Vertical angle determination

We try to determine the vertical angle of the magnetic field's direction by fitting the following formula to the data:

$$\phi = \arctan\left(\frac{\sqrt{(I_s + A)^2 + R^2 (I_l + B)^2}}{K(I_v + E)}\right)$$

Here I_s is the current in the small coil, I_l the current in the large coil and I_v the current in the vertical coil.

A, B, E, R, K are free parameters. A, B, E represent corrections in the all three directions of the components of the magnetic field; R and K are scaling factors.

The angle is counted as zero when the field is vertical.

We can get an estimate of the parameters, which we will use in fitting of the whole formula, by examining the data with only one of the planar coils switched on. Here, assuming the correction B (or A) is much smaller than the field generated by I_s or I_l , the above formula simplifies to:

$$\Phi = \arctan\left(\frac{(I_s + A)}{(K(I_v + E))}\right)$$

and

$$\Phi = \arctan\left(\kappa \frac{(I_l + B)}{(I_v + E)}\right)$$

where κ is R/K.

Measured data:

Small coil + vertical coil:

I_s	I_v	Compass	Compass - reverse
7	0	1990	11
5	10	1462.5	537
0	14	997.5	1002
-5	10	534	1466
-7	0	7	1994

The first column is I_s , second I_v; the third and fourth columns are reading taken from the compass, the second one after rotating it round the vertical axis by 180°.

We first use both readings to find the zero of the compass. Looking at our formula, we see that we must limit ourselves to the interval (0, π/2). The angle is then given by the deviation of the measurement from the compass' zero.

The (average) zero of the compass in this measurement is **1000.1** .

Data from the first measurement, with the angle in radians:

I_s	I_l	I_v	Angle (rad)
7	0	0	1.5543
5	0	10	0.7269
0	0	14	0.0035
-5	0	10	0.7320
-7	0	0	1.5606

(Here I wrote the values of all three currents: I_s , I_l, and I_v).

The error for the angle is **± 0.0007854** (roughly 0.05 °) for all values.

Large coil + vertical coil:

I_l	I_v	Compass	Compass - reverse
-7	0	3972	1959
-5	10	518	1411.5
0	14	983	947
5	10	1438	491
7	0	1955	3976

To avoid problems using mod(4000), we re-brand the measurements over 2000 as negative:

I_l	I_v	Compass	Compass - reverse
-7	0	-28	1959
-5	10	518	1411.5
0	14	983	947
5	10	1438	491
7	0	1955	-24

The (average) zero of the compass in this measurement is **965.5** .

Data from the second measurement:

I_s	I_l	I_v	Angle (rad)
0	-7	0	1.5610
0	-5	10	0.7018
0	0	14	0.0283
0	5	10	0.7438
0	7	0	1.5543

First, I tried fitting both the “partial” formulas to get an idea for the parameters' values.

Small coil fit:

A = -0.026297 +/- 0.03113 (118.4%)

K = 0.549914 +/- 0.004105 (0.7465%)

E = 0.175074 +/- 0.0401 (22.9%) Arial

Large coil fit:

κ = 1.79487 +/- 0.01537 (0.8565%)

B = 0.210944 +/- 0.03583 (16.98%)

E = 0.171157 +/- 0.04543 (26.54%)

Using these as starting points for the fit of the whole function (with R= κ * K), we get these values for the parameters:

A = -0.0195053 +/- 0.06669 (341.9%)

K = 0.550365 +/- 0.008133 (1.478%)

E = 0.168082 +/- 0.06048 (35.98%)

B = 0.157533 +/- 0.0419 (26.6%)

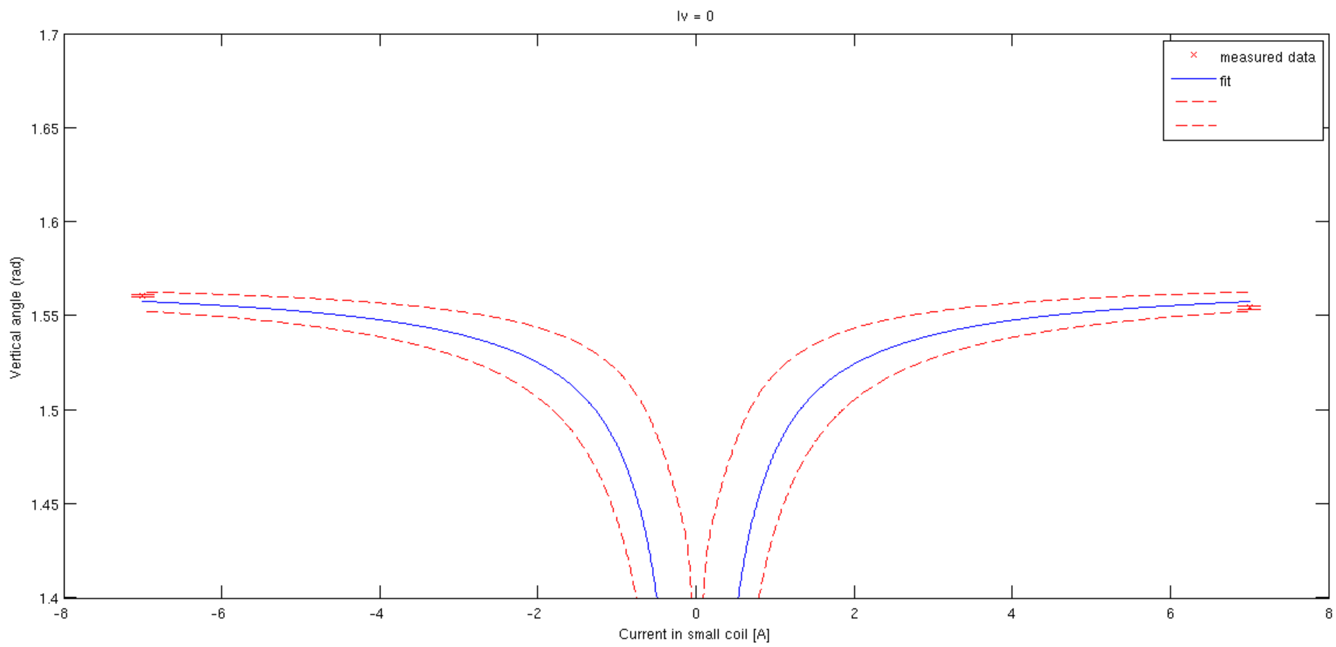
R = 0.987262 +/- 0.01873 (1.898%)

