The physics angles θ^* and ϕ^* for SANE with Maple V

The physics system for asymmetry calculation is defined as
$z =$ along incident electron momentum $\vec{k}$, i.e. along beam
$y =$ along cross product $\vec{k} \times \vec{k}'$ (points UP in lab, for $\vec{k}'$ horizontal and pointing towards BETA, to the left of the beam)
$x =$ normal to y, z (points towards BETA for SANE)
We want to find the angles θ^* and ϕ^* of the target spins relative to the momentum transfer vector $\vec{q} = \vec{k} - \vec{k}'$. For this purpose, we apply Euler angle rotations to the lab coordinate system into the $\vec{q}$ system.

First, rotate counterclockwise about z by the azimuthal scattering angle $\phi = \phi_e$ (There are no subscripts in Maple V, so we use plain theta and phi for $\theta \equiv \theta_q$ and $\phi \equiv \phi_e$):

```
Rzphi:=linalg[matrix]([[cos(phi),sin(phi),0], [-sin(phi),cos(phi),0],[0,0,1]])
```

$$Rzphi := \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For $\phi = \pi/2$ his rotation takes the x axis into the initial y axis.

Then, rotate about the physics y by the angle θ_q between $\vec{k}$ and $\vec{q}$ from $\vec{k}$ to $\vec{q}$.

This is a CLOCKWISE rotation, since $\vec{q}$ is to the right of $\vec{k}$ and y is UP, so sines are *(-1). This rotation is identical to RSS, because the SANE and RSS systems are just related by a π rotation about $\vec{k}$, so the relative directions of $\vec{k}$ and $\vec{q}$ stay the same.

```
>   Ryq:=linalg[matrix]([[cos(theta),0,sin(theta) ],[0,1,0],[-sin(theta),0,cos(theta)]]);
```

$$Ryq := \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

Final rotation, about the rotated z^* axis

```
Rzg:=linalg[matrix]([[cos(gamma),sin(gamma),0 ],[-sin(gamma),cos(gamma),0],[0,0,1]])
```

$$Rzg := \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> with(linalg):
> Rstar:=multiply(Rzg,Ryq,Rzphi);
```

$Rstar :=$

$[\cos(\gamma)\cos(\theta)\cos(\phi) - \sin(\gamma)\sin(\phi)\,,\ \cos(\gamma)\cos(\theta)\sin(\phi) + \sin(\gamma)\cos(\phi)\,,\ \cos(\gamma)\sin(\theta)]$

$[-\sin(\gamma)\cos(\theta)\cos(\phi) - \cos(\gamma)\sin(\phi)\,,\ -\sin(\gamma)\cos(\theta)\sin(\phi) + \cos(\gamma)\cos(\phi)\,,\ -\sin(\gamma)\sin(\theta)$
$]$

$[-\sin(\theta)\cos(\phi)\,,\ -\sin(\theta)\sin(\phi)\,,\ \cos(\theta)]$

Since there is no third rotation, i.e. γ=0,

```
Rzg1:=linalg[matrix](3,3,[1,0,0,0,1,0,0,0,1])
```

$$Rzg1 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> Rstar1:=evalm(Rzg1&*Ryq&*Rzphi);
```

$$Rstar1 := \begin{bmatrix} \cos(\theta)\cos(\phi) & \cos(\theta)\sin(\phi) & \sin(\theta) \\ -\sin(\phi) & \cos(\phi) & 0 \\ -\sin(\theta)\cos(\phi) & -\sin(\theta)\sin(\phi) & \cos(\theta) \end{bmatrix}$$

PARA:

B points along -z in all systems

```
> Bpara:=linalg[matrix](3,1,[0,0,-Bz]);
```

$$Bpara := \begin{bmatrix} 0 \\ 0 \\ -Bz \end{bmatrix}$$

```
> Bstarpara:=evalm(Rstar1 &* Bpara);
```

$$Bstarpara := \begin{bmatrix} -\sin(\theta)\,Bz \\ 0 \\ -\cos(\theta)\,Bz \end{bmatrix}$$

```
> cosstar:=Bstarpara[3,1]/Bz;
```

$$cosstar := -\cos(\theta)$$

```
> phistar:=arctan(Bstarpara[2,1]/Bstarpara[1,1] );
```

$$phistar := 0$$

$\phi^* = 0$ or 180, depending on the sign of the denominator. Here denominator is negative, so $\phi^* = 180$

80°: B points almost along x in physics system.

```
> B80:=linalg[matrix] (3,1,[Bf*sin(beta),0,Bf*cos(beta)]);
```

$$B80 := \begin{bmatrix} Bf\sin(\beta) \\ 0 \\ Bf\cos(\beta) \end{bmatrix}$$

```
> Bstar80:=evalm(Rstar1 &* B80);
```

$$Bstar80 := \begin{bmatrix} \cos(\theta)\cos(\phi)\,Bf\sin(\beta) + \sin(\theta)\,Bf\cos(\beta) \\ -\sin(\phi)\,Bf\sin(\beta) \\ -\sin(\theta)\cos(\phi)\,Bf\sin(\beta) + \cos(\theta)\,Bf\cos(\beta) \end{bmatrix}$$

```
> cosstar:= Bstar80[3,1]/Bf:cosStar:=simplify(cosstar);
```

$$cosStar := -\sin(\theta)\cos(\phi)\sin(\beta) + \cos(\theta)\cos(\beta)$$

```
> phistar:=arctan(Bstar80[2,1]/Bstar80[1,1]):  phiStar:=simplify(phistar);
```

$$phiStar := -\arctan\left(\frac{\sin(\phi)\sin(\beta)}{\cos(\theta)\cos(\phi)\sin(\beta) + \sin(\theta)\cos(\beta)}\right)$$

Sample result

Recall $\theta = \theta_q, \phi =$ out-of-plane ϕ_e for substitutions. β is field direction in the lab's (not always same as physics) x-z plane.

```
> thstar:=evalf(arccos( subs(theta=22.29*Pi/180,phi=(360-0)*Pi/180,
beta=80*Pi/180,cosstar))/Pi*180);
```

$$thstar := 102.290000$$

```
> phis:= subs(theta=22.29*Pi/180,phi=(360-0)*Pi/180,beta=90*Pi/180,phiStar):
> phis1:=evalf(phis*180./Pi);
```

$$phis1 := 0$$

So $\cos\theta^* = -\sin\theta_q \cos\phi_e \sin\beta + \cos\theta_q \cos\beta$,
and $\tan\phi^*$ is in fourth quadrant (− sine, + cosine).

For SANE's HMS data, the perp field is pointing almost along $-x$ in the physics system, so all terms with β change signs, and $\tan\phi^*$ is in the second quadrant, like RSS.