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Pion polarizabilities in Chiral Dynamics

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Introduction

Composite particle in external EM field

$$H = H_0(A) + 2\pi\alpha\ E^2 + 2\pi\beta\ B^2 + \cdots$$

$\alpha,\ \beta$ electric and magnetic dipole polarizabilities

in NR case: $\alpha >> \beta$

H atom: $\alpha_H \sim 3.8\mathring{A}^3$ $V_H = 0.6\mathring{A}^3$

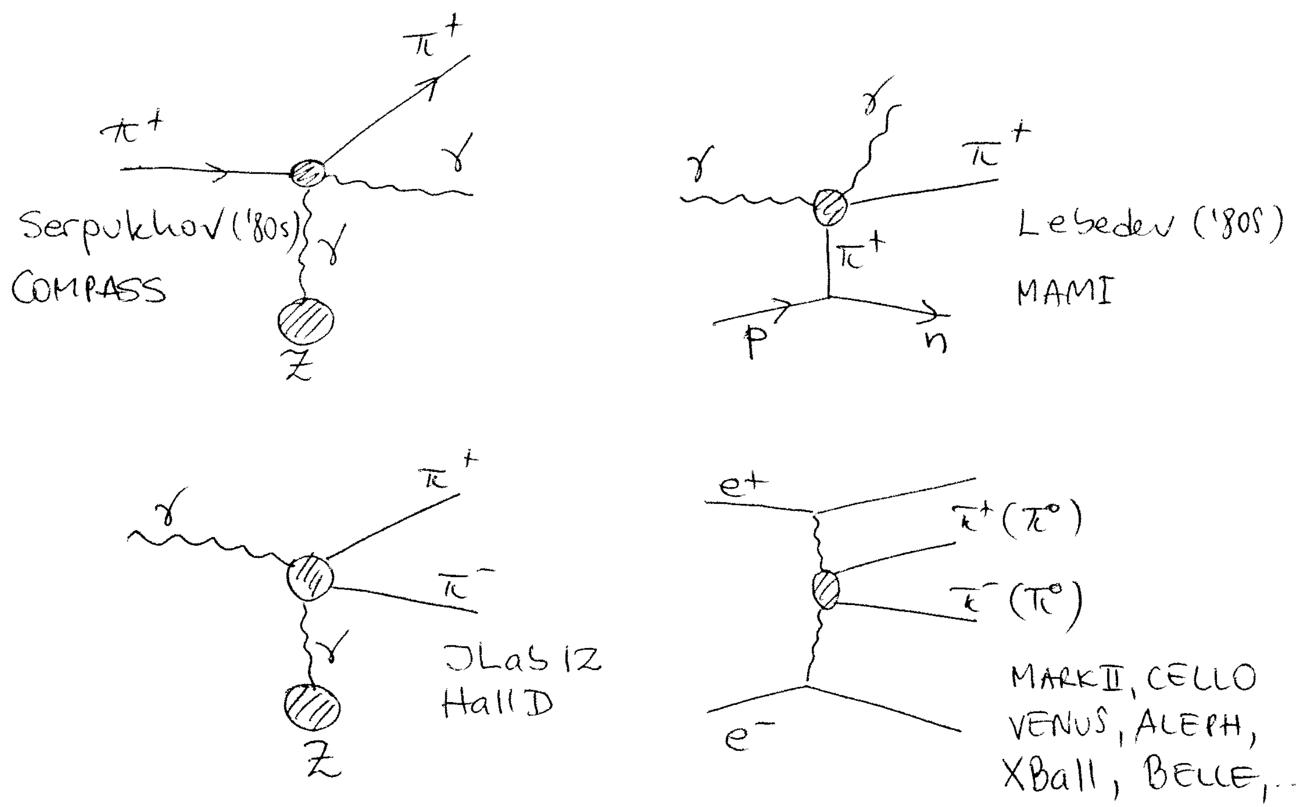
Nucleons

$\alpha_N \sim 11 \times 10^{-4}\ \mathrm{fm}^3$ $\beta_N \sim 3 \times 10^{-4}\ \mathrm{fm}$ $V_N \sim 2.5\ \mathrm{fm}^3$

Pion polarizabilities

- very challenging to measure/extract from measurements
- important tests of chiral dynamics