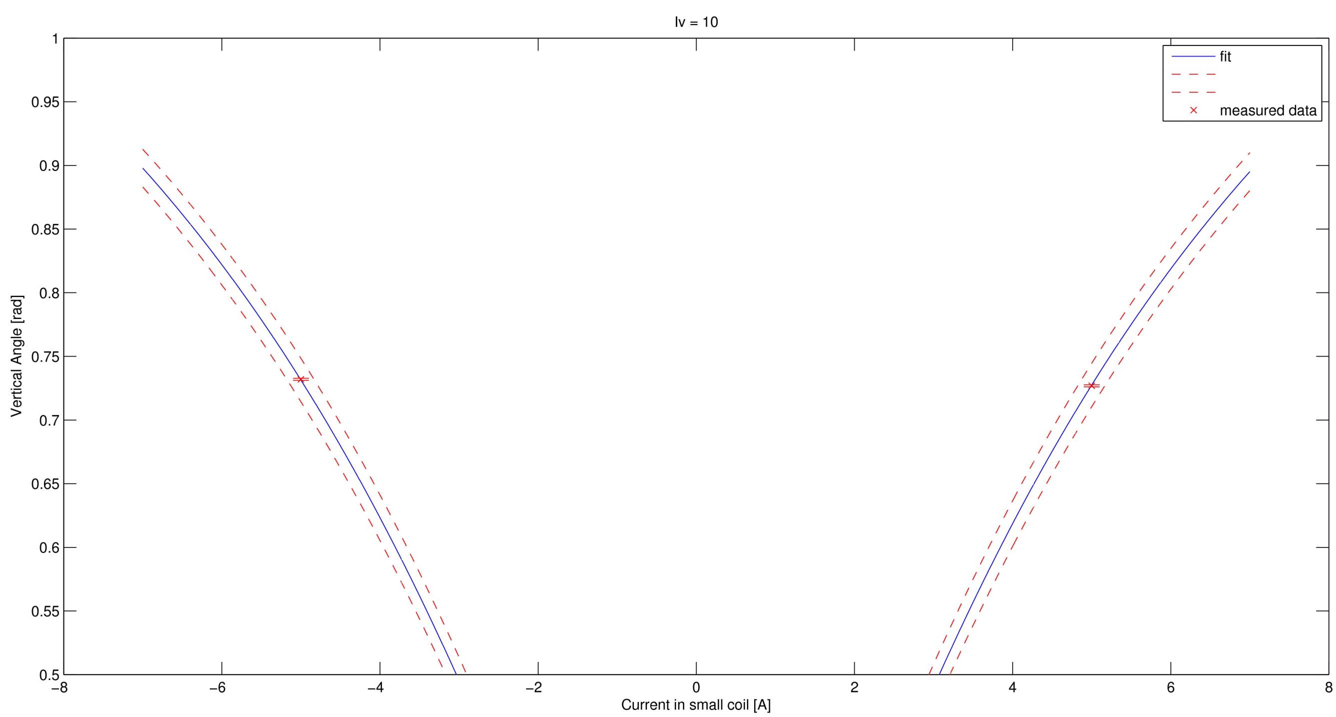
The total error of our fit is a function of the three currents with the values of parameters and their error as above. Here is the measured angle, calculated angle, their difference, the absolute error of the fit at that value, and the relative error of the fit, for every measurement. All angles are in **degrees**.

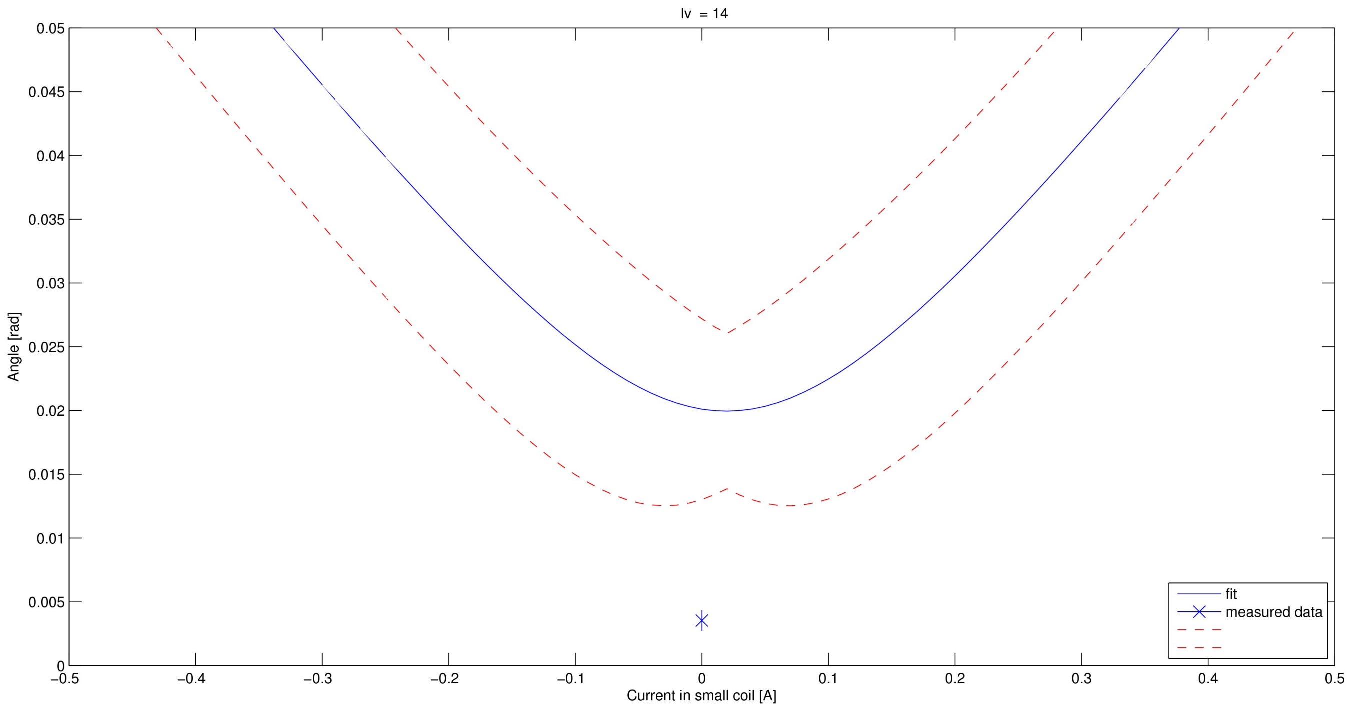
I_s	I_l	I_v	measured [°]	calculated [°]	Δ(meas.-calc.) [°]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
7	0	0	89.2409	89.0325	0.2085	0.2917	0.3276
5	0	10	41.6825	41.6371	0.0454	0.9783	2.3497
0	0	14	1.1522	0.2024	0.9498	0.4060	200.5440
-5	0	10	41.9044	41.9295	0.0251	0.9761	2.3280
-7	0	0	89.2452	89.3929	0.1477	0.2900	0.3244
0	-7	0	89.2159	89.3929	0.1770	0.3134	0.3506
0	-5	10	40.5229	40.1975	0.3254	1.3695	3.4070
0	0	14	1.1522	1.6190	0.4674	0.4060	25.0680
0	5	10	42.3142	42.6043	0.2901	1.3651	3.2041
0	7	0	89.2504	89.0325	0.2179	0.2994	0.3363

Graphs:

