Inverse transport of protons in BigBite experiment

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Abstract

We present rather fast algorithm to calculate particle's possible history after detecting it with BigBite spectrometer.

1 Problem

In the experiment E05-102 BigBite spectrometer was used for detection of protons. Quantities measured at multi-wire drift chambers (MWDC) were both coordinates on a plane (x_{Det}, y_{Det}) and two angles $(\Theta_{Det}, \Phi_{Det})$, used to determine the momentum's direction.

Our problem is to determine whether the particle came from the target or not. We want to get rid of unintentional events such as detection of some cosmic particle. In addition, if the particle is legit, we would like to know it's precise starting location and it's path through space.

With the field map of magnetic field inside the BigBite coil being known we can almost compute the entire trajectory of the particle's motion through space, since the position and rotation of MWDC are known too, of course. The only missing variable is the *magnitude* of particle's momentum.

2 Shooting method

Since we know the equation of motion, $\dot{\vec{v}} = \frac{e}{m\gamma}\vec{v} \times \vec{B}$, and five (which means all but one) initial conditions, we can try guessing the unknown initial value. Therefore we are going to use the so called *shooting method*.

We set unknown variable to be parameter and then start with some value of momentum's magnitude. If the value isn't right we proceed with increasing or decreasing it – whichever the result implies. Using bisection the method can be pretty effective. The latter shortens interesting interval (which increases precision) logarithmically. Anyway, in order to have a successful method, the observed variable has to be monotonously related to our independent parameter. This required property is nicely shown in figure 1.

