# Hard Exclusive Electroproduction of $\pi^+$ Mesons

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**Outline:**

- **handbag factorization for meson electroproduction**
- **pion pole, $\widetilde{H}$, $\widetilde{E}$,**
- **a twist-3 contribution**
- **results**
- **Summary**

# The leading-twist $\gamma^* p \to \pi^+ n$ amplitudes

$$\mathcal{M}_{0+,0+}(\pi^+) = \sqrt{1-\xi^2}\frac{e_0}{Q}\left\{\langle\widetilde{H}^{(3)}\rangle - \frac{\xi^2}{1-\xi^2}\langle\widetilde{E}^{(3)}\rangle - \frac{2\xi m Q}{1-\xi^2}\frac{\rho_\pi}{t-m_\pi^2}\right\},$$

$$\mathcal{M}_{0-,0+}(\pi^+) = \frac{e_0}{Q}\frac{\sqrt{-t}}{2m}\left\{\xi\langle\widetilde{E}^{(3)}\rangle + 2mQ^2\frac{\rho_\pi}{t-m_\pi^2}\right\},$$

$t' = t - t_0, \quad \xi \simeq x_{Bj}/2, \quad F^{(3)} = F^u - F^d$

convolution:

$$\langle F\rangle = \sum_\lambda \int_{-1}^{1} dx \mathcal{H}_{0\lambda,0\lambda}(x,\xi,Q^2,t=0)F(x,\xi,t)$$

subprocess amplitudes worked out in modified pert.approach (Sterman et al)
LO pQCD + quark trans. momenta and Sudakov suppressions
for $Q^2 \to \infty \Longrightarrow$ lead. twist (collinear approximation)

# Double distributions

integral representation (i= valence, sea quarks, gluons)

$$H_i(\bar{x},\xi,t') = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-\bar{x})\, f_i(\beta,\alpha,t') + D_i\, \Theta(\xi^2-\bar{x}^2)$$

$f_i$ double distributions Mueller *et al* (94), Radyushkin (99)

advantage - polynomiality automatically satisfied

$D_i(\bar{x},t)$ ($i=$gluon, sea) additional free function, support $-\xi < \bar{x} < \xi$

useful ansatz with relation to PDFs (reduction formula respected)

$$f_i(\beta,\alpha,t') = h_i(\beta)\, \exp[(b_i+\alpha_i' \ln(1/\beta))t]\, \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)}\, \frac{[(1-|\beta|)^2-\alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\begin{aligned} h_g(t=0) &= |\beta| g(|\beta|)\,, & n_g &= 2 & \alpha_g' &= 0.15\,\mathrm{GeV}^{-2} \\ h^q_{\mathrm{sea}}(t=0) &= q(|\beta|)\,\mathrm{sign}(\beta)\,, & n_{\mathrm{sea}} &= 2 & \alpha_{\mathrm{sea}} &= \alpha_g \\ h^q_{\mathrm{val}}(t=0) &= q_{\mathrm{val}}(\beta)\,\Theta(\beta)\,, & n_{\mathrm{val}} &= 1 & \alpha_v' &= 0.9\,\mathrm{GeV}^{-2} \end{aligned}$$

sea quarks mix with gluons under evolution

# Leading-twist calculations

Mankiewicz et al (98), Vanderhaeghen et al (99), Belitsky-Mueller (01),...
leading-twist calculation with double distr. model for $\widetilde{H}$ input $\Delta q$ and

$$\widetilde{E}_v^u = -\widetilde{E}_v^d = \frac{1}{\sqrt{2}}\Theta(|\,\bar{x}\,| \le \xi)\frac{\Phi_\pi(\tau)}{\xi}\frac{m g_{\pi NN} f_\pi}{m_\pi^2 - t} F_{\pi NN}(t)$$

provides l.t. result for pion FF
( about 1/3 of exp. value measured in same reaction CLAS (06))
fails with cross section by order of magnitude

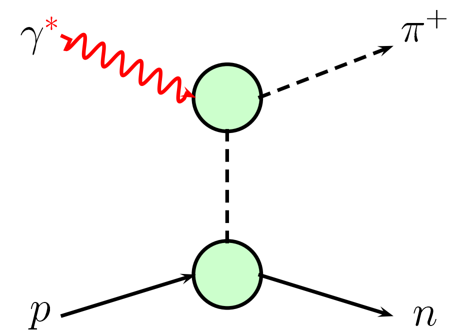
full pion FF needed and extra $\widetilde{E}$, see Goloskokov-K(09),Bechler-Mueller (09)