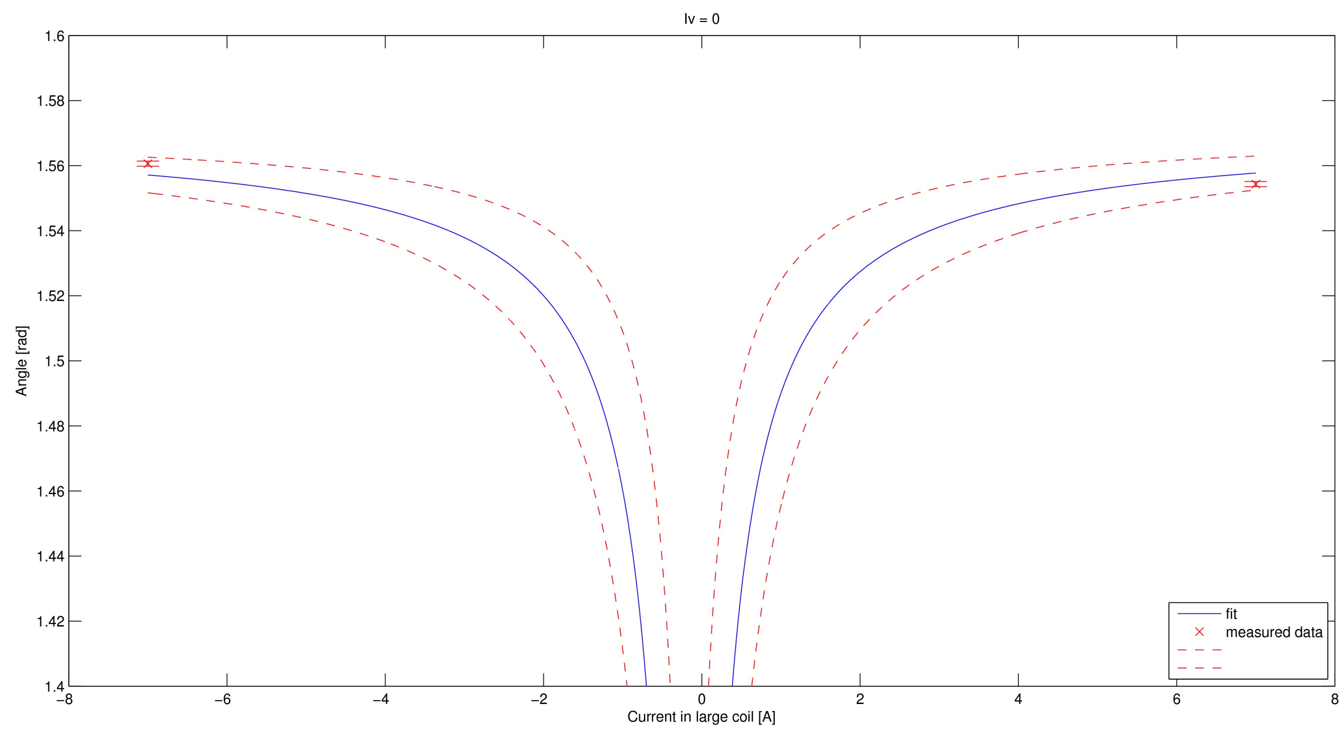
Small coil:

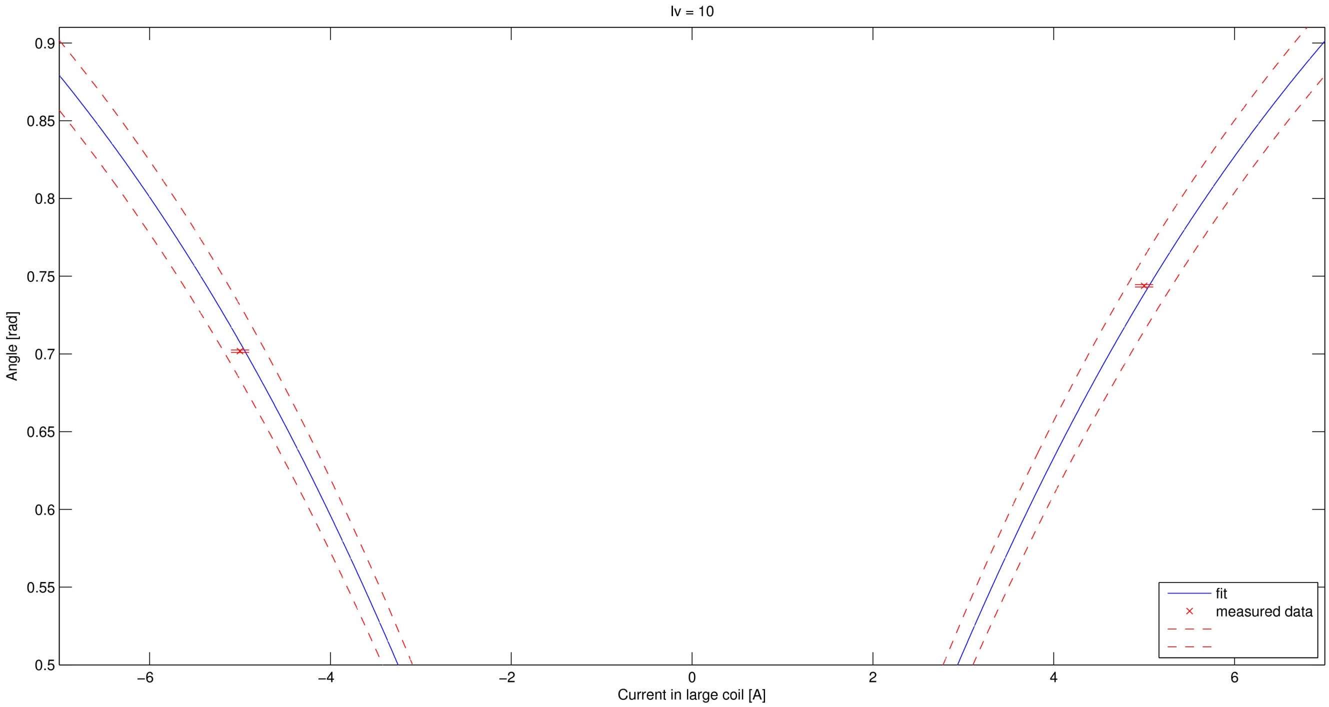


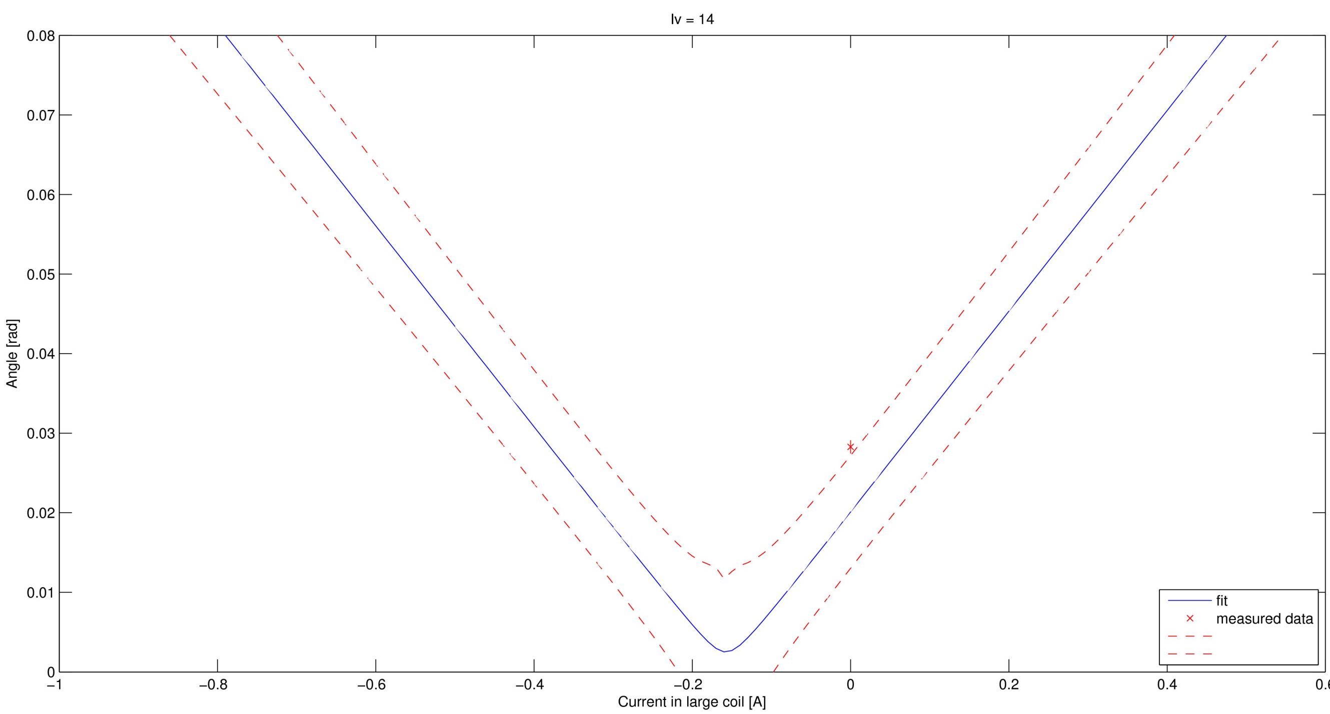



Iv = 14
Angle [rad]
Current in small coil [A]
fit
measured data

Large coil:


Iv = 0
Angle [rad]
Current in large coil [A]
fit
measured data


Iv = 10
Angle [rad]
Current in large coil [A]
fit
measured data


Iv = 14
Angle [rad]
Current in large coil [A]
fit
measured data

Planar angle determination

Here, the formula is:

$$\phi = \arctan\left(\frac{R\,(I_l + B)}{(I_s + A)}\right)$$

Parameters R, B, and A are the same as in the vertical case. (R is the scaling factor between the two coils, A and B are corrections in both directions).

We get the angle from the measurements, which are in the *Hall system* (i.e. Cartesian):

$$\tan(\Phi) = \frac{x}{z}$$

For least-squares fitting, the following expression is often more robust:

$$\frac{x}{z} = \frac{R\,(I_l + B)}{(I_s + A)}$$

I used both formulas, and will show the results from both.

Here, a mechanical compass (a needle) was used; (z, x) are the coordinates of the compass' endpoints, effectively defining two angles for every measurement.

The *coil system* is rotated from the *Hall system* by **143°**. ($-z_{coil}$ axis is rotated 37° degrees from the z axis). All data was transformed into the coil system prior to analysis.

Because I was using *atan2*, the angles were transformed to [-π, π] .

Measured data:

With BigBite ON:

I_s	I_l	Z	X
4.2	-5.6	-5.08	231.36
		8.53	-225.24
-1.0	-6.9	163.58	164.09
		-160.76	-157.66
-5.6	-4.2	230.12	-1.63
		-226.93	10.15
-6.9	1.0	124.34	-194.29
		-136.74	180.47
-4.2	5.6	-8.92	-235.87
		-3.86	221.07
1.0	6.9	-169.16	-168.16
		155.83	153.44
5.6	4.2	-234.87	-11.25
		222.11	-4.57
6.9	-1.0	-149.41	170.82
		135.21	-186.50

(The two sets of coordinates corresponding to each set of currents represent the coordinates of the endpoints of the needle. Each pair of coordinates in a set is always the same endpoint. So far, the currents are in the *Coil System* and the coordinates in the *Hall System*.)

With BigBite OFF:

I_s	I_l	Z	X
4.2	-5.6	-8.52	229.46
		1.88	-227.36
-5.6	-4.2	225.13	0.12
		-231.79	4.20
-4.2	5.6	-4.84	-238.88
		-3.00	217.99
5.6	4.2	-235.88	-3.15
		220.98	-3.36

To determine the angle, we must do the following:

- Find the zero of the compass – subtract the average value of a coordinate in a pair from each endpoint. Thus we get symmetric pairs, differing only in their sign.
- Use atan2 to get the angle, ranging from $-\pi$ to π.
- Rotate the angle by 143°, taking care of the discontinuity at 180°

The algorithm for deriving the data, used for fitting the simplified formula ("*the quotient*"), is a bit different:

- Find the zero of the compass – subtract the average value of a coordinate in a pair from each endpoint. Again we get symmetric pairs, differing only in their sign. (We only get one angle per pair of currents)
- Rotate the whole dataset of coordinates by 143°.
- Compute the quotient x'/z'.

<u>The final set of data:</u>

<u>With BigBite ON:</u>

I_s	I_l	Φ (rad) =atan2(x', z')	dΦ (rad)	x'/z' (the "quotient")	d(x'/z')
4.2	-5.6	-0.89523	0.00006	-1.24788	0.00016
-1.0	-6.9	-1.71443	0.00005	6.91414	0.00242
-5.6	-4.2	-2.52159	0.00006	0.71391	0.00009
-6.9	1.0	2.82505	0.00006	-0.32756	0.00006
-4.2	5.6	2.20549	0.00006	-1.35808	0.00017
1.0	6.9	1.42593	0.00005	6.85442	0.00238
5.6	4.2	0.66039	0.00006	0.77673	0.00010
6.9	-1.0	-0.25240	0.00005	-0.25790	0.00006

<u>With BigBite OFF:</u>

I_s	I_l	Φ (rad) =atan2(x', z')	dΦ (rad)	x'/z' (the "quotient")	d(x'/z')
4.2	-5.6	-0.90226	0.00006	-1.26603	0.00016
-5.6	-4.2	-2.50475	0.00006	0.73965	0.00009
-4.2	5.6	2.21254	0.00006	-1.33822	0.00017
5.6	4.2	0.64531	0.00006	0.75283	0.00010

The measurement error in coordinates is **0.01 mm**. The errors are computed using this value, and used as weights in least-square fitting.

<u>Data analysis:</u>

I used two approaches, the first one (called "*atan2*" from here on), was to calculate the measured angle and then fit the arctan formula. The second one (called "*the quotient"* from here on) fitted the ratio of both currents (plus corrections etc) to the quotient of the coordinates,

For fitting, I used two tools: *MATLAB* and *gnuplot*. The regression statistics they provide are a bit different. *MATLAB* also has the option of using robust non-linear regression. All the fits are displayed in the following table:

<u>BigBite ON:</u>

Approach	Tool	A	dA	B	dB	R	dR	STATS
atan2	*MATLAB*	**0.04421**	0.40520	**0.21440**	0.38940	**1.02100**	0.11220	SSE: 2.13e-06 R-square: 0.9985 Adjusted R-square: 0.9979 RMSE: 0.0006527
atan2 - LAR(robust)	*MATLAB*	**0.03747**	0.2912	**0.205**	0.2871	**0.994**	0.0796	SSE: 1.128e-06 R-square: 0.9992 Adjusted R-square: 0.9989 RMSE: 0.0004749
atan2	*gnuplot*	**0.04464**	0.31530	**0.19184**	0.31760	**1.03845**	0.10140	rms of residuals : 1618.39 variance of residuals (reduced chisquare) : 2.61917e+06
quotient	*MATLAB*	**0.03236**	0.02470	**0.19340**	0.16980	**0.99750**	0.00230	SSE: 2.853e-06 R-square: 0.9999 Adjusted R-square: 0.9999 RMSE: 0.0007554
quotient	*gnuplot*	**0.04008**	0.28730	**0.18507**	0.31360	**1.02990**	0.09350	rms of residuals : 1570.67 variance of residuals (reduced chisquare) : 2.46701e+06

BigBite OFF:

Approach	Tool	A	dA	B	dB	R	dR	STATS
atan2	*MATLAB*	**-0.01952**	0.7635	**0.1575**	0.7646	**0.9873**	0.159	SSE: 3.474e-08 R-square: 1 Adjusted R-square: 0.9999 RMSE: 0.0001864
atan2 - LAR(robust)	*MATLAB*	**0.04412**	0.6622	**0.08593**	0.6638	**0.981**	0.1367	SSE: 2.616e-08 R-square: 1 Adjusted R-square: 0.9999 RMSE: 0.0001618
atan2	*gnuplot*	**0.05679**	0.04545	**0.07974**	0.04552	**0.98555**	0.00944	rms of residuals: 149.841 variance of residuals (reduced chisquare) : 22452.2
quotient	*MATLAB*	**0.05389**	0.2662	**0.08174**	0.3240	**0.9793**	0.0442	SSE: 3.224e-08 R-square: 0.9999 Adjusted R-square: 0.9998 RMSE: 0.0001796
quotient	*gnuplot*	**0.05678**	0.04528	**0.07973**	0.04559	**0.98542**	0.00942	rms of residuals: 150.017 variance of residuals: 22505.2

Finally, to get a bigger sample size, we try fitting both datasets together:

<u>BigBite ON + BigBite OFF:</u>

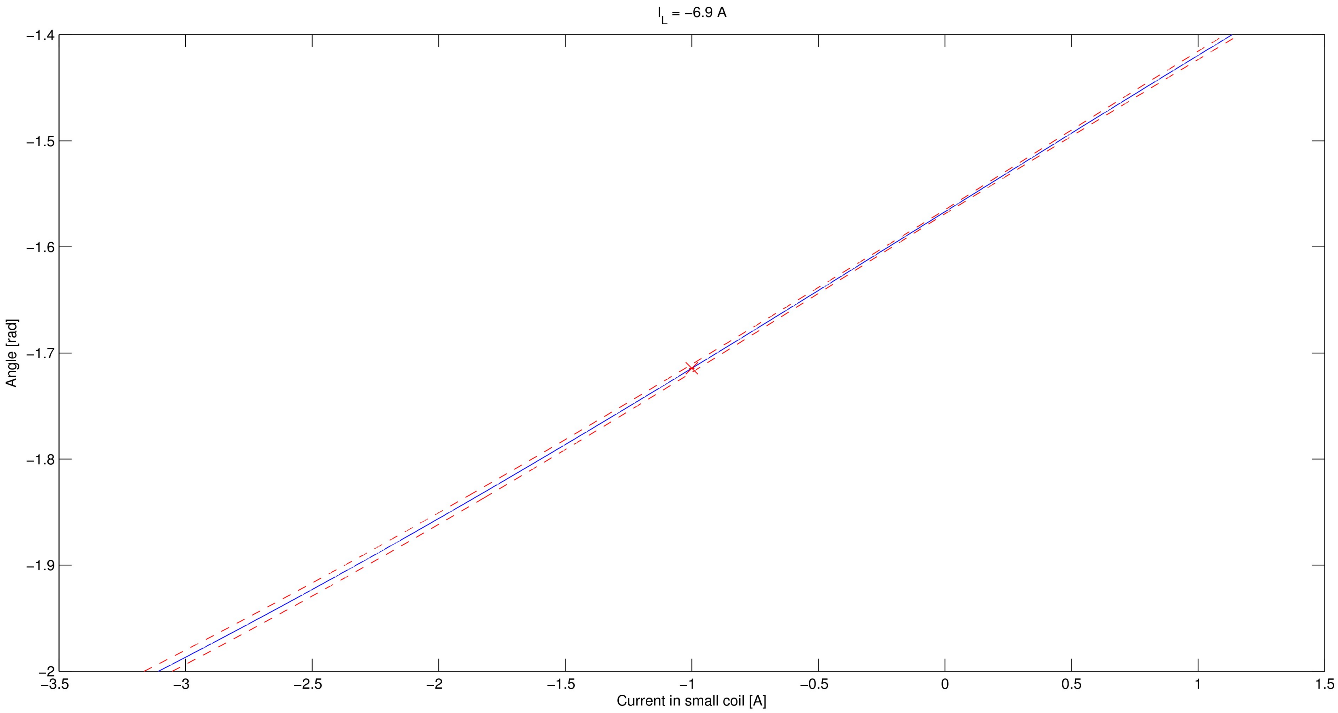
Approach	Tool	A	dA	B	dB	R	dR	STATS
atan2	*MATLAB*	**0.04917**	0.2155	**0.1683**	0.2120	**1.004**	0.0531	SSE: 2.206e-06 R-square: 0.999 Adjusted R-square: 0.9988 RMSE: 0.0004951
atan2 - LAR(robust)	*MATLAB*	**0.02925**	0.1401	**0.1575**	0.1398	**0.9838**	0.0338	SSE: 9.453e-07 R-square: 0.9996 Adjusted R-square: 0.9995 RMSE: 0.0003241
atan2	*gnuplot*	**0.0464188**	0.1992	**0.15999**	0.2041	**1.01274**	0.05453	rms of residuals : 1227.24 variance of residuals (reduced chisquare): 1.50613e+06
quotient	*MATLAB*	**0.02638**	0.0125	**0.152**	0.0859	**0.9975**	0.0013	SSE: 3.258e-06 R-square: 0.9999 Adjusted R-square: 0.9999 RMSE: 0.0006017
quotient	*gnuplot*	**0.04307**	0.1872	**0.15180**	0.19960	**1.00878**	0.05132	rms of residuals: 1187.95 variance of residuals (reduced chisquare): 1.41122e+06

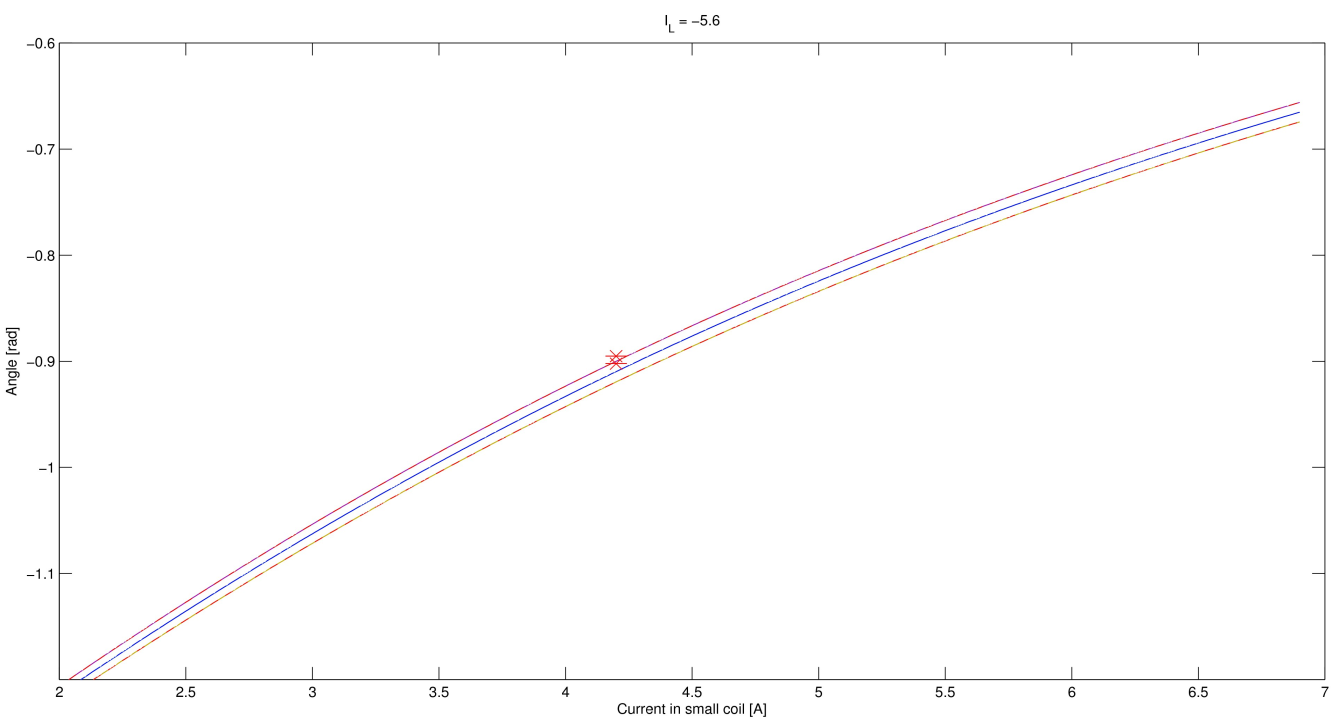
To get a better illustration, here's a table of measured angles, calculated angles, their difference, the absolute and the relative error in that point. (The error is a function of both currents).
I used both sets of measurements, and for the parameters' values I chose the "*quotient with MATLAB"* fit, which is second-to-last in the previous table. All angles are in **degrees**.

