Momentum resolution

For a particle with charge z of momentum p in a uniform magnetic field B with a pitch angle (dip angle) λ

$$p \cos\lambda = (0.3) zBR$$

p in GeV/c, B in Tesla, R in meters. The curvature

$$k = \frac{1}{R}$$

The error in k is

$$(\delta k)^2 = (\delta k_{\mathrm{res}})^2 + (\delta k_{\mathrm{ms}})^2$$

$$\delta k_{\mathrm{res}} = \frac{\epsilon}{L'^2}\sqrt{\frac{720}{N+4}}$$

where ϵ is the position resolution in meters, L' is the projected length of the track onto the bending plane in meters and N is the number of measurements.

$$\delta k_{\mathrm{ms}} = \frac{(0.016\ \mathrm{GeV}/c)z}{Lp\beta\cos^2\lambda}\sqrt{N_{\mathrm{rl}}}$$

where N_{rl} is the number of radiation lengths in the detector and L is the total track length in the detector.

Error on Slope and y-intercept of a Straight-Line Fit, Equally Spaced Measurements

$$\chi^2 = \sum \left[\frac{1}{\sigma_i^2} (y_i - a - bx_i)^2 \right]$$

χ^2 is minimized for a and b where

$$a = \frac{1}{\Delta} \left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} \right)$$

$$b = \frac{1}{\Delta} \left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} \right)$$

where

$$\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2} \right)^2$$

The variance of the parameters a and b are, for equal errors

$$\sigma_a^2 \approx \frac{\sigma^2}{\Delta'} \sum x_i^2$$

$$\sigma_b^2 \approx \frac{n\sigma^2}{\Delta'}$$

where

$$\Delta' = n \sum x_i^2 - \left(\sum x_i \right)^2$$

For n equally spaced measurements spanning the interval $[0, L]$,

$$x_i = \frac{L(i-1)}{n-1}$$

This gives

$$\sigma_a^2 = \frac{2\sigma^2(2n-1)}{n(n+1)}$$

$$\sigma_b^2 = \frac{12\sigma^2(n-1)}{L^2 n(n+1)}$$

Angular Uncertainty Due to Multiple Coulomb Scattering

Define

$$\theta_0 = \theta_{\rm plane}^{\rm rms} = \frac{1}{\sqrt{2}}\theta_{\rm space}^{\rm rms}.$$

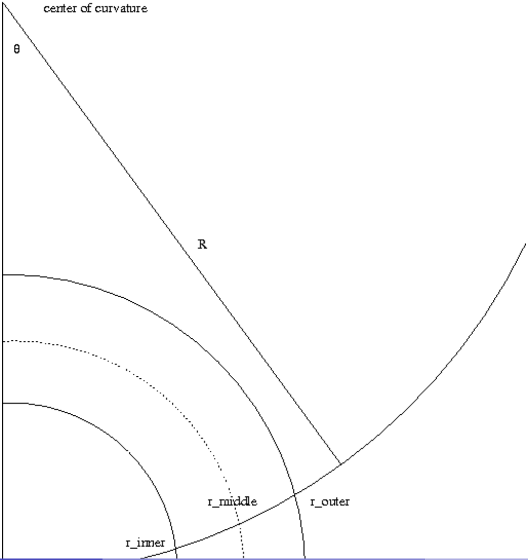
The central angular distribution is approximately Gaussian with a width given by

$$\theta_0 = \frac{(13.6\ \mathrm{MeV})}{\beta c p} z \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right].$$

The angle $\Psi_{\rm plane}$ is the angle between an unscattered trajectory and a line drawn from the entrance point of the detector to the exit point. We use this as an approximation to the contribution of multiple scattering to both the azimuthal and polar angles.

$$\Psi_{\rm plane}^{\rm rms} = \frac{1}{\sqrt{3}}\theta_0.$$

Contribution to Azimuthal Angle Resolution from Curvature Resolution



Curvature $k = 1/R$ and direction in plane is measured at some position rotated from the vertex by an angle θ about the center of curvature, not at the vertex. To infer the azimuthal angle ϕ at the vertex, track must be swum backward through angle θ. Determination of θ depends on R and thus on k. An error in θ translates directly into an error in ϕ.

$$\theta \approx \frac{r_{\text{middle}}}{R} = r_{\text{middle}} k$$

$$\delta\theta = r_{\text{middle}} \delta k$$

for now take $r_{\text{middle}} = (r_{\text{outer}} - r_{\text{inner}})/2$.

Geometry of the CDC

- as you go out of the end of the cdc:
  1. number of measurements gets reduced
  2. transverse length gets reduced
  3. reduction is different for axial layers and stereo layers

Geometry of the FDC

- as you go out the side of the fdc
  1. number of measurements gets reduced
  2. length of detector gets reduced
  3. below a certain momentum you don't go out the side of the fdc (ignored for now)

Combining the CDC and the FDC

For each quantity, the measurements in the CDC and FDC are statistically independent. They are therefore combined using:

$$\sigma_{\text{total}} = \frac{1}{\sqrt{\frac{1}{\sigma^2_{\text{CDC}}} + \frac{1}{\sigma^2_{\text{FDC}}}}}$$

to get the overall resolution.