# The pion pole contribution

pion exchange (small $-t$, large $Q^2$)

$$\rho_\pi = \sqrt{2} g_{\pi NN} F_\pi(Q^2) F_{\pi NN}(t')$$
$$F_\pi = [1 + Q^2/(0.50\mathrm{GeV}^2)]^{-1}$$
$$F_{\pi NN} = (\Lambda_N^2 - m_\pi^2)/(\Lambda_N^2 - t')$$

$$\mathcal{M}^{\rm pole}_{0+,0+} = -e_0 \frac{2m\xi Q}{\sqrt{1-\xi^2}} \frac{\rho_\pi}{t - m_\pi^2},$$
$$\mathcal{M}^{\rm pole}_{0-,0+} = +e_0 Q\sqrt{-t'} \frac{\rho_\pi}{t - m_\pi^2},$$
$$\mathcal{M}^{\rm pole}_{0+,\pm+} = \pm 2\sqrt{2} e_0 \xi m \sqrt{-t'} \frac{\rho_\pi}{t - m_\pi^2},$$
$$\mathcal{M}^{\rm pole}_{0-,\pm+} = \pm \sqrt{2} e_0 t' \sqrt{1-\xi^2} \frac{\rho_\pi}{t - m_\pi^2}.$$




ampls. for transv. pol. photons disappear for forward scattering
cross section for $\gamma p \to \pi^+ n$ (or $p\bar{p} \to n\bar{n}$), should show forward dip
but exhibit pronounced spike experimentally (width $\mathcal{O}(m_\pi^2)$)
ways out: conspirator (Phillips(67)), poor man's absorption model (Williams(70)), Regge cuts (Rhanama-Storrow(82)), nucleon exchange (gauge invariance)
modifies non-flip amplitude

$$\mathcal{M}^{\rm pole}_{0-,++} \Longrightarrow \sqrt{2} e_0 (t_0 - m_\pi^2) \sqrt{\frac{1+\xi}{1-\xi}} \frac{\rho_\pi}{t - m_\pi^2} \qquad \text{too small}$$

# Target asymmetries in electroproduction

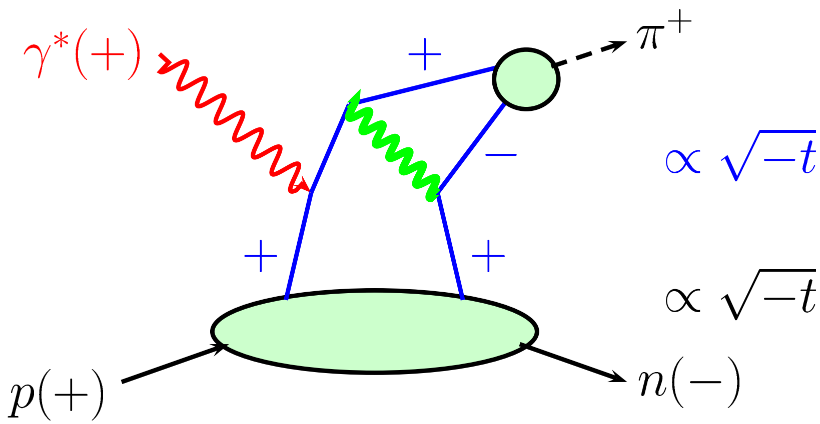
| observable | dominant interf. term | $\gamma^* p \to MB$ amplitudes | low $t'$ behavior |
|---|---|---|---|
| $A_{UT}^{\sin(\phi-\phi_s)}$ | LL | $\mathrm{Im}\big[\mathcal{M}^*_{0-,0+}\mathcal{M}_{0+,0+}\big]$ | $\propto \sqrt{-t'}$ |
| $A_{UT}^{\sin(\phi_s)}$ | LT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,0+}\big]$ | const. |
| $A_{UT}^{\sin(2\phi-\phi_s)}$ | LT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,-+}\mathcal{M}_{0+,0+}\big]$ | $\propto t'$ |
| $A_{UT}^{\sin(\phi+\phi_s)}$ | TT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,++}\big]$ | $\propto \sqrt{-t'}$ |
| $A_{UT}^{\sin(2\phi+\phi_s)}$ | TT | $\propto \sin\theta_\gamma$ | $\propto t'$ |
| $A_{UT}^{\sin(3\phi-\phi_s)}$ | TT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,-+}\mathcal{M}_{0+,-+}\big]$ | $\propto (-t')^{(3/2)}$ |
| $A_{UL}^{\sin(\phi)}$ | LT | $\mathrm{Im}\big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0-,0+}\big]$ | $\propto \sqrt{-t'}$ |