Experiments



Current status

$\pi^{\pm}$

Data	Reaction	Parameter	10^{-4} fm^3
Serpukhov ($\alpha_\pi + \beta_\pi = 0$) [12]	$\pi Z \longrightarrow \pi Z \gamma$	α_π	$6.8 \pm 1.4 \pm 1.2$
Serpukhov [13]		$\alpha_\pi + \beta_\pi$	$1.4 \pm 3.1 \pm 2.8$
		β_π	$-7.1 \pm 2.8 \pm 1.8$
Lebedev [7]	$\gamma N \longrightarrow \gamma N \pi$	α_π	20 ± 12
Mami A2 [14]	$\gamma p \longrightarrow \gamma \pi^+ n$	$\alpha_\pi - \beta_\pi$	$11.6 \pm 1.5 \pm 3.0 \pm 1.5$
PLUTO [8]	$\gamma\gamma \longrightarrow \pi^+\pi^-$	α_π	$19.1 \pm 4.8 \pm 5.7$
DM1 [9]	$\gamma\gamma \longrightarrow \pi^+\pi^-$	α_π	17.2 ± 4.6
DM2 [10]	$\gamma\gamma \longrightarrow \pi^+\pi^-$	α_π	26.3 ± 7.4
Mark II [11]	$\gamma\gamma \longrightarrow \pi^+\pi^-$	α_π	2.2 ± 1.6
Blobal fit: MARK II, VENUS, ALEPH, TPC/2γ, CELLO, BELLE (L. Fil'kov, V. Kashevarov) [15]	$\gamma\gamma \longrightarrow \pi^+\pi^-$	$\alpha_\pi - \beta_\pi$	$13.0^{+2.6}_{-1.9}$
		$\alpha_\pi + \beta_\pi$	$0.18^{+0.11}_{-0.02}$
Global fit: MARK II, Crystal ball (A. Kaloshin, V. Serebryakov [16]	$\gamma\gamma \longrightarrow \pi^+\pi^-$	$\alpha_\pi - \beta_\pi$	5.2 ± 0.95

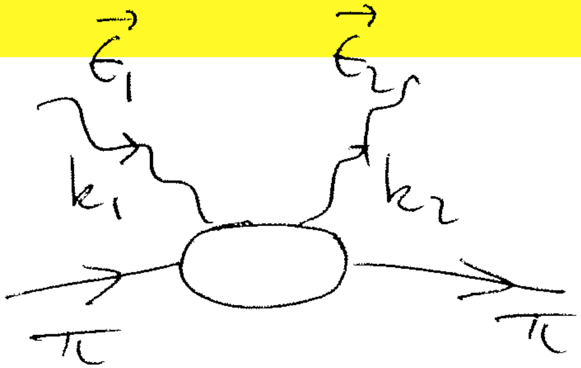
Model	Parameter	10^{-4} fm^3
χPT	$\alpha_\pi - \beta_\pi$	5.7 ± 1.0
	$\alpha_\pi + \beta_\pi$	0.16
QCM	$\alpha_\pi - \beta_\pi$	7.05
	$\alpha_\pi + \beta_\pi$	0.23
QCD sum rules	$\alpha_\pi - \beta_\pi$	11.2 ± 1.0
Dispersion sum rules	$\alpha_\pi - \beta_\pi$	13.60 ± 2.15
	$\alpha_\pi + \beta_\pi$	0.166 ± 0.024

COMPASS preliminary $\quad \alpha_{\pi^\pm} - \beta_{\pi^\pm} = 3.8 \pm 2.1$

π^0

	fit	DSRs [2]	ChPT
$(\alpha_1 - \beta_1)_{\pi^0}$	-1.6 ± 2.2 [3] -0.6 ± 1.8 [9]	-3.49 ± 2.13	-1.9 ± 0.2 [11]
[illegible]	[illegible]	[illegible]	[illegible]

Compton amplitude @ low energy



Lab frame amplitude

$$T = -\frac{e^2}{M_\pi} Q_\pi^2 \ \vec{\epsilon_1} \cdot \vec{\epsilon_2}^* + 4\pi(\bar{\alpha}_\pi \omega_1 \omega_2 \ \epsilon_1 \cdot \vec{\epsilon_2}^* + \bar{\beta}_\pi \ \epsilon_1 \times \vec{k}_1 \cdot \vec{\epsilon_2}^* \times \vec{k}_2) + \mathcal{O}(\omega^4)$$

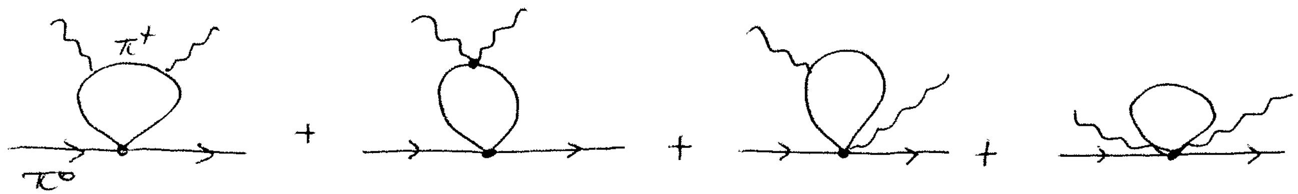
Dispersion relation: Baldin-Lapidus sum rule

