Vertical angle determination

We try to determine the vertical angle of the magnetic field's direction by fitting the following formula to the data:

$$\phi = \arctan\left(\frac{\sqrt{(I_s + A)^2 + R^2(I_l + B)^2}}{K(I_v + E)}\right)$$

Here I_s is the current in the small coil, I_l the current in the large coil and I_v the current in the vertical coil.

A, B, E, R, K are free parameters. A, B, E represent corrections in the all three directions of the components of the magnetic field; R and K are scaling factors.

The angle is counted as zero when the field is vertical.

We can get an estimate of the parameters, which we will use in fitting of the whole formula, by examining the data with only one of the planar coils switched on. Here, assuming the correction B (or A) is much smaller than the field generated by I_s or I_l , the above formula simplifies to:

$$\Phi = \arctan\left(\frac{(I_s + A)}{(K(I_v + E))}\right)$$

and

$$\Phi = \arctan\left(\kappa\frac{(I_l + B)}{(I_v + E)}\right)$$

where κ is R/K.

Measured data:

Small coil + vertical coil:

I_s	I_v	Compass	Compass - reverse
7	0	1990	11
5	10	1462.5	537
0	14	997.5	1002
-5	10	534	1466
-7	0	7	1994

The first column is I_s, second I_v; the third and fourth columns are reading taken from the compass, the second one after rotating it round the vertical axis by 180°.

We first use both readings to find the zero of the compass. Looking at our formula, we see that we must limit ourselves to the interval (0, π/2). The angle is then given by the deviation of the measurement from the compass' zero.

The (average) zero of the compass in this measurement is **1000.1** .

Data from the first measurement, with the angle in radians:

I_s	I_l	I_v	Angle (rad)
7	0	0	1.5543
5	0	10	0.7269
0	0	14	0.0035
-5	0	10	0.7320
-7	0	0	1.5606

(Here I wrote the values of all three currents: I_s, I_l, and I_v).

The error for the angle is **± 0.0007854** (roughly 0.05 °) for all values.

Large coil + vertical coil:

I_l	I_v	Compass	Compass - reverse
-7	0	3972	1959
-5	10	518	1411.5
0	14	983	947
5	10	1438	491
7	0	1955	3976

To avoid problems using mod(4000), we re-brand the measurements over 2000 as negative:

I_l	I_v	Compass	Compass - reverse
-7	0	-28	1959
-5	10	518	$\Phi = \arctan\left(\kappa \frac{(I_l + I$ 1411.5
0	14	983	947
5	10	1438	491
7	0	1955	-24

The (average) zero of the compass in this measurement is **965.5** .

Data from the second measurement:

I_s	I_l	I_v	Angle (rad)
0	-7	0	1.5610
0	-5	10	0.7018
0	0	14	0.0283
0	5	10	0.7438
0	7	0	1.5543

First, I tried fitting both the “partial” formulas to get an idea for the parameters' values.

Small coil fit:

A = -0.026297 +/- 0.03113 (118.4%)

K = 0.549914 +/- 0.004105 (0.7465%)

E = 0.175074 +/- 0.0401 (22.9%)

Large coil fit:

κ = 1.79487 +/- 0.01537 (0.8565%)

B = 0.210944 +/- 0.03583 (16.98%)

E = 0.171157 +/- 0.04543 (26.54%)

Using these as starting points for the fit of the whole function (with $R = \kappa * K$), we get these values for the parameters:

A = -0.0195053 +/- 0.06669 (341.9%)

K = 0.550365 +/- 0.008133 (1.478%)

E = 0.168082 +/- 0.06048 (35.98%)

B = 0.157533 +/- 0.0419 (26.6%)

