

## Vertical angle determination

We try to determine the vertical angle of the magnetic field's direction by fitting the following formula to the data:

$$\phi = \arctan \left( \frac{\sqrt{(I_s + A)^2 + R^2(I_l + B)^2}}{K(I_v + E)} \right)$$

Here  $I_s$  is the current in the small coil,  $I_l$  the current in the large coil and  $I_v$  the current in the vertical coil.

The coils are *Helmholtz coils*, which means the magnetic field strength ( $|\mathbf{B}|$ ) is linear with the current, provided we are close enough to the center of the coil system.

$$B = \text{const.} * I_{l,s,v}$$

Typically, each current would have a corresponding scaling constant. Because we have three currents, we can divide the whole expression with one of the scaling constants, leaving us with  $\mathbf{R}$  and  $\mathbf{K}$ .

The scaling constants are of course determined by the number of turns and the radius of the each coil.

Even when there's no current, we still have some "residual" magnetic field – contributions from the Earth's magnetic field, BigBite spectrometer etc. We brand these  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{E}$ . They are not meant to have any specific physical meaning, they just represent cumulative corrections in the x axis direction –  $\mathbf{A}$ , the y axis direction -  $\mathbf{B}$  and the z (vertical) direction –  $\mathbf{E}$ . (in the classic Cartesian system, z,x,y in the coil system).

The angle is counted as zero when the field is vertical.

The formula stems from the fact that the tangent of an angle is defined as the ratio between the total vertical component (only  $I_s$  "points" in this direction) and the total horizontal component (hence the square root).

We can get an estimate of the parameters, which we will use in fitting of the whole formula, by examining the data with only one of the planar coils switched on. Here, assuming the correction B (or A) is much smaller than the field generated by  $I_s$  or  $I_l$ , the above formula simplifies to:

$$\Phi = \arctan \left( \frac{(I_s + A)}{(K(I_v + E))} \right)$$

and

$$\Phi = \arctan \left( \kappa \frac{(I_l + B)}{(I_v + E)} \right)$$

where  $\kappa$  is  $R/K$ .

**Measured data:**

Small coil + vertical coil:

$I_s$	$I_v$	Compass	Compass - reverse
7	0	1990	11
5	10	1462.5	537
0	14	997.5	1002
-5	10	534	1466
-7	0	7	1994

The first column is  $I_s$ , second  $I_v$ ; the third and fourth columns are reading taken from the compass, the second one after rotating it around the vertical axis by  $180^\circ$ .

We first use both readings to find the zero of the compass. Looking at our formula, we see that we must limit ourselves to the interval  $(0, \pi/2)$ . The angle is then given by the deviation of the measurement from the compass' zero.

The (average) zero of the compass in this measurement is **1000.1** .

Data from the first measurement, with the angle in radians:

$I_s$	$I_l$	$I_v$	Angle (rad)
7	0	0	1.5543
5	0	10	0.7269
0	0	14	0.0035
-5	0	10	0.7320
-7	0	0	1.5606

(Here I wrote the values of all three currents:  $I_s$ ,  $I_l$ , and  $I_v$ ).

The **observational error for the angle** is  $\pm 1e-4$  (roughly  $0.05^\circ$ ) for all values.

Large coil + vertical coil:

$I_l$	$I_v$	Compass	Compass - reverse
-7	0	3972	1959
-5	10	518	1411.5
0	14	983	947
5	10	1438	491
7	0	1955	3976

To avoid problems using  $\text{mod}(4000)$ , we re-brand the measurements over 2000 as negative:

$l_i$	$l_v$	Compass	Compass - reverse
-7	0	-28	1959
-5	10	518	1411.5
0	14	983	947
5	10	1438	491
7	0	1955	-24

The (average) zero of the compass in this measurement is **965.5** .

Data from the second measurement:

$l_s$	$l_i$	$l_v$	Angle (rad)
0	-7	0	1.5610
0	-5	10	0.7018
0	0	14	0.0283
0	5	10	0.7438
0	7	0	1.5543

First, I tried fitting both the “partial” formulas to get an idea for the parameters' values.

Small coil fit:

<b>A = -0.026297</b>	<b>+/- 0.03113</b>	<b>(118.4%)</b>
<b>K = 0.549914</b>	<b>+/- 0.004105</b>	<b>(0.7465%)</b>
<b>E = 0.175074</b>	<b>+/- 0.0401</b>	<b>(22.9%) Arial</b>

Large coil fit:

<b><math>\kappa</math> = 1.79487</b>	<b>+/- 0.01537</b>	<b>(0.8565%)</b>
<b>B = 0.210944</b>	<b>+/- 0.03583</b>	<b>(16.98%)</b>
<b>E = 0.171157</b>	<b>+/- 0.04543</b>	<b>(26.54%)</b>

Using these as starting points for the fit of the whole function (with  $R = \kappa * K$ ), we get these values for the parameters:

<b>A = -0.0195053</b>	<b>+/- 0.06669</b>	<b>(341.9%)</b>
<b>K = 0.550365</b>	<b>+/- 0.008133</b>	<b>(1.478%)</b>
<b>E = 0.168082</b>	<b>+/- 0.06048</b>	<b>(35.98%)</b>
<b>B = 0.157533</b>	<b>+/- 0.0419</b>	<b>(26.6%)</b>
<b>R = 0.987262</b>	<b>+/- 0.01873</b>	<b>(1.898%)</b>

The total error of our fit is a function of the three currents with the values of parameters and their error as above. Here is the measured angle, calculated angle, their difference, the absolute error of the fit at that value, and the relative error of the fit, for every measurement.

All angles are in **degrees**.

$I_s$	$I_l$	$I_v$	measured [°]	calculated[°]	$\Delta(\text{meas.-}$ $\text{calc.})$ [°]	$\Delta(\text{meas.-}$ $\text{calc.})$ [%]	$d\phi$ [°]	$\phi = \arctan\left(\frac{R/(I+B)}{I+A}\right)$ [%]
7	0	0	89.0550	89.2409	0.1859	0.2088	0.2917	0.3275
5	0	10	41.6475	41.6825	0.0350	0.0840	0.9783	2.3491
0	0	14	0.2025	1.1516	0.9491	468.6828	0.4060	200.4940
-5	0	10	41.9400	41.9044	0.0356	0.0850	0.9761	2.3274
-7	0	0	89.4150	89.2452	0.1698	0.1899	0.2900	0.3243
0	-7	0	89.4387	89.2155	0.2233	0.2496	0.3134	0.3504
0	-5	10	40.2075	40.5074	0.2999	0.7459	1.3695	3.4062
0	0	14	1.6200	1.1516	0.4684	28.9146	0.4060	25.0618
0	5	10	42.6150	42.2987	0.3163	0.7423	1.3651	3.2033
0	7	0	89.0550	89.2500	0.0000	0.2189	0.2994	0.3362

$d\phi$  Represents the *absolute value of the total differential* of the fitted formula at that point with the parameters' values given, as well as their errors, from the fit:

$$d\Phi = \left| \frac{\partial \phi}{\partial A} \right| dA + \left| \frac{\partial \phi}{\partial R} \right| dR + \left| \frac{\partial \phi}{\partial B} \right| dB + \left| \frac{\partial \phi}{\partial K} \right| dK + \left| \frac{\partial \phi}{\partial E} \right| dE$$

where  $\phi = \arctan \left( \frac{\sqrt{(I_s + A)^2 + R^2(I_l + B)^2}}{K(I_v + E)} \right)$

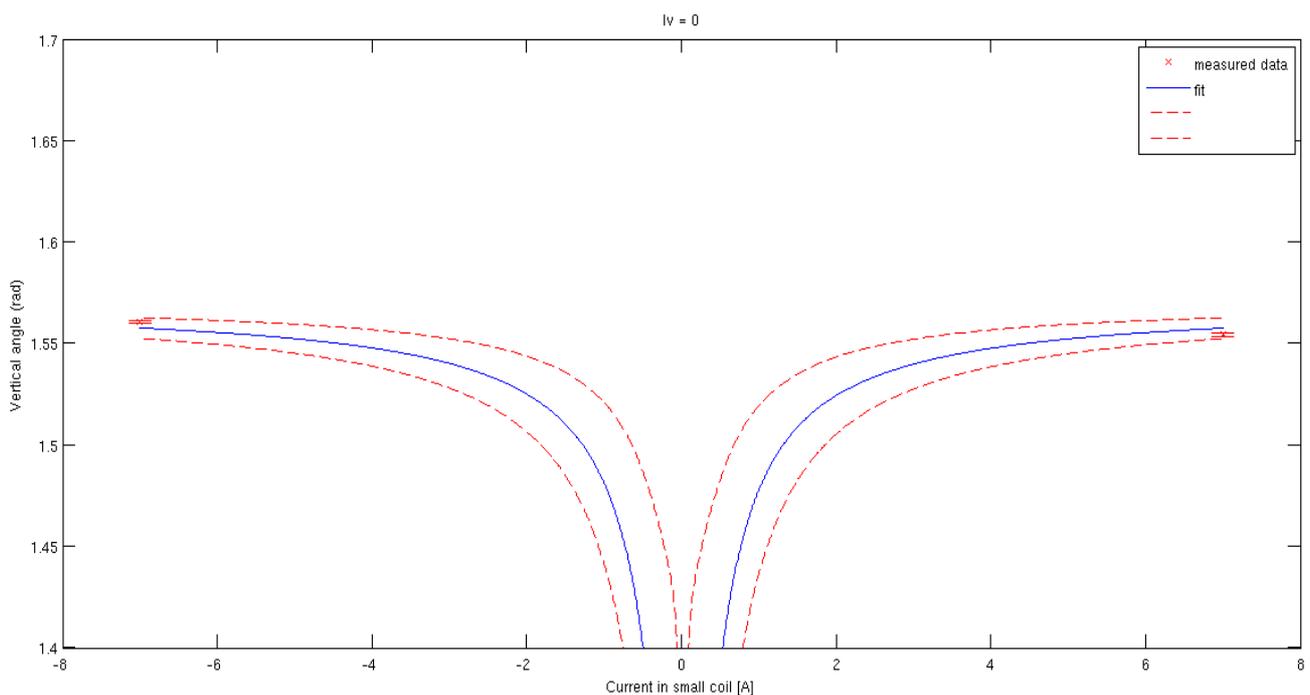
It is an estimate of the total error of the calculated value in a specific point ( $d\phi = d\phi(I_s, I_l, I_v)$ ).