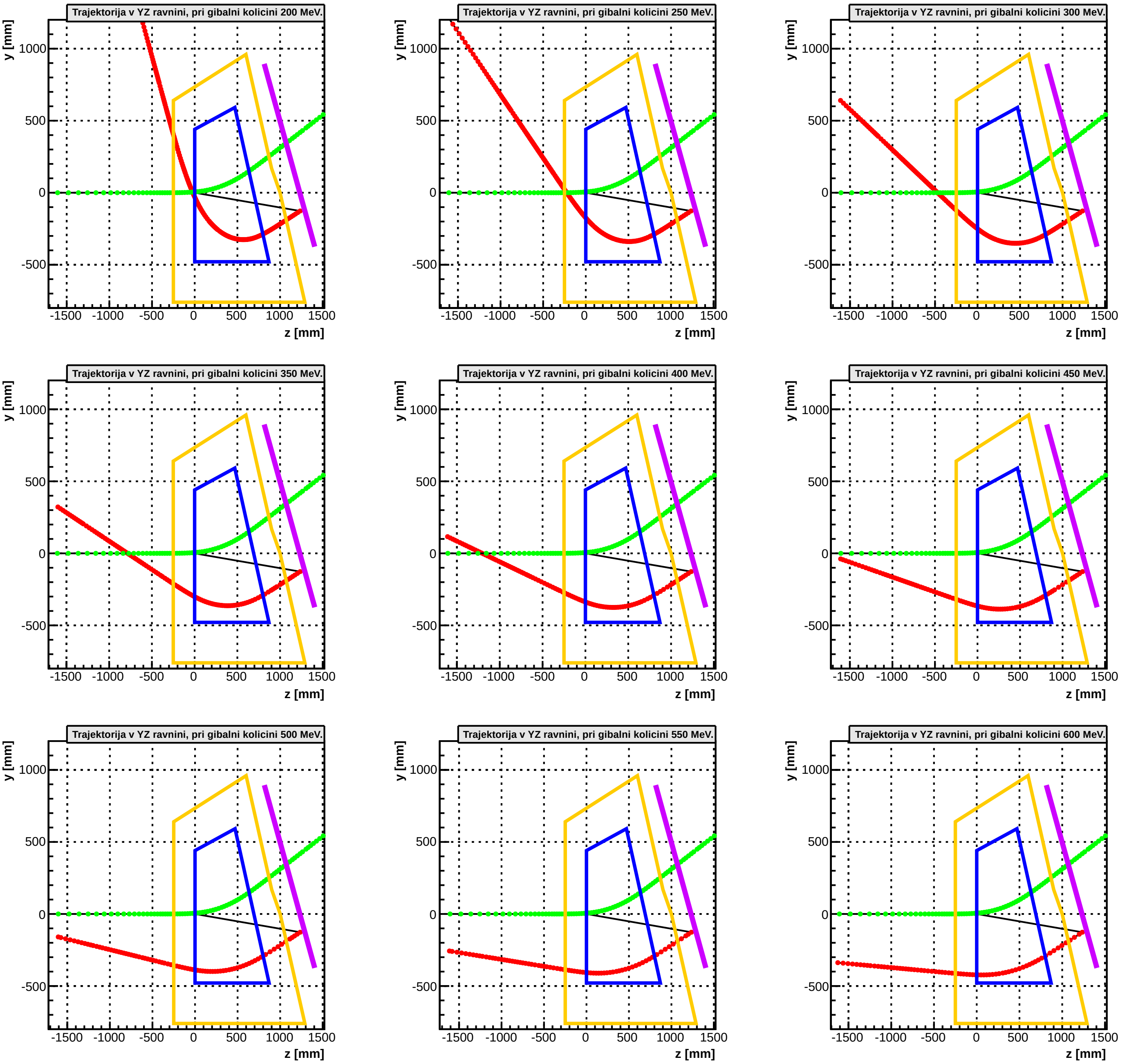

Figure 1: Monotonous relation between momentum's magnitude and y coordinate. Yellow and blue lines show the position of BB coil, violet line shows MWDC plane, green track is in a way central track and the red track shows the trajectory of proton in $YZ-$plane. Clearly the y coordinate in target plane decreases while momentum's magnitude increases. (The correct value for momentum would be around 426 MeV/c.)

3 Integration – calculating the trajectory

Magnetic field inside the BigBite coil is close to homogeneous. Also the field outside the effective field boundary is zero. That is why we choose the adaptive step size Runge-Kutta method for solving the ODE. It gives us precise and answer fast. The method is taken from [1, p. 714].

4 Calculating the field in given point

Field is given with field map – values are known in points of Cartesian mesh. They are only given for positive values of x. First we expand them using map $(B_x, B_y, B_z)_{(x,y,z)} \to (B_x, -B_y, -B_z)_{(-x,y,z)}$. The argument for this kind of mapping is that B_x is almost homogeneous inside the BB coil, while B_y and B_z are corrections of higher order.

Fast interpolation is of essential meaning as we need to compute field components many times during integration. We can close wanted point into a small cube – such that we know the field in its vertices. Since our data are already ordered, logarithmic search of these vertices is possible. Afterwards we use trilinear method (we find affine coordinates of desired point) to get good approximation for magnetic field in this wanted point.

We can also draw field plots fast enough to sample in a few thousand points per each plot. Plots are shown in figures 2 and 3. The orientation of coordinate system (our definition) is most easily seen from first plot in figure 3.

5 Bisection and results

In figure 4 we can see how bisection method works and gets closer and closer to target step by step.

I believe algorithm is fast enough with calculation time less than second per event.

References

[1] William H. Press et. al.: *Numerical recipes in C: the art of scientific computing.* Cambridge University Press, 1992.

