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Crystal Ball experiment on $\gamma\gamma\to\pi^0\pi^0$ and neutral pion polarizability

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We obtain the most general and simple unitary formulae for the amplitudes $\gamma\gamma\to\pi\pi$ near threshold which are parameterized using the pion polarizability. Describing the Crystal Ball data [Crystal Ball Collab., H. Marsiske et al., Phys. Rev. D 41 (1990) 3324] $\gamma\gamma\to\pi^0\pi^0$ by means of them we get $(\alpha-\beta)^{\pi^0}=(1.0\pm2.5)\times10^{-42}$ cm^3 for the π^0 polarizability.

1. Recently, the first data of the Crystal Ball Collaboration has been published [1] on the $\gamma\gamma\to\pi^0\pi^0$ cross section in a wide energy region, beginning at threshold [#1]. We would like to point out that additional information may be obtained from this data, concerning certain low-energy parameters – the so-called pion polarizabilities [3]. Polarizabilities of π^+ have already been measured [4,5] in experiment; however, for π^0, this is the first possibility to obtain such information.

Recall that hadron polarizabilities in the formulae for the Compton amplitudes define the corrections to the classical Thompson limit and play the role of threshold structure constants like the electromagnetic radius in a form factor. More exactly: in the laboratory system at the initial frequency $\omega\to0$ the amplitude is of the form

$$T(\gamma\pi\to\gamma\pi)=-2e^2(\boldsymbol{e}\,\boldsymbol{e}')+2\mu\{\alpha_\pi\omega\omega'(\boldsymbol{e}\,\boldsymbol{e}')+\beta_\pi[\boldsymbol{K}\boldsymbol{e}][\boldsymbol{K}'\boldsymbol{e}']\}\,, \qquad (1)$$

where μ is the pion mass, ω is the frequency, K is the momentum, $e^2/4\pi=\frac{1}{137}$, the prime refers to the final photon. The second order in the frequency term contains two structural constants α and β called the electric and magnetic polarizabilities.

Passing over to the helicity amplitudes $\gamma\gamma\to\pi\pi$ in the CMS we have in the vicinity of the point $s=0$, $t=u=\mu^2$

$$T_{++}=T^{\pi}_{++}+\tfrac{1}{2}s\mu(\alpha-\beta)^{\pi}\,,$$
$$T_{+-}=T^{\pi}_{+-}+(tu-\mu^4)(\alpha+\beta)^{\pi}/2\mu\,, \qquad (2)$$

where T^{π} is the QED contribution for π^+ and zero for π^0.

Pion polarizabilities may be calculated (see, e.g., the review of ref. [6]) in different low-energy models based usually on chiral symmetry. It was found that π^0 polarizability is the most model sensitive, so its experimental measurement is the most interesting.

2. For our purpose we can use directly the amplitudes $\gamma\gamma\to\pi\pi$ from ref. [3], but in the vicinity of threshold these formulae may be made more general and simpler [#2]. We consider here only the helicity amplitude $++$, which is the main contribution near threshold.

We make the following simplifications to the model of ref. [3].

(a) In the cross channel we retain only the lightest resonances ρ, ω.

(b) We unite all the contributions and consider polarizabilities as free parameters.

[#1] When this paper was finished, we learned about the appearance of JADE data [2], but JADE did not indicate the absolute value of the cross section.

[#2] As for other unitary models $\gamma\gamma\to\pi\pi$ (see for instance, ref. [17]), the polarizabilities are not controlled there. Note that all the models are quite similar at first sight, but may differ in details.

