

Determination of beam energy and spectrometer central momentum with the analysis of ^{12}C and ^1H elastic data

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Abstract

I received and analyzed data from the Jefferson Lab, that was obtained in experiment E05-102 in Hall A, using High Definition Spectrometer. It contained the characteristics of scattered electrons of the various elements. I focused on their energy.

1 Introduction

My project was to get the central momentum of the spectrometer and check the validity with the given one. Central momentum is used to calculate the total energy of the scattered electrons. Using my new calculated central momentum, i determine the starting beam electron energy.

I had data from different runs on the spectrometer. Two sets of runs for carbon and one run for hydrogen. They contained all kinds of data measured on the scattered electron. I used data for relative momentum (deviation from p_0) named δp , total energy p and the scattering angle .

2 Central momentum

p_0 , energy of the scattered electron and δp are linked via this equation:

$$p_0(1 + \delta p) = p' \tag{1}$$

I drew histograms for p_0 and δp . I examined the most noticeable peaks, and determine to what kind of electrons they belong. Scattered electrons can excite nuclei into higher states of energy. I used this equation to define the peaks:

$$p' = p_0(1 + \delta p) - p_{ESD} = \frac{p}{1 + \frac{p}{M}(1 - \cos \gamma)} - p_{ESD} \tag{2}$$

p_{ESD} stands for the difference of the energy of the primary state of the element and the state after the scattering. p stand for starting beam energy (electrons).

Then I fit gauss function on the peaks and compared peaks that belonged to the same electrons. I draw a graph of the points and fit a linear function. The coefficient of the function is equivalent to p_0 .

2.1 Carbon

I analyzed every run one by one, then I combined the data and did it again. I did that for every set, which had the same configurations. At the end I compared the results.

First chart shows us the energy levels of every electron, which was calculated at the JLab from the p_0 and δp . Second chart represents δp for every scattered electron. Third chart helped me get p_0 . I fit gauss function on the peaks. Compared peaks that belonged to the same electrons then draw a graph of that points and fit a linear function. The coefficient of the function is equivalent to p_0 .

I was able to define peaks of electrons that excited carbon into first and third energy state (they are marked on the first charts with a red line). When I combined the runs, I also found a peak that belonged to elastic electrons.

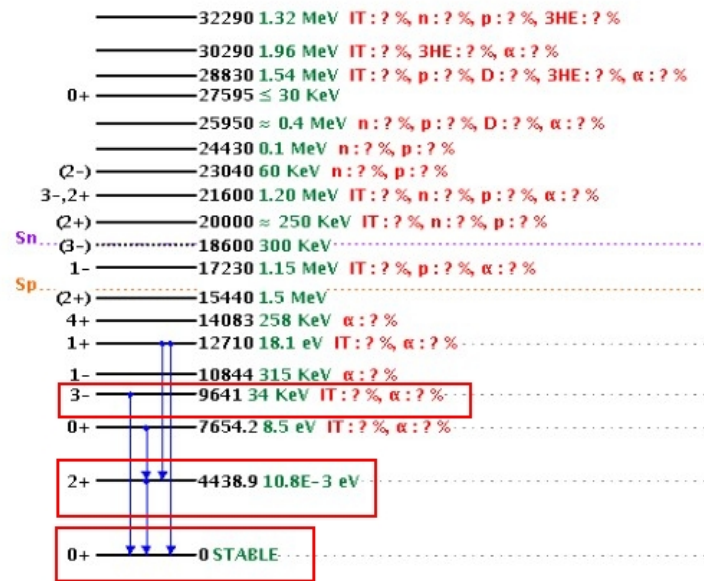


Figure 1: Carbon energy states. States marked with a red rectangle were seen.

