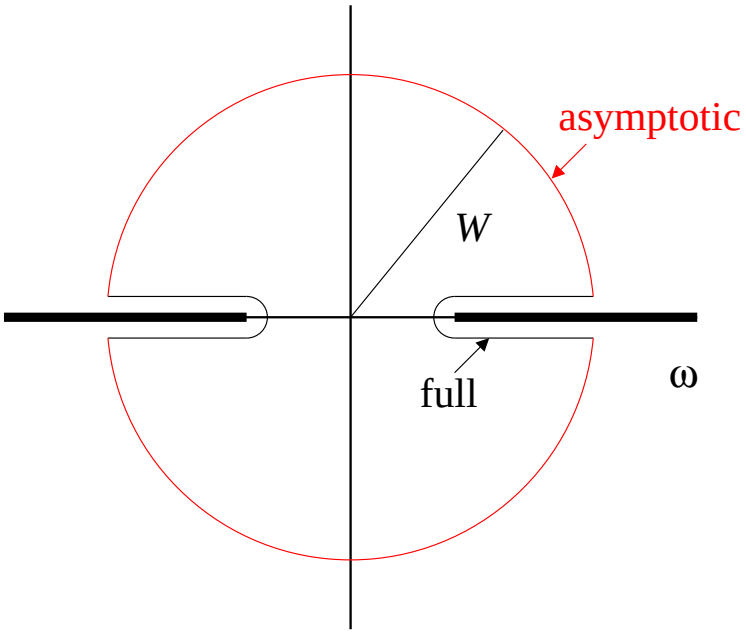
Finite–energy sum rules and duality in exclusive electroproduction

C. Weiss (JLab), Joint N*–GPD Meeting 11–Sep–09

- FESR relate integrals of $\mathrm{Im}(\mathrm{Amp})$ over finite energy range $\omega < W$ to high–energy behavior of amplitude
  - $\rightarrow$ Constraints on high–energy behavior or/and resonance parameters
  - $\rightarrow$ Duality of s–channel resonances and t–channel exchanges
  - $\rightarrow$ Tested extensively in πN, γN and low–Q^2 eN scattering with Regge exchange parametrizations at high energies
- Can we extend them to high–Q^2 exclusive processes?
  - $\rightarrow$ Partonic mechanism at high energies, GPDs!
  - $\rightarrow$ Experience with quark–hadron duality in inclusive DIS!
  - $\rightarrow$ Constraints on N^* reaction models?
  - $\rightarrow$ Implications for hard exclusive amplitudes?

Finite–energy sum rules: Basics



$$\int_{\text{thr}}^{W} d\omega\, \omega^n\, \text{Im}\, F(\omega)$$

$$= \sum_j \frac{c_j W^{\alpha_j+n+1}}{\alpha_j+n+1}$$

Dolen, Horn, Schmid 68

- Technique: Write Cauchy integral for function which vanishes at $\omega = 0$, e.g.

  $F(\omega) = F(-\omega)$ crossing–odd amplitude

  $\omega\, F(\omega)$ moment, etc.

- Asymptotic behavior of amplitude

  $F(\omega) = F_{\text{as}}(\omega) + \tilde{F}(\omega)$

  $F_{\text{as}}(\omega) \sim \sum_j c_j \xi_j \omega^{\alpha_j}$ Regge asymptotics

  $\tilde{F}(\omega)$ vanishes faster than ω^{-1}

- FESR relates dispersion integral over finite energy range $\omega < W$ to constants parametrizing high–energy behavior

  $\rightarrow$ powerful constraints

  $\rightarrow$ many applications!

Finite–energy sum rules: Applications I

- Simplest case: Infer leading Regge behavior of amplitude from ratios of finite–energy integrals

$$W^{m-n}\,\frac{\displaystyle\int_{\rm thr}^{W} d\omega\;\omega^n\;\mathrm{Im}\,F(\omega)}{\displaystyle\int_{\rm thr}^{W} d\omega\;\omega^m\;\mathrm{Im}\,F(\omega)} \;=\; \frac{\alpha+m+1}{\alpha+n+1}$$

constant, related to α!

- High n: Integrals more and more sensitive to energies $\omega \sim W$

Finite–energy sum rules: Applications II

I) ω, ρ only

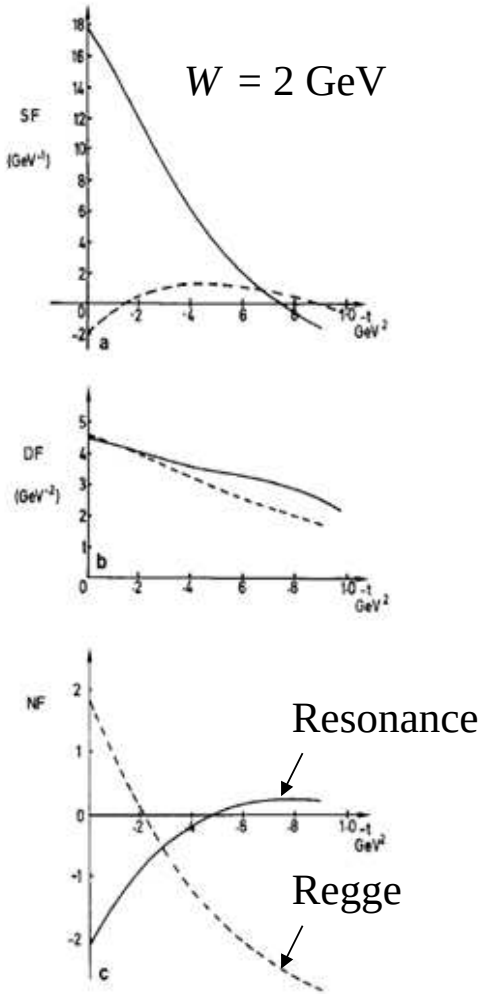


Fig. 2a—c. The photoproduction FESR integrals. The dashes are the integral (2.7) of the Regge amplitude and the solid curves are the integral (2.8) of the resonances for **a** the single-flip amplitude, **b** the double flip amplitude **c** the non-flip amplitude