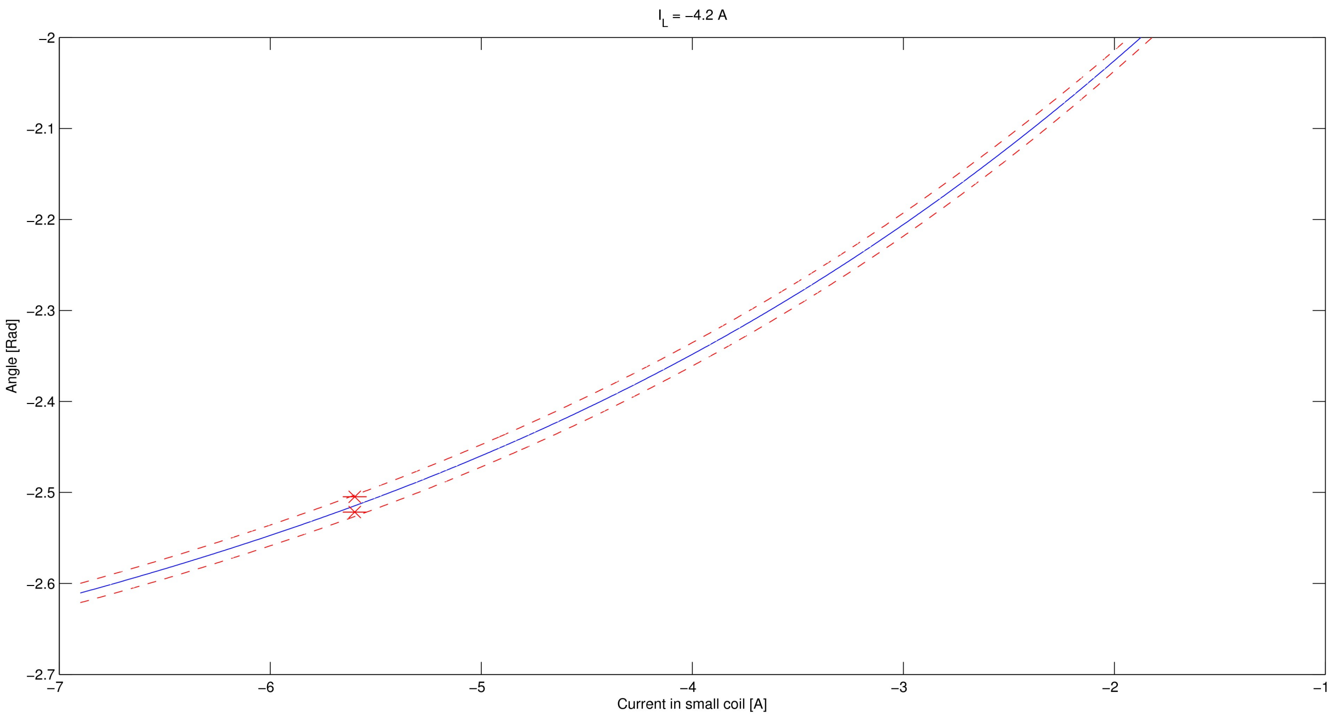
I_s	I_l	measured [°]	calculated[°]	Δ(meas.-calc.) [°]	$d\phi$ [°]	$\frac{d\phi}{\phi}$ [%]
4.2	-5.6	-51.6958	-52.1274	0.4316	0.5561	1.0668
4.2	-5.6	-51.2927	-52.1274	0.8347	0.5561	1.0668
-1.0	-6.9	-98.2297	-98.2304	0.0007	0.2181	0.2221
-5.6	-4.2	-144.4764	-144.0781	0.3983	0.6742	0.4679
-5.6	-4.2	-143.5116	-144.0781	0.5665	0.6742	0.4679
-6.9	1.0	161.8634	170.5092	8.6458	0.7239	0.4246
-4.2	5.6	126.3656	126.0326	0.3329	0.5242	0.4159
-4.2	5.6	126.7692	126.0326	0.7366	0.5242	0.4159
1.0	6.9	81.6996	81.6986	0.0010	0.2101	0.2571
5.6	4.2	37.8375	37.6525	0.1849	0.6446	1.7121
5.6	4.2	36.9737	37.6525	0.6789	0.6446	1.7121
6.9	-1.0	-14.4612	-6.9627	7.4985	0.7198	10.3380

Graphs:

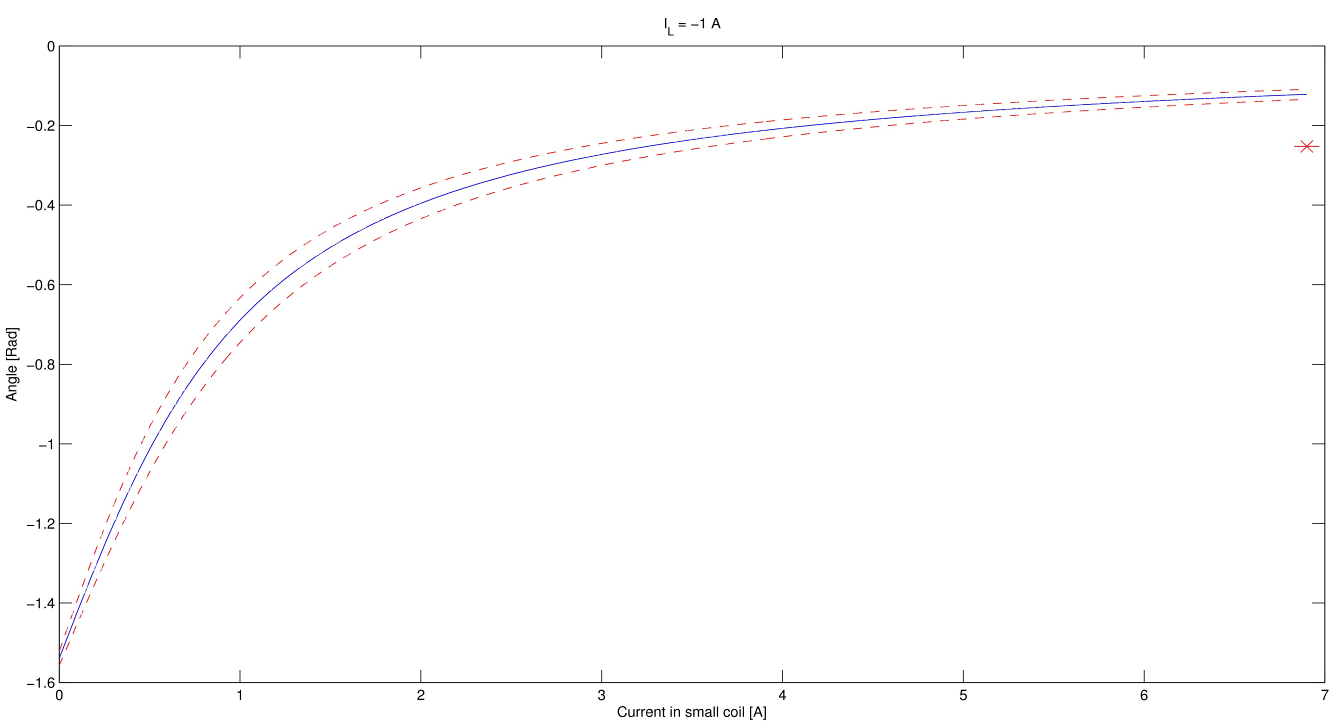
Both currents go through these values: (-6.9, -5.6, -4.2, -1, 1, 4.2, 5.6, 6.9). I decided to plot the angle as a function of the current in the small coil, keeping the current in the large coil fixed, resulting in 8 graphs total.