2.1.1 Carbon (set 1456, 1457, 1458)

Target type	Multi Carbon
Beam e_0	1.245 GeV/c
p_0	1.238 GeV/c
θ	17°

http://hallaweb.jlab.org/halog/log/html/0904_archive/logdir.html

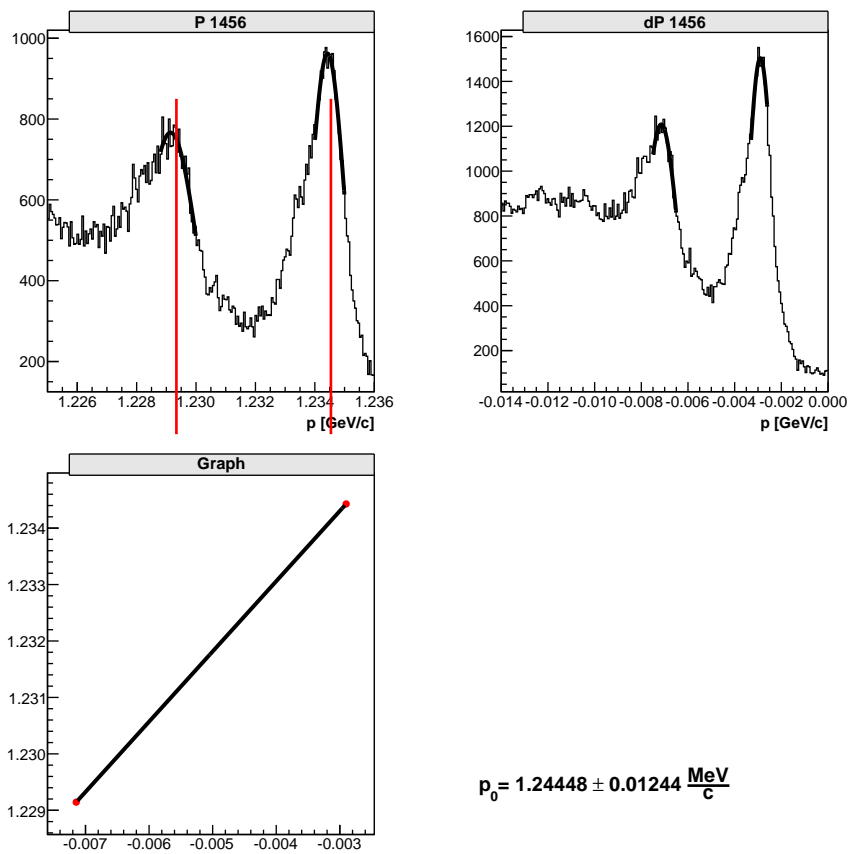
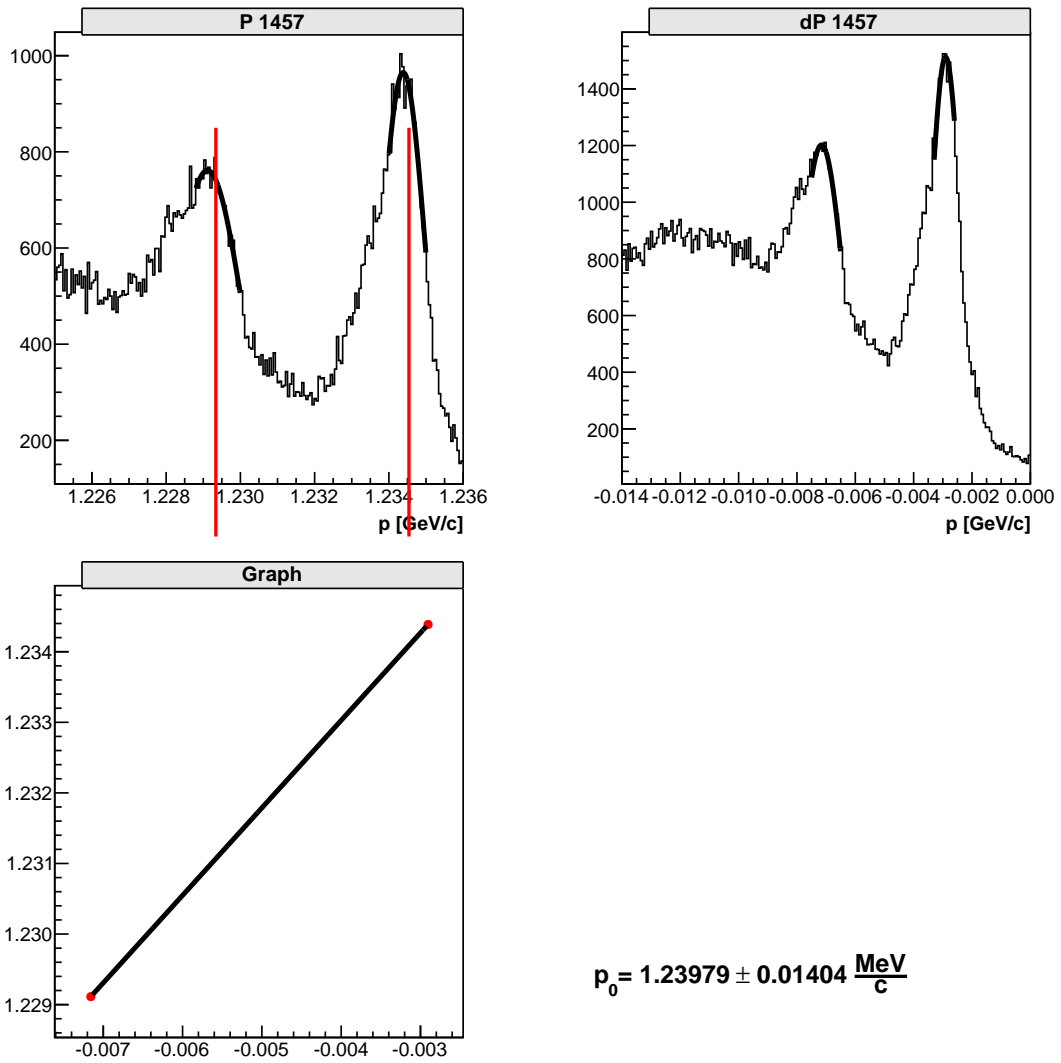
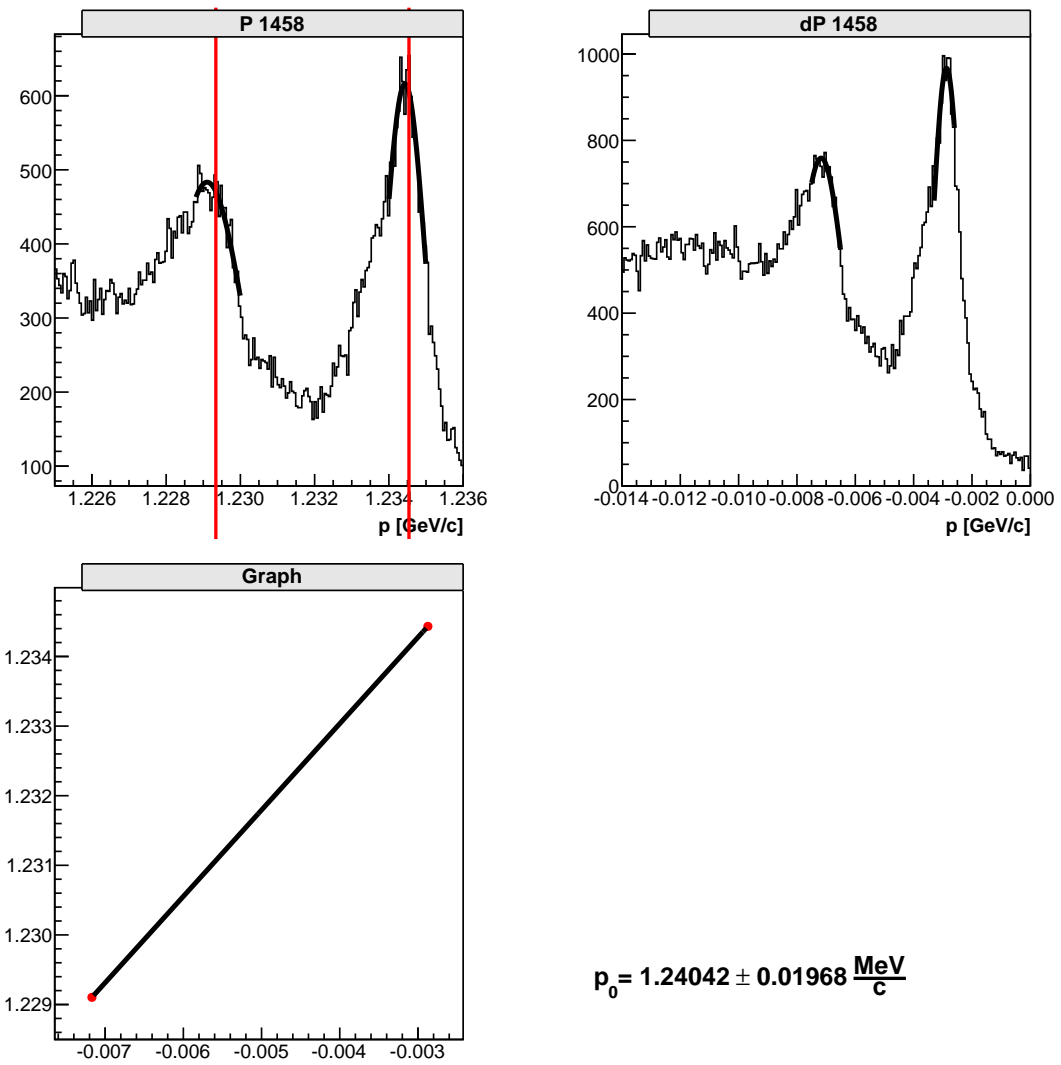


Figure 2: Run 1456.



$$p_0 = 1.23979 \pm 0.01404 \frac{\text{MeV}}{c}$$

Figure 3: Run 1457.



$$p_0 = 1.24042 \pm 0.01968 \frac{\text{MeV}}{c}$$

Figure 4: Run 1458.