$\phi$ azimuthal angle between lepton and hadron plane; $\phi_s$ orientation of target spin vector; $\theta_\gamma$ rotation from direction of incoming lepton to virtual photon one

$\pi^+$: all measured; detailed info. on amplitudes

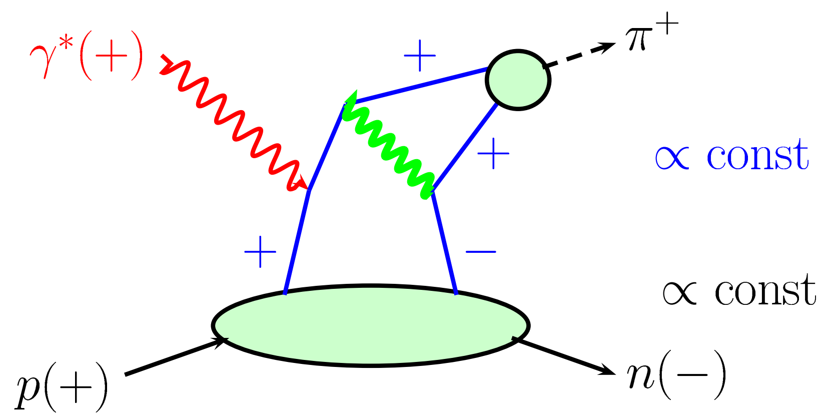
$\sin\phi_s$ moment very large, no indication for vanishing in forw. direction

# Can $\mathcal{M}_{0-,++}$ be fed by ordinay GPDs?




lead. twist pion wave fct. $\propto q' \cdot \gamma\gamma_5$
(perhaps including $\mathbf{k}_\perp$)

$\mathcal{M}_{0-,++} \propto t'$




twist-3 w.f.

$\mathcal{M}_{0-,++} \propto$ const

helicity flip GPDs ($H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$) required
Hoodbhoy-Ji (98), Diehl (01)

# A twist-3 contribution

$$\mathcal{M}^{\rm twist-3}_{0-,\mu+} = e_0\sqrt{1-\xi^2}\int_{-1}^{1} d\bar{x}\,\Big\{\mathcal{H}_{0-,\mu+}\big[H_T^{(3)} - \frac{\xi}{1-\xi^2}(\xi E_T^{(3)} - \widetilde{E}_T^{(3)})\big]\Big\} + \mathcal{O}(\frac{t'}{m^2})$$

$$\mathcal{M}^{\rm twist-3}_{0+,\mu+} \propto t'$$

twist-3 ( 3-part. contr. neglected: $\tau\Phi_P = \Phi_\sigma/N_c - \tau\Phi'_\sigma/(2N_c)$

solution: $\Phi_P = 1, \Phi_\sigma = \Phi_{AS}$ Braun-Halperin (90))

$\sim q'\cdot\gamma\gamma_5\Phi + \mu_\pi\gamma_5\Big[\Phi_P - \imath\sigma_{\mu\nu}(\ldots\Phi'_\sigma + \ldots\Phi_\sigma\partial/\partial\mathbf{k}_{\perp\nu})\Big]$ Beneke-Feldmann (01)

$\mathcal{H}_{0-,++}\neq 0$, $\Phi_P$ dominant, $\Phi_\sigma$ contr. $\propto t'/Q^2$