II) ω, ρ + daughter

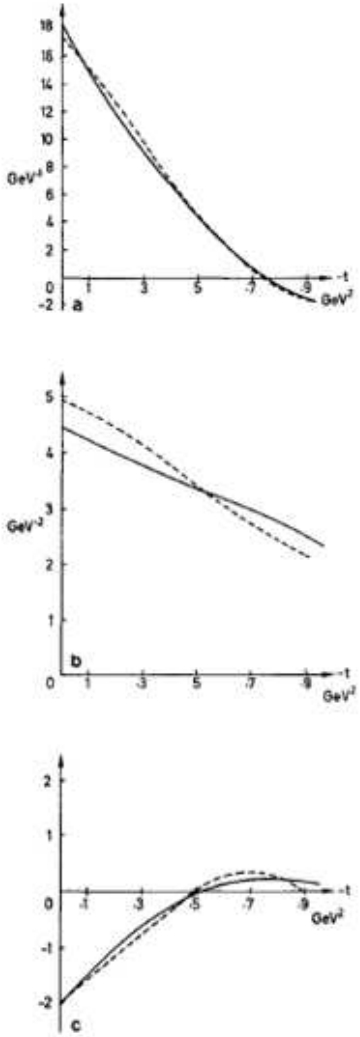


Fig. 5a—c. Fit to the resonance FESR of the Regge model including the daughter term (2.14) with the same conventions as Fig. 2. It will be seen that the agreement is very much improved

- Use FESR to constrain Regge parametrization

  Example: π^0 photo– and electroproduction

  I) leading trajectories ω, ρ only

  II) including daughter $\alpha_d = \alpha_\omega - 2$

  Collins P, Wilkie 81

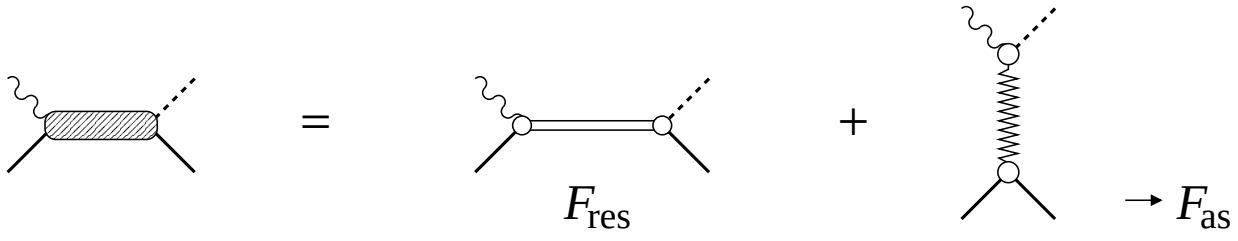
  . . . many other examples!

Finite–energy sum rules: Duality

- Sum rule for $n = 0$ can equivalently be written as

$$\int_0^W d\omega \, \mathrm{Im}\, F(\omega) \stackrel{!}{=} \int_0^W d\omega \, \mathrm{Im}\, F_{\mathrm{as}}(\omega) \qquad \text{“global duality”}$$

- Phenomenological models: Add resonance and exchange contributions

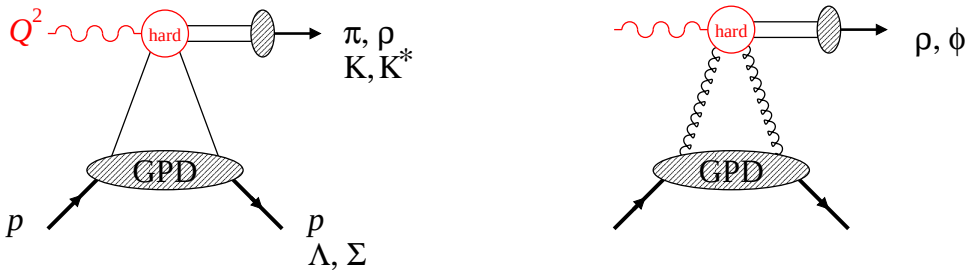


$$\int_0^W d\omega \, \mathrm{Im}\, F_{\mathrm{res}}(\omega) = 0 \qquad \text{resonance contributions “average out!”}$$

  - Model consistent only if strong cancelletions between resonances, or if one or other term dominates
  - Modified version: Pomeron as background, resonances dual to Reggeons [Harari 68]

FESR: Exclusive meson production at high Q^2

- Large $Q^2 \sim W^2$: Partonic description



- Partonic description *dual* to resonances: Adding them would raise the same double counting issues as in soft processes. . .

- Topics for discussion
  - Can we combine resonances and partonic mechanism in a dispersion representation of high–Q^2 amplitudes?
    Cf. recent work on dispersion representation of hard exclusive amplitudes
  - What can we learn from experience with quark–hadron duality in inclusive DIS?