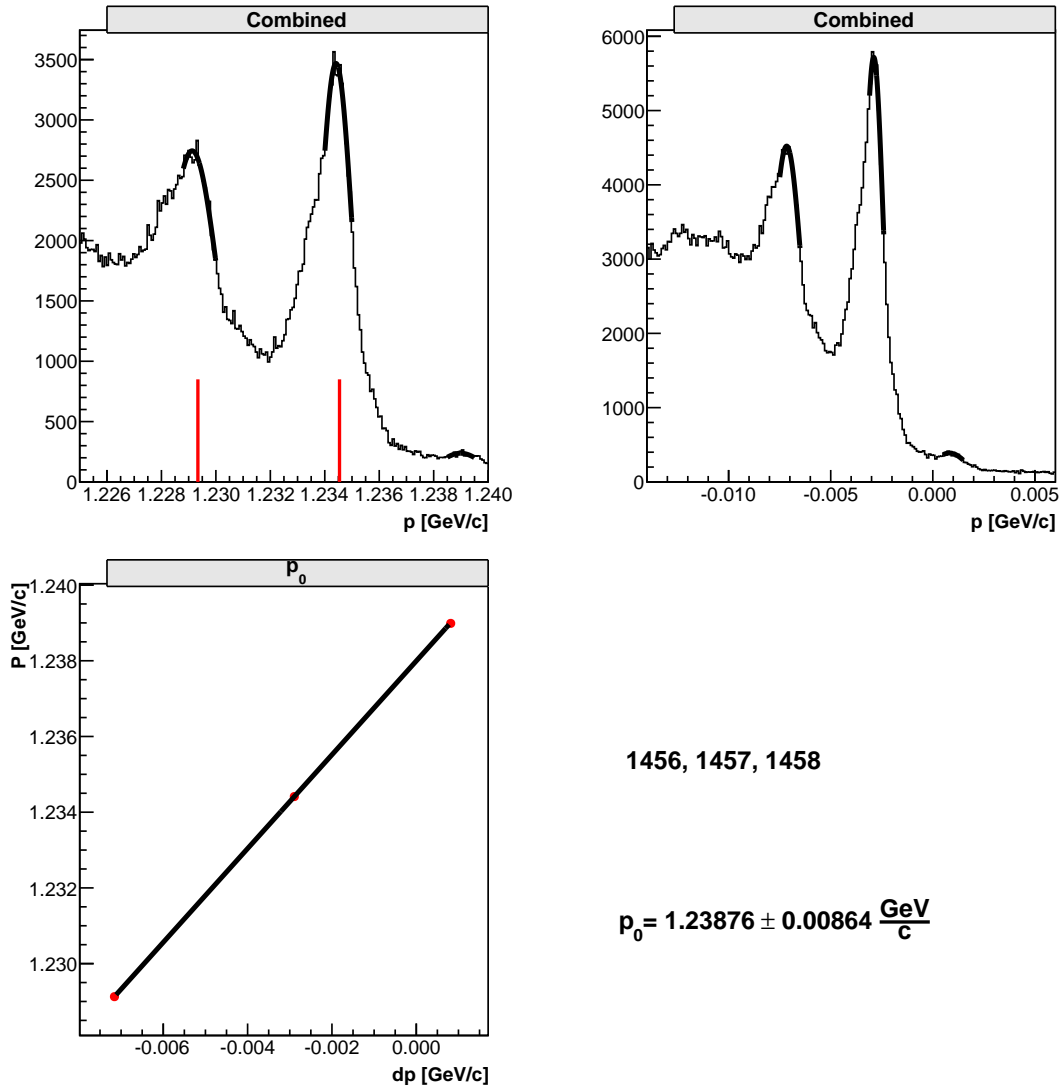


Figure 5: Combined (1456, 1457, 1458)

2.1.2 Carbon (set 1460, 1461, 1462)

Target type	Multi Carbon
Beam e_0	1.245 GeV/c
p_0	1.2 GeV/c
θ	17°

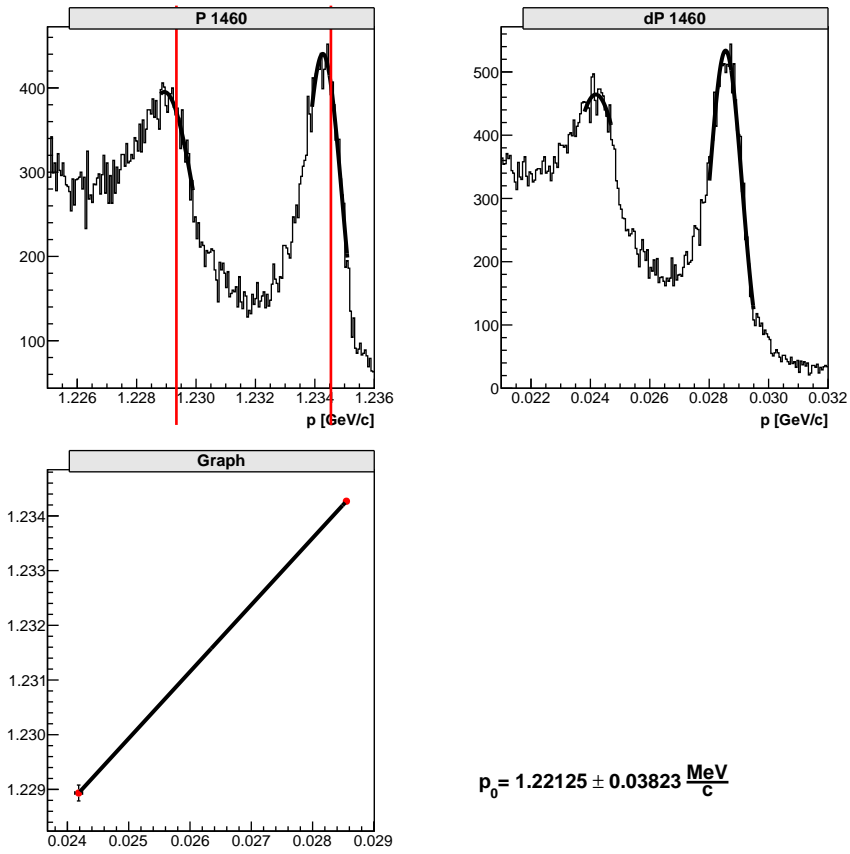
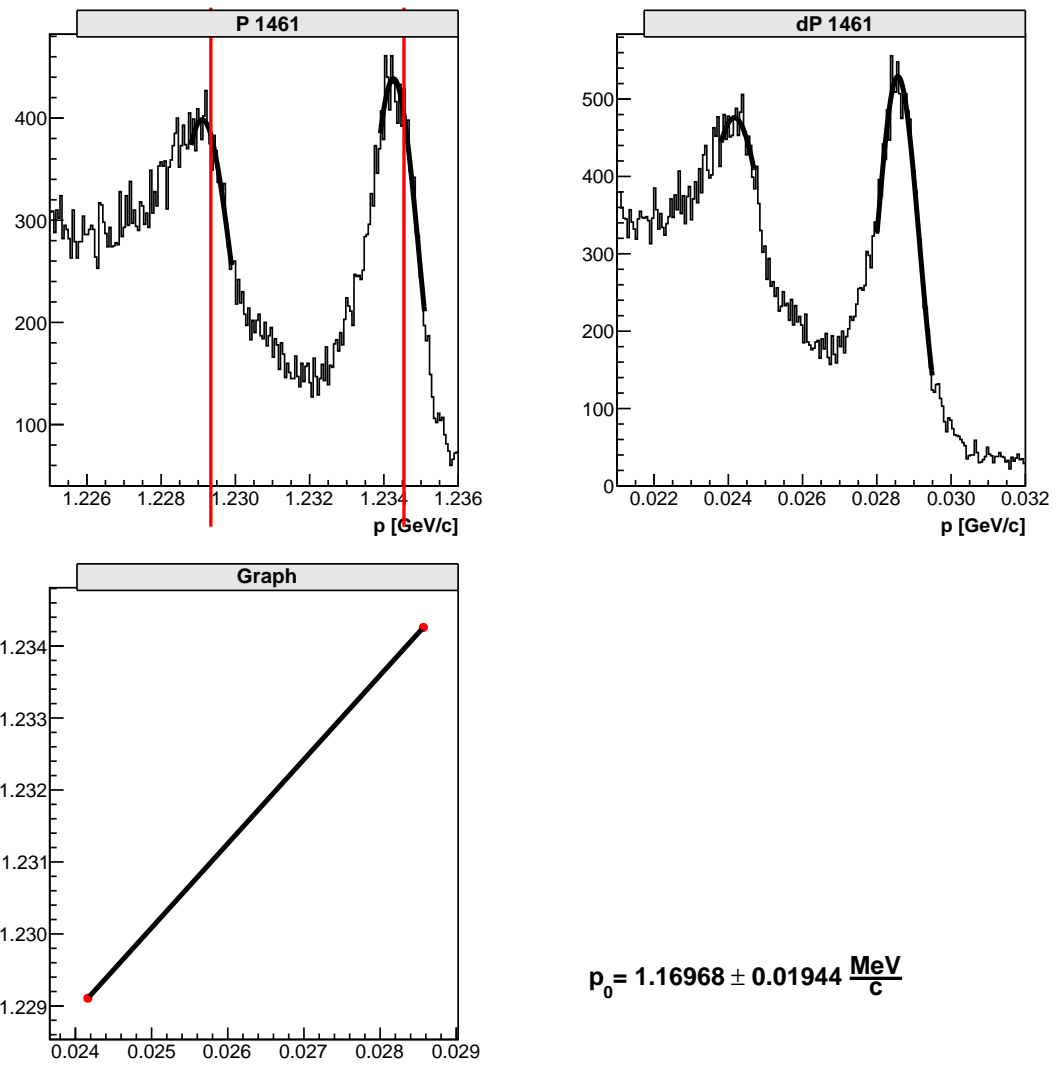