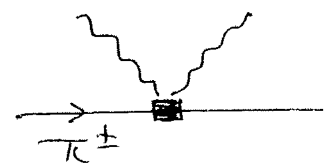
$$\bar{\alpha} + \bar{\beta} = \frac{1}{2\pi^2} \int_{\omega_{th}}^{\infty} d\omega \ \frac{\sigma(\gamma\pi \to X)}{\omega^2} \geq 0$$

gives fundamental constraint

Polarizabilities in ChPT

originate at order p^4 in chiral expansion





predicted at this order: $\bar{\alpha}_\pi + \bar{\beta}_\pi = 0$

$$\bar{\alpha}_{\pi^0} - \bar{\beta}_{\pi^0} = -\frac{\alpha}{48\pi^2 F_\pi^2 M_\pi} \sim -1.0$$ [Bijnens & Cornet]

$$\bar{\alpha}_{\pi^+} - \bar{\beta}_{\pi^+} = \frac{\alpha}{24\pi^2 F_\pi^2 M_\pi}(\ell_6 - \ell_5) \sim 5.6$$ [Teren'tev; Donoghue & Holstein]

LECs from $< r^2 >_{\pi^+}$ **and** $\pi^+ \to e^+ \nu \gamma$

$$\ell_6 - \ell_5 = 3.0 \pm 0.3$$

$\gamma\gamma \to \pi\pi$

$$\bar{M}_{++}(s, t=0) = 2\pi\sqrt{s}(\bar{\alpha}_\pi - \bar{\beta}_\pi)$$

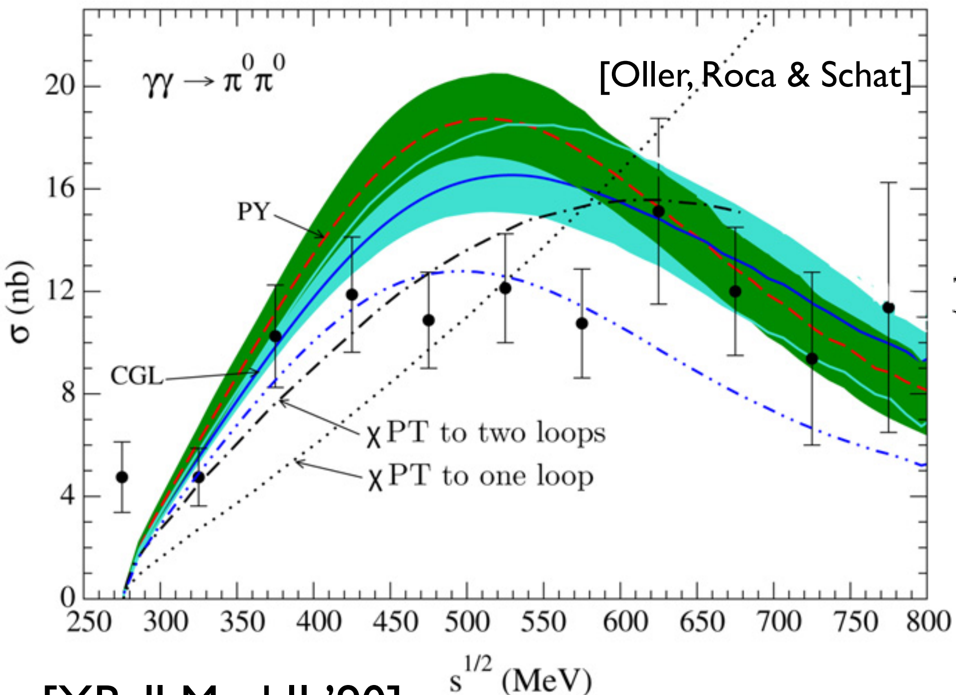
$$\bar{M}_{+-}(s, t=0) = 2\pi\sqrt{s}(\bar{\alpha}_\pi + \bar{\beta}_\pi)$$

$$\bar{\alpha}_\pi \pm \bar{\beta}_\pi = \frac{1}{2\pi M_\pi} \left(M_{+\mp} - M_{\mathrm{Born}}\right)\Big|_{s=0,\, t=M_\pi^2}$$

