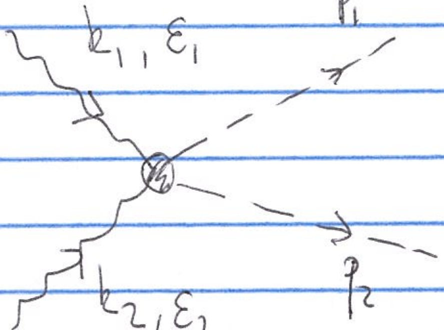
$\gamma\gamma \to \pi^0 \pi^0$

Amplitude for pion production in $\gamma\gamma$ collision:



most general form of the amplitude.

$$A_{\gamma\gamma\to\pi^0\pi^0} = e^2 \, \varepsilon_1^\mu \varepsilon_2^\nu V_{\mu\nu}$$

$$V_{\mu\nu} = A(s,t,u)\, T_{1\mu\nu} + B(s,t,u)\, T_{2\mu\nu} + C(s,t,u)\, T_{3\mu\nu} + D(s,t,u)\, T_{4\mu\nu}$$

where:

$$\Delta_\mu = (p_1 - p_2)_\mu .$$

$$T_{1\mu\nu} = \frac{s}{2} g_{\mu\nu} - k_{1\nu} k_{2\mu}$$

$$T_{2\mu\nu} = 2s\, \Delta_\mu \Delta_\nu - (t-u)^2 g_{\mu\nu} - 2(t-u)(k_{1\nu}\Delta_\mu - k_{2\mu}\Delta_\nu)$$

$$T_{3\mu\nu} = k_{1\mu} k_{2\nu}$$

$$T_{4\mu\nu} = s(k_{1\mu}\Delta_\nu - k_{2\nu}\Delta_\mu) - (t-u)(k_{1\mu}k_{1\nu} + k_{2\mu}k_{2\nu})$$

C and D terms do not contribute and are neglected.

A and B are symmetric under crossing $(t,u) \to (u,t)$.

Helicity amplitudes (defined by the photon helicities):

$$H_{++} = A + 2(4M_\pi^2 - s)B$$

$$H_{+-} = \frac{8(M_\pi^4 - tu)}{s} B.$$

They have partial wave expansions:

$$H_{++}(s,t,u) = \sum_{J=0,2,4,\ldots} h_+^J(s)\, d_{00}^J(\cos\theta)$$

$$H_{+-}(s,t,u) = \sum_{J=2,4,\ldots} h_-^J(s)\, d_{20}^J(\cos\theta)$$

θ defined by $\vec{k}_1 \cdot \vec{p}_1 = |\vec{k}_1||\vec{p}_1| \cos\theta$ in CM frame

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 s}{64} \beta(s) H(s,t)$$

$$H(s,t) = |H_{++}|^2 + |H_{+-}|^2$$

$$\beta(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}$$

Low energy chiral expansion

Lowest order result:

perhaps this is already good enough to do simulations:

$$A = \frac{4 G_\pi(s)}{s F_\pi^2} (s - M_\pi^2)$$

$$B = 0$$

$$G_\pi(s) = -\frac{1}{16\pi^2}\left(1 + \frac{2M_\pi^2}{s} \int_0^1 \frac{dx}{x} \log\left(1 - \frac{s}{M_\pi^2} x(1-x)\right)\right)$$

Bellucci, Sainio and Gasser give A and B to higher order in ChPT.

FINAL FORM OF AMPLITUDE @ LOWEST CHIRAL ORDER

$$A_{\gamma\gamma\to\pi^0\pi^0} = e^2\, \varepsilon_1^{\mu}\, \varepsilon_2^{\nu}\, \frac{4\, G_\pi(s)}{s\, F_\pi^2}\,(s - M_\pi^2) \cdot \left(\frac{s}{2}\, g_{\mu\nu} - k_{1_\nu}\, k_{2_\mu}\right)$$

(see fig. in page (1))