Figure 6: Run 1460.



$$p_0 = 1.16968 \pm 0.01944 \frac{\text{MeV}}{c}$$

Figure 7: Run 1461.

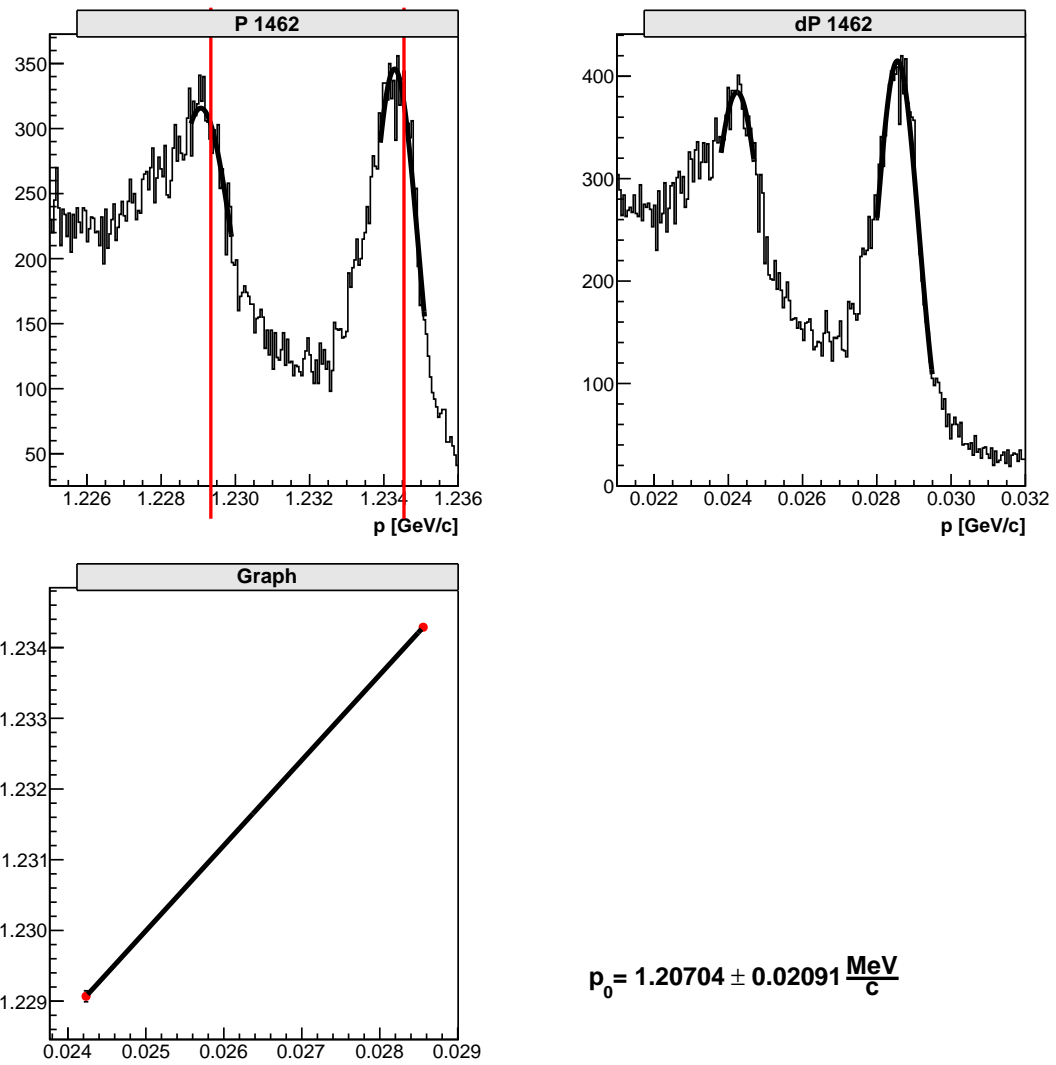
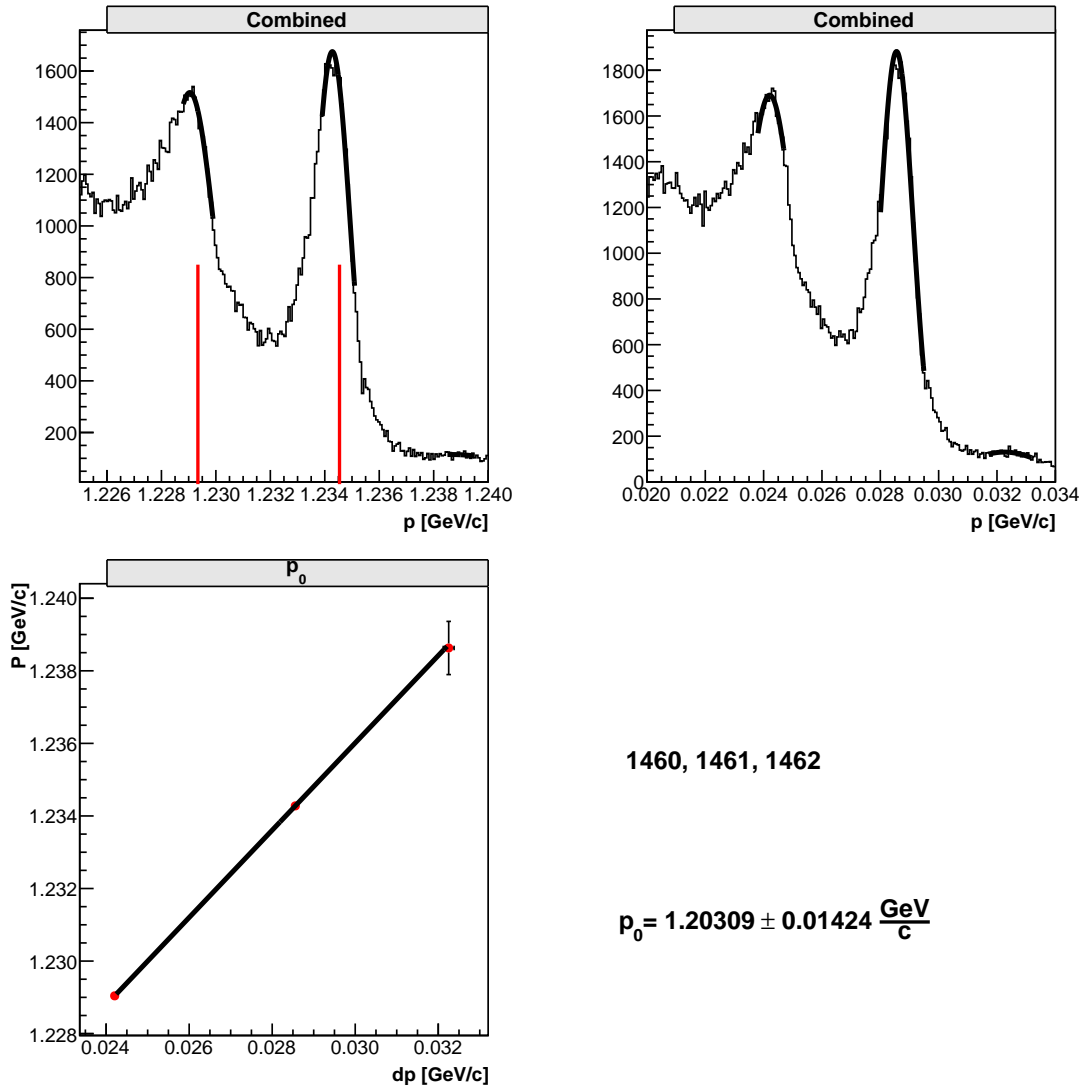