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Thus, we have simple and practically model-independent expressions, which work well for $\sqrt{s}<0.7$ GeV [#3]:

$$M^{\rm C}_{++}=\frac{\Omega^0(s)}{\sqrt{3}}\,[C^0+H^0_V(s)]$$

$$+\frac{\Omega^2(s)}{\sqrt{6}}\,[C^2+H^2_V(s)]+W^{\rm C}\,,$$

$$M^{\rm N}_{++}=\frac{\Omega^0(s)}{\sqrt{3}}\,[C^0+H^0_V(s)]$$

$$-\sqrt{\tfrac{2}{3}}\,\Omega^2(s)\,[C^2+H^2_V(s)]+W^{\rm N}\,. \qquad (3)$$

Here $\Omega^{\rm I}(s)$ is the Omnes function, corresponding to the phase shift $\delta^{\rm I}_0$, $\Omega^{\rm I}(0)=1$, and

$$W^{\rm C}=M^{\pi}_{++}(s,t)-Z^{\rho}\left(\frac{t}{M^2_\rho-t}+\frac{u}{M^2_\rho-u}\right)+a^{\rm C}\,,$$

$$W^{\rm N}=-Z^{\rho}\left(\frac{t}{M^2_\rho-t}+\frac{u}{M^2_\rho-u}\right)$$

$$-Z^{\omega}\left(\frac{t}{M^2_\omega-t}+\frac{u}{M^2_\omega-u}\right)+a^{\rm N}\,. \qquad (4)$$

Here $\mu=m_\pi$, $Z^{\rm R}=\frac{1}{4}g^2_{{\rm R}\pi\gamma}\approx 24\pi\Gamma({\rm R}\to\pi\gamma)/M^3_{\rm R}$, $a^{\rm C}$, $a^{\rm N}$ are constants, corresponding to any high-energy contributions, M^{π}_{++} denotes π-exchange (QED). The $H^{\rm I}_V$ are rescattering integrals given by

$$H^{\rm I}_V(s)=\frac{s}{\pi}\int\frac{{\rm d}s'}{s'(s'-s)}\,\frac{\sin\delta^0_0(s')}{|\Omega^{\rm I}(s')|}\,V^{\rm I}(s')\,.$$

Unitarity only requires

$$V^{\rm I}(s)=W^{{\rm I},J=0}(s)\,, \qquad (5)$$

where $W^{\rm I}$ are isotopical combinations of $W^{\rm C}$ and $W^{\rm N}$. Comparing the amplitudes (3) with definition (2), we obtain

$$\tfrac{1}{2}\mu(\alpha-\beta)^{\rm C}=a^{\rm C}+\frac{C^0}{\sqrt{3}}+\frac{C^2}{\sqrt{6}}-Z^{\rho}\frac{2\mu^2}{M^2_\rho}\,,$$

$$\tfrac{1}{2}\mu(\alpha-\beta)^{\rm N}=a^{\rm N}+\frac{C^0}{\sqrt{3}}-\sqrt{\tfrac{2}{3}}\,C^2$$

$$-Z^{\rho}\frac{2\mu^2}{M^2_\rho}-Z^{\omega}\frac{2\mu^2}{M^2_\omega}\,. \qquad (6)$$

[#3] M is the helicity amplitude with the kinematical factor extracted: $T_{++}=sM_{++}$.

Eqs. (3)–(6) contain four free parameters: $(\alpha-\beta)^{\rm C}$, $(\alpha-\beta)^{\rm N}$, C^0, C^2. The dependence on C^0, C^2 enters owing to the final state interaction and considering this amplitude not in the $\gamma\pi\to\gamma\pi$ channel, where the polarizability is defined. But it is seen from our calculations that the experimental smallness of δ^2_0 leads to $\Omega^2(s)\approx 1$ and the term $C^2\Omega^2$ does not differ from the constants $a^{\rm C}$, $a^{\rm N}$.

So we may put $C^2=0$ in (3), absorbing $C^2\Omega^2$ by constants with good accuracy. Besides, one may be convinced of the very weak dependence of $H^{\rm I}_V(s)$ on the constants $a^{\rm C}$, $a^{\rm N}$ because the main contribution to $V^{\rm I}$ is generated by QED. So we neglect them inside $H^{\rm I}_V$ and arrive at the amplitude $\gamma\gamma\to\pi^0\pi^0$, which depends on $(\alpha-\beta)^{\rm N}$ and C^0:

$$M^{\rm N}_{++}=\frac{\Omega^0(s)}{\sqrt{3}}\,H^0_V(s)-\sqrt{\tfrac{2}{3}}\,\Omega^2(s)H^2_V(s)$$

$$+\frac{C^0}{\sqrt{3}}\,(\Omega^0-1)+\tfrac{1}{2}\mu(\alpha-\beta)^{\rm N}+W^{\rm N}\,,$$

$$W^{\rm C}=M^{\pi}_{++}(s,t)-Z^{\rho}\left(\frac{t-\mu^2}{M^2_\rho-t}+\frac{u-\mu^2}{M^2_\rho-u}\right),$$

$$W^{\rm N}=-Z^{\rho}\left(\frac{t-\mu^2}{M^2_\rho-t}+\frac{u-\mu^2}{M^2_\rho-u}\right)$$

$$-Z^{\omega}\left(\frac{t-\mu^2}{M^2_\omega-t}+\frac{u-\mu^2}{M^2_\omega-u}\right),$$

$$V^{\rm I}(s)=W^{{\rm I},J=0}(s)\,. \qquad (7)$$

3. Some words about the $\pi\pi$-interaction, which is a necessary input in (7). We use the same approximation of experimental data on $\pi\pi$ as in ref. [3] with the scattering lengths from the experiment of ref. [8]:

$$a^0_0\mu=0.24\pm 0.04\,,\quad a^2_0\mu=-0.04\pm 0.04\,.$$

δ^0_0 in (3) is in the same manner indeed only a smooth part of the experimental phase shift (without the $f_0(975)$ effect) because of the very small coupling $f_0\gamma\gamma$ (see the results of the coupled-channel investigation of ref. [9]). Our calculations show that the amplitudes are more sensitive to the near-threshold behavior of the phase shifts than to their other properties. We think that the influence of the $\pi\pi$-interaction form on the value $\alpha-\beta$ needs a more detailed investigation but here we restrict ourselves by the one variant of the $\pi\pi$-interaction that roughly corresponds to experiment.

4. Here we fit the cross section $\gamma\gamma\to\pi^0\pi^0$ [1] in the region $\sqrt{s}<0.7$ GeV by formula (7), considering $\alpha-\beta$ and C^0 as free parameters. As a result we obtain [#4]

$$(\alpha-\beta)^{\mathrm{N}}=(1.0\pm2.5)\times10^{-42}\ \mathrm{cm}^3\,,$$

$$C^0=0.115\pm0.030\ \mathrm{GeV}^{-2}\,,$$

$$\chi^2=11.4\quad \mathrm{at\ ND}=7\,. \tag{8}$$

The corresponding cross section is shown in fig. 1. Note that we have taken into account statistical errors only of ref. [1].

The question arises whether the ρ, ω contributions are of importance in amplitude (7). One may be convinced that neglecting them leads to a marked difference in the cross section at $\sqrt{s}>0.5$ GeV. Using such an oversimplified model for describing the data we obtain instead of (8)

$$(\alpha-\beta)^{\mathrm{N}}=(5.5\pm1.7)\times10^{-42}\ \mathrm{cm}^3\,,$$

[#4] We use the system of units $e^2=4\pi\alpha$, where $\alpha\approx\frac{1}{137}$. Note, to avoid mistakes, that some analyses use the system $e^2=\alpha$, and the corresponding values of polarizabilities differ by a factor 4π.

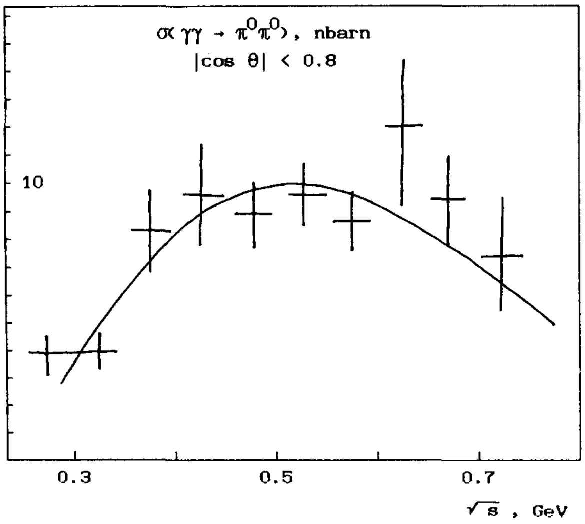

Fig. 1. Comparison of CB data [1] near the threshold with our predicted cross section using the parameter values of eq. (8).

