DIS Contribution to the Integrals of Structure Functions

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1 Introduction

From JLab Experiment 94-010, high precision data for spin structure functions of the neutron are available and various integrals of these structure functions have been evaluated. However, experimental data do not cover full kinematic range in general and it is necessary to estimate contributions from the non-measured region, especially Deep Inelastic Scattering (DIS) region. This report presents one of the possible ways to evaluate these contribions from DIS region.

2 Quantities of Interest

Using measured $g_1(x,Q^2)$ and $g_2(x,Q^2)$, the following integrals are of interest.

$$\Gamma_1 = \int_0^1 g_1(x,Q^2)\,dx \tag{1}$$

$$\Gamma_2 = \int_0^1 g_2(x,Q^2)\,dx \tag{2}$$

$$d_2 = 3\int_0^1 x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx \tag{3}$$

$$= \int_0^1 x^2(2g_1(x,Q^2) + 3g_2(x,Q^2))\,dx, \tag{4}$$

with $g_2^{\mathrm{WW}}(x,Q^2)$ defined as

$$g_2^{\mathrm{WW}}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y}\,dy. \tag{5}$$

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to apply ... techniques.

First of all, it should be emphasized that Eqs. 3 and 4 are no longer identity when the lower limit of the integral is not 0. Starting from Eq. 3,

$$d_2 = 3\int_\epsilon^1 x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx \tag{16}$$

Please note that Eqs. 3 and 4 are mathematical identity due to Eq. 5 as will be discussed later in this report.

Because of the definition of $g_2^{\mathrm{WW}}(x,Q^2)$ by Eq. 5, it has pretty interesting property under integration. For a general discussion, let's consider the weighted integral of $g_2^{\mathrm{WW}}(x,Q^2)$ such as

$$\int_0^{x_0} x^n g_2^{\mathrm{WW}}(x,Q^2)\,dx. \tag{6}$$

Here, the upper bound of the integration has been changed to x_0 instead of 1, which can be set to 1 when it is necessary. Using its definition (Eq. 5),

$$\begin{aligned}
\int_0^{x_0} x^n g_2^{\mathrm{WW}}(x,Q^2)\,dx &= -\int_0^{x_0} x^n g_1(x,Q^2)\,dx \\
&\quad + \int_0^{x_0} x^n \int_x^1 \frac{g_1(y,Q^2)}{y}\,dy\,dx \qquad (7)\\
&= -\int_0^{x_0} x^n g_1(x,Q^2)\,dx \\
&\quad + \left[\frac{x^{n+1}}{n+1}\int_x^1 \frac{g_1(y,Q^2)}{y}\,dy\right]_0^{x_0} \\
&\quad + \int_0^{x_0} \frac{x^{n+1}}{n+1}\frac{g_1(x,Q^2)}{x}\,dx \qquad (8)\\
&= -\int_0^{x_0}\left(\frac{n}{n+1}\right) x^n g_1(x,Q^2)\,dx \\
&\quad + \frac{x_0^{n+1}}{n+1}\int_{x_0}^1 \frac{g_1(x,Q^2)}{x}\,dx. \qquad (9)
\end{aligned}$$

$d[fg] = g\,df + f\,dg$

$g\,df = d[fg] - f\,dg$

$x^n \int_x^1 \frac{g_1}{y}\,dy = d\left[\frac{x^{n+1}}{n+1}\int_x^1 \frac{g_1}{y}\,dy\right] + \frac{x^{n+1}}{n+1}\cdot\frac{g_1}{x}$

In the derivation, Eq. 8 was obtained using integration by parts. A special case is obtained when $x_0 = 1$,

$$\int_0^1 x^n g_2^{\mathrm{WW}}(x,Q^2)\,dx = -\int_0^1 \left(\frac{n}{n+1}\right) x^n g_1(x,Q^2)\,dx \tag{10}$$

When $n = 2$, we have $n=0$ $\int_0^1 g_2^{WW}\,dx = -\int_0^1 \frac{0}{1} g_1\,dx = 0$ ✓

$$\int_0^1 3x^2 g_2^{\mathrm{WW}}(x,Q^2)\,dx = -\int_0^1 2x^2 g_1(x,Q^2)\,dx, \tag{11}$$

which can be used to relate Eqs. 3 and 4.