### Graphs:

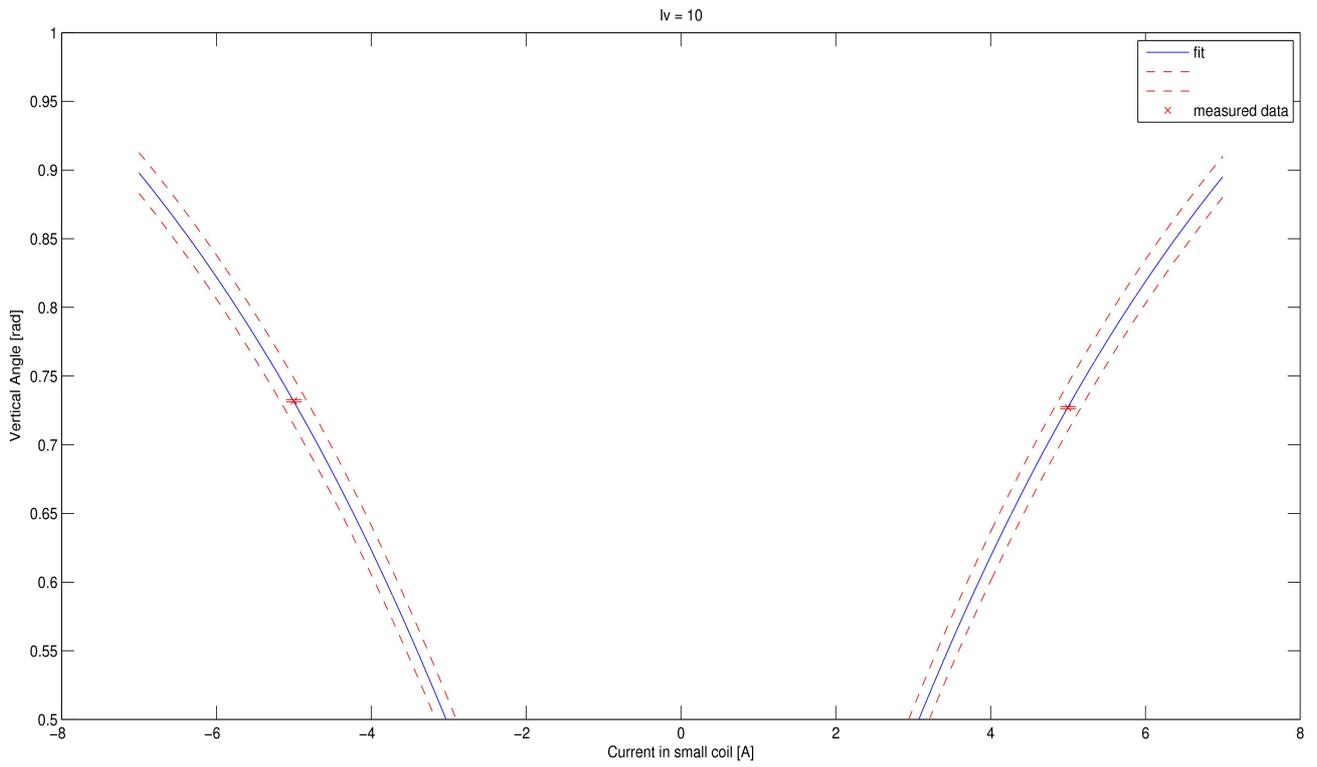
We only have two currents switched on at the same time. I decided to fix the vertical current at measured values and plot the angle as a function of the other current (i.e. small coil or large coil.) This gives us six graphs in total. I also plotted the calculated error bounds and the measured values with their errors.

#### Small coil:

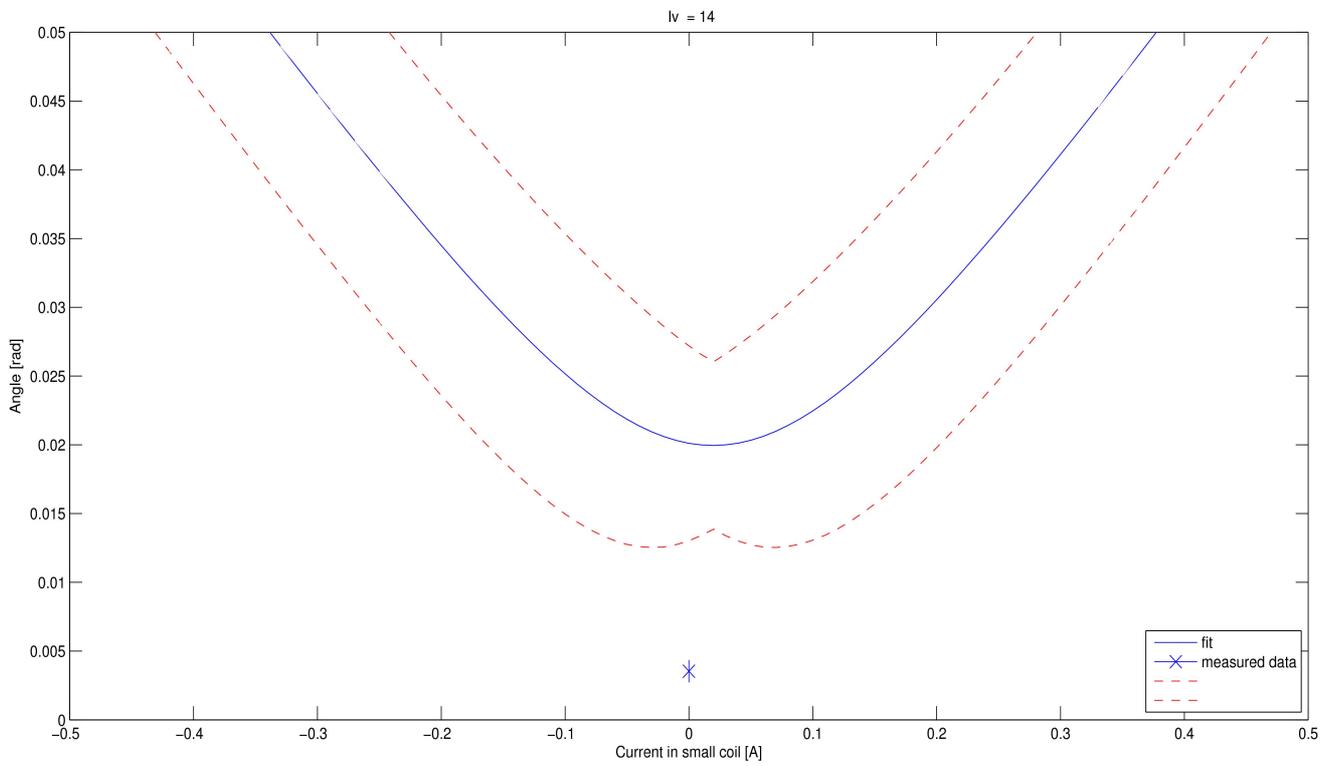
$I_v = 0$ :