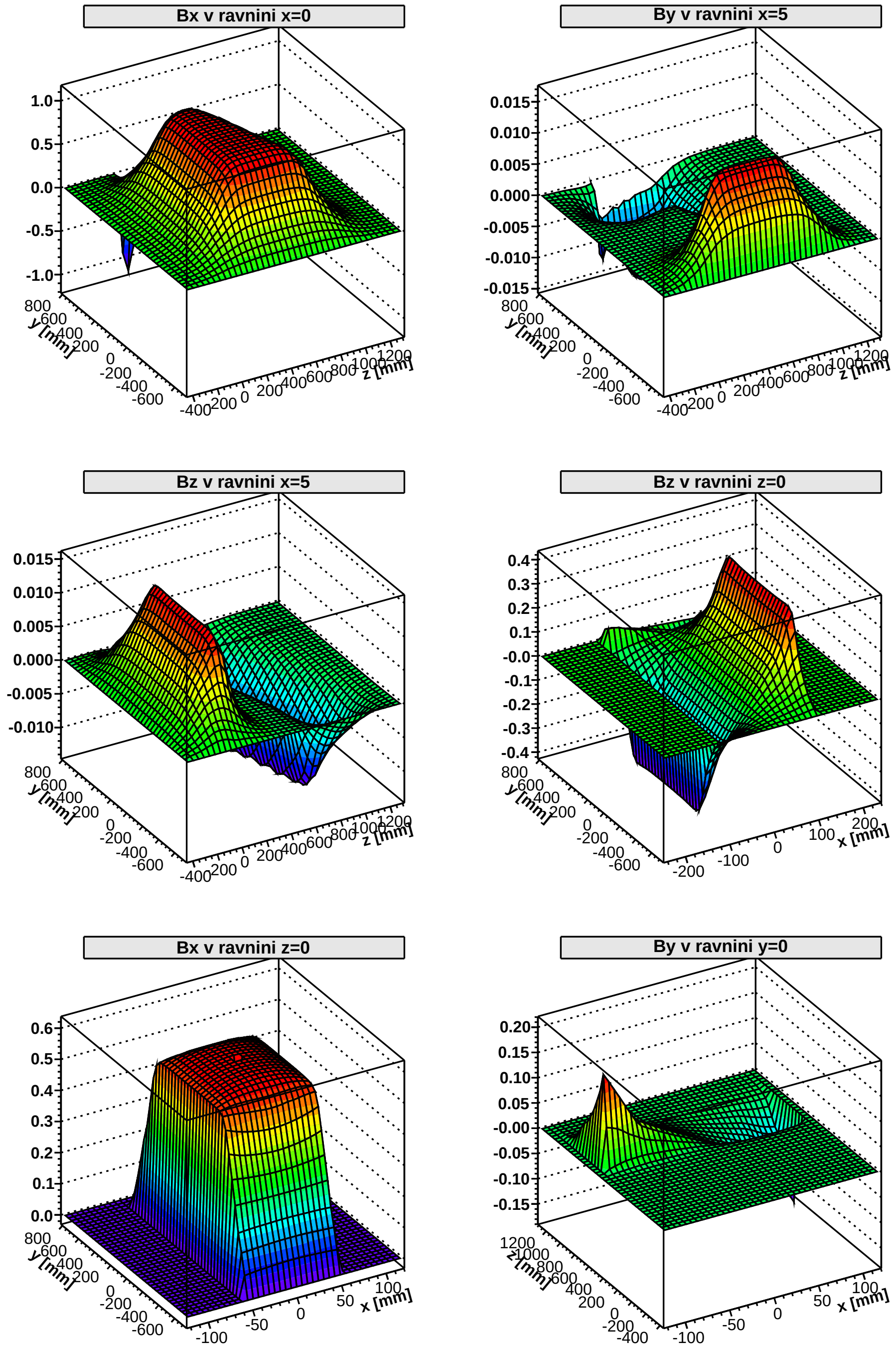

Figure 2: Field components on some planes, shown 3D.