$\mu_\pi = m_\pi^2/(m_u+m_d) \simeq 2\,\mathrm{GeV}$ at scale $2\,\mathrm{GeV}$

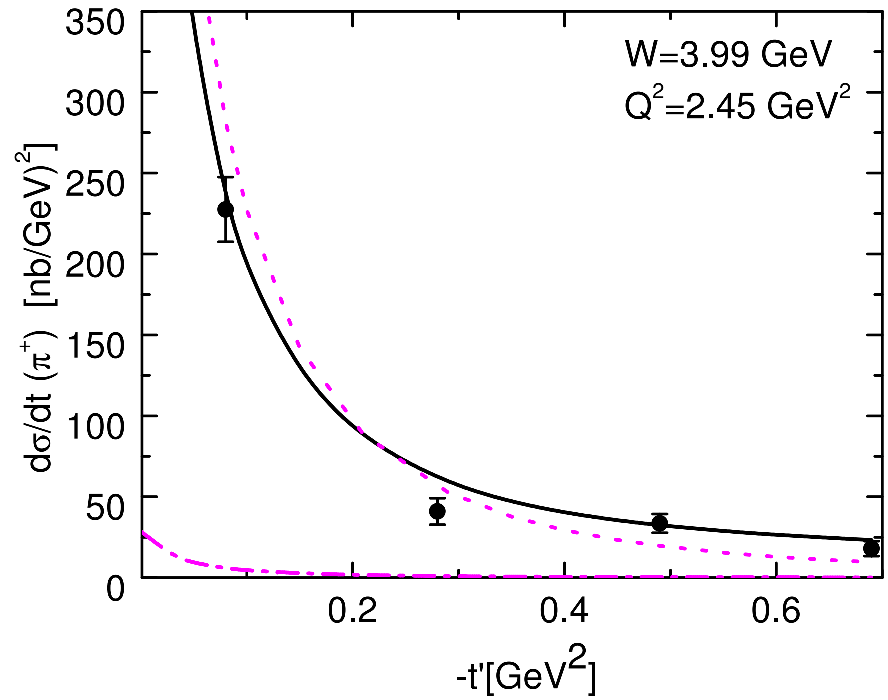
in coll. appr.: $\mathcal{H}_{0-,++}$ infr. sing. and double pole $1/(x-\xi)^2$ m.p.a. regular

small $\xi$: $H_T$ should dominate; take transversity PDF from Anselmino et al (07)