$-\int_0^{x_0} x^n g_1\,dx + \left[\frac{x_0^{n+1}}{n+1}\int_x^1 \frac{g_1(y)}{y}\,dy\right]_0^{x_0} + \int_0^{x_0}\frac{x^{n+1}}{n+1}\frac{g_1}{x}\,dx$ $\qquad \frac{n+1-1}{n+1}$

$-\int_0^{x_0} x^n g_1\,dx + \int_0^{x_0}\frac{x^n}{n+1} g_1\,dx + \frac{x_0^{n+1}}{n+1}\int_{x_0}^1 \frac{g_1}{y}\,dy = -\int_0^{x_0} x^n g_1\left(1 - \frac{1}{n+1}\right)$

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3 DIS Contribution to Γ_1

For the structure function $g_1(x, Q^2)$, relatively large amount of data exists for DIS region and their contribution to Γ_1 can be quite reliably evaluated. One example is using the parametrization of DIS data by Bianchi and Thomas[1, 2].

Bianchi & Thomas gives σ_{TT}
$\sigma_{TT} = f(g_1, g_2)$
so he assumes g_2 and extracts g_1 from σ_{TT}

4 DIS Contribution to Γ_2

For the structure function $g_2(x, Q^2)$, quite little is known compared to $g_1(x, Q^2)$, especially for the neutron. One possible approach is to use $g_2^{\mathrm{WW}}(x, Q^2)$ to approximate $g_2(x, Q^2)$, since $g_2^{\mathrm{WW}}(x, Q^2)$ can be calculated with the knowledge of $g_1(x, Q^2)$. This approximation may be valid at high Q^2 where higher twist effect is small, but its validity is questionable at low Q^2 region. Anyway, with this approximation, Γ_2 can be evaluated as

$$\begin{aligned}
\int_0^1 g_2(x, Q^2)\, dx &= \int_\varepsilon^1 g_2(x, Q^2)\, dx + \int_0^\varepsilon g_2(x, Q^2)\, dx && (12) \\
&\simeq \int_\varepsilon^1 g_2(x, Q^2)\, dx + \int_0^\varepsilon g_2^{\mathrm{WW}}(x, Q^2)\, dx && (13) \\
&= \int_\varepsilon^1 g_2(x, Q^2)\, dx + \varepsilon \int_\varepsilon^1 \frac{g_1(x, Q^2)}{x}\, dx. && (14)
\end{aligned}$$

Now both integrals can be evaluated using measured quantities for $g_1(x, Q^2)$ and $g_2(x, Q^2)$. The two integrals can even be combined into one as

$$\int_\varepsilon^1 \left[g_2(x, Q^2) + \frac{\varepsilon g_1(x, Q^2)}{x} \right] dx. \qquad (15)$$

5 DIS Contribution to d_2

The contributions of DIS region to d_2 are highly suppressed by x^2 weighting in the integral. As a result, it can be ignored for the first order. Nevertheless, let's try to apply similar techniques.

First of all, it should be emphasized that Eqs. 3 and 4 are no longer identity when the lower limit of the integral is not 0. Starting from Eq. 3,

$$d_2 = 3 \int_\varepsilon^1 x^2 (g_2(x, Q^2) - g_2^{\mathrm{WW}}(x, Q^2))\, dx \qquad (16)$$