Figure 8: Run 1462.



1460, 1461, 1462

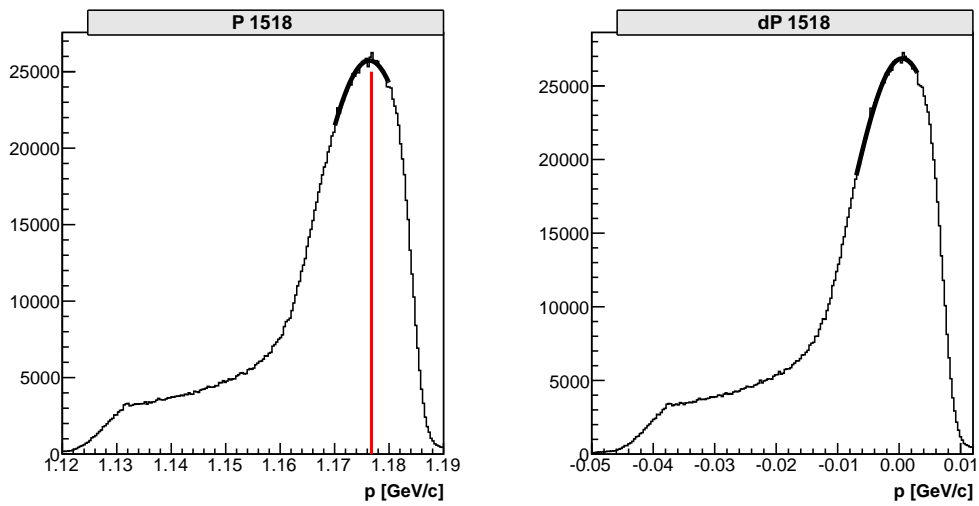
$$p_0 = 1.20309 \pm 0.01424 \frac{\text{GeV}}{c}$$

Figure 9: Combined (1460, 1461, 1462)

2.2 Hydrogen

I did everything very similar to carbon. The only difference was that I was able to analyze just one peak and that were elastic electrons. I used the mean of the peak from total energy chart and mean of the peak from δp chart. I used this equation to determine p_o and its error :

$$p_0 = \frac{p}{1 + \delta p} \quad (3)$$



$$p_0 = 1.17576 \pm 0.00015 \frac{\text{GeV}}{\text{c}}$$

Figure 10: Run 1518

2.3 Results

2.3.1 Carbon (set 1456, 1457, 1458)

Average of the individual run analysis:

$$p_0 = 1.24187 \pm 0.00841 \frac{GeV}{c}$$

Combined data:

$$p_0 = 1.23876 \pm 0.00786 \frac{GeV}{c}$$

2.3.2 Carbon (set 1460, 1461, 1462)

Average of the individual run analysis:

$$p_0 = 1.19117 \pm 0.01334 \frac{GeV}{c}$$

Combined data:

$$p_0 = 1.20306 \pm 0.01424 \frac{GeV}{c}$$

2.3.3 Hydrogen (1518)

$$p_0 = 1.17576 \pm 0.00015 \frac{GeV}{c}$$

3 Beam energy (electrons)

From here on I will use just combined data, because I could easily recognized peaks and also fit them better. This equation was used:

$$p' = p_0(1 + \delta p) = \frac{p}{1 + \frac{p}{M}(1 - \cos \gamma)} \quad (4)$$

$$p = \frac{p_0(1 + \delta p)}{1 + \frac{p_0(1 + \delta p)}{M}(1 - \cos \gamma)} \quad (5)$$

For every single entry I calculated beam energy from its δp in θ with the held of the upper equation. θ varied for 0.023° around 17° . This was actually an estimation, because I used plane angle, not space. Then I also fit gauss function on the known peaks. The mean from the peak of elastic electrons was already the result. But for those electrons which exited the element, I had to add p_{ESD} cording to the equation (2).

