Energy Losses of Heavy Ions

Bethe - Bloch formula for Heavy Ions (Please see Leo.):

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left(\mathrm{Log}\left[\frac{2 m_e \gamma^2 v^2 W_{max}}{I^2}\right] - 2\beta^2 - \delta - 2\frac{C}{Z}\right)$$

This formula gives us mean energy losses.

Most probable energy losses :

In Landau regime $\kappa < 0.01$ the most probable energy loss is given by the formula :

$$\Delta_{MP} = \xi\left(\mathrm{Log}\left[\frac{\xi}{\epsilon}\right] + 0.201 - \delta\right)$$

$$\xi = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}$$

$$\kappa = \frac{\xi}{W_{max}}$$

For $0.01 < \kappa < 10$ the Most Probable energy loss is determined by the Vavilov distribution. I don' t know much about it.

For $\kappa > 10$ The most probable energy loss equals the mean energy loss. In this case the energy loss distribution is Gaussian function.

The difference between the most probable energy loss and most probable energy loss for κ < 0.01:

$$\Delta_{MP} - |\Delta_{Bethe}| = \xi\left(\beta^2 + \mathrm{Log}\left[\frac{\xi}{W_{max}}\right] + 0.201\right) < 0$$

This formula is valid only for energy losses of heavy ions in Landau regime.

How should we calculate the most probable energy loss in the Vavilov regime ??

MCEEP uses the same formula (from above) to calculate the MP energy losses in the Vavilov regime. This approach is not valid for all κ in give region ($0.01<\kappa<10$). When $\left(\beta^2 + \mathrm{Log}\left[\frac{\xi}{W_{max}}\right] + 0.201\right)$ becomes > 0 the MP energy losses become bigger that MEAN energy losses. This can not be.

I have decided to use this formula only until $\left(\beta^2 + \mathrm{Log}\left[\frac{\xi}{W_{max}}\right] + 0.201\right) < 0$. Afterwards I have assumed that the energy-loss distribution function is almost Gaussian, ergo $\Delta_{MP} = |\Delta_{Bethe-Bloch}|$.

Energy losses for Electrons

In this case I have used the correct Bethe - Bloch formula for electrons to calculate their mean energy losses and than used the same formulas and approximations as above to calculate the most probable energy losses. With this approximation I bypassed
the problem with the Vavilov distribution.

Bethe - Bloch formula for electrons:

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left(\mathrm{Log}\left[\frac{\tau^2 (\tau+2)}{2\left(\frac{I}{m_e c^2}\right)^2}\right] + F[\tau] - \delta - 2\frac{C}{Z}\right)$$

$$W_{max} = \frac{\text{Kinetic Energy}}{2}$$

$$\xi = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}$$

Most probable energy losses for electrons (in my approximation):

While $\left(\beta^2 + \mathrm{Log}\left[\frac{\xi}{W_{max}}\right] + 0.201\right) < 0$:

$$\Delta_{MP} = \left|\Delta_{Bethe}^{El}\right| + \xi\left(\beta^2 + \mathrm{Log}\left[\frac{\xi}{W_{max}}\right] + 0.201\right) < 0$$

If $\left(\beta^2 + \mathrm{Log}\left[\frac{\xi}{W_{max}}\right] + 0.201\right) > 0$:

$$\Delta_{MP} = \left|\Delta_{Bethe}^{El}\right|$$