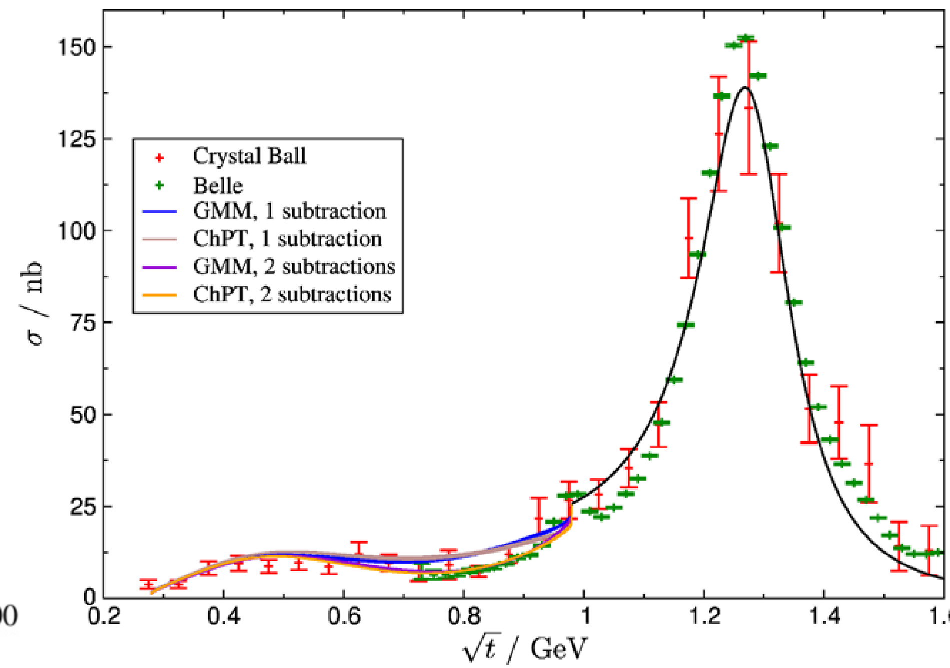
$$\bar{\alpha}_\pi - \bar{\beta}_\pi \quad \mathrm{S-wave}$$

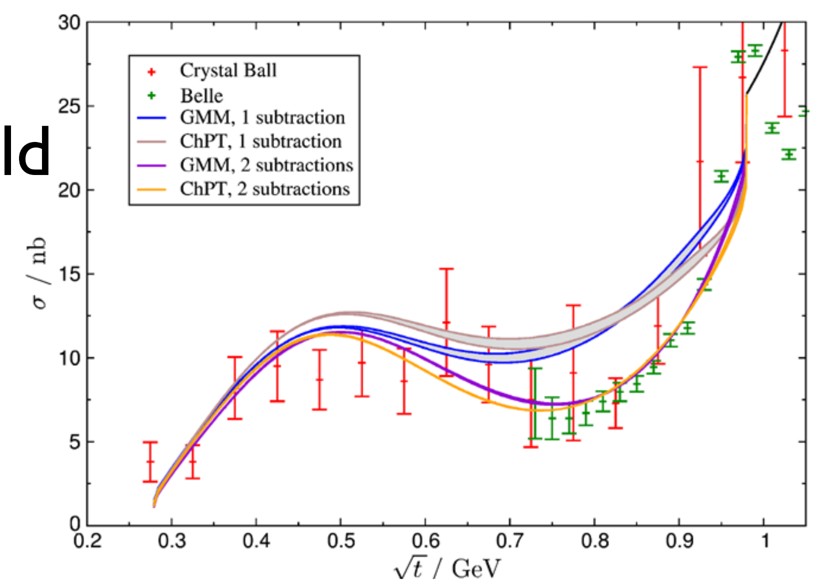
$$\bar{\alpha}_\pi + \bar{\beta}_\pi \quad \mathrm{D-wave}$$

Problem with $\gamma\gamma \to \pi^0\pi^0$ need for higher order in ChPT



[XBall MarkII '90]





[Hoferichter et al]

leading order poor even near threshold

need polarizabilities @ 2 loops

[Bellucci, Gasser & Sainio; Gasser, Ivanov & Sainio]

$$\bar{\alpha}_{\pi^0} \pm \bar{\beta}_{\pi^0} = \frac{\alpha}{16\pi^2 F_\pi^2 M_\pi}\left(c_\pm + \frac{M_\pi^2}{16\pi^2 F_\pi^2}\, d_\pm\right) \quad c_+ = 0 \quad c_- = -1/3 \quad d_+ \sim 1.4 \quad d_- \sim -1.1$$