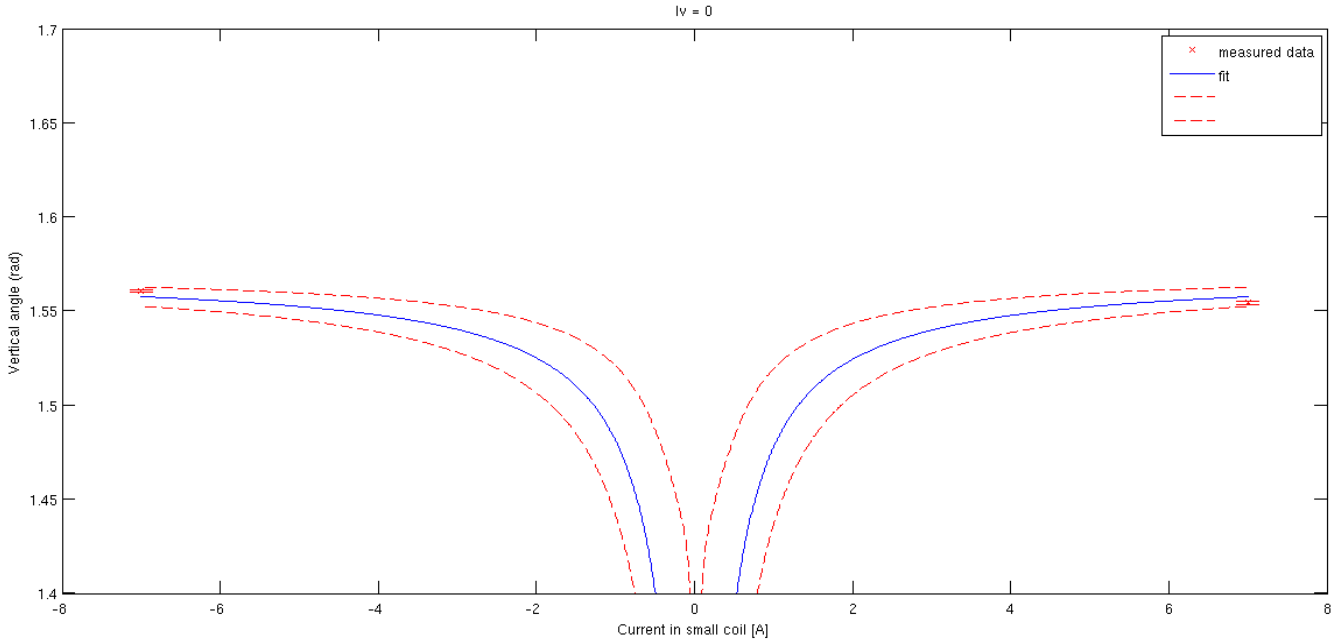
R = 0.987262 +/- 0.01873 (1.898%)