$I_V = 10:$

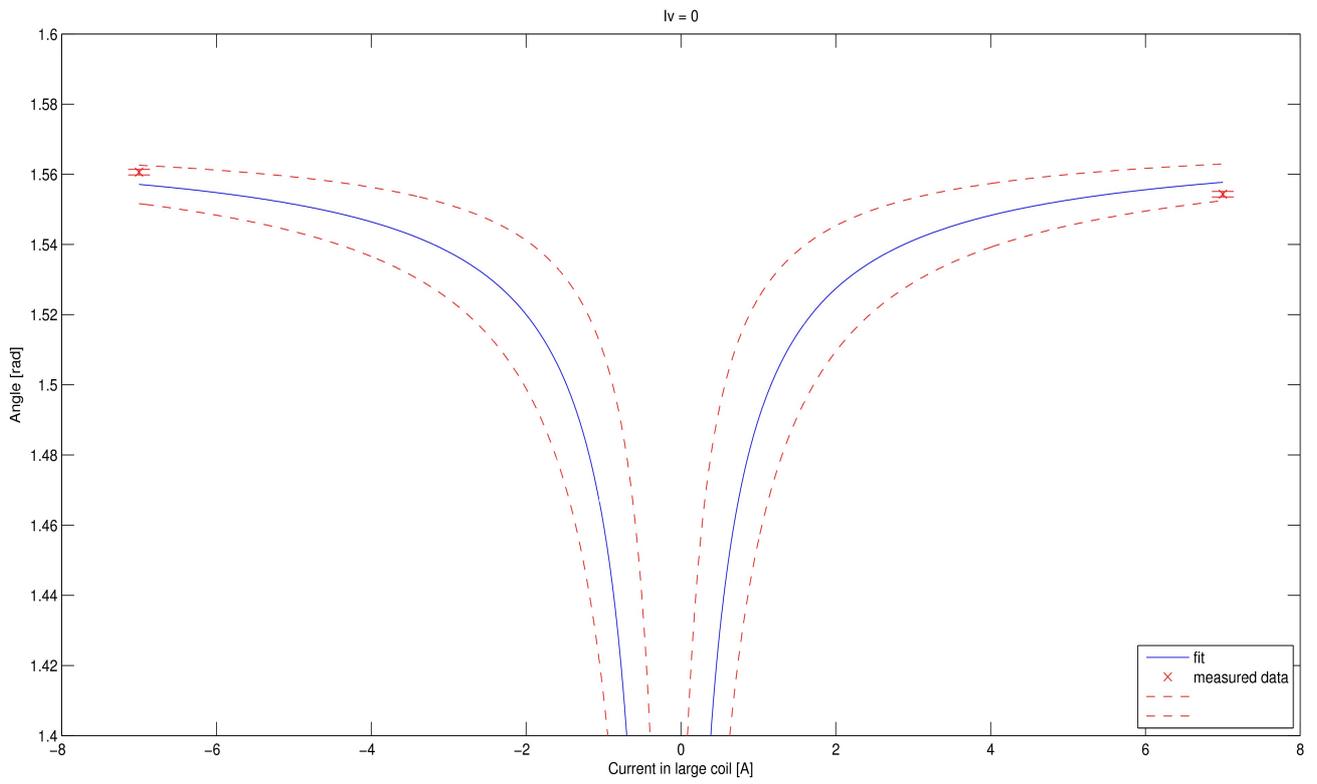


$I_V = 14:$

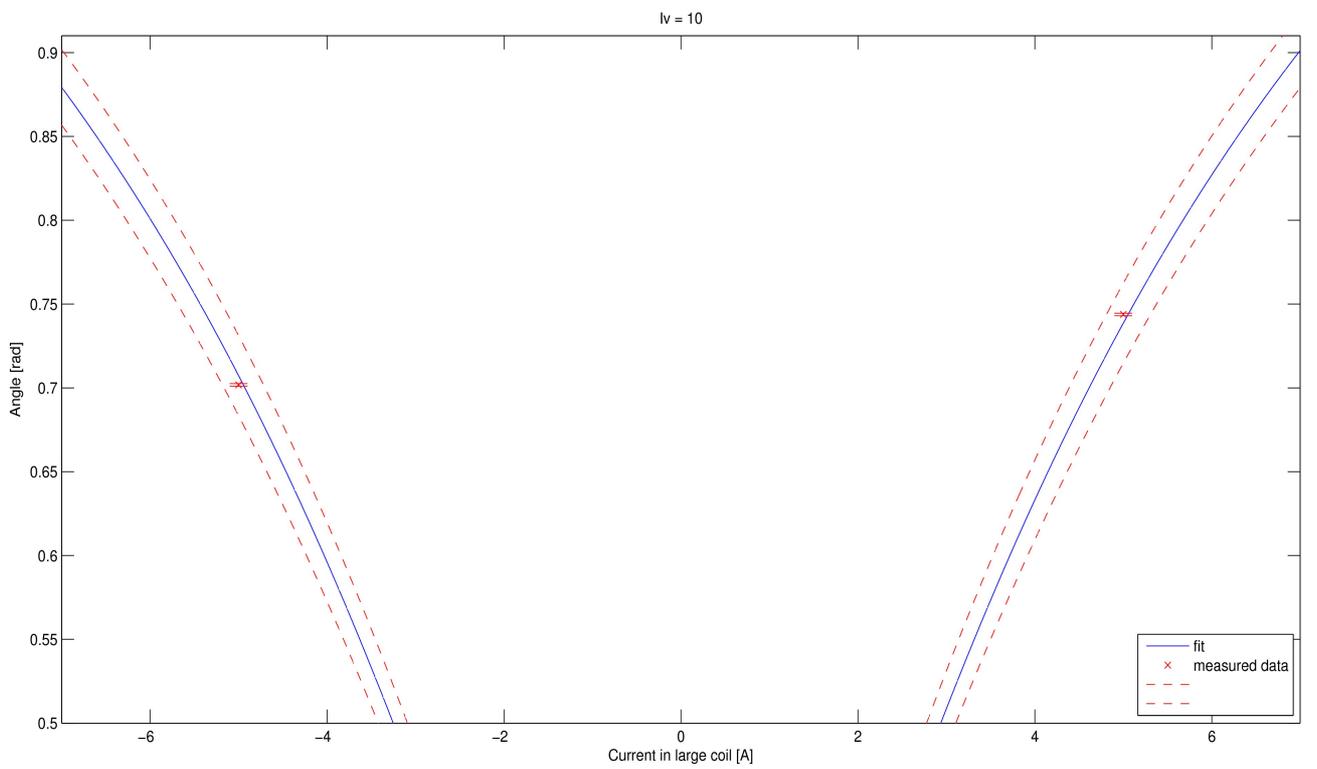


Large coil:

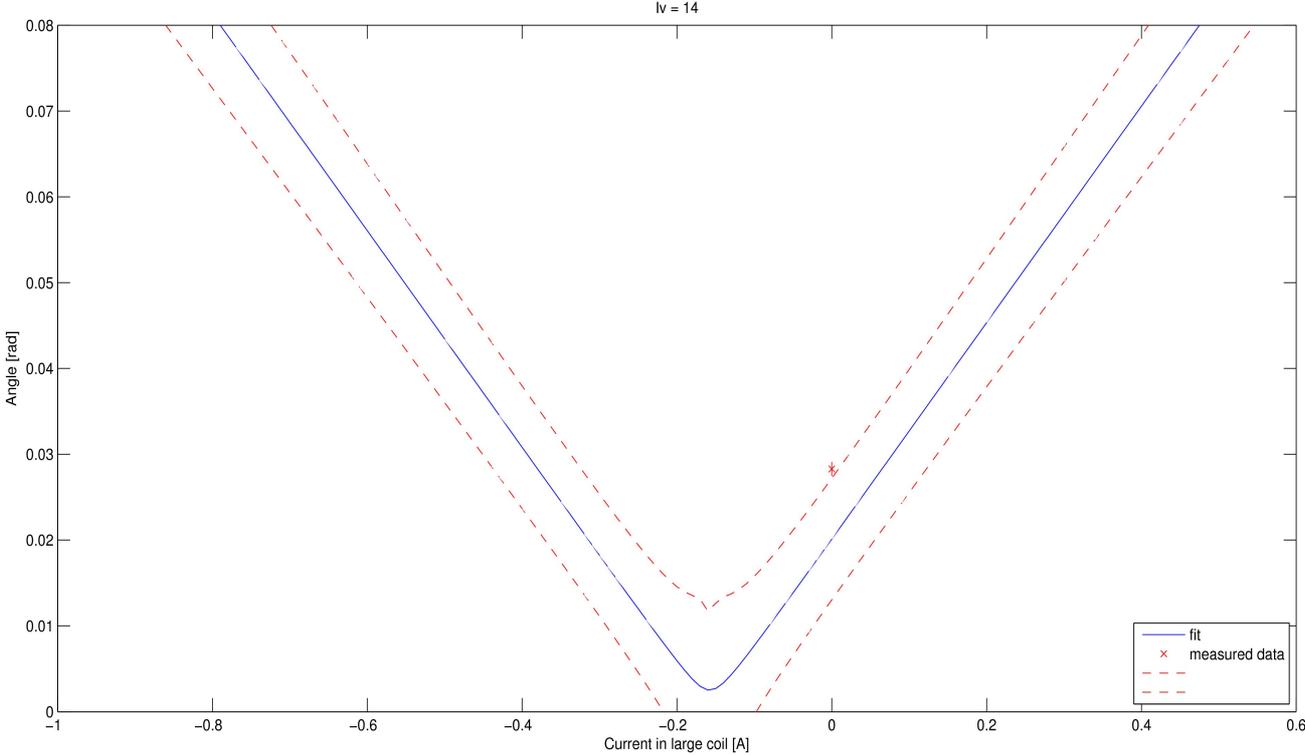
$I_v = 0$ :



$I_v = 10$ :



$I_V = 14:$



## Horizontal angle determination

Here, the formula is:

$$\phi = \arctan \left( \frac{R (I_l + B)}{(I_s + A)} \right)$$

I will call this formula the “*atan formula*”.

Parameters R, B, and A are the same as in the vertical case. (R is the scaling factor between the two coils, A and B are corrections in both directions).

We get the angle from the measurements, which are in the *Hall system* (i.e. Cartesian):

$$\tan(\Phi) = \frac{x}{z}$$

For least-squares fitting, the following expression is often more robust:

$$\frac{x}{z} = \frac{R (I_l + B)}{(I_s + A)}$$

This formula I will call the “*quotient formula*”.

I used both formulas, and will show the results from both.

Here, a mechanical compass (a needle) was used; (z, x) are the coordinates of the compass' endpoints, effectively defining two angles for every measurement.

The *coil system* is rotated from the *Hall system* by **143°**. ( - z<sub>coil</sub> axis is rotated 37° degrees from the z axis). All data was transformed into the coil system prior to analysis.

Because I was using *atan2*, the angles ranged from [-π, π] .

**Measured data:**

The two sets of coordinates corresponding to each set of currents represent the coordinates of the endpoints of the needle. Each pair of coordinates in a set is always the same endpoint. So far, the currents are in the *Coil System* and the coordinates in the *Hall System*. We have measurements with both *BigBite* spectrometer switched on and switched off.