3.1 Carbon (set 1456, 1457, 1458)

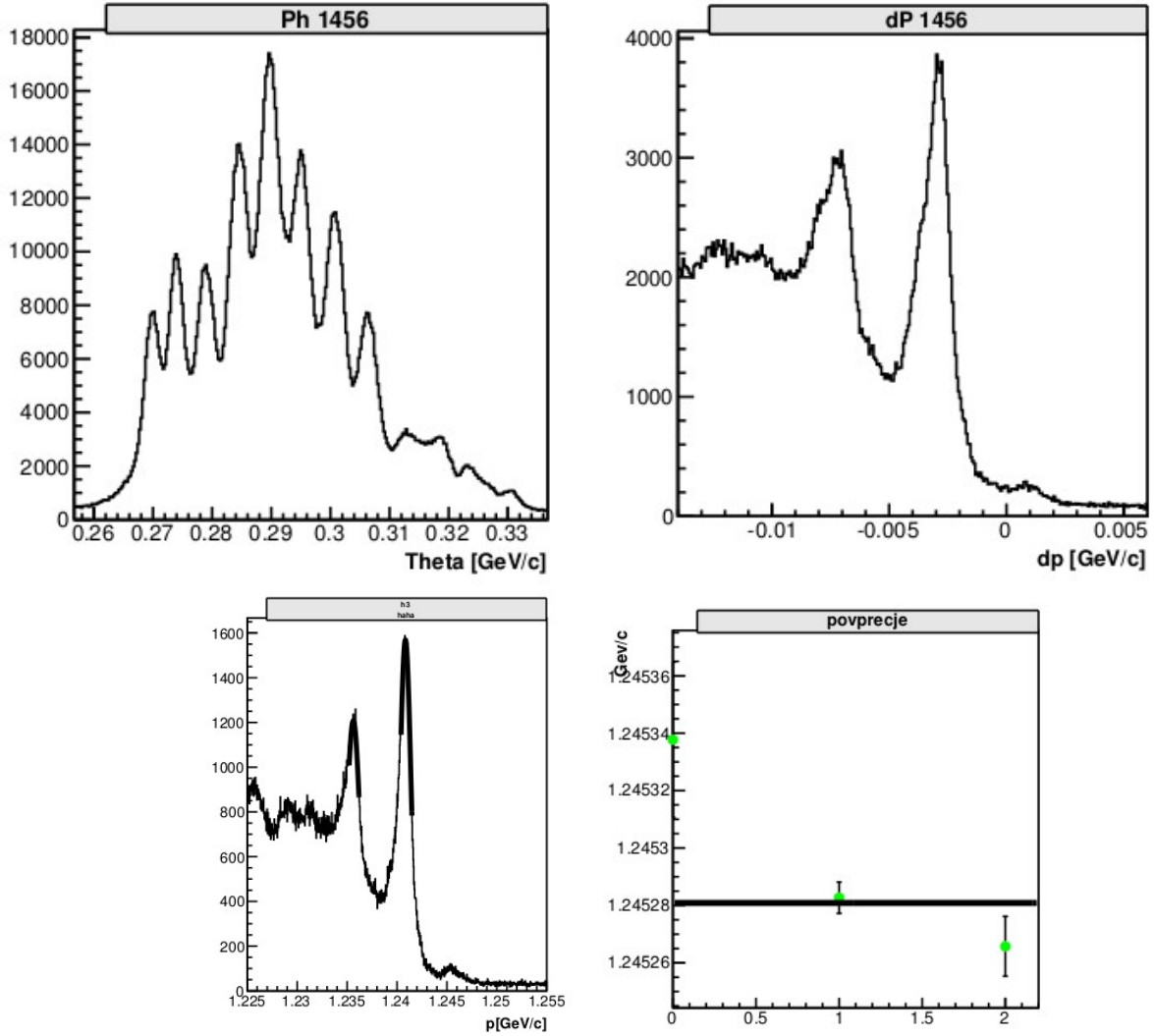


Figure 11: Combined 1456, 1457, 1458

First peak: $p_1 = 1.23562 \pm 0.00001 + 0.009641 \frac{GeV}{c} = 1.24527 \frac{GeV}{c}$
 Second peak: $p_2 = 1.24084 \pm 0.00001 + 0.0044389 \frac{GeV}{c} = 1.24528 \frac{GeV}{c}$
 Third peak: $p_3 = 1.24084 \pm 0.00003 \frac{GeV}{c}$
 Average: $p = 1.245281 \pm 0.000005 \frac{GeV}{c}$