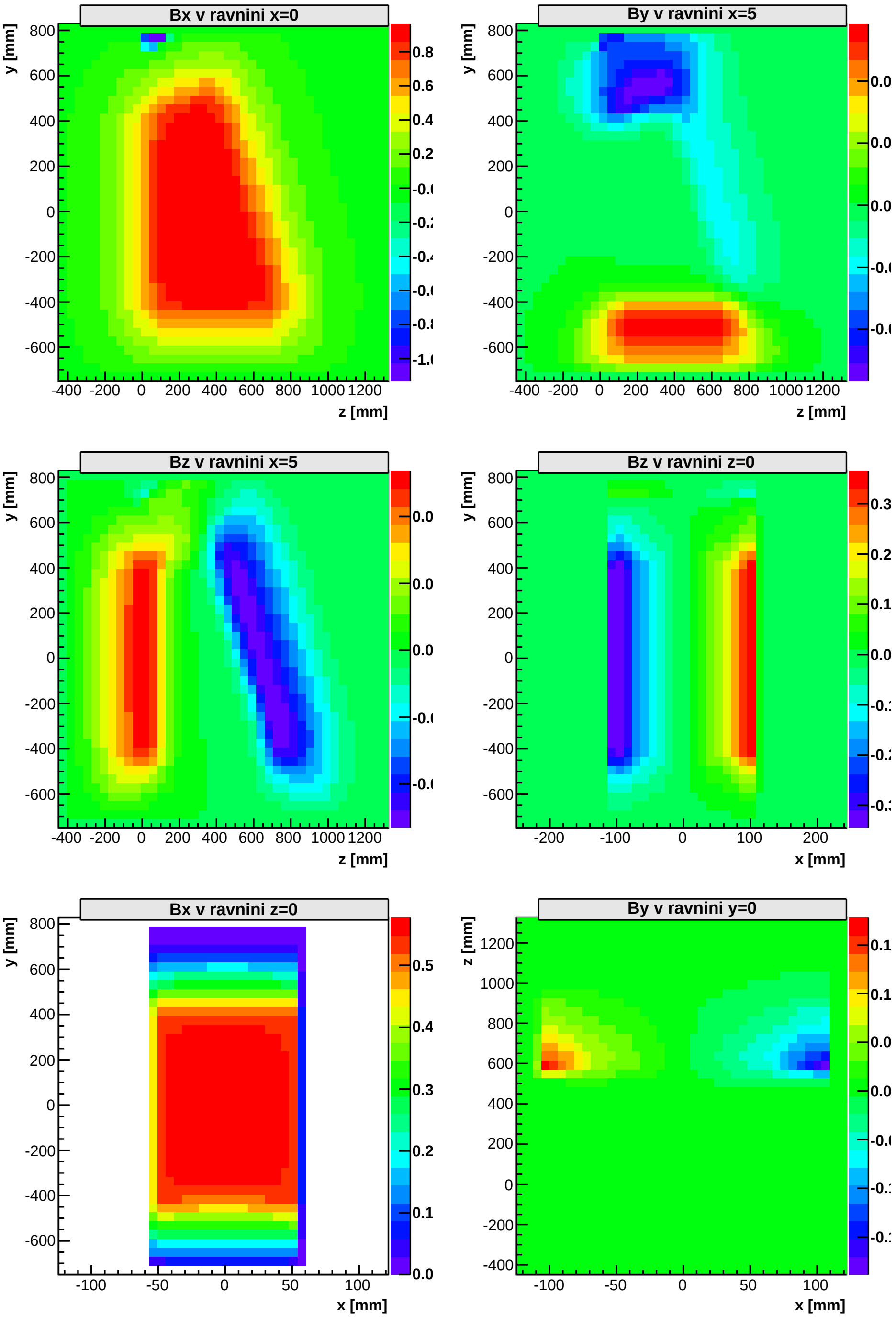

Figure 3: Field components on some planes, shown with contour plotting. We can easily see the shape of BB coil, for example in first picture.