**With BigBite ON:**

$I_s$	$I_l$	Z	X
4.2	-5.6	-5.08	231.36
		8.53	-225.24
-1.0	-6.9	163.58	164.09
		-160.76	-157.66
-5.6	-4.2	230.12	-1.63
		-226.93	10.15
-6.9	1.0	124.34	-194.29
		-136.74	180.47
-4.2	5.6	-8.92	-235.87
		-3.86	221.07
1.0	6.9	-169.16	-168.16
		155.83	153.44
5.6	4.2	-234.87	-11.25
		222.11	-4.57
6.9	-1.0	-149.41	170.82
		135.21	-186.50

With BigBite OFF:

$I_s$	$I_l$	Z	X
4.2	-5.6	-8.52	229.46
		1.88	-227.36
-5.6	-4.2	225.13	0.12
		-231.79	4.20
-4.2	5.6	-4.84	-238.88
		-3.00	217.99
5.6	4.2	-235.88	-3.15
		220.98	-3.36

### ***Determining the angle:***

As said before, we have two options for least-squares fitting: a direct formula using the *atan* of the angle, and fitting the ratio of currents with corrections directly to the ratio of the Cartesian coordinates that gives us the same angle.

But before we can do anything, we must transform the data into the same coordinate system. I chose to transform the angles into the coil system, but I could as well transform the currents ( really the magnetic field components) into the Hall system.

When using the angle, the transformation is simply rotation. The whole process for getting the angle in the *coil system* is as follows:

- Fix the midpoint of the compass' needle – subtract the average value of a coordinate in a set(of two endpoints) from each endpoint. Thus we get symmetric pairs, differing only in their sign. We have artificially pinned the compass down.
- Use *atan2* to get the angle, ranging from  $-\pi$  to  $\pi$ .  $\phi = \text{atan2}(x, z)$  This angle is in *Hall system*.
- Rotate the angle by  $143^\circ$ , taking care of the discontinuity at  $180^\circ$

To get a set of data for the *quotient formula*, the procedure is a bit different:

- Fix the midpoint of the compass' needle – subtract the average value of a coordinate in a set from each endpoint. Again we get symmetric pairs, differing only in their sign. (We only get one angle per set).
- When we have pinned the compass down, we rotate the whole dataset by **143°**, using the standard rotation matrix. Now we have the coordinates of the fixed compass' endpoints in the *Coil system*.
- Now we just compute the quotient  $x'/z'$  ( ' denotes coordinates in the *Coil system*).

**The final set of data:**

With BigBite ON:

$I_s$	$I_l$	$\Phi$ (rad) =atan2(x', z')	d $\Phi$ (rad)	x'/z' (the "quotient")	d(x'/z')
4.2	-5.6	-0.89523	6.15E-05	-1.24788	0.00016
-1.0	-6.9	-1.71443	4.96E-05	6.91414	0.00242
-5.6	-4.2	-2.52159	6.10E-05	0.71391	0.00009
-6.9	1.0	2.82505	5.52E-05	-0.32756	0.00006
-4.2	5.6	2.20549	6.12E-05	-1.35808	0.00017
1.0	6.9	1.42593	4.96E-05	6.85442	0.00238
5.6	4.2	0.66039	6.14E-05	0.77673	0.00010
6.9	-1.0	-0.25240	5.33E-05	-0.25790	0.00006

With BigBite OFF:

$I_s$	$I_l$	$\Phi$ (rad) =atan2(x', z')	d $\Phi$ (rad)	x'/z' (the "quotient")	d(x'/z')
4.2	-5.6	-0.90226	6.15E-05	-1.26603	0.00016
-5.6	-4.2	-2.50475	6.12E-05	0.73965	0.00009
-4.2	5.6	2.21254	6.13E-05	-1.33822	0.00017
5.6	4.2	0.64531	6.13E-05	0.75283	0.00010

The observational error in coordinates is **0.01 mm**. The compound observational errors in the angle and quotient were computed as a total differential using the error in coordinates as  $dz$  or  $dx$ .

The compound errors were then used as weights in least-squares fitting.

### **Data analysis:**

As already explained, I used two approaches, the first one (called "*atan2*" in the following tables), was to calculate the measured angle and then fit the arc tangent formula. The second one (called "*the quotient*" in the tables) fitted the ratio of both currents (plus corrections, etc.) to the quotient of the coordinates.

For the fitting itself, I used two tools: *MATLAB* and *gnuplot*. The regression statistics they provide are a bit different. *MATLAB* also has the option of using robust non-linear regression, which helps with the *atan2* approach.

We try to get an idea of the magnitude of *BigBite*'s influence (in the horizontal direction) by comparing the parameters' values for both sets, although the *BigBite OFF* set has only four entries (for 2 variables and 3 parameters), so the results are only a rough approximation.

At the end, we compare the values of the parameters  $A, B$  and  $R$ , which are physically the same in both the vertical and horizontal case.

All the fits are displayed in the following tables:

BigBite ON:

Approach	Tool	A	dA	dA/A [%]	B	dB	dB/B [%]	R	dR	dR/R [%]	STATS
<i>atan2</i>	<i>MATLAB</i>	<b>0.04421</b>	0.40520	916	<b>0.21440</b>	0.38940	181	<b>1.02100</b>	0.11220	11	SSE: 2.13e-06 R-square: 0.9985 Adjusted R-square: 0.9979 RMSE: 0.0006527
<i>atan2 – LAR (robust)</i>	<i>MATLAB</i>	<b>0.03747</b>	0.2912	777	<b>0.205</b>	0.2871	140	<b>0.994</b>	0.0796	8	SSE: 1.128e-06 R-square: 0.9992 Adjusted R-square: 0.9989 RMSE: 0.0004749
<i>atan2</i>	<i>gnuplot</i>	<b>0.04464</b>	0.31530	706	<b>0.19184</b>	0.31760	166	<b>1.03845</b>	0.10140	10	rms of residuals : 1618.39 variance of residuals (reduced chisquare) : 2.61917e+06
<i>quotient</i>	<i>MATLAB</i>	<b>0.03236</b>	0.02470	76	<b>0.19340</b>	0.16980	88	<b>0.99750</b>	0.00230	0.2	SSE: 2.853e-06 R-square: 0.9999 Adjusted R-square: 0.9999 RMSE: 0.0007554
<i>quotient</i>	<i>gnuplot</i>	<b>0.04008</b>	0.28730	717	<b>0.18507</b>	0.31360	169	<b>1.02990</b>	0.09350	9	rms of residuals : 1570.67 variance of residuals (reduced chisquare) : 2.46701e+06

BigBite OFF:

Approach	Tool	A	dA	dA/A [%]	B	dB	dB/B [%]	R	dR	dR/R [%]	STATS
<i>atan2</i>	<i>MATLAB</i>	- <b>0.01952</b>	0.7635	3911	<b>0.1575</b>	0.7646	485	<b>0.9873</b>	0.159	16	SSE: 3.474e-08 R-square: 1 Adjusted R-square: 0.9999 RMSE: 0.0001864
<i>atan2 – LAR (robust)</i>	<i>MATLAB</i>	<b>0.04412</b>	0.6622	1500	<b>0.08593</b>	0.6638	772	<b>0.981</b>	0.1367	14	SSE: 2.616e-08 R-square: 1 Adjusted R-square: 0.9999 RMSE: 0.0001618
<i>atan2</i>	<i>gnuplot</i>	<b>0.05679</b>	0.04545	80	<b>0.07974</b>	0.04552	57	<b>0.98555</b>	0.00944	1	rms of residuals: 149.841 variance of residuals (reduced chisquare) : 22452.2
<i>quotient</i>	<i>MATLAB</i>	<b>0.05389</b>	0.2662	493	<b>0.08174</b>	0.3240	396	<b>0.9793</b>	0.0442	5	SSE: 3.224e-08 R-square: 0.9999 Adjusted R-square: 0.9998 RMSE: 0.0001796
<i>quotient</i>	<i>gnuplot</i>	<b>0.05678</b>	0.04528	79	<b>0.07973</b>	0.04559	57	<b>0.98542</b>	0.00942	1	rms of residuals: 150.017 variance of residuals: 22505.2

BigBite ON + BigBite OFF:

Approach	Tool	A	dA	dA/A [%]	B	dB	dB/B [%]	R	dR	dR/R [%]	STATS
<i>atan2</i>	<i>MATLAB</i>	<b>0.04917</b>	0.2155	438	<b>0.1683</b>	0.2120	125	<b>1.004</b>	0.0531	5	SSE: 2.206e-06 R-square: 0.999 Adjusted R-square: 0.9988 RMSE: 0.0004951
<i>atan2 – LAR (robust)</i>	<i>MATLAB</i>	<b>0.02925</b>	0.1401	479	<b>0.1575</b>	0.1398	88	<b>0.9838</b>	0.0338	3	SSE: 9.453e-07 R-square: 0.9996 Adjusted R-square: 0.9995 RMSE: 0.0003241
<i>atan2</i>	<i>gnuplot</i>	<b>0.04642</b>	0.1992	429	<b>0.15999</b>	0.2041	128	<b>1.01274</b>	0.05453	5	rms of residuals : 1227.24 variance of residuals (reduced chisquare): 1.50613e+06
<i>quotient</i>	<i>MATLAB</i>	<b>0.02638</b>	0.0125	47	<b>0.152</b>	0.0859	57	<b>0.9975</b>	0.0013	0.1	SSE: 3.258e-06 R-square: 0.9999 Adjusted R-square: 0.9999 RMSE: 0.0006017
<i>quotient</i>	<i>gnuplot</i>	<b>0.04307</b>	0.1872	434	<b>0.15180</b>	0.19960	131	<b>1.00878</b>	0.05132	15	rms of residuals: 1187.95 variance of residuals (reduced chisquare): 1.41122e+06

### BigBite's influence:

Choosing the fitting method that “looks the best”, that is the *quotient-MATLAB* fit (the second-to-last one in the tables), we can make some assumptions about the influence of *BigBite* being turned on:

	<b>A</b>	<b>dA</b>	<b>B</b>	<b>dB</b>	<b>R</b>	<b>dR</b>
<i>BigBite</i> ON	<b>0.03236</b>	0.02470	<b>0.19340</b>	0.16980	<b>0.99750</b>	0.00230
<i>BigBite</i> OFF	<b>0.05389</b>	0.26620	<b>0.08174</b>	0.32400	<b>0.97930</b>	0.04420
<b>Difference</b>	<b>0.00969</b>	0.29090	<b>0.11166</b>	0.49380	<b>0.01820</b>	0.04650
<b>Relative</b>	<b>30%</b>		<b>58%</b>		<b>2%</b>	

BigBite mostly affects in the direction of the large coil, as the results in the parameter A cannot be trusted because of the error.