3.2 Carbon (set 1460, 1461, 1462)

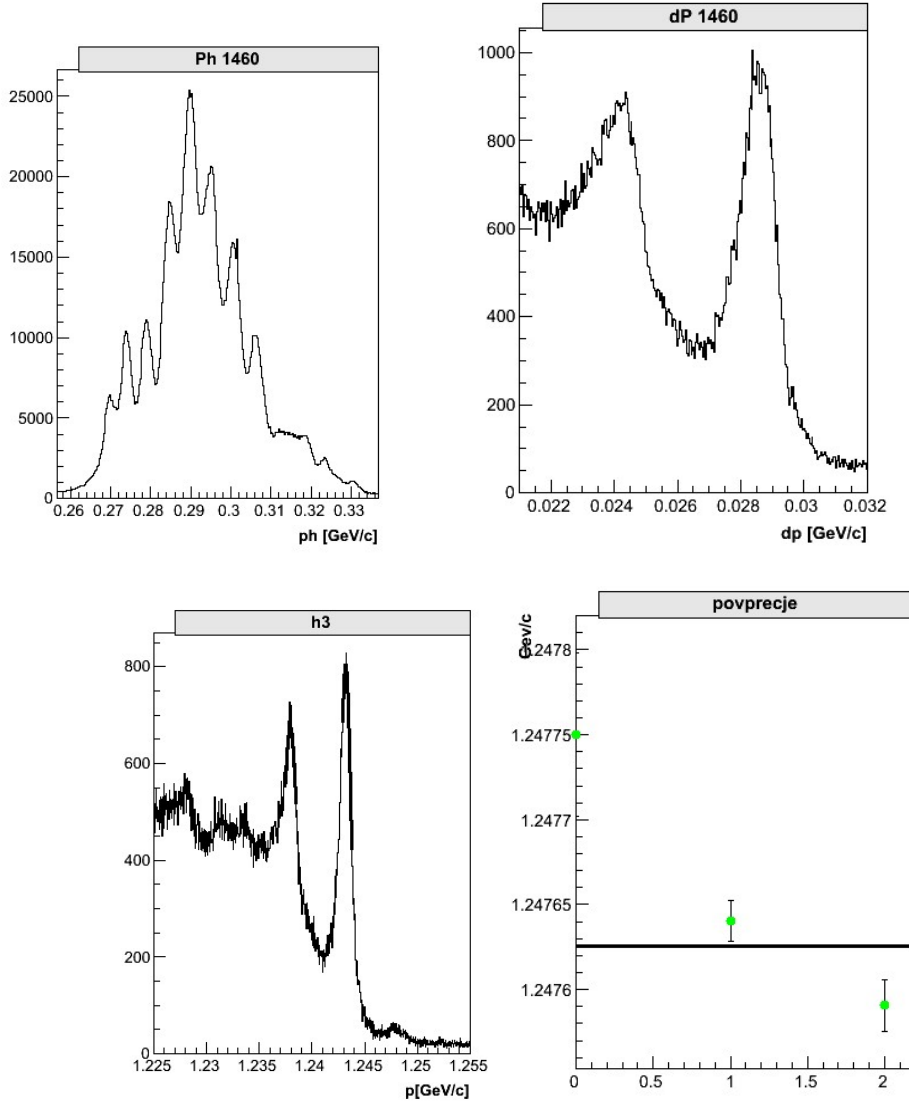


Figure 12: Combined 1460, 1461, 1462

First peak: $p_1 = 1.23795 \pm 0.00002 + 0.009641 \frac{GeV}{c} = 1.24759 \frac{GeV}{c}$
 Second peak: $p_2 = 1.24320 \pm 0.00001 + 0.0044389 \frac{GeV}{c} = 1.24764 \frac{GeV}{c}$
 Third peak: $p_3 = 1.24775 \pm 0.00005 \frac{GeV}{c}$
 Average: $p = 1.24763 \pm 0.00001 \frac{GeV}{c}$

3.3 Hydrogen (1518)

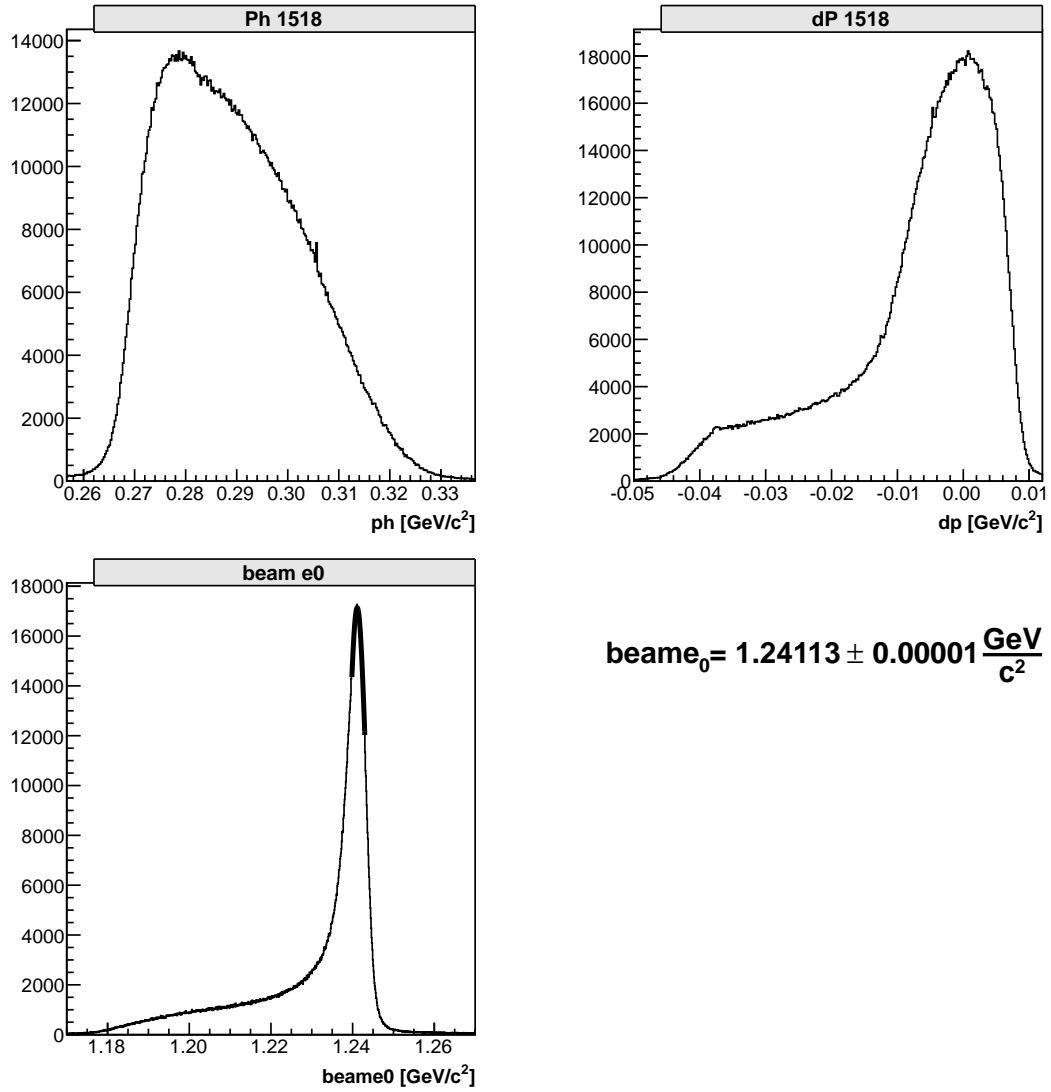


Figure 13: Run 1518

4 Results

4.1 Beam energy error

Fit error for beam energy was very little, because I didn't take into account error from p_0 . I examined graph of the first derivative of the upper function and determined the error from its maximum value. This is the first derivative of the function:

$$\Delta p = \frac{1 + \delta p}{\left(1 - p_0 \frac{(1 + \delta p)(1 - \cos \gamma)}{M}\right)} \quad (6)$$

$$\Delta p = \frac{a}{\left(1 - p_0 \frac{ab}{M}\right)} \quad (7)$$

4.1.1 Carbon (set 1456, 1457, 1458)

a goes from 0.9900 to 1.0001, b goes from 0.0336 to 0.0590

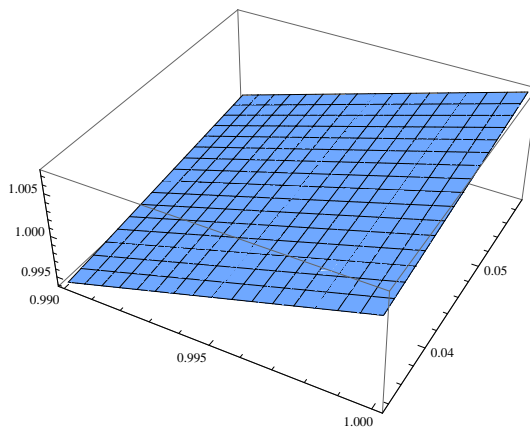


Figure 14: Graph for the error function for Carbon (set 1456, 1457, 1458)

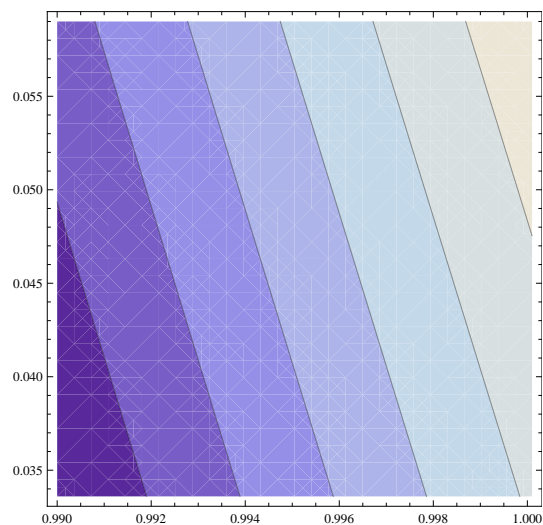


Figure 15: Another graph for the error function for Carbon (set 1456, 1457, 1458)

We see that we get the biggest value, if we enter maximum values of a in b.