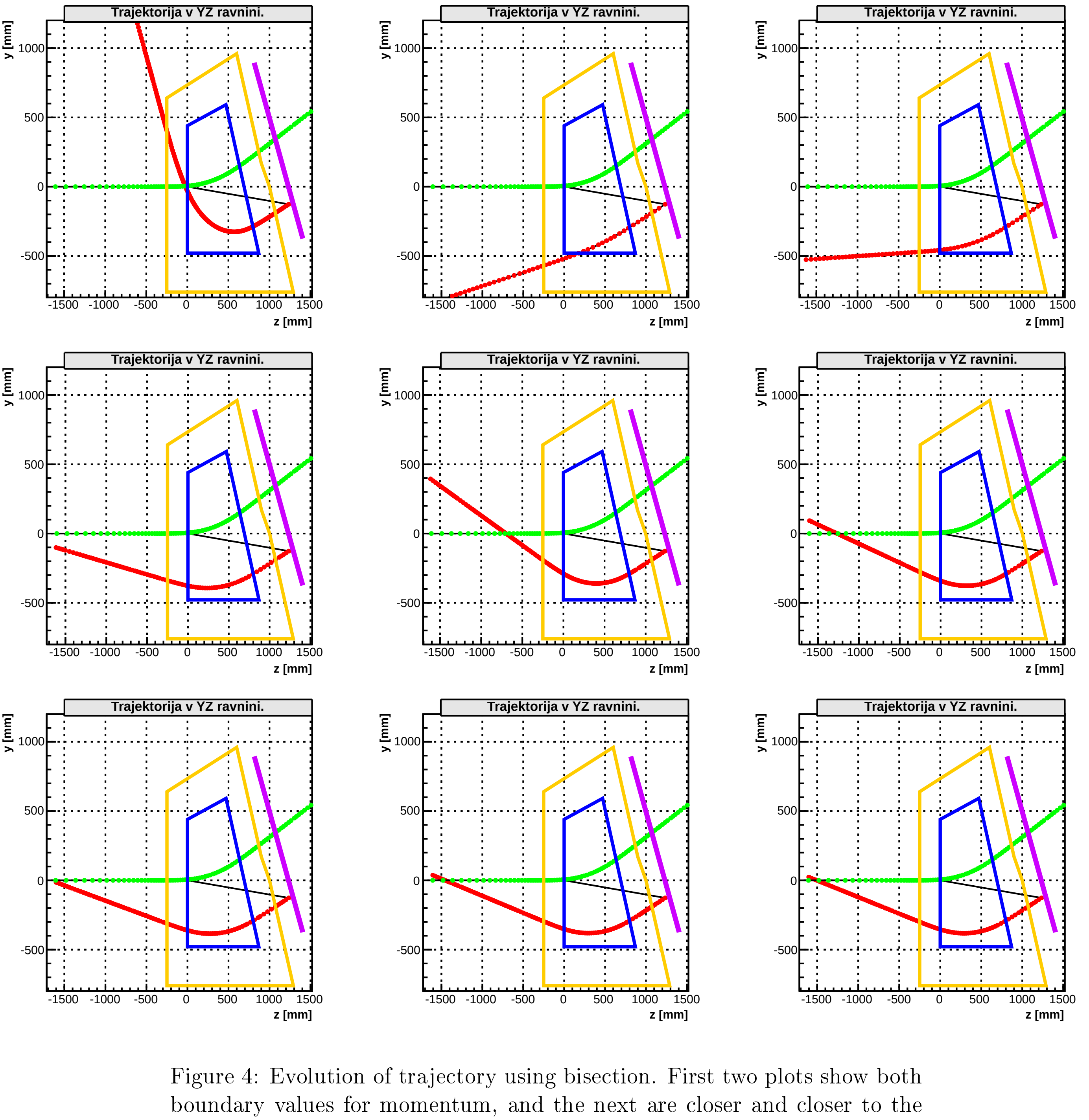

Figure 4: Evolution of trajectory using bisection. First two plots show both boundary values for momentum, and the next are closer and closer to the target at $(0, 0, -1500)$. Meaning of other lines is same as in figure 1.