$$C^0=0.115\pm0.030\ \mathrm{GeV}^2\,,$$

$$\chi^2=10.3\quad \mathrm{at\ ND}=7\,.$$

One should note that the main contribution to $\chi^2(8)$ comes from the two experimental points near threshold and their influence is opposite. To demonstrate this fact we can unite these two points into one, with the result

$$(\alpha-\beta)^{\mathrm{N}}=(4.4^{+3.0}_{-2.5})\times10^{-42}\ \mathrm{cm}^3\,,$$

$$C^0=0.144\pm0.016\ \mathrm{GeV}^{-2}$$

$$\chi^2=3.7\quad \mathrm{at\ ND}=6\,.$$

Having the numbers (8) we can estimate the charged pion polarizability by means of the model of ref. [3]. As a result we have $(\alpha-\beta)^{\mathrm{C}}=(9.3\pm2.5)\times10^{-42}\ \mathrm{cm}^3$. To derive this, information about the decays $\mathrm{R}\to\pi\gamma$ is necessary, so it is difficult to estimate the total error here.

Recall that the experiment on $\pi^+\mathrm{Z}\to\pi^+\mathrm{Z}\gamma$ [4] gives $(\alpha-\beta)^{\mathrm{C}}=17.0\pm3.6$ in the same units, and $\gamma\mathrm{p}\to\gamma\pi^+\mathrm{n}$ [5] gives $(\alpha-\beta)^{\mathrm{C}}=50\pm30$. Different theoretical predictions for π^+ [6] are in the interval $(8\text{–}16)\times10^{-42}$ cm^3, and one can expect $|(\alpha-\beta)^{\mathrm{N}}|<3\times10^{-42}$ cm^3 from different theoretical predictions.

5. We conclude that the experimental cross section $\gamma\gamma\to\pi^0\pi^0$ [1] allows one to obtain information about π^0 polarizability from the near-threshold region. The use of a $\pi\pi$ interaction close to the experimental one allowing for chiral zeros leads to the value (8), which gives a reasonable estimate for $(\alpha-\beta)^{\mathrm{C}}$, also. For better confidence it is necessary to investigate the influence of the $\pi\pi$ interaction on the values $\alpha-\beta$ obtained and to take the systematical errors in the experiment into account. Certainly, the formulae obtained may be applied to the $\gamma\gamma\to\pi^+\pi^+$ also, and a joint analysis of data on $\gamma\gamma\to\pi^+\pi^-$ and $\pi^0\pi^0$ will be more reliable and preferable.

References

[1] Crystal Ball Collab., H. Marsiske et al., Phys. Rev. D 41 (1990) 3324.
[2] JADE Collab., T. Oest et al., preprint DESY 90-025 (1990).
[3] A.E. Kaloshin and V.V. Serebryakov, Z. Phys. C 32 (1986) 279.

[4] Yu.M. Antipov et al., Phys. Lett. B 121 (1983) 445; Z. Phys. C 24 (1984) 39.
[5] T.A. Aibergenov et al., Czech. J. Phys. B 36 (1986) 948.
[6] V.A. Petrun'kin, Fiz. Elem. Chastits At. Yadra 12 (1981) 692.
[7] G. Mennessier, Z. Phys. C 16 (1983) 241;
D.N. Lyth, J. Phys. G 10 (1984) 39;
K.L. Au, D. Morgan and M.R. Pennington, Phys. Rev. D 35 (1987) 1633.
[8] E.A. Alekseeva et al., Sov. Phys. JETP 55 (1982) 591.
[9] A.E. Kaloshin and V.V. Serebryakov, Yad. Fiz. 47 (1988) 179.