4.1.2 Carbon (set 1460, 1461, 1462)

I have done the same as the above, just different numbers.

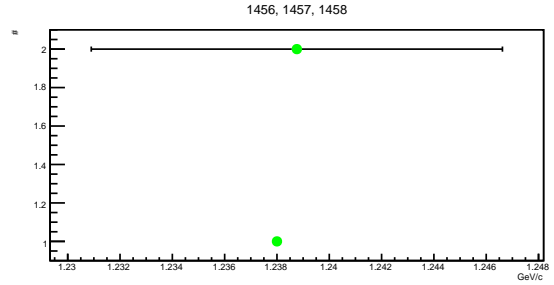
4.1.3 Hydrogen (1518)

I have done the same as the above, just different numbers.

4.2 Final results

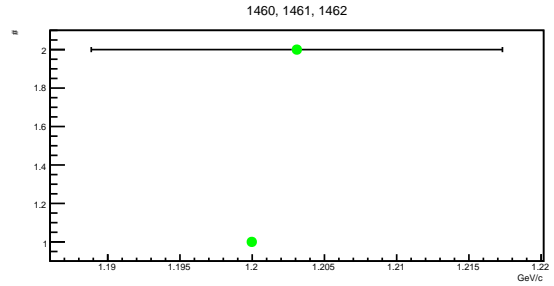
$$p'_{01} = 1.23800 \pm 0.00001 \frac{\text{GeV}}{\text{c}}$$

$$p'_{02} = 1.23876 \pm 0.00786 \frac{\text{GeV}}{\text{c}} \leftarrow \text{Carbon (1456,...)}$$



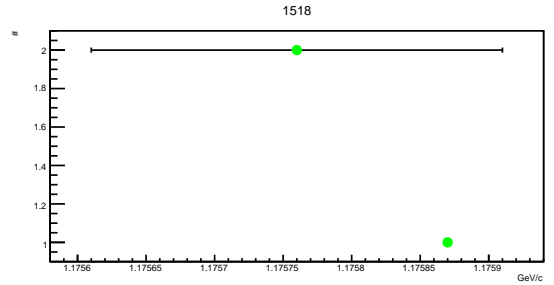
$$p'_{01} = 1.19997 \frac{\text{GeV}}{\text{c}}$$

$$p'_{02} = 1.20309 \pm 0.01423 \frac{\text{GeV}}{\text{c}} \leftarrow \text{Carbon (1460,...)}$$



$$p'_{01} = 1.17587 \frac{\text{GeV}}{\text{c}}$$

$$p'_{02} = 1.17576 \pm 0.00015 \frac{\text{GeV}}{\text{c}} \leftarrow \text{Hydrogen (1518)}$$



$$p_1 = 1.24523 \frac{\text{GeV}}{\text{c}} \leftarrow \text{given}$$

$$p_2 = 1.24528 \pm 0.00797 \frac{\text{GeV}}{\text{c}} \leftarrow \text{Carbon (1456, 1457, 1458)}$$

$$p_3 = 1.24763 \pm 0.01490 \frac{\text{GeV}}{\text{c}} \leftarrow \text{Carbon (1460, 1461, 1462)}$$

$$p_4 = 1.24113 \pm 0.00017 \frac{\text{GeV}}{\text{c}} \leftarrow \text{Hydrogen (1518)}$$

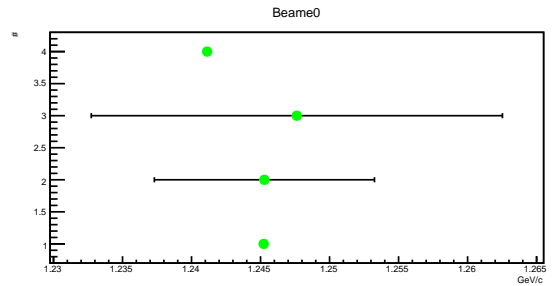
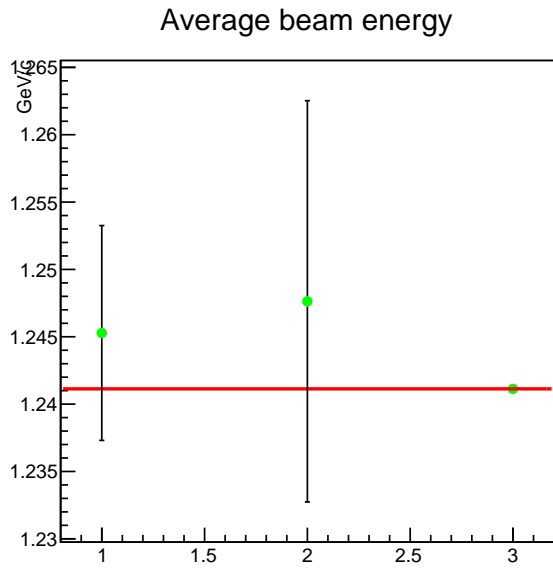


Figure 16: Final results (p_{01} are values, that were given, p_{02} are my results)



$$p = 1.24523 \frac{\text{GeV}}{c}$$

$$p_1 = 1.24113 \pm 0.00017 \frac{\text{GeV}}{c}$$

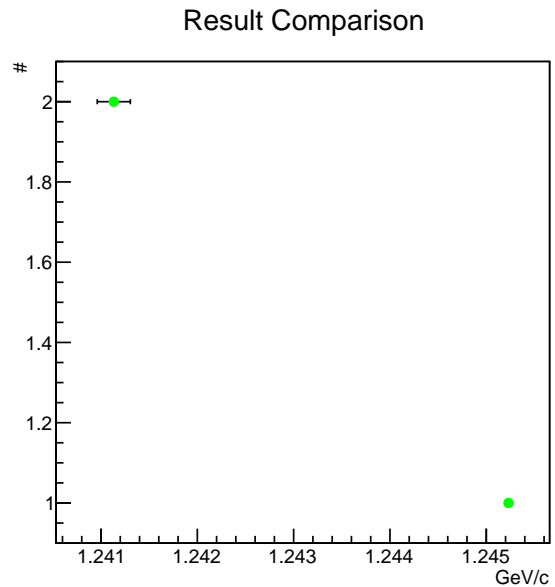


Figure 17: Comparison between given and calculated beam energy

5 Conclusion

We can see, that the result from hydrogen had a great effect on the average value of beam energy. That was because its error was very small and the errors from the other was much bigger. I'm glad that my

result value is smaller than the given one, because then can I assume that this mistake must be a result of energy losses.