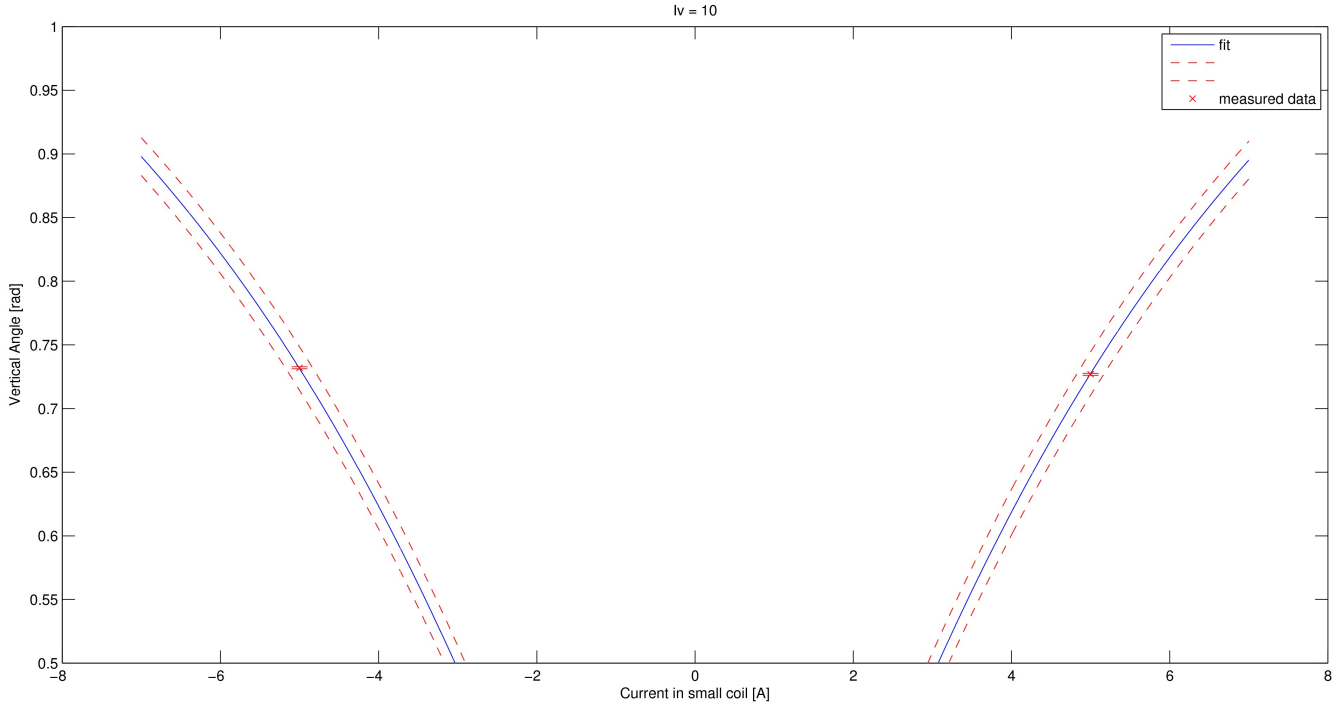
The total error of our fit is a function of the three currents with the values of parameters and their error as above. Here is the measured angle, calculated angle, their difference, the absolute error of the fit at that value, and the relative error of the fit, for every measurement. All angles are in **degrees**.

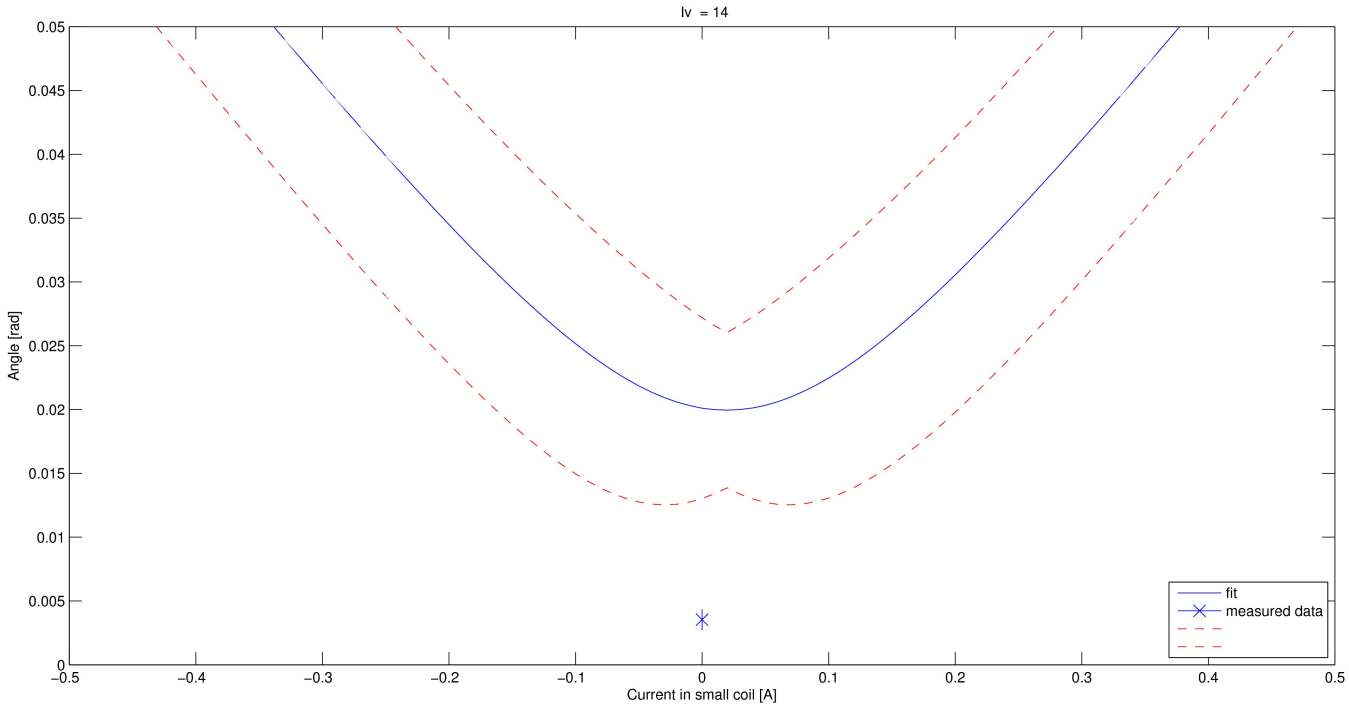
I_s	I_l	I_v	measured [°]	calculated [°]	Δ(meas.-calc.) [°]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
7	0	0	89.2409	89.0325	0.2085	0.2917	0.3276
5	0	10	41.6825	41.6371	0.0454	0.9783	2.3497
0	0	14	1.1522	0.2024	0.9498	0.4060	200.5440
-5	0	10	41.9044	41.9295	0.0251	0.9761	2.3280
-7	0	0	89.2452	89.3929	0.1477	0.2900	0.3244
0	-7	0	89.2159	89.3929	0.1770	0.3134	0.3506
0	-5	10	40.5229	40.1975	0.3254	1.3695	3.4070
0	0	14	1.1522	1.6190	0.4674	0.4060	25.0680
0	5	10	42.3142	42.6043	0.2901	1.3651	3.2041
0	7	0	89.2504	89.0325	0.2179	0.2994	0.3363

