parameter x_{bar} to 0.5cm, then the x-like neutral axis is found at (Θ_0, x_0) = (-1 mrad, 2.4 mm)[2]. I believe the MCC operators could safely steer the beam to that location as long as the raster size were not dramatically increased. So while the neutral axis would be accessible, it would be cramping our style.

2. Neutral Axis Searches

If Jim's model is close enough to reality, then we should be able to predict the neutral x-like axis with only 5 linearly independent measurements, either:

i. 2 measurements of the x_0 position sensitivity (parameters b, d, and e), plus 3 measurements of the Θ_0 angle sensitivity (parameters c, d, and f), **or**
ii. 3 measurements of the x_0 position sensitivity, plus 2 measurements of the Θ_0 angle sensitivity.

Since the y-like variables also require another 5 linearly independent measurements, a minimum of 10 carefully planned measurements are needed to determine the neutral axis (Θ_0 , x_0 , Φ_0 , y_0) according to the formula in the preceding section. One simply has to solve a system of 5 linear equations in 5 variables (twice), so I won't elaborate.[3] Note, that this procedure is almost guaranteed to produce a result even if the model is too simple, and since

[2] One must bear in mind however that Jim's fitted b term is currently so small as to be indistinguishable from statistical noise.

[3] I've implicitly been assuming that there are no cross terms between x-like and y-like variables. Well, when Jim starts to worry about it, then I'll start to worry about it.

nothing is over-determined there would be no residuals and hence no error estimates. So I think we need to take additional measurements to test the model and estimate errors.

The following series of 8 measurements should allow determination of the x-like neutral axis while permitting cross-checks:

Measurement No.	Θ_0	X_0 (cm)	Modulation variable
1,5	+0.0057 deg (+0.1 mrad)	+0.1	X, Θ
2,6	“”	-0.1	“”,“”
3,7	-0.0057 deg (-0.1 mrad)	+0.1	“”,“”
4,8	“”	-0.1	“”,“”

Together with similar measurements for the y-like variables, a total of 16 measurements is suggested. The beam time required does not appear to be excessive. Assuming the nominal dynamic range of the planned downstream beam modulation system (+-50 µm in position, +-50 µrad in angle), at the nominal Qweak luminosity we could make 16 measurements of the sensitivity with 10% statistical error in about 40 minutes. Taking into account the need for MCC operators to resteer the beam, plus beam trips, it appears a realistic search for the neutral axis without automated procedures could still be completed in a single shift. That’s good news, because we’ll probably have to do it a handful of times.

Summary

Jim’s recent update on sensitivities suggests that we can find the neutral axis (Θ_0 ,x_0 ,Φ_0 ,y_0) in only 10 measurements, but I suggest more (16?) in order to test the model and determine residuals. The time required does not appear to be excessive. If the data suggest that the model is not complete, then we can either make the model more sophisticated, or improve the symmetry of the experiment.