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$$
\begin{aligned}
&= 3\int_\varepsilon^1 x^2 g_2(x,Q^2)\,dx - 3\int_\varepsilon^1 x^2 g_2^{\mathrm{WW}}(x,Q^2)\,dx && (17)\\
&= 3\int_\varepsilon^1 x^2 g_2(x,Q^2)\,dx \\
&\quad -3\left(\int_0^1 x^2 g_2^{\mathrm{WW}}(x,Q^2)\,dx - \int_0^\varepsilon x^2 g_2^{\mathrm{WW}}(x,Q^2)\,dx\right) && (18)\\
&= 3\int_\varepsilon^1 x^2 g_2(x,Q^2)\,dx \qquad \text{from (11)} && (19)\\
&\quad +2\int_0^1 x^2 g_1(x,Q^2)\,dx - 2\int_0^\varepsilon x^2 g_1(x,Q^2)\,dx \\
&\quad +\varepsilon^3\int_\varepsilon^1 \frac{g_1(x,Q^2)}{x}\,dx \qquad 3\int_0^\varepsilon x^2 \frac{g_1}{x}\,dx && (20)\\
&= \int_\varepsilon^1 x^2(2g_1(x,Q^2)+3g_2(x,Q^2))\,dx \\
&\quad +\varepsilon^3\int_\varepsilon^1 \frac{g_1(x,Q^2)}{x}\,dx. && (21)
\end{aligned}
$$

The end result is not equal to Eq. 4 with lower bound set to ε. This is a consequence of non-equality of the integrands,

$$
2g_1(x,Q^2)+3g_2(x,Q^2) \neq 3(g_2(x,Q^2)-g_2^{\mathrm{WW}}(x,Q^2)). \qquad (22)
$$

Now if we try to evaluate

$$
3\int_0^\varepsilon x^2(g_2(x,Q^2)-g_2^{\mathrm{WW}}(x,Q^2))\,dx \qquad (23)
$$

using approximation $g_2(x,Q^2) \simeq g_2^{\mathrm{WW}}(x,Q^2)$ at small x region, the result is identically zero since the integrand vanishes. This makes sense since the difference $g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2)$ is a measure of higher twist effect and if $g_2(x,Q^2) = g_2^{\mathrm{WW}}(x,Q^2)$, there is none, giving zero contribution to the integral.

For completeness, let's consider DIS contribution to d_2 with Eq. 4:

$$
\begin{aligned}
d_2 &= \int_0^1 x^2(2g_1(x,Q^2)+3g_2(x,Q^2))\,dx && (24)\\
&= \int_\varepsilon^1 x^2(2g_1(x,Q^2)+3g_2(x,Q^2))\,dx \\
&\quad +\int_0^\varepsilon x^2(2g_1(x,Q^2)+3g_2(x,Q^2))\,dx. && (25)
\end{aligned}
$$

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$\int_0^\varepsilon x^2 2g_1 + 3\int_0^\varepsilon x^2 g_2^{ww} = 2\int_0^\varepsilon x^2 g_1 - 3\int_0^\varepsilon \frac{2}{3} x^2 g_1 + \frac{3}{3}\varepsilon^3 \int_\varepsilon^1 \frac{g_1}{x} dx$

Now if we use the approximation $g_2(x, Q^2) \simeq g_2^{\mathrm{WW}}(x, Q^2)$ at small x region for the second integral, it can be shown that,

$$\int_0^\epsilon x^2(2g_1(x,Q^2) + 3g_2^{\mathrm{WW}}(x,Q^2))\,dx = \epsilon^3 \int_\epsilon^1 \frac{g_1(x,Q^2)}{x}\,dx, \tag{26}$$

which is equal to Eq. 21.

In summary, d_2 can be evaluated in two ways:

Method 1

$$\begin{aligned} d_2 &= 3\int_0^1 x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx && (27)\\ &= 3\int_\epsilon^1 x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx \\ &\quad +3\int_0^\epsilon x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx && (28)\\ &= \int_\epsilon^1 x^2(2g_1(x,Q^2) + 3g_2(x,Q^2))\,dx \\ &\quad +\epsilon^3\int_\epsilon^1 \frac{g_1(x,Q^2)}{x}\,dx \\ &\quad +3\int_0^\epsilon x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx && (29)\end{aligned}$$