[Bellucci, Gasser & Sainio]

$$\bar{\alpha}_{\pi^0} + \bar{\beta}_{\pi^0} = 1.15 \pm 0.30$$

$$\bar{\alpha}_{\pi^0} - \bar{\beta}_{\pi^0} = -1.90 \pm 0.20$$

$$\bar{\beta}_{\pi^0} > 0 \quad \pi^0 \ is\ paramagnetic$$

3 LECs @ $\mathcal{O}(p^6)$ resonance saturation estimates [Bellucci et al]

l^r	l^R: ω	ρ^0	ϕ	$A(1^{+-})$	$\sum_R l^R$	l^R: $S(0^{++})$	f_2
a_1^r	−33.2	−6.1	−0.1	0.0	−39	±0.8	∓4.1
a_2^r	12.5	2.3	≃ 0	−1.3	13	±1.3	±1.0
b^r	2.1	0.4	≃ 0	0.7	3	0.0	±0.5

	$O(E^{-1})$: 1 loop	$O(E)$: $h_\pm^r$	2 loops	chiral logs	Total	Uncertainty
$(\alpha+\beta)^N$	0.00	1.00	0.17	[0.21]	≃ 1.15	±0.30
$(\alpha-\beta)^N$	−1.01	−0.58	−0.31	[−0.18]	≃ −1.90	±0.20
$\bar{\alpha}_{\pi^0}$	−0.50	0.21	−0.07	[0.01]	≃ −0.35	±0.10
$\bar{\beta}_{\pi^0}$	0.50	0.79	0.24	[0.20]	≃ 1.50	±0.20

large NLO corrections required by data at low energy
and predicted by resonance saturation

ChPT at $\mathcal{O}(p^6)$ matched to unitarity gives good description up to $\sqrt{s} \sim 1\ \mathrm{GeV}$

[Portoles & Pennington; Donoghue & Holstein; Fil'kov & Kashevarov; Pennington; Oller, Roca & Schat; Hofferichter, Phillips & Schat; many others]

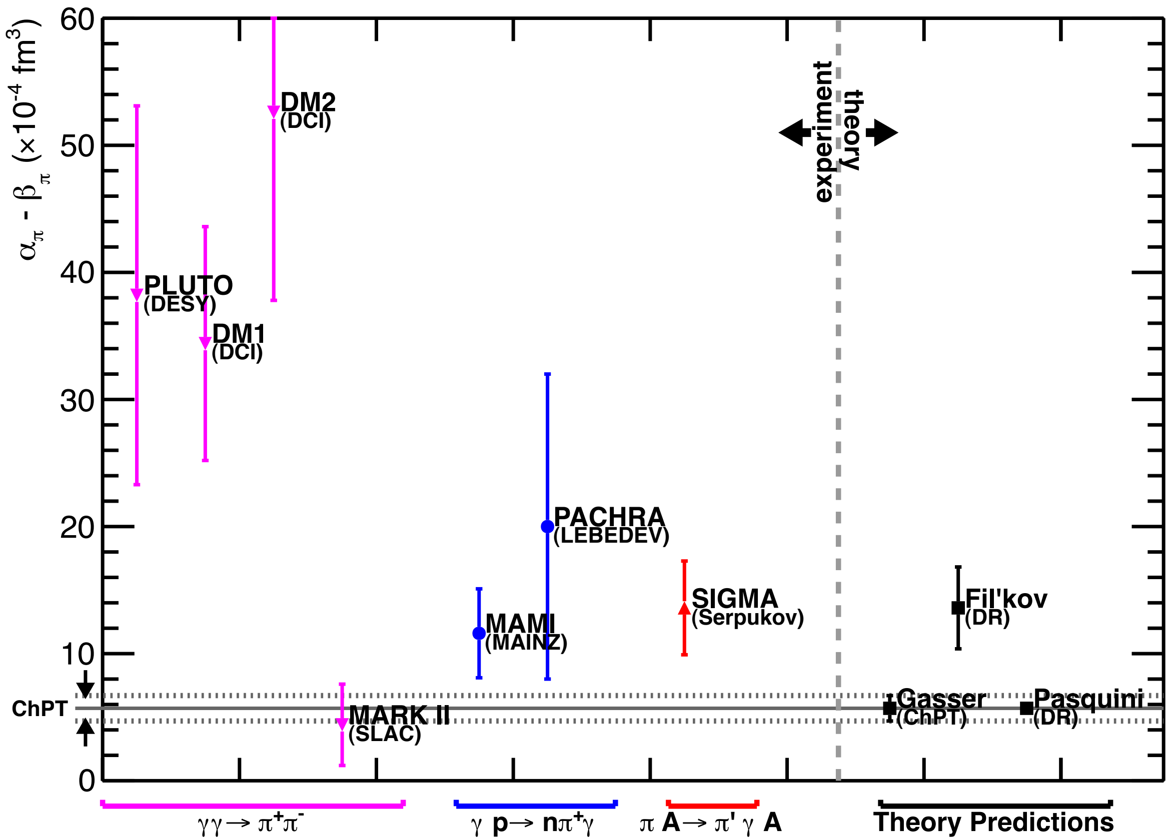
Different methods: DRs, N/D, Roy equations, explicit resonances,...