### Comparative table:

To get a better illustration, here's a table of measured angles, calculated angles, their difference, the absolute and the relative error in that point. (The error is a function of both currents). I included both *BigBite ON* and *BigBite OFF* sets of measurements,

I used both sets of measurements, and for the parameters' values I chose the “*quotient with MATLAB*” fit, which is second-to-last in all the tables. All angles are in **degrees**.

As before,  $d\phi$  represents the *absolute value of the total differential* of the fitted formula at that point with the parameters' values given, as well as their errors, from the fit:

$$d\phi = \left| \left( \frac{\partial \phi}{\partial R} \right) \right| dR + \left| \left( \frac{\partial \phi}{\partial B} \right) \right| dB + \left| \left( \frac{\partial \phi}{\partial A} \right) \right| dA$$

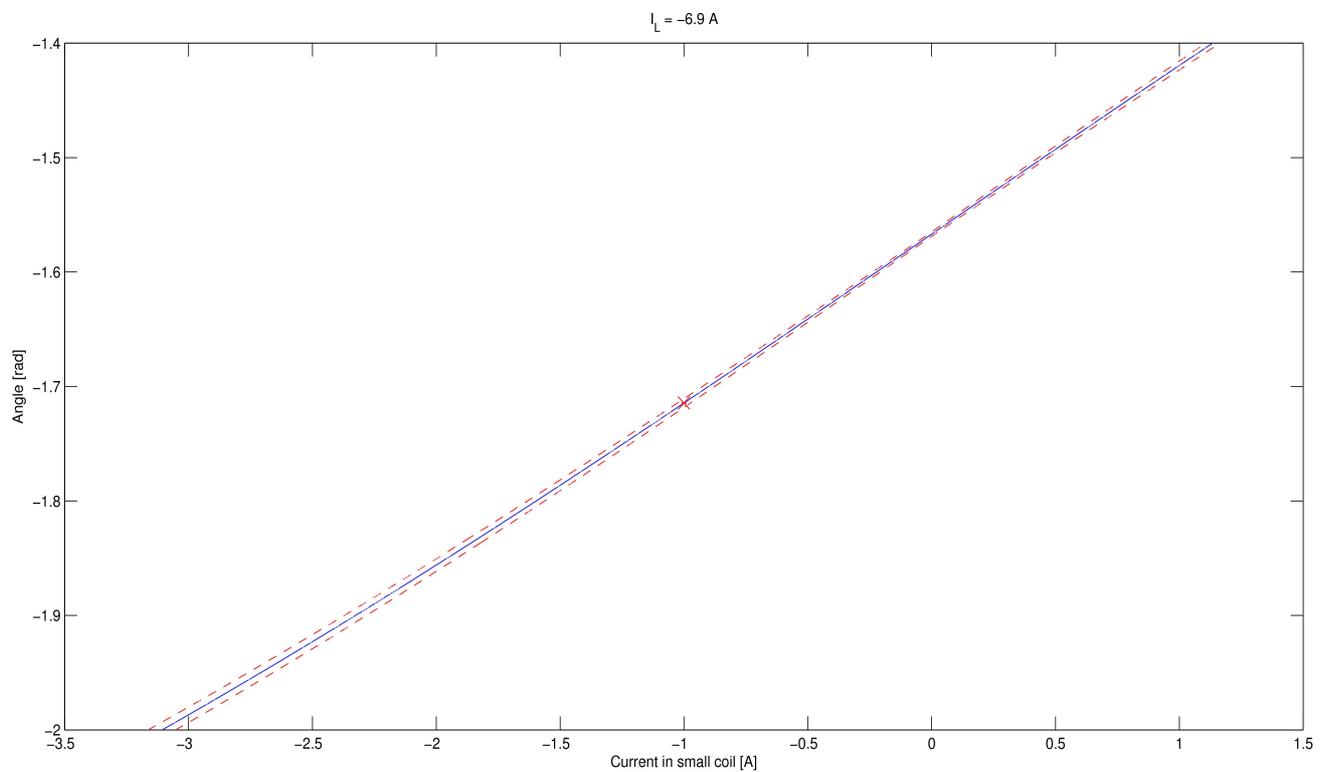
where  $\phi = \arctan \left( \frac{R(I_l + B)}{(I_s + A)} \right)$ . It is an estimate of the total error of the fit.

$I_s$	$I_l$	measured [°]	calculated[°]	$\Delta(\text{meas.} - \text{calc.})$ [°]	$\Delta(\text{meas.} - \text{calc.})$ [%]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
4.2	-5.6	-51.6958	-52.1274	0.4316	0.83488	0.5561	1.0668
4.2	-5.6	-51.2927	-52.1274	0.8347	1.62733	0.5561	1.0668
-1.0	-6.9	-98.2297	-98.2304	0.0007	0.00071	0.2181	0.2221
-5.6	-4.2	-144.4764	-144.0781	0.3983	0.27569	0.6742	0.4679
-5.6	-4.2	-143.5116	-144.0781	0.5665	0.39474	0.6742	0.4679
-6.9	1.0	161.8634	170.5092	8.6458	5.34142	0.7239	0.4246
-4.2	5.6	126.3656	126.0326	0.3329	0.26344	0.5242	0.4159
-4.2	5.6	126.7692	126.0326	0.7366	0.58106	0.5242	0.4159
1.0	6.9	81.6996	81.6986	0.0010	0.00122	0.2101	0.2571
5.6	4.2	37.8375	37.6525	0.1849	0.48900	0.6446	1.7121
5.6	4.2	36.9737	37.6525	0.6789	1.83617	0.6446	1.7121
6.9	-1.0	-14.4612	-6.9627	7.4985	51.85254	0.7198	10.3380

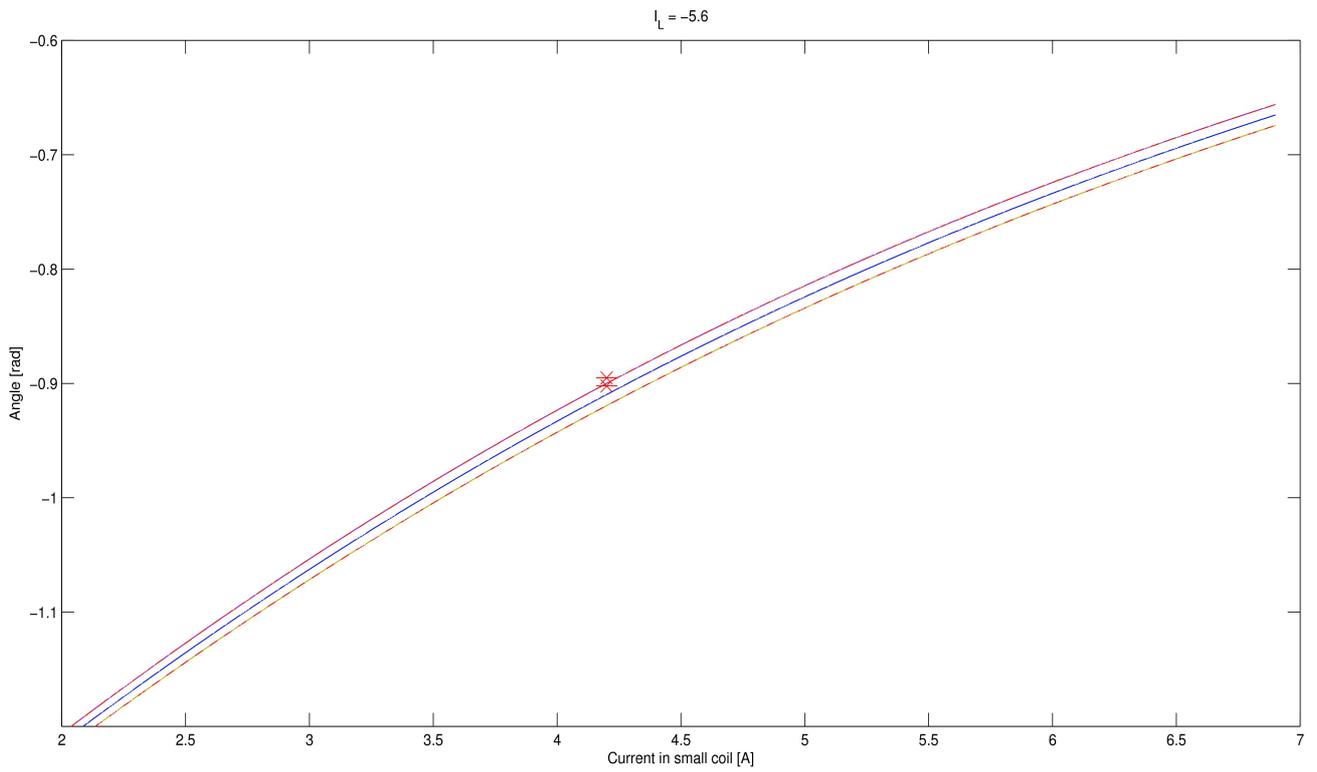
## Graphs:

Both currents go through these values: (-6.9, -5.6, -4.2, -1, 1, 4.2, 5.6, 6.9). I decided to plot the angle as a function of the current in the small coil, keeping the current in the large coil fixed, resulting in 8 graphs total. Also plotted are the error bounds and the appropriate measured points with error bars.