$\delta^a = 7.46 N_T^a(1-x)^5[q(x)+\Delta q(x)]$ $N_T^u = 0.5$ $N_T^d = -0.6$

input to double distr. ansatz

# Results on unseparated $\pi^+$ cross section

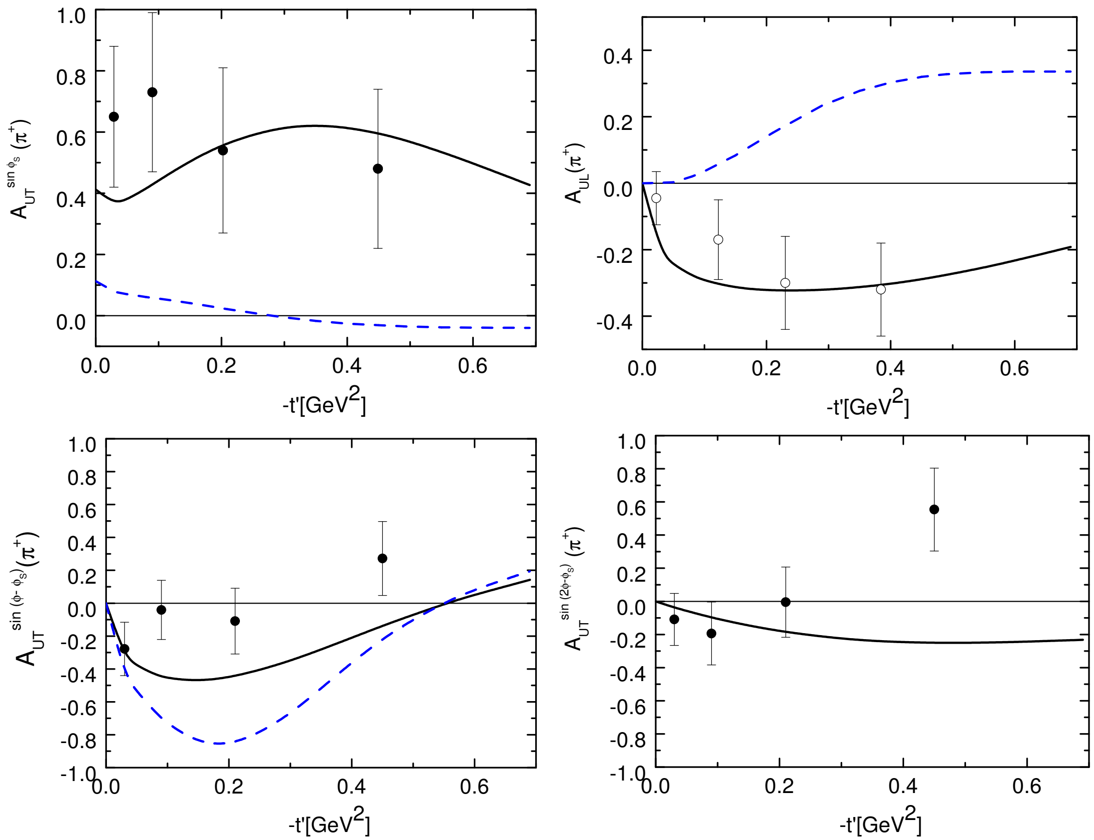



data from HERMES 07 Goloskokov-K (09)

magenta lines: pion pole contr. (unseparated and transverse cross sections)

new data on $F_{\pi\gamma}$ may change our understanding of pion DA

# Results on target asymetries




Goloskokov-K (09)

$Q^2 = 2.5\,\mathrm{GeV}^2$

$W = 3.99\,\mathrm{GeV}$

prel. data on $A_{UT}$ HERMES (08); $A_{UL}$ HERMES(02)

upper (lower) blue: without twist-3 contr. (only LL)

other asym.: $|A_{UT}| < 0.1$ agree with exp.

# Comparison with $F_\pi - 2$ data

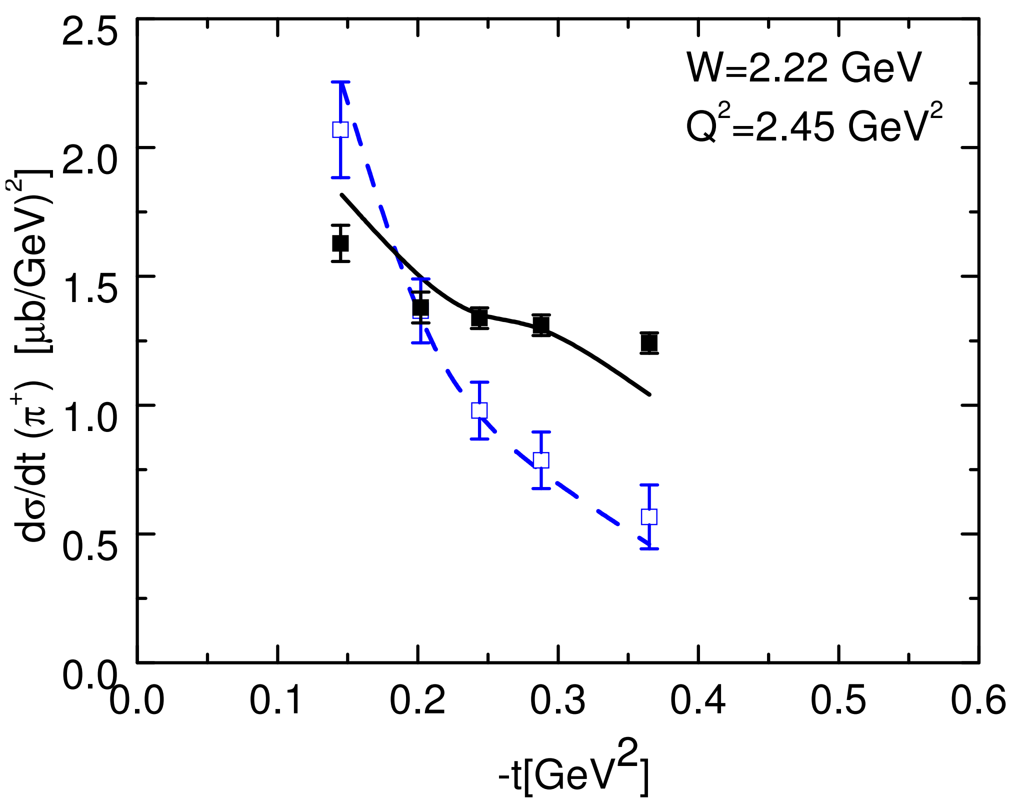
Can we apply our approach for $W \geq 4\,\mathrm{GeV}$? (cf. $\rho^0$ production, val.quarks)