	fit	DSRs [2]	ChPT
$(\alpha_1 - \beta_1)_{\pi^0}$	-1.6 ± 2.2 [3] -0.6 ± 1.8 [9]	-3.49 ± 2.13	-1.9 ± 0.2 [11]

[F&K]

Priority: improvement over the old XBall measurements at low energy

The $\pi^{\pm}$ polarizabilities AD 2013



[from PAC40 proposal]

$\gamma\gamma \to \pi^+\pi^-$

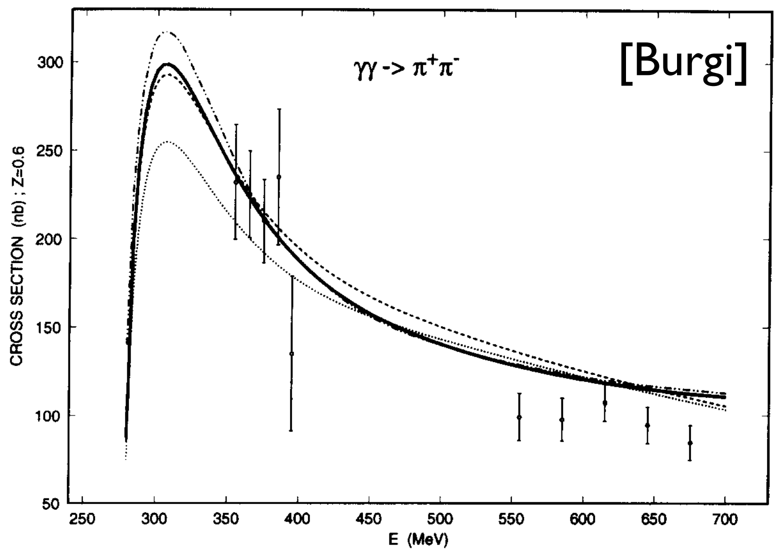
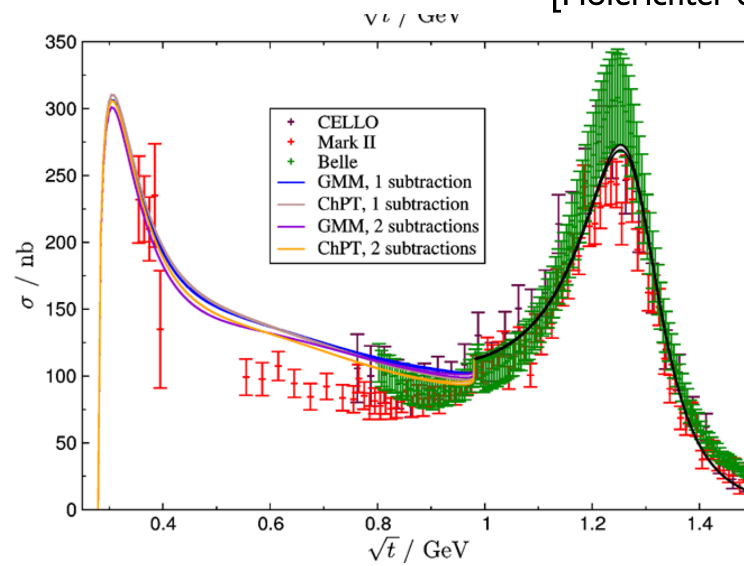
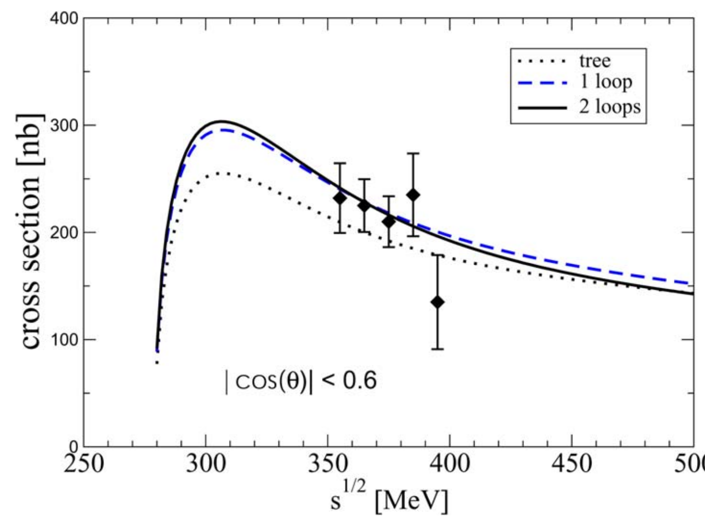


Fig. 9. The $\gamma\gamma \to \pi^+\pi^-$ cross section $\sigma(s; |\cos\theta| \leqslant Z = 0.6)$ as a function of the center-of-mass energy E, together with the data from the Mark II collaboration [21]. We have added in quadrature the tabulated statistical and systematical errors. In addition, there is an overall normalization uncertainty of 7% in the data [21]. The solid line is the full two-loop result, the dashed line corresponds to the one-loop approximation [18] and the dotted line is the Born contribution. The dashed-double dotted line is the result of a dispersive calculation performed by Donoghue and Holstein (Fig. 7 in Ref. [11]).