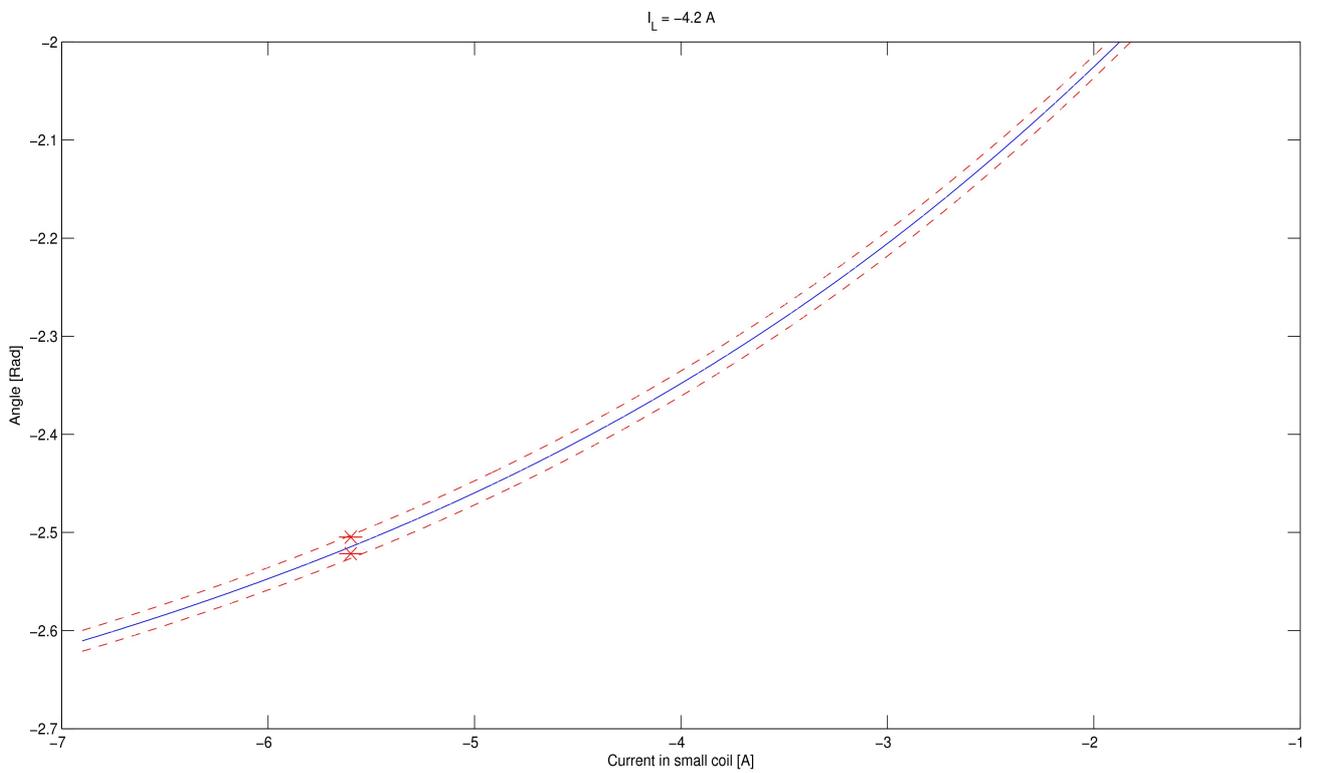
$I_L = -6.9 \text{ A}$



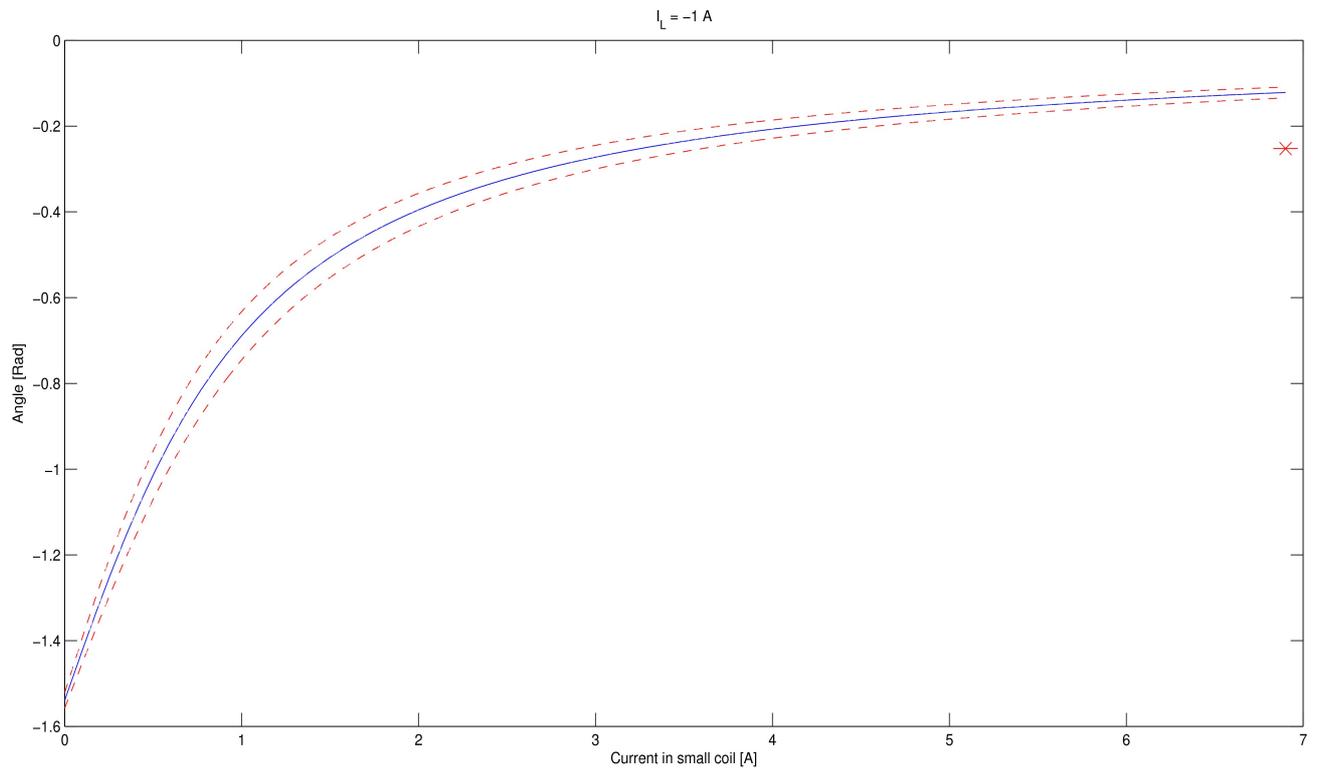
$I_l = -5.6 \text{ A}$



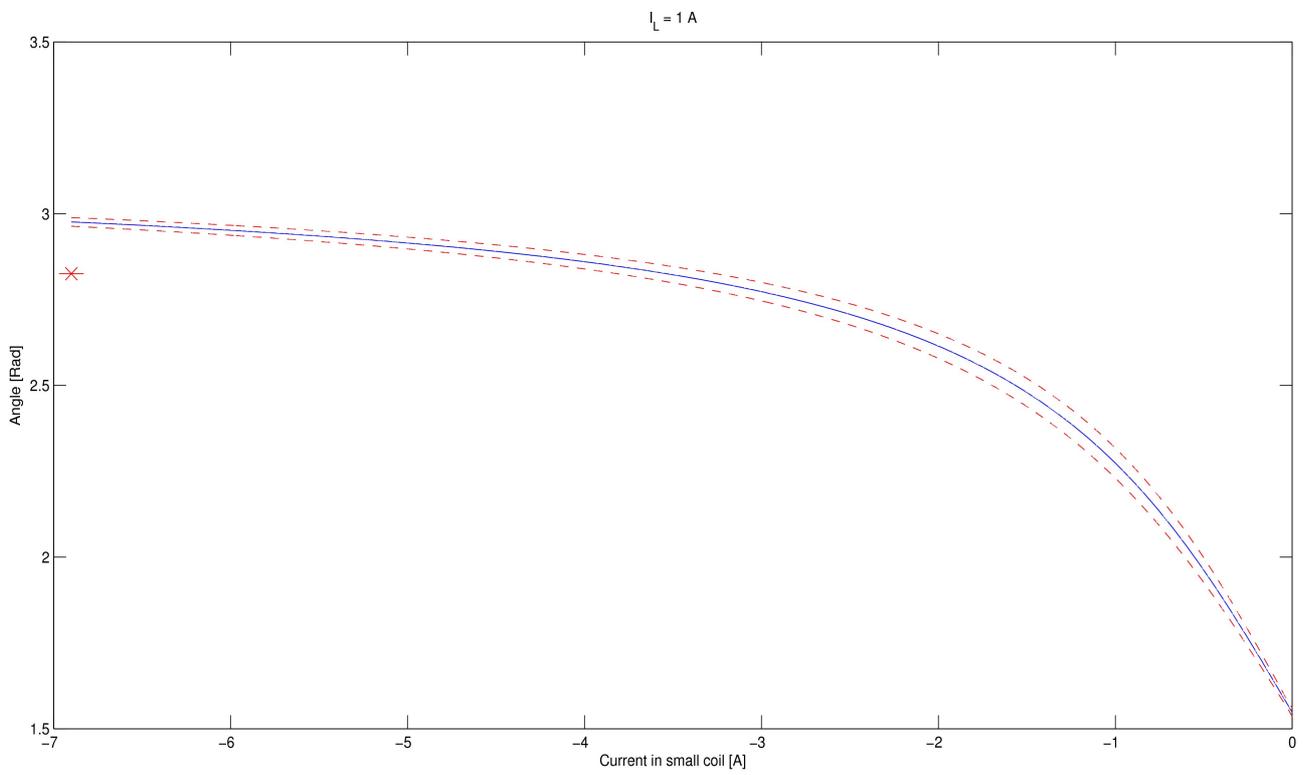
$I_l = -4.2 \text{ A}$



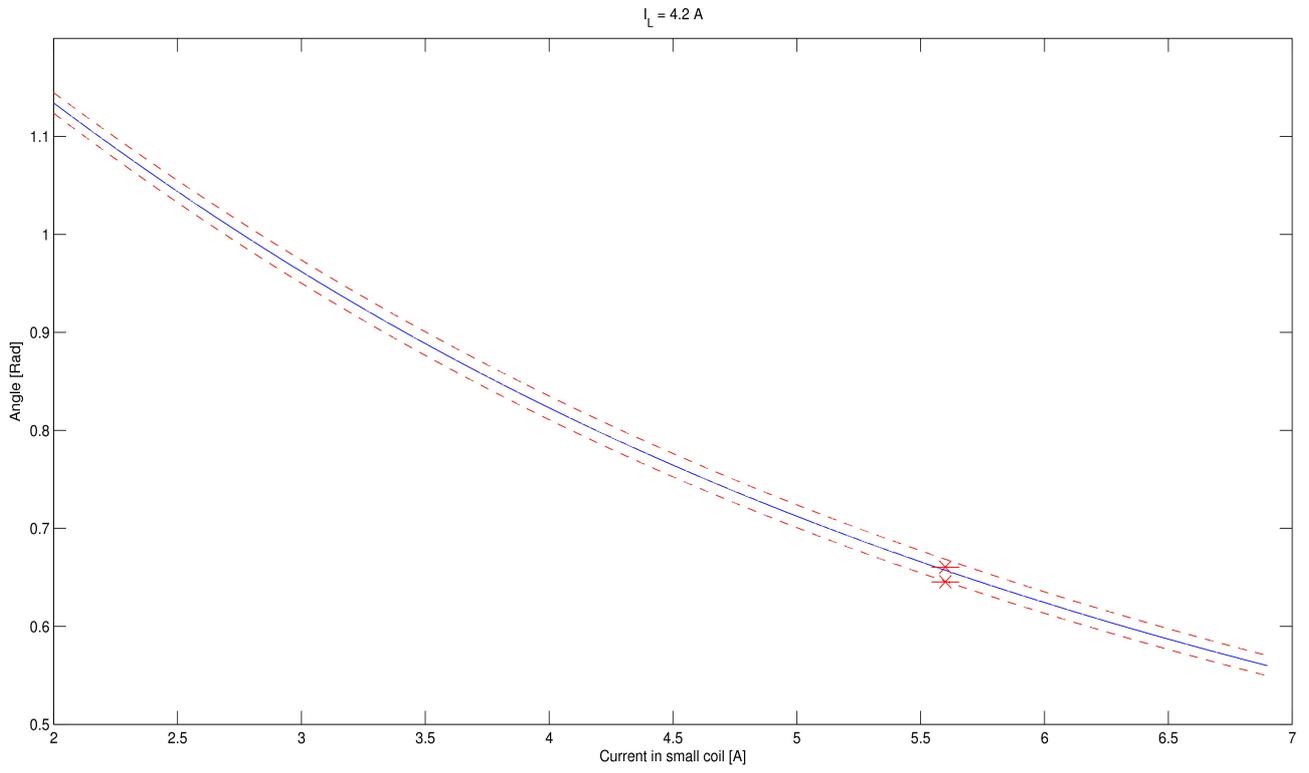
$I_l = -1 \text{ A}$



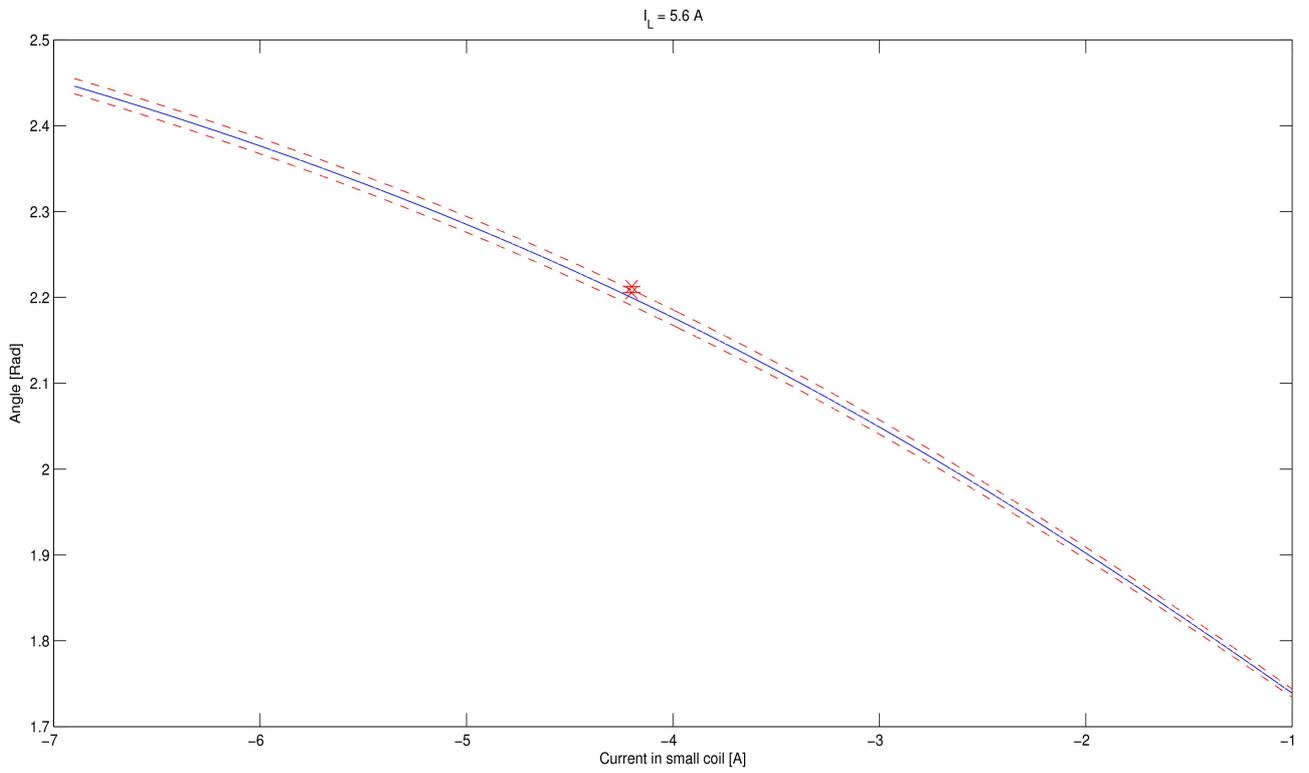
$I_l = 1 \text{ A}$



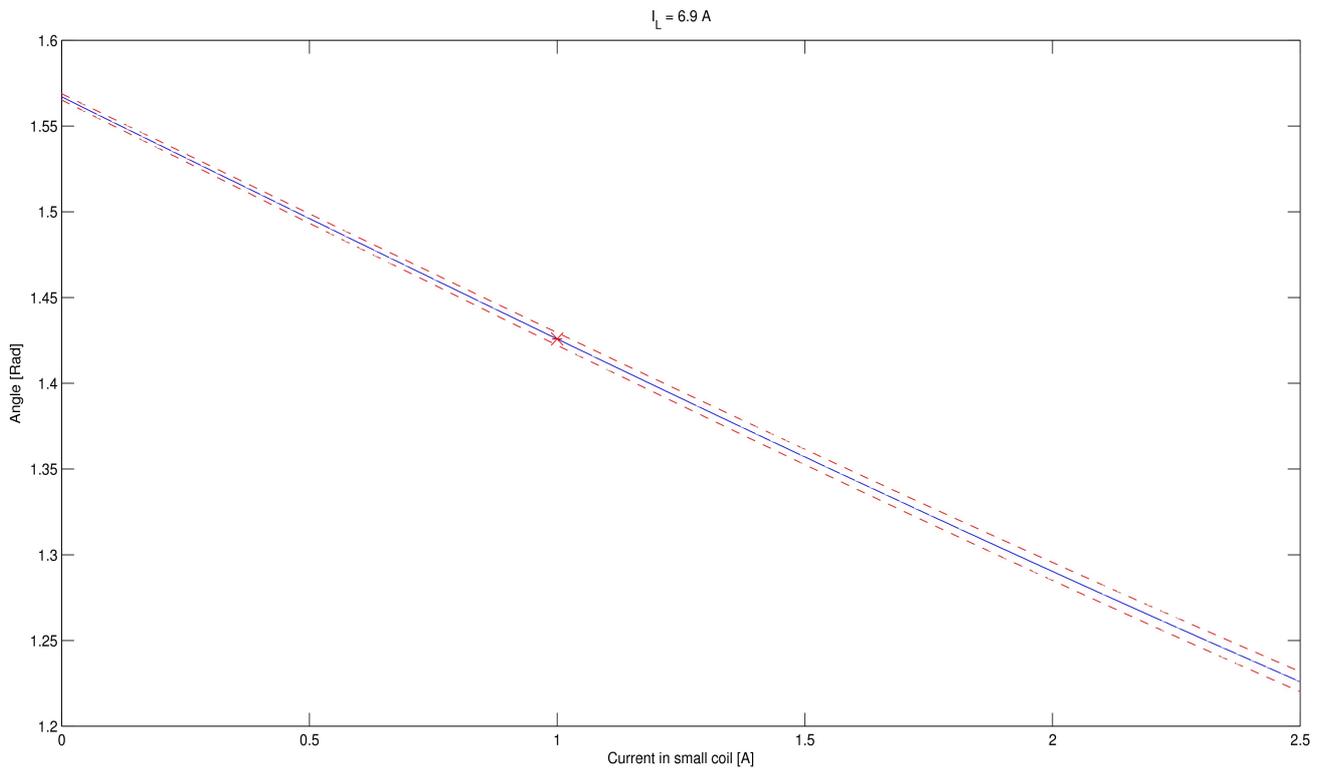
$I_l = 4.2 \text{ A}$



$I_l = 5.6 \text{ A}$



$I_L = 6.9 \text{ A}$



### Comparison between the vertical and horizontal values for parameters:

We compare the parameters that are common to both cases. For the horizontal case I used the same fit as before, the *MATLAB quotient* one. I used the values from the fit over both BigBite ON and OFF datasets.