We find: transverse cross section not bad, longitudinal one somewhat small

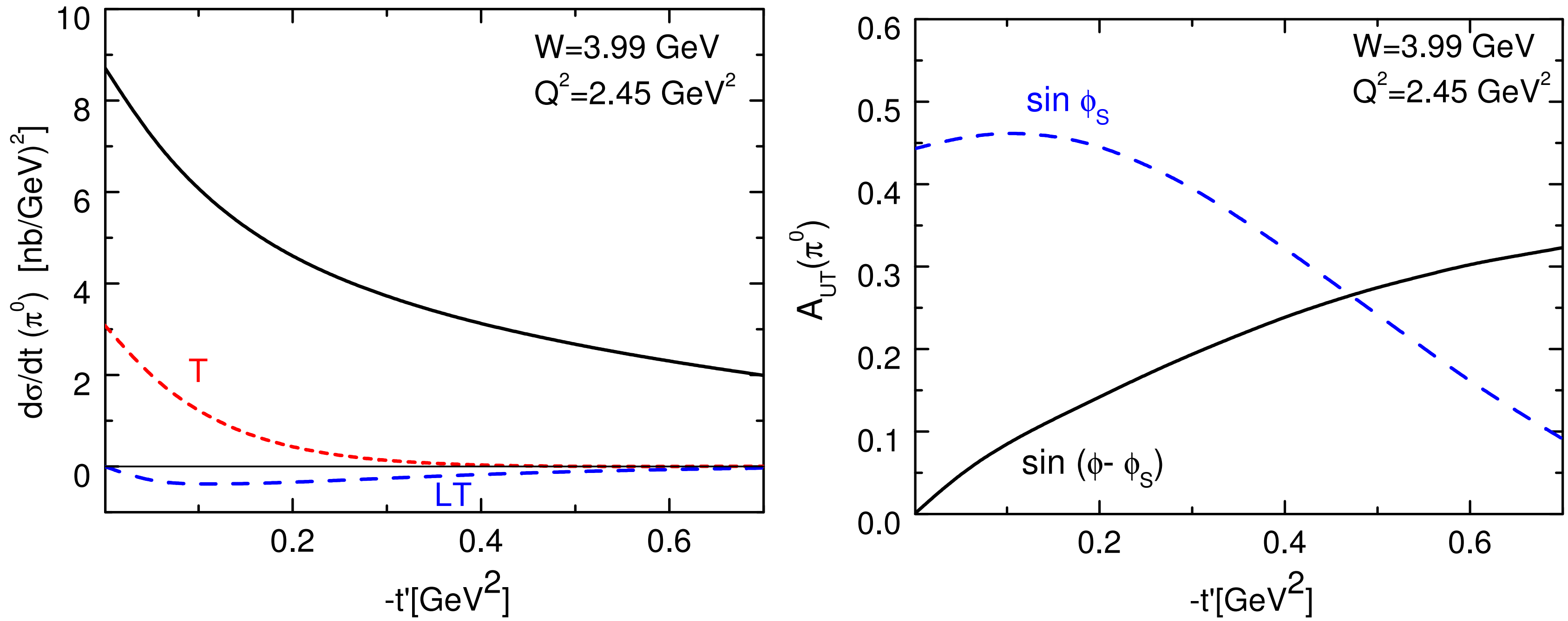
Our approach optimized for small $\xi(\lesssim 0.1)$

large $x(\geq 0.6)$ behaviour of GPDs not probed

With little modifications of large $x$ behaviour of GPDs $\widetilde{E}$ and $H_T$:



# Results on $\pi^0$ electroproduction




pion exchange absent Goloskokov-K(09)

Other twist-3 estimate: Ahmad et al (08)

(subprocess viewed as form factors for $\gamma - \pi$ transitions under the action of vector and axial-vector currents)

# Summary

- transversely polarized photons play an important role in $\pi^+$ production at accessible values of $Q^2$
- clear signal for amplitude $\mathcal{M}_{0-,++}$; in handbag approach described by a twist-3 effect consisting of (lead. twist) helicity-flip GPDs and a twist-3 pion wave function
- GPDs $\widetilde{H}$, $\widetilde{E}$ and $H_T$ modeled through reggeized double distributions and subprocess calculated within mod. pert. approach
- fair agreement with cross section data from HERMES
- fair agreement with HERMES data on polarized target asymmetries