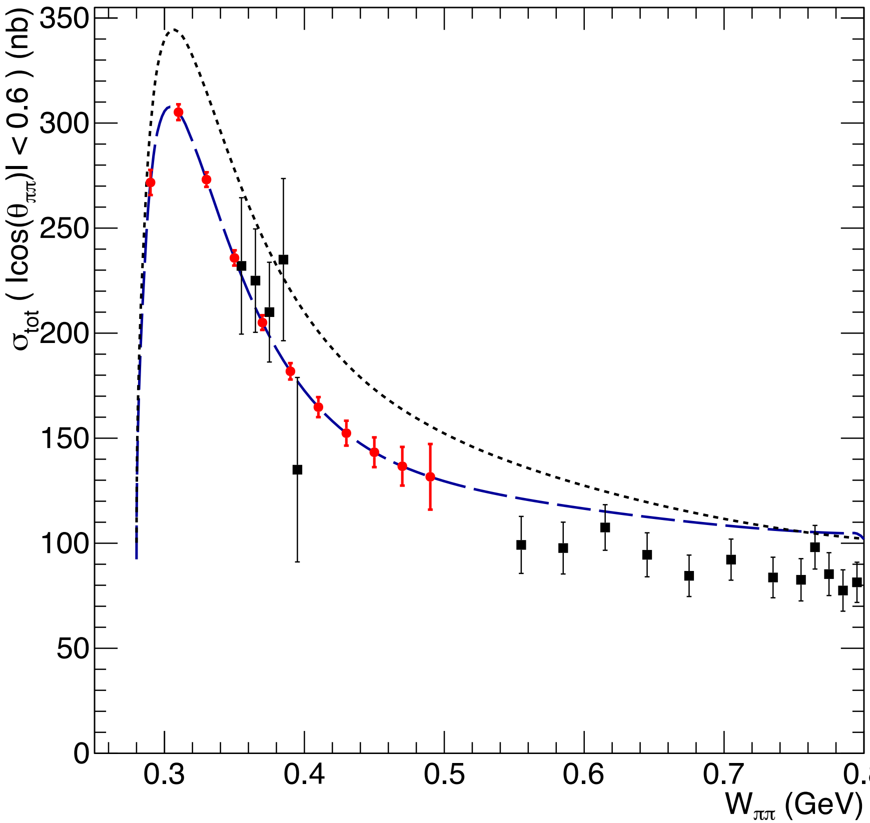
[Hoferichter et al]





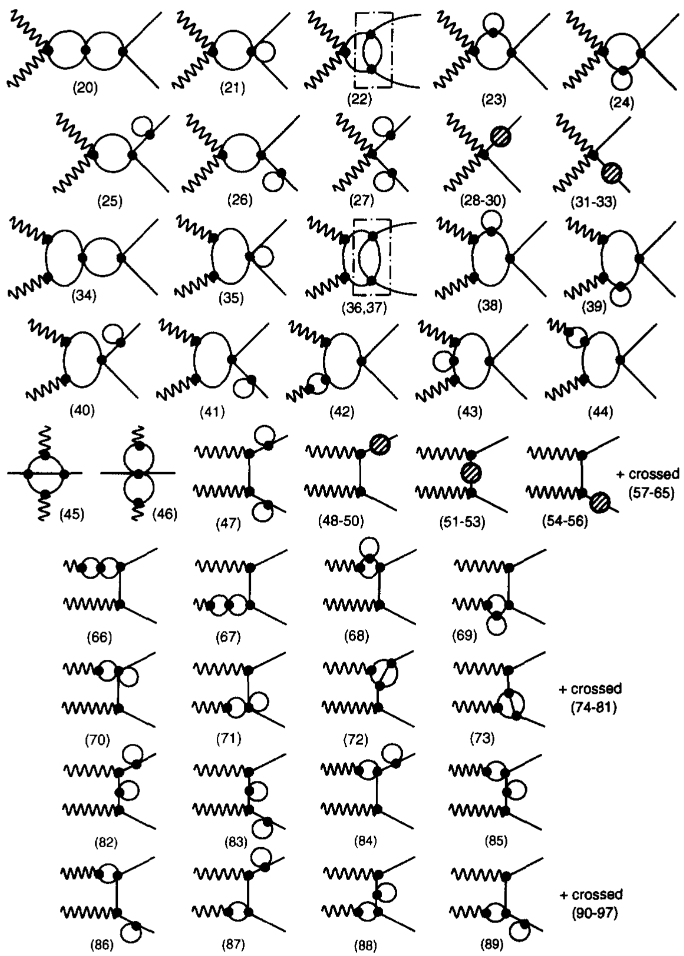
Large contribution of Born term makes experimental access to polarizability effects more difficult