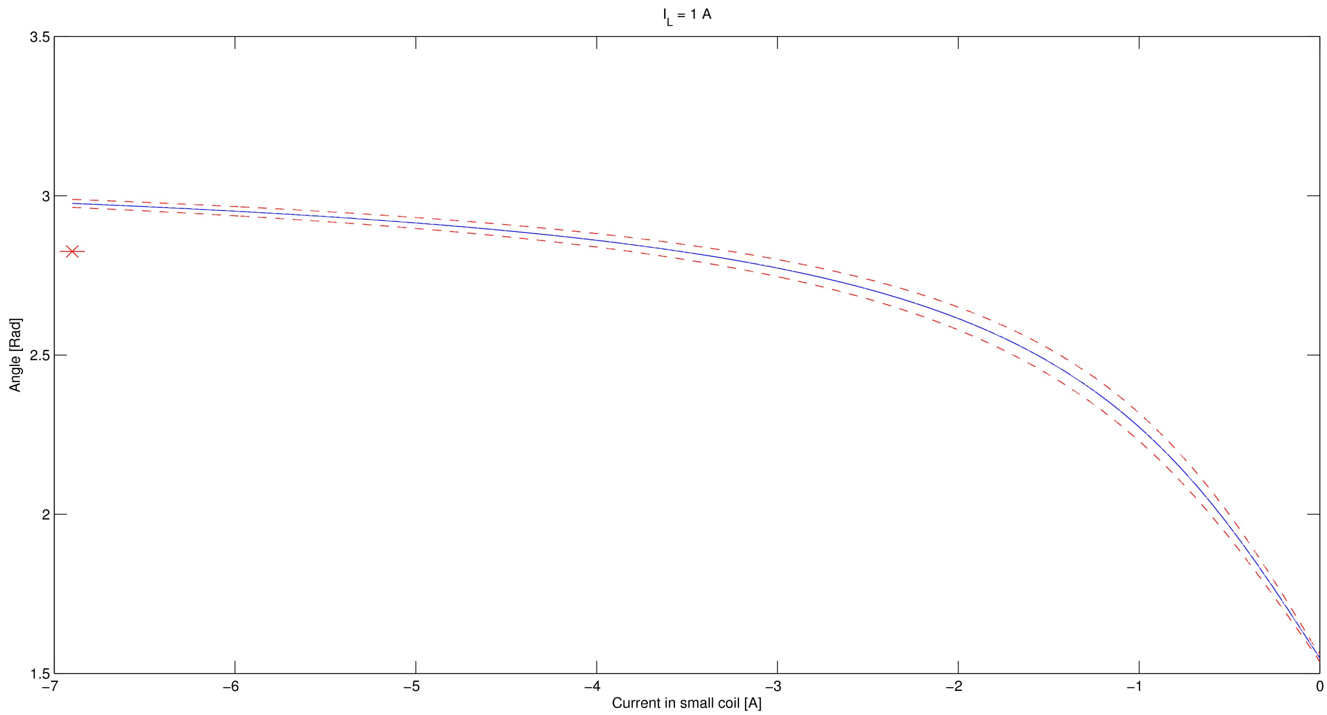




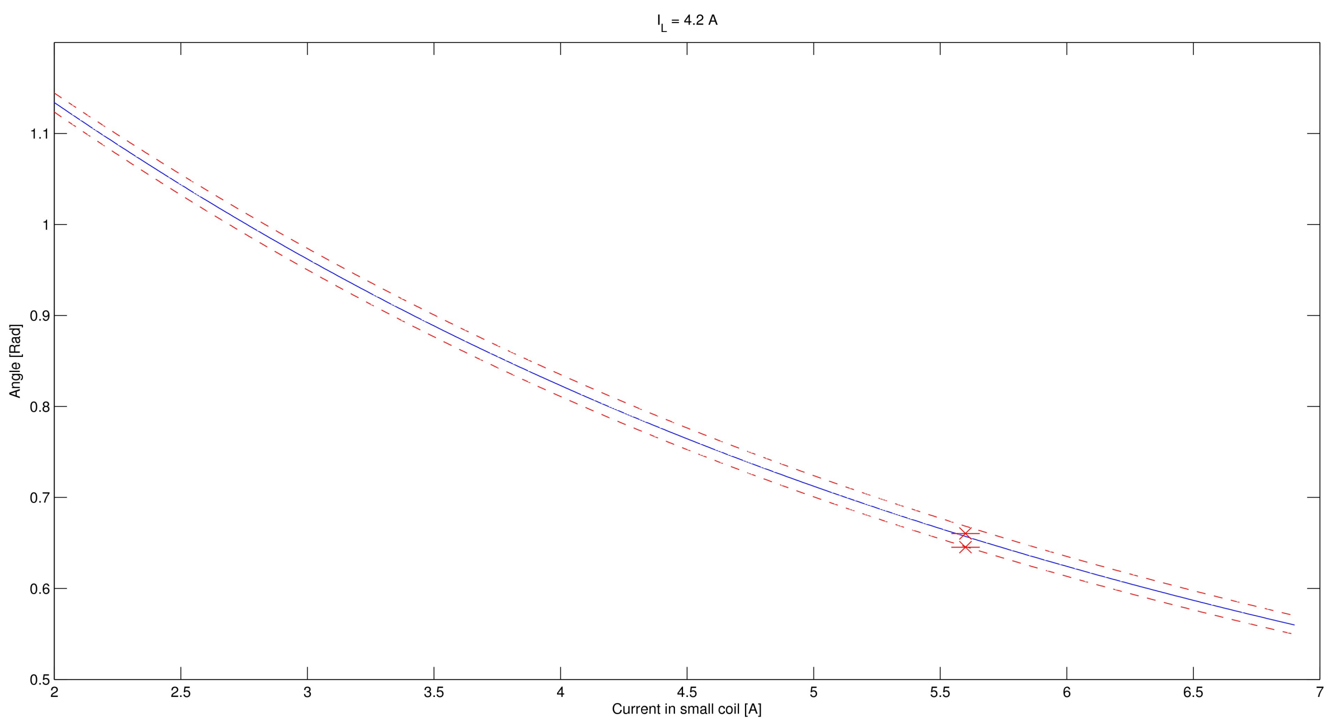
I_L = −4.2 A
Angle [Rad]
Current in small coil [A]
−2
−2.1
−2.2
−2.3
−2.4
−2.5
−2.6
−2.7
−7
−6
−5
−4
−3
−2
−1