Graphs:

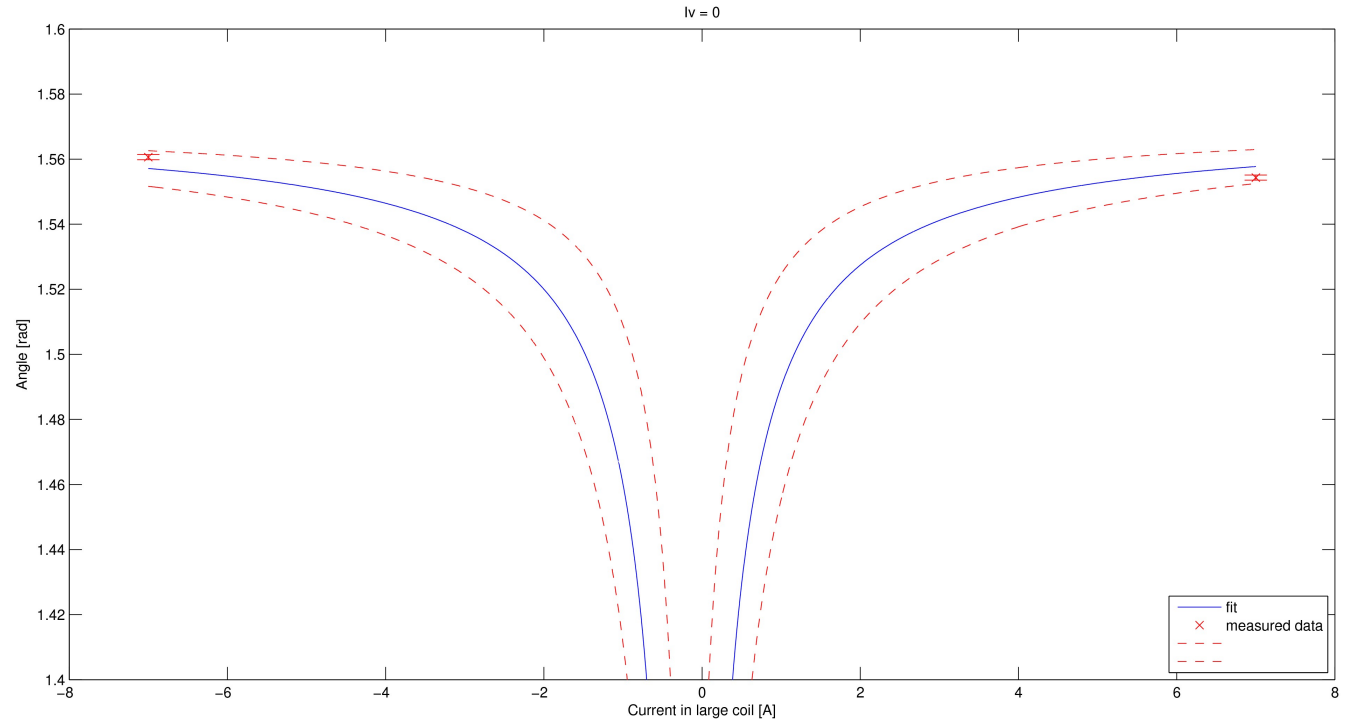
Small coil:

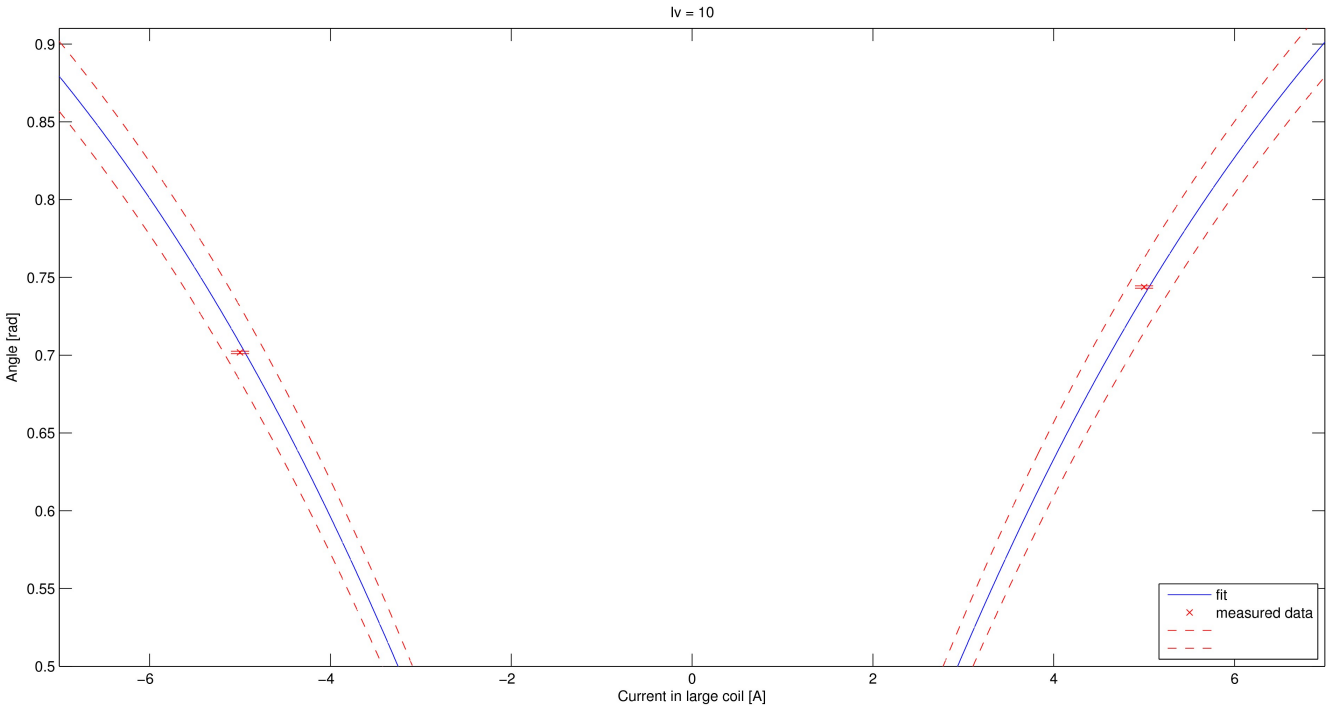


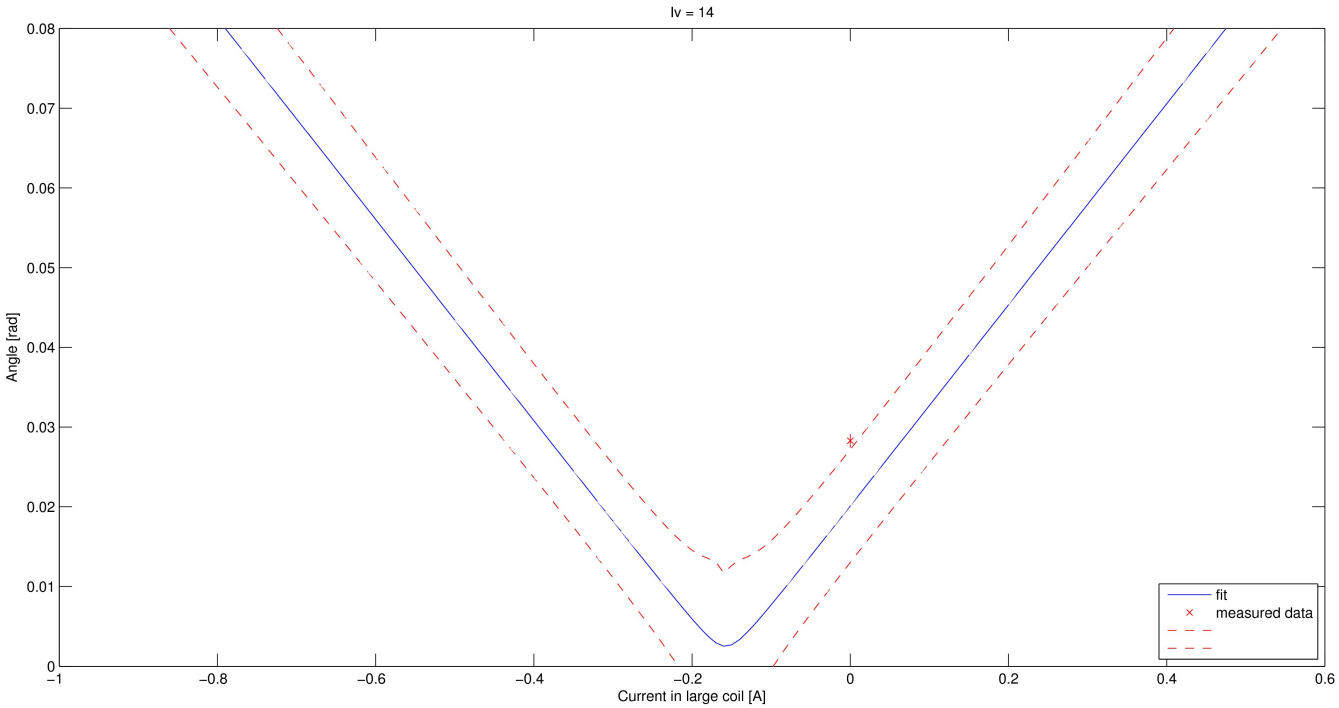



Iv = 14
Angle [rad]
Current in small coil [A]
fit
measured data

Large coil:


Iv = 0
Angle [rad]
Current in large coil [A]
fit
measured data


Iv = 10
Angle [rad]
Current in large coil [A]
fit
measured data


Iv = 14
Angle [rad]
Current in large coil [A]
fit
measured data