Method 2

$$\begin{aligned} d_2 &= \int_0^1 x^2(2g_1(x,Q^2) + 3g_2(x,Q^2))\,dx && (30)\\ &= \int_\epsilon^1 x^2(2g_1(x,Q^2) + 3g_2(x,Q^2))\,dx \\ &\quad +\int_0^\epsilon x^2(2g_1(x,Q^2) + 3g_2(x,Q^2))\,dx && (31)\\ &\simeq \int_\epsilon^1 x^2(2g_1(x,Q^2) + 3g_2(x,Q^2))\,dx \\ &\quad +\epsilon^3\int_\epsilon^1 \frac{g_1(x,Q^2)}{x}\,dx. && (32)\end{aligned}$$

Please note that in Method 1, no approximation has been used. Only if we make an approximation $g_2(x,Q^2) \simeq g_2^{\mathrm{WW}}(x,Q^2)$ at the last step, we get the same result as Method 2.

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Since there is no reliable way to approximate $g_2(x, Q^2)$ in DIS region, it would be reasonable to estimate maximum contribution in small x region. This can be done in the following way:

$$
\begin{aligned}
d_2(\mathrm{DIS}) &= 3\int_0^{\epsilon} x^2(g_2(x,Q^2) - g_2^{\mathrm{WW}}(x,Q^2))\,dx && (33)\\
&\leq 3\int_0^{\epsilon} x^2(|g_2(x,Q^2)| + |g_2^{\mathrm{WW}}(x,Q^2)|)\,dx && (34)\\
&\simeq 6\int_0^{\epsilon} x^2|g_2^{\mathrm{WW}}(x,Q^2)|\,dx && (35)\\
&= 6\left|\int_0^{\epsilon} x^2 g_2^{\mathrm{WW}}(x,Q^2)\,dx\right| && (36)\\
&= 6\left|-\int_0^{\epsilon}\frac{2}{3}x^2 g_1(x,Q^2)\,dx + \frac{\epsilon^3}{3}\int_{\epsilon}^{1}\frac{g_1(x,Q^2)}{x}\,dx\right| && (37)\\
&= \left|2\epsilon^3\int_{\epsilon}^{1}\frac{g_1(x,Q^2)}{x}\,dx - 4\int_0^{\epsilon} x^2 g_1(x,Q^2)\,dx\right|. && (38)
\end{aligned}
$$

from (9) $n=2$

Note that to get Eq. 35, an approximation $g_2(x, Q^2) \simeq g_2^{\mathrm{WW}}(x, Q^2)$ has been made. This approximation is not good for $g_2(x, Q^2) - g_2^{\mathrm{WW}}(x, Q^2)$ but should be reasonable for $|g_2(x, Q^2)| + |g_2^{\mathrm{WW}}(x, Q^2)|$. And then in going from Eq. 35 to Eq. 36, an assumption has been made that $g_2^{\mathrm{WW}}(x, Q^2)$ does not change sign between $x = 0$ and $x = \epsilon$, which is not totally unreasonable in DIS region. The two pieces in Eq. 38 can be evaluated using measured $g_1(x, Q^2)$ and parametrization of existing data for $g_1(x, Q^2)$.

6 Conclusion

In this report, a method has been suggested to evaluate the contribution from DIS region for various integrals of structure functions. For the integral of $g_1(x, Q^2)$, parametrization of DIS data has been used, while for the integral of $g_2(x, Q^2)$ or d_2, an approximation $g_2(x, Q^2) \simeq g_2^{\mathrm{WW}}(x, Q^2)$ at small x region has been used. Using special property of $g_2^{\mathrm{WW}}(x, Q^2)$, it is possible to evaluate the contribution at small x using measured quantity $g_1(x, Q^2)$. Also with reasonable approximation and assumption, it is possible to estimate upper bound of DIS contribution to d_2 matrix element.

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References

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[2] N. Bianchi and E. Thomas, Nucl. Phys. **B82** (2000) 256.

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