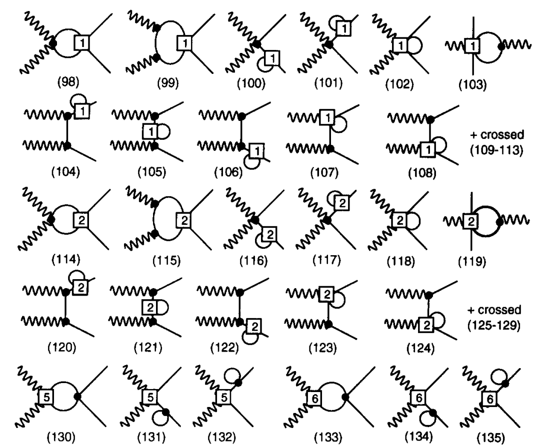
$\gamma + \gamma \rightarrow \pi^+ + \pi^-$



[JLab Hall D proposal]

$\bar{\alpha}_{\pi^+}$, $\bar{\beta}_{\pi^+}$ @ 2-loops [Burgi ’96; Gasser, Ivanov & Sainio ’06]





Insights into the ChPT calculation [Burgi]

- $\mathcal{O}(p^6)$ ChPT

- 3 LECs @ $\mathcal{O}(p^6)$

l^r	l^R			$\sum_R l^R$
	ρ	a_1	b_1	
$a_1^{r,c}$	−3.28	0	0	-3.3 ± 1.65
$a_2^{r,c}$	1.23	−0.35	−0.13	0.75 ± 0.65
$b^{r,c}$	0.20	0.18	0.06	0.45 ± 0.15

$$\bar{\alpha}_{\pi^+} + \bar{\beta}_{\pi^+} = 0.3 \pm 0.1 \quad (0)$$

$$\bar{\alpha}_{\pi^+} - \bar{\beta}_{\pi^+} = 4.4 \pm 1.0 \quad (5.6 \pm 0.8)$$

NLO corrections of natural size

$$\bar{\beta}_{\pi^\pm} < 0 \quad \pi^{\pm}\text{is diamagnetic}$$

	fit [5]	DSRs [2]	ChPT [34]	
			to one-loop	to two-loops
$(\alpha_1 - \beta_1)_{\pi^\pm}$	$13.0^{+2.6}_{-1.9}$	13.60 ± 2.15	6.0	5.7 [5.5]
$(\alpha_1 + \beta_1)_{\pi^\pm}$	$0.18^{+0.11}_{-0.02}$	0.166 ± 0.024	0	0.16 [0.16]

[F&K]

$\alpha - \beta$	Born		Gen. Born		Vector mesons		Sum
	$I = 0$	$I = 2$	$I = 0$	$I = 2$	$I = 0$	$I = 2$	
π^+	5.65	−0.69	6.30	−0.54	−0.065	0	5.70

[Pasquini, Drechsel & Scherrer]

Barbara Pasquini's talk

ChPT assessment for $\pi^{\pm}$ polarizabilities

Rather constrained predictions based on natural size estimate of NLO LECs

Significant deviations would be very surprising, but tested must be

Given the spread in experimental determinations, including very large deviations from the ChPT predictions, the JLab experiment is of extreme importance

Summary

- Pion polarizabilities are rigorous ChPT predictions in chiral limit
- Significant corrections at NLO in ChPT for the neutral pion
- NLO corrections for charged pion are of natural size (modulo assumptions on NLO LECs)
- Experimental extractions of polarizabilities still open problem, in particular for charged pion due to conflicting results
- Hall D @ JLab 12: unique opportunity to measure $\gamma\gamma \to \pi^+\pi^-$ for $\sqrt{s} < 500 \text{ MeV}$ to extract $\bar{\alpha}_{\pi^\pm} - \bar{\beta}_{\pi^\pm}$ with unprecedented accuracy
- Similar experiment for neutral pion seems to be necessary and a natural future step with Primakoff production
- Impact in particle physics: Michael Ramsey-Musolf's talk