The relative difference is calculated with respect to the value from the horizontal case, since this is the more trustworthy one.

	<b>A</b>	<b>dA</b>	<b>B</b>	<b>dB</b>	<b>R</b>	<b>dR</b>
Vertical	<b>-0.01951</b>	0.06669	<b>0.15753</b>	0.04190	<b>0.98726</b>	0.01873
Horizontal	<b>0.02638</b>	0.01250	<b>0.15200</b>	0.08590	<b>0.99750</b>	0.00130
<b>Difference</b>	<b>0.04589</b>		<b>0.00553</b>		<b>0.01024</b>	
<b>Relative</b>	<b>174%</b>		<b>4%</b>		<b>1%</b>	

## Fits without the parameter A:

Looking at the table above, we see that the parameter A is very small in size with a large dispersion. We suspected it might be effectively zero, and is used in the fits as some sort of “dummy variable”, taking any value necessary to minimize the *chi squared*.

If that were the case, fitting both formulas without this parameter should give us better results, as the system is thus better defined.

### Values for parameters:

#### Vertical:

<b>K = 0.550371</b>	<b>+/- 0.007509</b>	<b>(1.364%)</b>
<b>E = 0.16803</b>	<b>+/- 0.05584</b>	<b>(33.23%)</b>
<b>B = 0.158136</b>	<b>+/- 0.0382</b>	<b>(24.16%)</b>
<b>R = 0.987281</b>	<b>+/- 0.0173</b>	<b>(1.752%)</b>

For comparison, the old set is:

A = -0.0195053	+/- 0.06669	(341.9%) )
K = 0.550365	+/- 0.008133	(1.478%)
E = 0.168082	+/- 0.06048	(35.98%)
B = 0.157533	+/- 0.0419	(26.6%)
R = 0.987262	+/- 0.01873	(1.898%)

#### Horizontal:

<b>B = -0.02776</b>	<b>+/- 0.0120</b>	<b>(43%)</b>
<b>R = 0.9976</b>	<b>+/- 0.0017</b>	<b>(0.2%)</b>

The old set:

A = 0.02638	+/- 0.01250	(47%)
B = 0.152	+/- 0.08590	(57%)
R = 0.9975	+/- 0.00130	(0.1%)

**Vertical fit WITHOUT parameter A:**

$l_s$	$l_l$	$l_v$	measured [°]	calculated[°]	$\Delta(\text{meas.-}$ $\text{calc.})$ [°]	$\Delta(\text{meas.-}$ $\text{calc.})$ [%]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
7	0	0	89.05500	89.24330	0.18830	0.21140	0.25260	0.28360
5	0	10	41.64750	41.79360	0.14610	0.35070	0.20240	0.48590
0	0	14	0.20250	1.14710	0.94460	466.48720	0.30320	149.71990
-5	0	10	41.94000	41.79360	0.14640	0.34920	0.20240	0.48250
-7	0	0	89.41500	89.24330	0.17170	0.19200	0.25260	0.28250
0	-7	0	89.43870	89.21570	0.22300	0.24930	0.27980	0.31290
0	-5	10	40.20750	40.50690	0.29940	0.74470	0.91310	2.27090
0	0	14	1.62000	1.14710	0.47290	29.18910	0.30320	18.71500
0	5	10	42.61500	42.30510	0.30990	0.72730	0.90650	2.12730
0	7	0	89.05500	89.25040	0.19540	0.21940	0.26730	0.30010

**For comparison, vertical fit WITH A (from a couple of pages up)**

$l_s$	$l_l$	$l_v$	measured [°]	calculated[°]	$\Delta(\text{meas.}-\text{calc.})$ [°]	$\Delta(\text{meas.}-\text{calc.})$ [%]	$d\phi$ [°]	$\phi = \arctan\left(\frac{R/(L+B)}{(L+A)}\right)$ [%]
7	0	0	89.0550	89.2409	0.1859	0.2088	0.2917	0.3275
5	0	10	41.6475	41.6825	0.0350	0.0840	0.9783	2.3491
0	0	14	0.2025	1.1516	0.9491	468.6828	0.4060	200.4940
-5	0	10	41.9400	41.9044	0.0356	0.0850	0.9761	2.3274
-7	0	0	89.4150	89.2452	0.1698	0.1899	0.2900	0.3243
0	-7	0	89.4387	89.2155	0.2233	0.2496	0.3134	0.3504
0	-5	10	40.2075	40.5074	0.2999	0.7459	1.3695	3.4062
0	0	14	1.6200	1.1516	0.4684	28.9146	0.4060	25.0618
0	5	10	42.6150	42.2987	0.3163	0.7423	1.3651	3.2033
0	7	0	89.0550	89.2500	0.1950	0.2189	0.2994	0.3362

**Horizontal fit WITHOUT A:**

$l_s$	$l_l$	measured [°]	calculated[°]	$\Delta(\text{meas.}-\text{calc.})$ [°]	$\Delta(\text{meas.}-\text{calc.})$ [%]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
4.2	-5.6	-51.69580	-53.20000	1.50420	2.90960	0.10540	0.20390
4.2	-5.6	-51.29270	-53.20000	1.90730	3.71850	0.10540	0.20560
-1.0	-6.9	-98.22970	-98.23320	0.00350	0.00360	0.02790	0.02840
-5.6	-4.2	-144.47640	-143.01490	1.46150	1.01160	0.12510	0.08660
-5.6	-4.2	-143.51160	-143.01490	0.49670	0.34610	0.12510	0.08720
-6.9	1.0	161.86340	171.99860	10.13520	6.26160	0.11090	0.06850
-4.2	5.6	126.36560	127.07290	0.70730	0.55970	0.10630	0.08410
-4.2	5.6	126.76920	127.07290	0.30360	0.23950	0.10630	0.08390
1.0	6.9	81.69960	81.70120	0.00160	0.00190	0.02820	0.03460
5.6	4.2	37.83750	37.83750	1.21570	3.21300	0.12560	0.33210
5.6	4.2	36.97366	36.62180	0.35190	0.95180	0.12560	0.33980
6.9	-1.0	-14.46124	-8.45190	6.00930	41.55470	0.11150	0.77070

**Horizontal fit WITH A:**

$l_s$	$l_l$	measured [°]	calculated[°]	$\Delta(\text{meas.} - \text{calc.})$ [°]	$\Delta(\text{meas.} - \text{calc.})$ [%]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
4.2	-5.6	-51.6958	-52.1274	0.4316	0.83488	0.5561	1.0668
4.2	-5.6	-51.2927	-52.1274	0.8347	1.62733	0.5561	1.0668
-1.0	-6.9	-98.2297	-98.2304	0.0007	0.00071	0.2181	0.2221
-5.6	-4.2	-144.4764	-144.0781	0.3983	0.27569	0.6742	0.4679
-5.6	-4.2	-143.5116	-144.0781	0.5665	0.39474	0.6742	0.4679
-6.9	1.0	161.8634	170.5092	8.6458	5.34142	0.7239	0.4246
-4.2	5.6	126.3656	126.0326	0.3329	0.26344	0.5242	0.4159
-4.2	5.6	126.7692	126.0326	0.7366	0.58106	0.5242	0.4159
1.0	6.9	81.6996	81.6986	0.0010	0.00122	0.2101	0.2571
5.6	4.2	37.8375	37.6525	0.1849	0.48900	0.6446	1.7121
5.6	4.2	36.9737	37.6525	0.6789	1.83617	0.6446	1.7121
6.9	-1.0	-14.4612	-6.9627	7.4985	51.85254	0.7198	10.3380

While we can “get away” without using the parameter A in the vertical case, the results are about twice as inaccurate in the horizontal case. The only exception are the four measurements without BigBite, where the fit without A is comparably good, which is understandable, since the main component in A (theoretically) is BigBite's fringe fields.

### **Final results:**

For the final values of the parameters I used the weighted average of parameter values from both the vertical and horizontal measurements, except for K and E, which are only measurable in the vertical case. I did not use the fits without parameter A; those were only a test whether A isn't exactly zero.

### **Final values:**

$$\mathbf{A = 0.0248 \quad +/- 0.0123 \quad (50\%)}$$

$$\mathbf{B = 0.1565 \quad +/- 0.0377 \quad (24\%)}$$

$$\mathbf{R = 0.99745 \quad +/- 0.00130 \quad (0.1\%)}$$

$$\mathbf{K = 0.550365 \quad +/- 0.008133 \quad (1.478\%)}$$

$$\mathbf{E = 0.168082 \quad +/- 0.06048 \quad (35.98\%)}$$