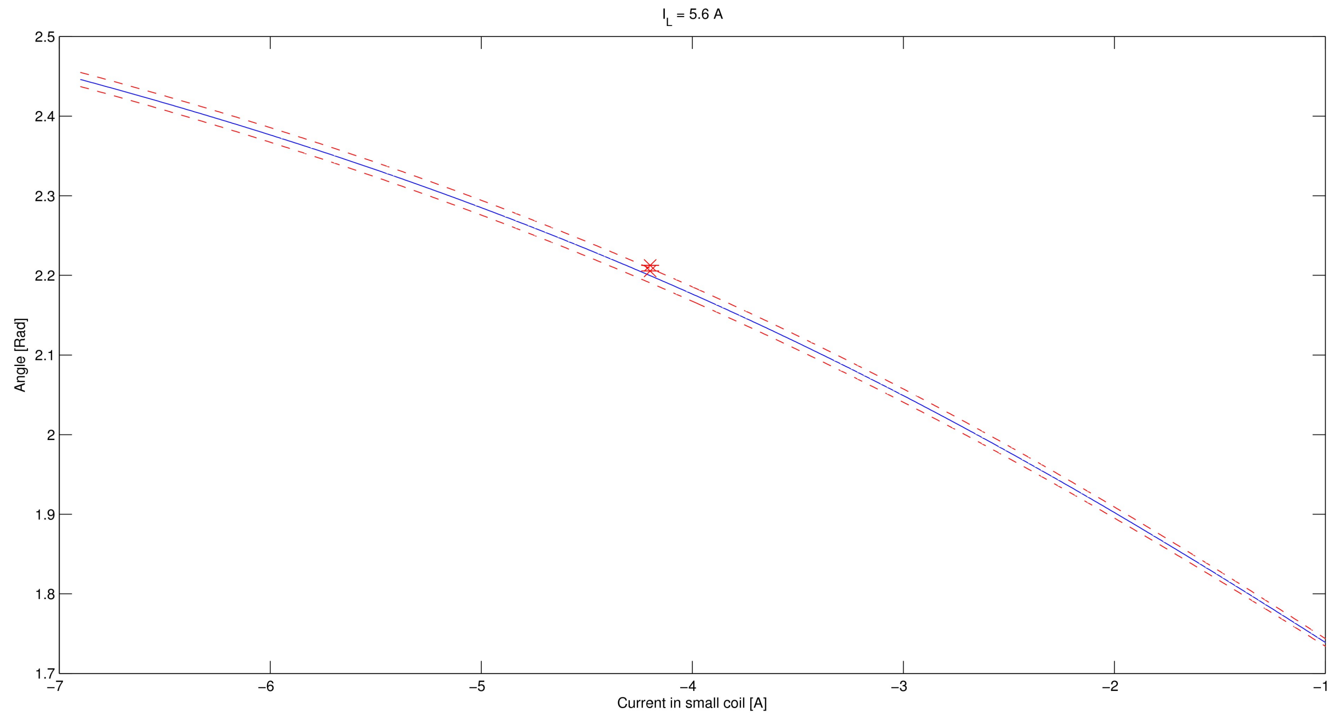
I_L = −1 A
Angle [Rad]
Current in small coil [A]
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
0
1
2
3
4
5
6
7



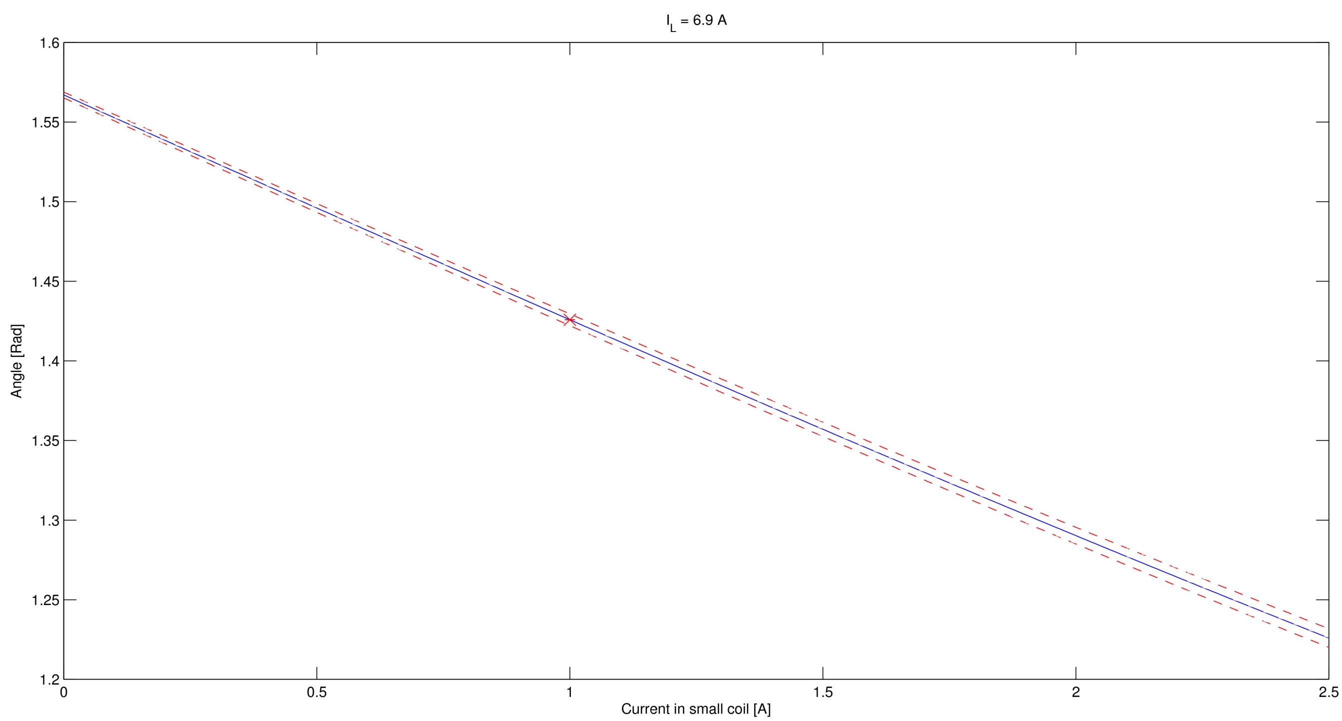
I_L = 1 A
Angle [Rad]
Current in small coil [A]
3.5
3
2.5
2
1.5
−7
−6
−5
−4
−3
−2
−1
0



I_L = 4.2 A
Angle [Rad]
Current in small coil [A]
1.1
1
0.9
0.8
0.7
0.6
0.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7



I_L = 5.6 A
Angle [Rad]
Current in small coil [A]
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
1.7
−7
−6
−5
−4
−3
−2
−1



I_L = 6.9 A
Angle [Rad]
Current in small coil [A]
1.6
1.55
1.5
1.45
1.4
1.35
1.3
1.25
1.2
0
0.5
1
1.5
2
2.5